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CORRELATIONS OF THE POLARIZATIONS OF TWO PARTICLES WITH SPIN 1/2 IN THE FINAL STATE

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This report is based on the works fulfilled together with Professor $\boxed{\text{M.I.Podgoretsky}}$ [1,2].

1. INTRODUCTION. NONFACTORIZABLE TWO-PARTICLE STATES

Spin correlations in two-particle quantum systems, which will be discussed, are related to the important class of interference correlations that arise when the two-particle wave function is not reduced to the simple product of one-particle wave functions. It can be represented only as a sum of products of one-particle wave functions. Thus, we will consider nonfactorizable two-particle states which are the coherent superpositions of pairs of one-particle states:

$$|\phi\rangle^{(1,2)} = \sum_i \sum_k c_{ik} \, |i\rangle^{(1)} |k\rangle^{(2)}, \tag{1}$$

where c_{ik} are constants, $\sum_i \sum_k |c_{ik}|^2 = 1$.

If a two-particle system itself is a part of a more complicated system, which is described by the two-particle density matrix, the nonfactorizability means that this density matrix is not reduced to the direct product of one-particle density matrices, it can be represented only as a sum of such products:

$$\hat{\rho}^{(1,2)} = \sum_i \sum_k b_{ik} \; \hat{\rho}_i^{(1)} \otimes \hat{\rho}_k^{(2)}, \tag{2}$$

where the symbol $\otimes$ denotes the direct product of matrices, $\sum_i \sum_k b_{ik} = 1$.

Generally, correlations at the registration of nonfactorizable two-particle states by one-particle detectors should be considered as the manifestation of the quantum mechanical effect predicted, at first, by Einstein, Podolsky, and Rosen [3]. The essence of this effect is as follows. If the two-particle state is not factorizable, the character of measurements performed for the first particle determines the readings of the detector that analyzes the state of the second particle, although both the particles may prove to be at a large distance after their creation. In this case the amplitude of the registration of a two-particle state (1) by two one-particle detectors, selecting the states $|L\rangle^{(1)}$ and $|M\rangle^{(2)}$, is a result of the interference of pairs of one-particle states:

$$A_{LM} = \sum_i \sum_k c_{ik} \langle L|i\rangle^{(1)} \langle M|k\rangle^{(2)}. \tag{3}$$

With this, due to the correlations, the selection of different states $|L\rangle^{(1)}$ and $|M\rangle^{(1)}$ only for the first particle leads to the different states of the second particle:

$$|\Psi\rangle_L^{(2)} = \sum_i \sum_k c_{ik} \langle L|i\rangle\, |k\rangle^{(2)},$$
$$|\Psi\rangle_M^{(2)} = \sum_i \sum_k c_{ik} \langle M|i\rangle\, |k\rangle^{(2)}. \tag{4}$$

Let us note that the states $|\Psi\rangle_L^{(2)}$ and $|\Psi\rangle_M^{(2)}$ can be the eigenfunctions of noncommuting operators. As a result, in the presence of the correlations the one-particle state is not pure, and it should be described by the density matrix but not by the wave function. We deal with the «management» by the state of one of two particles without the direct force action on it. A.Einstein considered this situation as a paradox testifying to the incompleteness of the quantum-mechanical description [3]. Now it is clear that here we have the correlation effect connected with coherent properties of quantum-mechanical superpositions. The properties of $K^0\overline{K}{}^0$-pairs provide an impressive example: the registration of one of two neutral kaons at its decay or its interaction determines the internal state of the second kaon [4–7]. The polarization correlations, which are discussed in this report, are from the same group of phenomena. It should be emphasized that precisely in these cases the so-called Bell inequalities are violated. These inequalities were derived at the probability level without taking into account the coherent properties of the quantum-mechanical superpositions [8–10].

2. TWO-PARTICLE DENSITY MATRIX AND SPIN CORRELATIONS

For two spin-1/2 particles, the spin density matrix with the sum of diagonal elements («trace»)

$$\mathrm{tr}_{(1,2)}\, \hat{\rho}^{(1,2)} = 1 \tag{5}$$

has the following general structure [2]:

$$\hat{\rho}^{(1,2)} = \frac{1}{4}\,[\hat{I}^{(1)} \otimes \hat{I}^{(2)} + (\hat{\sigma}^{(1)}\mathbf{P}_1) \otimes \hat{I}^{(2)} + \hat{I}^{(1)} \otimes (\hat{\sigma}^{(2)}\mathbf{P}_2) +$$
$$+ \sum_{i=1}^{3} \sum_{k=1}^{3} T_{ik}\, \hat{\sigma}_i^{(1)} \otimes \hat{\sigma}_k^{(2)}]. \tag{6}$$

Here $\hat{I}$ is the two-row unit matrix, $\hat{\sigma} = \{\hat{\sigma}_1, \hat{\sigma}_2, \hat{\sigma}_3\}$ is the Pauli vector operator, $\mathbf{P}_1 = \langle\hat{\sigma}^{(1)}\rangle$ and $\mathbf{P}_2 = \langle\hat{\sigma}^{(2)}\rangle$ are the polarization vectors, $T_{ik} = \langle\hat{\sigma}_i^{(1)} \otimes \hat{\sigma}_k^{(2)}\rangle$ is the correlation tensor. The corresponding one-particle matrices contain the polarization vectors only:

$$\hat{\rho}^{(1)} = \frac{1}{2}\,(\hat{I} + \hat{\sigma}\mathbf{P}_1), \quad \hat{\rho}^{(2)} = \frac{1}{2}\,(\hat{I} + \hat{\sigma}\mathbf{P}_2). \tag{7}$$

In the absence of correlations the factorization takes place:

$$T_{ik} = P_{1i}\, P_{2k}, \quad \hat{\rho}^{(1,2)} = \hat{\rho}^{(1)} \otimes \hat{\rho}^{(2)}. \tag{8}$$

Let two analyzers select the states of the first and the second particles with the polarization vectors $\zeta^{(1)}$ and $\zeta^{(2)}$. Then the detection probability depends linearly on polarization parameters of the two-particle system as well as on the final polarization parameters fixed by detectors, and it can be obtained by the replacement of the matrices $\hat{\sigma}_i^{(1)}$ and $\hat{\sigma}_k^{(2)}$ in the expression (6) with the spin projections $\zeta_i^{(1)}$ and $\zeta_k^{(2)}$, respectively. As a result

$$W = \frac{1}{4}\left[\, I + \mathbf{P}_1\zeta^{(1)} + \mathbf{P}_2\zeta^{(2)} + \sum_{i=1}^{3} \sum_{k=1}^{3} T_{ik}\, \zeta_i^{(1)}\, \zeta_k^{(2)} \right]. \tag{9}$$

Let only the polarization vector $\zeta^{(1)}$ of the first particle be measured. Then, due to the correlations, the spin state of the second particle, produced together with the first one, is described by the normalized density matrix

$$\hat{\tilde{\rho}}^{(2)} = \frac{1}{2}\,(1 + \zeta^{(1)}\mathbf{P}_1)^{-1} \left[(1 + \zeta^{(1)}\mathbf{P}_1)\,\hat{I} + \sigma\mathbf{P}_2 + \sum_{i=1}^{3} \sum_{k=1}^{3} T_{ik}\, \zeta_i^{(1)}\, \hat{\sigma}_k \right]. \tag{10}$$

In this case the polarization vector of the second particle has the components

$$\tilde{\zeta}_k^{(2)} = \frac{P_{2k} + \sum_{i=1}^{3} T_{ik}\, \zeta_i^{(1)}}{1 + \zeta^{(1)}\, \mathbf{P}_1}. \tag{11}$$

In the case of independent particles, when the factorization takes place, the detection of spin state of the first particle does not influence the polarization of the second particle: $\tilde{\zeta}^{(1)} = \mathbf{P}_2$.

The situation is of interest when both the polarization vectors $\mathbf{P}_1$ and $\mathbf{P}_2$ equal zero, i.e., one-particle states are unpolarized. Then spin effects are completely determined by the correlation tensor T_{ik}, and in accordance with Eq.(11)

$$\tilde{\zeta}_k^{(2)} = \sum_{i=1}^{3} T_{ik}\, \zeta_i^{(1)}. \tag{12}$$

If the one-particle states are unpolarized and the spin correlations are absent, then $\tilde{\zeta}^{(2)} = 0$ at any selection of the vector $\zeta^{(1)}$.

3. THE SECONDARY SCATTERING AS THE ANALYZER OF THE SPIN POLARIZATION

It is known that the scattering of a particle with spin 1/2 on a spinless or unpolarized target selects the states with the spin projections along the normal to the scattering plane.

Let the events of secondary scattering of two created particles with momenta $\mathbf{p}_1$ and $\mathbf{p}_2$ through the angles θ_1 and θ_2 play the role of spin analyzers. Then the final polarization vectors in Eq.(9) appear as the analyzing powers:

$$\zeta^{(1)} = \alpha_1(\mathbf{p}_1, \theta_1)\, \mathbf{n}, \qquad \zeta^{(2)} = \alpha_2(\mathbf{p}_2, \theta_2)\, \mathbf{m}. \tag{13}$$

Here $\mathbf{n}$ and $\mathbf{m}$ are the unit vectors along the normals to the scattering planes, α_1 and α_2 are the left-right azimuthal asymmetry factors, which equal zero at zero scattering angles. According to the Wolfenstein theorem [11,12], the analyzing power coincides with the polarization vector that arises as a result of scattering of the unpolarized particle on the same target. Taking into account Eq.(9), the probability of the simultaneous detection of two particles, produced in the same collision, after their scattering events is proportional to the quantity

$$W(\mathbf{n}, \mathbf{m}) = 1 + \alpha_1(\mathbf{p}_1, \theta_1)\, (\mathbf{P}_1\mathbf{n}) + \alpha_2(\mathbf{p}_2, \theta_2)\, (\mathbf{P}_2\mathbf{m}) + \\ + \alpha_1(\mathbf{p}_1, \theta_1)\, \alpha_2(\mathbf{p}_2, \theta_2) \sum_{i=1}^{3} \sum_{k=1}^{3} T_{ik}\, m_i\, m_k . \tag{14}$$

The formula (14) describes the correlation of the scattering planes.

Let two unpolarized particles be produced in the same nuclear collision, and subsequently one of them is scattered on a spinless or unpolarized target. Then the spin correlation results in the polarization of the other (unscattered) particle created together with the scattered one in the same collision event:

$$\tilde{\zeta}_k^{(2)} = \alpha_1(\mathbf{p}_1, \theta_1) \sum_{k=1}^{3} T_{ik}\, n_i . \tag{15}$$

This phenomenon makes it possible, in principle, to prepare particle beams with regulated spin polarization without acting directly on the particles to be polarized.

4. POLARIZATION CORRELATIONS IN THE SINGLET AND TRIPLET STATES

a) The internal state of the system of two spin-1/2 particles with total spin $S = 0$, or the singlet state, is the typical example of nonfactorizable two-particle states. It is described by the spin wave function

$$|\Psi\rangle_{S=0} = \frac{1}{\sqrt{2}} \left(|+1/2\rangle_z^{(1)}\, |-1/2\rangle_z^{(2)} - |-1/2\rangle_z^{(1)}\, |+1/2\rangle_z^{(2)} \right). \tag{16}$$

In the singlet state the spins are rigidly correlated: the spin projections, equalling $+1/2$ and $-1/2$, are opposite for any choice of the quantization axis z, while the polarization vector of each of the particles is equal to zero. In this case the spin two-particle density matrix has the form

$$\hat{\rho}^{(s)} = \frac{1}{4} [\hat{I}^{(1)} \otimes \hat{I}^{(2)} - \hat{\sigma}^{(1)} \otimes \hat{\sigma}^{(2)}]. \tag{17}$$

This corresponds to the polarization parameters

$$\mathbf{P}_1 = \mathbf{P}_2 = 0, \qquad T_{ik} = -\delta_{ik} \tag{18}$$

in the general expression (6).

In accordance with Eqs.(15) and (18), if one of two particles, produced in the singlet state, is scattered on a spinless or unpolarized target, and it acquires, as a result of scattering, the polarization

$$\zeta^{(1)} = \alpha(\mathbf{p}, \theta)\, \mathbf{n}$$

along the normal $\mathbf{n}$ to the scattering plane ($|\mathbf{n}| = 1$), the second (unscattered) particle, created together with the scattered one, acquires the opposite polarization depending on the scattering angle:

$$\hat{\zeta}^{(2)} = -\zeta^{(1)} = -\alpha(\mathbf{p}, \theta)\, \mathbf{n}, \tag{19}$$

where $\alpha(\mathbf{p}, \theta)$ is the left-right asymmetry factor.

In accordance with Eq.(14), the distribution over the angle φ between the secondary scattering planes of two particles, produced in the singlet state, has the form [1,2]

$$W(\mathbf{n}, \mathbf{m}) = 1 - \alpha_1(\mathbf{p}_1, \theta_1)\, \alpha_2(\mathbf{p}_2, \theta_2) \cos\varphi. \tag{20}$$

b) Now let us consider the triplet states (total spin $S = 1$), which are polarized and aligned along the spin quantization axis $\mathbf{l}$ ($|\mathbf{l}| = 1$). The states with spin projections onto the axis $\mathbf{l}$ equalling $+1$, -1, and 0, respectively, can be represented in the form

$$|\Psi\rangle_{+1} = |+1/2\rangle_l^{(1)} |+1/2\rangle_l^{(2)}, \quad |\Psi\rangle_{-1} = |-1/2\rangle_l^{(1)} |-1/2\rangle_l^{(2)},$$

$$|\Psi\rangle_0 = \frac{1}{\sqrt{2}} \left(|+1/2\rangle_l^{(1)} |-1/2\rangle_l^{(2)} + |-1/2\rangle_l^{(1)} |+1/2\rangle_l^{(2)} \right). \tag{21}$$

We denote the corresponding occupancies as W_+, W_-, and W_0. The two-particle spin density matrix is described by the expression

$$\hat{\rho}^{(1,2)} = W_+ \hat{\rho}_+ + W_- \hat{\rho}_- + W_0 \hat{\rho}_0, \tag{22}$$

where $W_+ + W_- + W_0 = 1$, and

$$\hat{\rho}_\pm = \frac{1}{4} [\hat{I}^{(1)} \otimes \hat{I}^{(2)} \pm (\hat{\sigma}^{(1)}\mathbf{l}) \otimes \hat{I}^{(2)} \pm \hat{I}^{(1)} \otimes (\hat{\sigma}^{(2)}\mathbf{l}) + (\hat{\sigma}^{(1)}\mathbf{l}) \otimes (\hat{\sigma}^{(2)}\mathbf{l})],$$

$$\hat{\rho}_0 = \frac{1}{4} [\hat{I}^{(1)} \otimes \hat{I}^{(2)} + \hat{\sigma}^{(1)} \otimes \hat{\sigma}^{(2)} - 2(\hat{\sigma}^{(1)}\mathbf{l}) \otimes (\hat{\sigma}^{(2)}\mathbf{l})]. \tag{23}$$

With this, the polarization vectors and the polarization tensor are

$$\mathbf{P}_1 = \mathbf{P}_2 = (W_+ - W_-)\,\mathbf{l}, \tag{24}$$

$$T_{ik} = (W_+ + W_- - 2W_0)\, l_i l_k + W_0 \delta_{ik}. \tag{25}$$

If $W_+ = W_- = W_0 = 1/3$, we have the unpolarized triplet with the density matrix

$$\hat{\rho}^{(t)} = \frac{1}{4} \left[\hat{I}^{(1)} \otimes \hat{I}^{(2)} + \frac{1}{3} \hat{\sigma}^{(1)} \otimes \hat{\sigma}^{(2)} \right]. \tag{26}$$

In this case

$$\mathbf{P}_1 = \mathbf{P}_2 = 0, \quad T_{ik} = \frac{1}{3} \delta_{ik}. \tag{27}$$

The following equality is valid:

$$\frac{1}{4} \hat{I}^{(1)} \otimes \hat{I}^{(2)} = \frac{1}{4} \hat{\rho}^{(s)} + \frac{3}{4} \hat{\rho}^{(t)}. \tag{28}$$

It shows that in the absence of spin correlations the system of two unpolarized particles with spin $1/2$ is an incoherent mixture of the singlet and triplet states with the statistical weights $1/4$ and $3/4$, respectively.

c) The case of the unpolarized triplet is realized in the peripherical breakup of an unpolarized deuteron, if the contribution of *D*-wave is neglected (at low momentum transfer and low excitation energies of the *np*-system). Due to the spin correlation, the events of secondary scattering of protons on the ^{12}C-target lead to the preparation of a beam of polarized neutrons produced together with the scattered protons. These neutrons should be polarized along the normal to the scattering plane of protons [1,2]:

$$\zeta^{(n)} = \frac{1}{3} \zeta^{(p)} = \frac{1}{3} \alpha_p(\mathbf{p}_p, \theta_p)\, \mathbf{n}, \qquad |\mathbf{n}| = 1. \tag{29}$$

Here $\zeta^{(p)}$ is the analyzing power for the proton.

The angular correlation of the scattering planes for the proton and neutron, produced at the peripherical breakup of an unpolarized deuteron, is described by the formula

$$W(\mathbf{n}, \mathbf{m}) = 1 + \frac{1}{3} \alpha_p(\mathbf{p}_p, \theta_p)\, \alpha_n(\mathbf{p}_n, \theta_n)\, (\mathbf{nm}), \tag{30}$$

where α_p and α_n are the left-right asymmetry factors for the proton and neutron, respectively. It should be noted that the relations (29) and (30) are valid even with taking into account the *D*-wave contribution, if one performs the averaging over the directions of the proton and neutron relative momenta in the deuteron rest frame [2].

d) Another example is given by the correlations between the polarizations of the proton and the ^{3}He nucleus from the reaction

$$\pi^+ + {}^4\mathrm{He} \to {}^3\mathrm{He} + p. \tag{31}$$

It follows from parity conservation that in this reaction the (^{3}He-p)-system is produced in the triplet states, independently of the emission angle [13]. If the ^{3}He nucleus or the proton is emitted at the zero angle with respect to the reaction axis $\mathbf{l}(|\mathbf{l}| = 1)$, then the states with spin projections $(+1)$ and (-1) onto this axis are forbidden (due to the conservation of the angular momentum and to the fact that the π-meson and the ^{3}He nucleus are spinless). Thus, the (^{3}He-p)-system is created in the triplet state with the zero spin projection onto the vector $\mathbf{l}$.

In accordance with Eqs.(24) and (25), in this case ($W_+ = W_- = 0$, $W_0 = 1$) the one-particle polarizations are zero and the correlation tensor has the form

$$T_{ik} = \delta_{ik} - 2 l_i l_k. \tag{32}$$

In particular, if the proton is scattered on a spinless or unpolarized target (for example, on the ^{12}C nucleus) and the corresponding analyzing power is

$$\zeta^{(p)} = \alpha_p(\mathbf{p}_p, \theta_p)\, \mathbf{n},$$

where $\mathbf{n}$ is the unit vector along the normal to the scattering plane, then the unscattered nucleus ^{3}He, created together with the unscattered proton, acquires the polarization

$$\zeta^{(\mathrm{He-3})} = \alpha_p(\mathbf{p}_p, \theta_p)\, (\mathbf{n} - 2(\mathbf{ln})\, \mathbf{l}). \tag{33}$$

Thus, the polarization vector $\zeta^{(\mathrm{He-3})}$ is constructed according to the reflection law in the plane $(\mathbf{n}, \mathbf{l})$; with this, $|\zeta^{(\mathrm{He-3})}| = |\zeta^{(p)}|$.

This can serve as a basis for preparing a beam of polarized ^{3}He nuclei without acting directly on these nuclei.

5. SPIN CORRELATIONS OF TWO IDENTICAL NUCLEONS AT SMALL RELATIVE MOMENTA

The effect of Bose or Fermi statistics leads to the correlation of spins of identical particles at small relative momenta. This is obvious due to the fact that the total spin S of the system of two identical particles and its orbital angular momentum L satisfy the well-known equality [14]

$$(-1)^{S+L} = 1. \tag{34}$$

When the momentum difference q approaches zero, states with nonzero orbital angular momenta disappear, and only the states with $L = 0$ and even total spin S remain. As a result, two identical particles with spin 1/2 (in particular, two protons or two neutrons) can be produced only in the singlet state, when the relative momentum difference tends to zero [1].

At nonzero values of 4-momentum difference q the triplet states ($S = 1$) of two protons or two neutrons are generated in nuclear collisions together with the singlet state ($S = 0$). The analysis shows that in the framework of the model of independent one-particle sources emitting unpolarized particles, which is used usually for the description of the momentum-energy correlations of identical particles with close momenta [15,16,17], the relative occupancies of the singlet and unpolarized triplet states are [2]

$$W^{(s)} = \frac{1}{4}\,(1 + |F(q)|^2 + 2B_{\text{int}}(q)), \tag{35}$$

$$W^{(t)} = \frac{3}{4}\,(1 - |F(q)|^2), \tag{36}$$

where $|F(q)|^2$ is the contribution of Fermi-statistics for noninteracting particles, $B_{\text{int}}(q)$ is the contribution of the s-wave final state interaction [16,17]. Both the quantities depend on space-time parameters of the multiple generation region and tend to zero at sufficiently large values of relative momentum q; the function $F(q)$ is expressed directly through the space-time distribution of 4-coordinates of sources [15,17]:

$$F(q) = \int W(x)\, e^{iqx}\, d^4x, \qquad F(0) = 1. \tag{37}$$

According to Eqs.(17), (26), (35), and (36), the normalized spin density matrix of two protons or two neutrons at small relative momenta has the following structure:

$$\hat{\rho}^{(1,2)} = \frac{1}{4}\,[\hat{I}^{(1)} \otimes \hat{I}^{(2)} - K\,\hat{\sigma}^{(1)} \otimes \hat{\sigma}^{(2)}],$$

where

$$K = \frac{|F(q)|^2 + B_{\text{int}}(q)}{2 - |F(q)|^2 + B_{\text{int}}(q)}. \tag{38}$$

In this situation the correlation tensor is

$$T_{ik} = -K\delta_{ik}. \tag{39}$$

In correspondence with this, at the secondary scattering of one of two identical nucleons with the momentum $\mathbf{p}_1$ the other (unscattered) nucleon with the momentum $\mathbf{p}_2$, which is produced together with the scattered nucleon in the same collision event, acquires the polarization

$$\zeta^{(2)} = -K\alpha(\mathbf{p}_1, \theta_1)\,\mathbf{n} \tag{40}$$

along the normal to the scattering plane of the first nucleon ($|\mathbf{n}| = 1$). The angular correlation of the scattering planes of two final identical nucleons has the form

$$W(\mathbf{n}, \mathbf{m}) = 1 - \alpha_1(\mathbf{p}_1, \theta_1)\,\alpha_2(\mathbf{p}_2, \theta_2)\,K\cos\varphi. \tag{41}$$

Here $\alpha(\mathbf{p}, \theta)$ is the left-right azimuthal asymmetry factor, $\cos\varphi = \mathbf{nm}$.

It should be emphasized that the correlation of the polarizations of two particles with spin 1/2, conditioned by their identity, is maximal for $q \to 0$ ($K = 1$, the singlet state). The s-wave interaction of final nucleons intensifies the relative contribution of the singlet [1,2]. In the framework of the model of independent one-particle sources [15–17] spin correlations vanish at sufficiently large q ($K = 0$).

6. VIOLATION OF THE BELL INEQUALITIES IN THE CASE OF NONFACTORIZABLE TWO-PARTICLE STATES

The analysis of the correlations between the scattering planes of two particles with spin 1/2 makes it possible to determine the quantity

$$\sum_i \sum_k T_{ik}\, n_i\, m_k = \langle(\hat{\sigma}^{(1)}\mathbf{n}) \otimes (\hat{\sigma}^{(2)}\mathbf{m})\rangle, \tag{42}$$

which is the average product of the double spin projections of the first and second particles onto different axes ($\mathbf{m}$ and $\mathbf{n}$ are the unit vectors), and to verify, in this way, the Bell inequalities [8,9]. These inequalities were obtained at the probability level in the framework of the concept of hidden parameters related to the common past of particles, separated from each other in space during the detection. With this, the coherent properties of the quantum-mechanical superpositions of two-particle states were not taken into consideration. One of these inequalities, as applied to particles with spin 1/2, has the form [10]

$$Q = \left| \langle(\hat{\sigma}^{(1)}\mathbf{n}) \otimes (\hat{\sigma}^{(2)}\mathbf{m})\rangle + \langle(\hat{\sigma}^{(1)}\mathbf{n}) \otimes (\hat{\sigma}^{(2)}\mathbf{m}')\rangle + \langle(\hat{\sigma}^{(1)}\mathbf{n}') \otimes (\hat{\sigma}^{(2)}\mathbf{m})\rangle - \langle(\hat{\sigma}^{(1)}\mathbf{n}') \otimes (\hat{\sigma}^{(2)}\mathbf{m}')\rangle \right| \le 2, \tag{43}$$

where $\mathbf{n}$, $\mathbf{m}$, $\mathbf{m}'$ and $\mathbf{n}'$ are arbitrary unit vectors. In quantum mechanics this inequality may be violated. In particular, that happens in the case of the singlet state, if the unit vectors are chosen as follows:

$$\mathbf{n} = \mathbf{m}, \quad \mathbf{n}'\mathbf{m} = \mathbf{m}'\mathbf{n} = \cos\varphi, \quad \mathbf{n}'\mathbf{m}' = \cos 2\varphi.$$

In accordance with Eqs.(42) and (17), in the singlet state

$$\langle(\hat{\sigma}^{(1)}\mathbf{n}) \otimes (\hat{\sigma}^{(2)}\mathbf{m})\rangle = -\,\mathbf{nm}.$$

As a result, we have at $0 < \varphi < \dfrac{\pi}{2}$

$$Q = \left|-1 - 2\cos\varphi + \cos 2\varphi\right| = 2 + 2\cos\varphi\,(1 - \cos\varphi) > 2$$

contrary to the inequality (43).

It should be stressed that there exists the difference of principle between the singlet state in quantum mechanics and the incoherent mixture of two-particle states with opposite projections onto axes, distributed isotropically in space. In the last case

$$\langle(\hat{\sigma}^{(1)}\mathbf{n}) \otimes (\hat{\sigma}^{(2)}\mathbf{m})\rangle = -\frac{1}{3}\,\mathbf{nm},$$

and the Bell inequality (43) holds.

7. SUMMARY

1. The analysis of spin correlations at the detection of nonfactorizable spin states of two particles with spin $1/2$ is performed.

2. Such correlations are connected with the general quantum-mechanical effect predicted by Einstein, Podolsky, and Rosen. Their study allows one to test the basic principles of quantum mechanics (in particular, to establish the violation of the classical Bell inequalities).

3. The correlation of spins leads to the angular correlation of the scattering planes for two final particles produced in the same event of collision and subsequently scattered on the spinless or unpolarized targets.

4. Due to the spin correlations, the secondary scattering of one of two unpolarized particles results in the polarization of the other (unscattered) particle produced in the same collision event. This effect makes it possible, in principle, to prepare particle beams with regulated spin polarization without the direct action on the particles to be polarized.

5. It is shown that the spins of the ^{3}He nucleus and the proton in the reaction $\pi^+ + {}^4\mathrm{He} \to {}^3\mathrm{He} + p$ are strongly correlated.

6. The effect of Fermi statistics leads to the spin correlations of two protons or two neutrons created with small relative momenta. These polarization correlations depend on the space parameters of the generation region.

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Корреляции поляризаций двух частиц со спином $1/2$ в конечном состоянии

Анализируются спиновые корреляции при регистрации нефакторизуемых двухчастичных спиновых состояний. Появление таких корреляций связано с общим квантово-механическим эффектом, предсказанным Эйнштейном, Подольским и Розеном. При наличии спиновых корреляций рассеяние одной из двух неполяризованных конечных частиц приводит к поляризации другой (нерассеянной) частицы. Это делает возможным приготовление пучков частиц с контролируемой спиновой поляризацией без прямого воздействия на поляризуемые частицы. Обсуждаются особенности корреляций в синглетном и триплетном состояниях двух частиц со спином $1/2$. Рассмотрены корреляции поляризаций двух тождественных нуклонов (протонов, нейтронов) с малыми относительными импульсами и нейтрон-протонные спиновые корреляции при развале дейтрона. Показано, что спины конечных частиц в реакции $\pi^+ + {}^4\mathrm{He} \to {}^3\mathrm{He} + p$ сильно скоррелированы. Исследуются корреляции плоскостей вторичного рассеяния двух частиц со спином $1/2$ на бесспиновой или неполяризованной мишени.

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Correlations of the Polarizations of Two Particles with Spin $1/2$ in the Final State

Spin correlations at the detection of nonfactorizable two-particle spin states are analyzed. The appearance of such correlations is connected with the general quantum-mechanical effect predicted by Einstein, Podolsky, and Rosen. In the presence of the spin correlations the scattering of one of two unpolarized final particles results in the polarization of the other (unscattered) particle. That makes it possible to prepare particle beams with controlled spin polarization without the direct action on the particle to be polarized. Specific features of correlations in singlet and triplet states of two particles with spin $1/2$ are discussed. The correlations of the polarizations of two identical nucleons (protons, neutrons) with small relative momenta and neutron-proton spin correlations at the deuteron breakup are considered. It is shown that the spins of the final particles from the reaction $\pi^+ + {}^4\mathrm{He} \to {}^3\mathrm{He} + p$ are strongly correlated. The correlations of the secondary scattering planes for two particles with spin $1/2$, scattered on spinless or unpolarized target, are studied.

The investigation has been performed at the Laboratory of High Energies, JINR.

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