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POSSIBLE PHYSICAL MANIFESTATION OF THE WEYL NON-ABELIAN GAUGE FIELD

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1.In ref.[1] it has been shown that the congruent transference introduced by Weyl [2] in 1922 defines a non-Abelian gauge field. In ref.[3], we have constructed a spinor representation of the Weyl gauge group and found the corresponding spinor current that is a source of the above gauge field. Since there exists the spinor source of a non-Abelian gauge field, it makes sense to question about observable manifestations of that sort of interactions on macroscopic scales. One possibility is considered in this note. It is shown that if a rotating body undergoes neither electromagnetic nor gravitational forces, the precession of the intrinsic angular momentum can be caused by the non-Abelian gauge field under consideration.

As it is known (see, e.g.,[4]), the most general law of parallel transference of vectors along a given curve $x^i = x^i(\tau)$ is defined by the system of ordinary differential equations of the form

$$\frac{dS^i}{d\tau} = -\Gamma^i_{jk} u^j S^k,$$

where $u^i = dx^i/d\tau$. From the condition for the length of a vector being conserved in the parallel transference it follows that the coefficients of linear connection Γ^i_{jk} should obey the system of algebraic equations

$$\partial_j g_{ik} - \Gamma^l_{ji} g_{lk} - \Gamma^l_{jk} g_{il} = 0.$$

It is assumed that the metric tensor g_{ij} is given, and the scalar product is defined by $g(S,S) = g_{ij} S^i S^j$. The general solution to this system of equations can be written in the form [2]

$$\Gamma^i_{jk} = \left\{ {i \atop jk} \right\} + G^i_{jk}, \tag{1}$$

where $G^i_{jk} = g^{il} G_{jkl}$ and G_{jkl} is a second-rank tensor field skew-symmetric in the last two indices $G_{jkl} + G_{jlk} = 0$. It is just the Weyl non-Abelian gauge field whose properties have been considered in the above-mentioned papers. In formula (1), $\left\{ {i \atop jk} \right\}$ denote, as usual, the components of Levi-Civita connection of the metrics g_{ij}

$$\left\{ {i \atop jk} \right\} = \frac{1}{2} g^{il} (\partial_j g_{kl} + \partial_k g_{jl} - \partial_l g_{jk}).$$

Connection (1) defines the congruent transference introduced by Weyl according to which a vector field is changed by that transference as follows:

$$\frac{dS^i}{d\tau} + \{^{\,i}_{jk}\}u^j S^k = -G^i_{jk} u^j S^k. \qquad (2)$$

The law of transference (2) includes the displacement belonging to the Riemannian geometry and rotation defined by the metric g_{ij} and bivector $G_{jkl}u^j$. The Weyl gauge theory is a realization of the abstract theory of gauge fields within the differential geometry whose specific feature is that it does not distinguish between space-time and a gauge space [1]. In this note, we discuss a possible physical manifestation of the non-Abelian gauge field G^i_{jk}.

2. Let us consider a rotating body, for instance, a gyroscope, that is accelerated by forces applied to its center of mass. Forces of this sort produce no torque, therefore, they do not change the vector length S^i of the proper angular momentum, but appear as its precession called the Thomas precession. We will apply equations (2) to describe the Thomas precession of a gyroscope initiated by the Weyl gauge field. The produced rotation should preserve the orthogonality of the vector of the intrinsic angular momentum of a body S^i and 4-velocity u^i. In view of (2) and the gauge tensor field G_{ijk} being skew-symmetric in the last two indices, we obtain

$$\frac{\delta}{\delta\tau}(u_i S^i) = \frac{\delta u^i}{\delta\tau} S_i + u_i \frac{\delta S^i}{\delta\tau} = \frac{\delta u^i}{\delta\tau} S_i - u_i G^i_{jk} u^j S^k = (\frac{\delta u^i}{\delta\tau} + G^i_{jk} u^j u^k) S_i,$$

where $\delta/\delta\tau$ is an absolute derivative. Then it follows that if the center of gravity of a gyroscope moves along the trajectory that is given by the equations of geodesics in connection (1)

$$\frac{\delta u^i}{\delta\tau} + G^i_{jk} u^j u^k = 0, \qquad (3)$$

then

$$\frac{\delta}{\delta\tau}(u_i S^i) = 0.$$

Thus, equations (2) are capable of describing precession of the proper angular momentum of a body in the framework of dynamics given by equations (3). Let us discuss them in greater detail.

3. Of great importance are quantities irreducible with respect to the fundamental group of symmetry. If this principle is applied to the field G_{ijk}, it is necessary to consider global transformations of the Weyl gauge group that is structured like the Lorentz group. The tensor G_{ijk}, with respect to this group is decomposable into irreducible components. According to the results of ref. [5] this expansions looks as follows:

$$G_{ijk} = \frac{2}{3}(T_{ijk} - T_{ikj}) + \frac{1}{3}(g_{ij}F_k - g_{ik}F_j) + e_{ijkl}A^l. \qquad (4)$$

where e_{ijkl} is a completely antisymmetric Levi-Civita tensor and other quantities are defined in the following way:

$$F_i = g^{jk}G_{jki}, \quad A_i = \frac{1}{6}e_{ijkl}G^{jkl},$$

$$T_{ijk} = \frac{1}{2}(G_{ijk} + G_{jik}) + \frac{1}{6}(g_{ki}F_j + g_{kj}F_i) - \frac{1}{3}g_{ij}F_k.$$

Consider the case when irreducible components $T_{ijk} = 0$. $A_i = 0$. Then it can be set

$$G_{ijk} = g_{ij}F_k - g_{ik}F_j. \qquad (5)$$

Inserting (5) into (3), we get

$$\frac{\delta u^i}{\delta\tau} + F^i - u^i(u_k F^k) = 0, \qquad (6)$$

since $u_i u^i = 1$. Within the relativistic mechanics [6], an accelerated motion of a particle with mass m is determined by equations

$$a^i = \frac{\delta u^i}{\delta\tau} = \frac{d^2x^i}{d\tau^2} + \{^{\,i}_{jk}\}\frac{dx^j}{d\tau}\frac{dx^k}{d\tau} = \frac{1}{m}N^i, \qquad (7)$$

where N^i is a 4-force orthogonal to a 4-velocity. So, an irreducible component given by the polar vector F^i, permits physical interpretation in the framework of relativistic mechanics provided that $\frac{1}{m}N^i = -F^i + u^i(u_k F^k)$.

Consider now how substitution (5) is consistent with the equations of motion of the field G_{ijk}, proposed in [1], [3]. Equations of the gauge field G_{ijk} are of the form

$$\nabla_i B^{ijkl} + G^k_{im} B^{ijml} - G^l_{im} B^{ijmk} + S^{jkl} = 0, \tag{8}$$

where

$$B_{ijkl} = \nabla_i G_{jkl} - \nabla_j G_{ikl} + G_{ikm} G^m_{jl} - G_{jkm} G^m_{il} + R_{ijkl} \tag{9}$$

is the strength tensor and the tensor S^{jkl} is the current, a source of the gauge field under consideration. It is assumed that the current S^{ijk} is given and obeys the equations

$$\nabla_i S^{ijk} + G^j_{im} S^{imk} - G^k_{im} S^{imj} = 0$$

that are analogous to the local conservation law of energy-momentum in General relativity [3].

Inserting (5) into (9), we get

$$B_{ijkl} =$$

$$g_{ijkl}(F_m F^m) + g_{ijmk}(\nabla^m + F^m)F_l - g_{ijml}(\nabla^m + F^m)F_k + R_{ijkl},$$

where $g_{ijkl} = g_{ik}g_{jl} - g_{il}g_{jk}$. The system of equations (5) and (8) for the field F_i is complicated and, besides, the number of equations is much larger than the number of unknown functions. Therefore, one should verify whether system (5) and (8) is consistent or not. Contracting equations (8) in indices j and l, we arrive at the equations

$$\nabla_i B^{ik} + G^k_{im} B^{im} + G_{ijm} B^{ijmk} + S^k = 0, \tag{10}$$

where $B^{ik} = g_{jl} B^{ijkl}$, $S^k = g_{jl} S^{jkl}$. Contraction in indices j and k, produces analogous equations; whereas that in k and l, an identity. So, any solution to eqs. (8) is a solution to eqs. (10). Inserting (5) into (10), we get

$$\nabla_i B^{ik} - F_m(B^{mk} + B^{km}) + F^k B + S^k = 0, \tag{11}$$

where $B = g_{ij} B^{ij}$. Like the strength tensor B_{ijkl} we can express the tensors B_{ij}, B in terms of the field F_i:

$$B_{ij} = 2g_{ij} F_m F^m - 2(\nabla_i + F_i)F_j - g_{ij} \nabla_m F^m + R_{ij}, \tag{12}$$

$$B = 6(F_m F^m - \nabla_m F^m) + R, \tag{13}$$

where R_{ij} is the Ricci tensor, the R-scalar curvature of space-time. Substituting (12) and (13) into (11), we find that any solution to equations (5) and (8) is a solution to the equations

$$\nabla^i \nabla_i F^k + 3F^k(\nabla_i F^i - F_i F^i) + \frac{1}{2}\nabla^k(\nabla_i F^i - 3F_i F^i) = \frac{1}{2} S^k, \tag{14}$$

where we put $R_{ijkl} = 0$.

To prove the consistency of system of equations (5) and (8) we show that it possesses nontrivial solutions at $S^{ijk} = 0, R_{ijkl} = 0$. One of the solutions looks as follows:

$$F_i = \frac{p_i}{p_0 x^0 + p_1 x^1 + p_2 x^2 + p_3 x^3} = \frac{p_i}{p_m x^m},$$

where p_i is a constant non-isotropic vector field, $p_i p^i \neq 0$.

It can be verified that eqs. (14) are fulfilled with that solution. Calculating the strength tensor we obtain

$$B_{ijkl} = g_{ijkl} \frac{p_m p^m}{(p_n x^n)^2}.$$

and inserting this expression into field equations (8) we see that they also hold valid.

Another solution to eqs. (5) and (8) without sources is a plane wave of an arbitrary shape:

$$F_i = k_i \Phi(kx),$$

where k_i is an isotropic (a null) vector, $k_i k^i = 0$, $kx = k_i x^i$. In this case, the strength tensor is given by

$$B_{ijnl} = (\Phi' + \Phi^2)(g_{ijmn} k_l - g_{ijml} k_n) k^m,$$

where Φ' is the derivative with respect to argument. Thus, system of equations (5) and (8) is consistent.

If in equations (14) we omit the terms with self-action and put $S^k = 0$, we obtain the equations which are a particular case (at $\lambda = 3/2$) of the equations

$$\nabla_i \nabla^i A^k - (1 - \lambda)\nabla^k \nabla_i A^i = 0,$$

that are employed in quantum electrodynamics for fixing gauge of the vector potential A^i [7].

4. To summarize, we dwell on a possible physical interpretation of our consideration. If a rotating body which does not undergo any torque but suffers acceleration, the direction of its rotation with respect to the inertial system changes in accordance with the rule of transference (2). Then it follows that we can qualitatively try to discover the Weyl non-Abelian gauge field by observing the behaviour of a gyroscope at a cosmic station. Since gravitational and electromagnetic forces can there be assumed zero, the gyroscope precession, if discovered, could be induced by that gauge field.

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