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Z-SCALING IN PROTON-NUCLEUS COLLISIONS AT HIGH ENERGIES

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1 Introduction

Nuclear reactions with high momentum transfer at relativistic energies present great source of information for studying the properties of quark and gluon interactions in the nuclear matter environment. The development of universal approach to the description of the processes is important for detail understanding of the influence of nuclei on the physical phenomena underlying secondary particle production. Numerous experimental data on relativistic nuclear interactions show that the general tendencies are manifested mostly in the high energy and high transverse momentum $q_\perp$-regions. They reflect specific characteristics of the elementary constituent interactions. This is especially actual with connection of the commissions of the large accelerators of hadrons and nuclei such as Relativistic Heavy Ion Collider (RHIC) at Brookhaven or Large Hadron Collider (LHC) at CERN [1, 2, 3]. The main physical goal of the investigations on these machines is to search for quark-gluon plasma (QGP), the hot and superdense phase of the nuclear matter [4]-[12].

At present there is no single clearly established signature of the QGP. Therefore, search for new regularities which are sensitive to the nature of phase transition from hadron to quark-gluon degrees of freedom are of special interest. Up to date, the investigations of properties of high energy nuclear interactions have revealed widely known scaling laws. Some of the most popular and famous are the Feynman scaling [13] for inclusive particle production, y-scaling observed in deep inelastic scattering on nuclei [14], limiting fragmentation found for nuclei fragmentation [15], $m_\perp$ scaling [16, 17], scaling in cumulative particle production [18, 19, 20], KNO scaling [21] and others. However, detailed experimental study has shown certain violations of these. It can be connected with the dynamics concerning the transition from the perturbative QCD quarks and gluons to the observed hadrons.

The inclusive cross section for particle production is considered as possible experimental observable for studying of unusual properties of nuclear matter at extreme conditions. In limited regions of transverse momenta ($q_\perp < 2$ GeV/c) the particle spectra are often presented as a function of the transverse mass, $m_\perp = \sqrt{q_\perp^2 + m^2}$. This is motivated by the experimental fact that the cross section for the production of a particle is described by the exponential in $m_\perp$ rather than $q_\perp$ [16, 17]. Furthermore, the shapes of the spectra are similar for different types of particles, when plotted against $m_\perp$. Explanations for the '$m_\perp$-scaling' usually assume some form of thermal equilibrium relating the inverse slope parameter to a temperature [22]. A thermal or Boltzmann model predicts that the number of particles per unit phase space is given by $d^3N/dq^3 \sim exp(-E/T)$. The T is a temperature of source and the E is center-of-mass energy associated with phase volume dq^3. Expressed in terms of the multiplicity, rapidity and the transverse mass one finds $m_\perp^{-1} d^2N/dm_\perp dy \sim m_\perp exp(-m_\perp/T_B(y))$, where $T_B(y)$ is the rapidity dependent Boltzmann temperature. It has been suggested by Hagedorn to present experimental data in terms of invariant cross section $(2\pi m_\perp)^{-1} d^2\sigma/dm_\perp dy$ and compare them with the thermal prediction by fitting the exact expression in soft $q_\perp$ region. Numerous experimental results on particle spectra measured in pp, pA, and AA collisions at BNL, CERN, and Fermilab [23, 24, 25] in wide energy and transverse momentum range show that the shapes of the distributions are not simple exponential in any representation. The deviations from pure exponential in $m_\perp$-representation are discussed in Refs. [26, 27, 28]. The slope constants that are used to characterize the spectra depend on particle type, rapidity, centrality and the energy of the collisions.

The common feature of the particle production at high energy $\sqrt{s}$ and high transverse momentum ($q_\perp > 1$ GeV/c) indicate the local character of hadron interactions. It leads to the conclusion about dimensionless of the constituents taking part in the interactions. The fact that the interaction is local founds its natural manifestation in the scale-invariance of the hadron interactions' cross sections. The invariance is an expression of self-similarity principle [18, 29]. This principle reflects the dropping of certain dimensional quantities or parameters out of the physical picture of the processes.

In the paper we exploit the concept based on the self-similarity of the elementary interactions complemented by considerations about fractal structure of the colliding objects. The ideas are implemented into the construction of new scaling, the z-scaling, for the description of inclusive particle production in pA interactions at high energies. The scaling was applied for the analysis of pp and $\bar{p}p$ collisions in the energy range $\sqrt{s} > 23$ GeV in Ref. [30]. The scaling function $H(z)$ is expressed via the invariant inclusive cross section $Ed^3\sigma/dq^3$ and the multiplicity density of charged particles $dN/d\eta \equiv \rho(s)$ produced at the pseudorapidity $\eta = 0$. It was found that the $H(z)$ is independent of colliding energy $\sqrt{s}$ and angle θ of the inclusive particle. In the case of hadron production the scaling function $H(z)$ is interpreted as the probability to form hadrons with a formation length z. The universality of $H(z)$ means that the hadronization mechanism is of universal nature. We suggest that the difference between the $H(z)$ for pp and/or $H_A(z)$ for pA collisions on one side and the $H_{AA}(z)$ for AA interactions on the other side can give definite evidence about the character of nuclear matter influence on the process of particle production. We propose that the dependence of $H_{AA}(z)$ on z for hadronic and QGP phases of nuclear matter can be quantitatively distinguished.

The paper is organized as follows. The method of constructing of the scaling function $H_A(z)$ for the $p + A \to h + X$ process is described in Sec. II. Some consequences of the method concerning fractality and scale relativity in particle production are discussed in the part C. In Sec. III, we show that the available high energy experimental data for the pA collisions (A=d, Be, Ti, W) confirm the z-scaling. It is found that the functions $H_A(z)$ demonstrate energy independence and A-universality in the considered energy region. Thus, besides the pp case, the variable z reflects self-similar behaviour of particle production in the pA interactions as well. The leading principle of self-similarity is in agreement with ideas about fractal character of the objects taking part in the interactions. In Sec. IV, we present physical interpretation of the scaling function $H(z)$ and the variable z.

2 General Principles of the z-Scaling

We start with the investigation of the inclusive process

$$M_1 + M_2 \to m_1 + X, \tag{1}$$

where M_1 and M_2 are masses of the colliding nuclei (or hadrons) and m_1 is the mass of the inclusive particle. In accordance with Stavinsky's ideas [19] the gross features of the inclusive particle distributions for the reaction (1) at high energies can be described in terms of the corresponding kinematical characteristics of the exclusive subprocess

$$(x_1M_1) + (x_2M_2) \to m_1 + (x_1M_1 + x_2M_2 + m_2). \tag{2}$$

The parameter m_2 is introduced in connection with internal conservation laws (for isospin, baryon number, and strangeness). The x_1 and x_2 are the scale-invariant fractions of the incoming four-momenta P_1 and P_2 of the colliding objects. The energy of the parton subprocess defined as

$$\hat{s}_x^{1/2} = \sqrt{(x_1P_1 + x_2P_2)^2} \tag{3}$$

represents the center-of-mass energy of the constituents taking part in the collision. In accordance with the space-time picture of hadron interactions at the parton level, the cross section for the production of the inclusive particle is governed by the minimal energy of colliding partons

$$d\sigma/dt \approx 1/\hat{s}^2_{min}(x_1, x_2). \tag{4}$$

The corresponding energy $\hat{s}^{1/2}_{min}$ is fixed as minimum of Eq. (3) which is necessary for creation of the secondary particle with mass m_1 and a four-momentum q. In the next, we present a scheme from which a more general structure of the variables x_1 and x_2 follows. We would like to emphasize two main points of this approach. First one is fractal character of the parton content of the composite structures involved. Second one is based on the self-similarity of the mechanism underlying the particle production on the level of the elementary constituent interactions.

2.1 Momentum fractions x_1 and x_2

Let us consider the elementary parton-parton collision as a binary subprocess which is a subject to the condition

$$(x_1P_1 + x_2P_2 - q)^2 = (x_1M_1 + x_2M_2 + m_2)^2. \tag{5}$$

The relationship between x_1 and x_2 can be conveniently written in the form

$$x_1x_2 - x_1\lambda_2 - x_2\lambda_1 = \lambda_0, \tag{6}$$

where

$$\lambda_1 = \frac{(P_2q) + M_2m_2}{(P_1P_2) - M_1M_2}, \quad \lambda_2 = \frac{(P_1q) + M_1m_2}{(P_1P_2) - M_1M_2}, \quad \lambda_0 = \frac{0.5(m_2^2 - m_1^2)}{(P_1P_2) - M_1M_2}. \tag{7}$$

Considering the process (2) as a parton-parton collision, we introduce the coefficient Ω which connects kinematical with dynamical characteristics of the interaction. The coefficient is chosen in the form

$$\Omega(x_1, x_2) = m(1 - x_1)^{\delta_1}(1 - x_2)^{\delta_2}, \tag{8}$$

where m is a mass constant and δ_1 and δ_2 are factors relating the fractal structure of the colliding objects. Physical interpretation of the coefficient Ω is given in Sec. IV. We determine the fractions x_1 and x_2 in a way to maximize the value of $\Omega(x_1, x_2)$,

$$\frac{d\Omega(x_1, x_2)}{dx_1} = 0, \tag{9}$$

simultaneously fulfilling the condition (6). This gives

$$x_1 = \lambda_1 + \chi_1, \qquad x_2 = \lambda_2 + \chi_2, \tag{10}$$

where

$$\chi_1 = \sqrt{\mu_1^2 + \omega_1^2} - \omega_1, \qquad \chi_2 = \sqrt{\mu_2^2 + \omega_2^2} + \omega_2. \tag{11}$$

Here we have used the notation

$$\mu_1^2 = (\lambda_1\lambda_2 + \lambda_0)\alpha\frac{(1 - \lambda_1)}{(1 - \lambda_2)}, \qquad \mu_2^2 = (\lambda_1\lambda_2 + \lambda_0)\frac{1}{\alpha}\frac{(1 - \lambda_2)}{(1 - \lambda_1)}, \tag{12}$$

$$\omega_1 = (\lambda_1\lambda_2 + \lambda_0)\frac{(\alpha - 1)}{2(1 - \lambda_2)}, \qquad \omega_2 = (\lambda_1\lambda_2 + \lambda_0)\frac{(\alpha - 1)}{2\alpha(1 - \lambda_1)}. \tag{13}$$

The parameter $\alpha = \delta_2/\delta_1$ is the ratio of the fractal dimensions δ_2 and δ_1. More detailed description concerning the physical interpretation of the coefficients is given in the part C. In the considered case of proton-nucleus interactions, the nucleus is labeled by index 2, the value of α is chosen to be atomic number A. The choice is justified by our analysis of experimental data. The relation reflects essential feature of the fractal structure of nuclei in the z-scaling scheme.

The variables x_1 and x_2 posses the interesting symmetry properties. They satisfy the hypothesis [19] of minimum recoil mass (5) in the elementary constituent interaction. Both are equal to unity along the phase space limit. From the conditions $x_i \le 1$ we get the restriction

$$\lambda_1 + \lambda_2 + \lambda_0 \le 1. \tag{14}$$

This inequality can be rewritten to the threshold condition

$$M_1 + M_2 + m_1 + m_2 \le \sqrt{s_A} \tag{15}$$

and to the condition

$$(M_1 + M_2 + m_2)^2 + E^2 - m_1^2 \le (\sqrt{s_A} - E)^2. \tag{16}$$

The symbol $\sqrt{s_A}$ stands for the center-of-mass energy of the pA system. The last inequality bounds kinematically the maximal possible energy E of the inclusive particle m_1 in the c.m.s. of the reaction (1). From the relation it follows that the variables x_1 and x_2 cover the full phase space ($0 \le x_1$, $x_2 \le 1$) accessible at any energy. The meaning of the parameter m_2 as the threshold for the production of the inclusive particle m_1 emerges here in the natural way. Further properties of the fractions x_1 and x_2 are described in more detail in the Appendix A.

2.2 Scaling variable z and scaling function $H(z)$

In accordance with the self-similarity principle we search for the solution depending on a single scaling variable z in the form

$$\frac{1}{<N>\sigma_{inel}}\frac{d\sigma}{dz} \equiv \psi(z). \quad (17)$$

Here σ_{inel} is the inelastic cross section, $<N>$ is the average multiplicity and $\psi(z)$ has to be a scaling function. The quantities refer to the pA interactions. The invariant differential cross section for the production of the inclusive particle m_1 depends on transverse and longitudinal momenta $q_\perp$ and $q_\parallel$, respectively. In terms of

$$y = \frac{1}{2}\ln\frac{\lambda_2}{\lambda_1}, \quad (18)$$

and the scaling variable $z = z(\lambda_1, \lambda_2)$, the invariant cross section can be expressed as

$$E\frac{d^3\sigma}{dq^3} = \frac{1}{\pi s A}\left(\frac{1}{\lambda_2}\frac{\partial z}{\partial\lambda_1} + \frac{1}{\lambda_1}\frac{\partial z}{\partial\lambda_2}\right)\frac{d^2\sigma}{dzdy}. \quad (19)$$

The s is square of the center-of-mass energy of the corresponding NN system and the A is atomic number. We introduce the scaling function $H(z)$ in the way

$$H(z) \equiv \frac{1}{2\pi z}\psi(z) = \frac{1}{g(\lambda_1,\lambda_2)\rho_A\sigma_{inel}}E\frac{d^3\sigma}{dq^3}, \quad (20)$$

where, the factor g is given by

$$g(\lambda_1, \lambda_2) \equiv (sA)^{-1}\left(\frac{z}{\lambda_2}\frac{\partial z}{\partial\lambda_1} + \frac{z}{\lambda_1}\frac{\partial z}{\partial\lambda_2}\right), \quad (21)$$

as follows from Eqs. (17) and (19). The relation (20) connects the inclusive differential cross section and the multiplicity density $\rho_A(s,\eta) = d<N>/d\eta$ with the scaling function $H(z)$. As usual, the combination (18) is approximated with (pseudo)rapidity η at high energies. The properties of the $\psi(z)$ and $H(z)$ under scale transformations of their argument z are given by the relations

$$z \to z' = az, \quad (22)$$

$$\psi(z) \to \psi'(z') = \frac{1}{a}\cdot\psi\left(\frac{z'}{a}\right), \quad (23)$$

$$H(z) \to H'(z') = \frac{1}{a^2}\cdot H\left(\frac{z'}{a}\right). \quad (24)$$

Next, we choose z as a physically meaningful variable which could reflect the self-similarity (scale invariance) as a general pattern of the hadron production. If we put for z an asymptotic value

$$z = \frac{\hat{s}_x^{1/2}}{Q} \to \frac{2\sqrt{\lambda_1\lambda_2}\sqrt{s_A}}{Q}, \quad (25)$$

with Q as a scale which in a first approximation does not depend on λ_1 and λ_2, we get the expression

This leads us to conclude that while the composition of velocities follows Einstein-Lorenz law, the composition of the corresponding dimensions of scale follows the multiplicative group law. The correspondence is a particular expression of scale relativity. Really, the resolution α with which measurements have been performed may be defined as a relative state of scale of reference system [31]. In the considered perspective, the principle of scale relativity states that Einstein-Lorenz composition law of velocities applies to the systems of reference whatever their state of scale.

3 Z-scaling in pA-collisions

Before analyzing the results on z-scaling in pA systems, we would like to remind main features of the scaling concerning the inclusive particle production in nucleon-nucleon interactions. In Fig. 1(a) we present the function $H(z)$ for charged hadrons produced in the central region of pp and $\bar{p}p$ collisions at $\sqrt{s} = 19 - 1800$ GeV. The scaling variable z and the $H(z)$ were constructed according to the formulae given in Sec. II. The result demonstrates the universality of $H(z)$, the independence of scaling function on colliding energy $\sqrt{s}$ in the considered energy region. Note that the data at $\sqrt{s} = 630$ GeV cover the kinematic range of the transverse momenta of secondary particles up to $q_\perp = 24$ GeV/c. For the comparison with the z-presentation, the $m_\perp$-dependence of the same data is shown in Fig. 1(b). One can see that the invariant cross section does not indicate any universality as a function of $m_\perp$ when plotted for different energies $\sqrt{s}$. Similar dependence of $H(z)$ for π^--meson production at $\sqrt{s} = 53$ GeV and $\theta = 2.86^0 - 90^0$ c.m.s. is shown in Fig. 2(a). Here we have used the value of $\rho_p(2.86^0) = 0.3$ for the angle $\theta = 2.86^0$ in Eq. (20) which corresponds to the data [32] on multiplicity densities in the fragmentation region. The approximate angular independence of the scaling function is in contrast with Fig. 2(b) where the same data are plotted as a function of the transverse mass $m_\perp$. Distinctive differences between the fragmentation and the central region represent various normalizations and slopes of the spectra.

We would like to note that the $m_\perp$-presentation is traditionally used to describe the particle spectra in a restricted transverse momentum range (say $p_\perp < 1 - 2$ GeV/c). Though all curves have non-exponential behaviour in larger transverse momentum regions ($q_\perp$ up to 10 or more GeV/c), there always exists good $m_\perp$-presentation for limited $m_\perp$ or $q_\perp$ intervals. In particular case of the central region for the collisions with $q_\perp \ll \sqrt{s}$, the relation between $m_\perp$ and z representation is as follows

$$z \propto \frac{m_\perp}{\rho(s)}, \quad (38)$$

$$H(z) \propto \frac{1}{\rho_A\sigma_{inel}}\cdot\frac{\rho^2(s)}{2\pi m_\perp}\frac{d^2\sigma}{dm_\perp dy}. \quad (39)$$

In this sense, the z scaling represents a generalization of the $m_\perp$ scaling in the high energy region. Let us stress that the energy and the angular universality of the z-scaling for pp and $\bar{p}p$ collisions was achieved with the same value of the fractal dimension $\delta_1 \sim 0.8$. We will use, therefore, this number in the analysis of the z scaling in the case of the pA systems. It was found also that there is a strong sensitivity of the scaling behaviour on the energy dependence of the scale $\rho(s)$. The relevant multiplicity densities of charged particles produced in the central pseudorapidity region in pp and $\bar{p}p$ collisions are shown in Fig. 3. The values of $dN(0)/d\eta \equiv \rho(s)$ resulting from the requirement of the z scaling are denoted by the crosses. The results of Monte Carlo simulations of multiplicity density $\rho_A(s)$ of secondaries produced in pA collisions for different nuclei are also shown in Fig. 3.

Let us proceed to the study of z-scaling in pA interactions. We have examined the experimental data on four nuclear targets D, Be, Ti, and W covering a wide range of atomic weight. In the considered experiment [33] the measurements were made at a laboratory angle of 77 mrad, which corresponds to the angles near 90^0 in the center-of-mass system of the corresponding nucleon - nucleon collisions. First, we exploit the data on the inclusive $\pi^{\pm}$-meson production in the $p+d \to h+X$ process at incoming momentum $p = 400$ GeV/c. The data are expressed in terms of the function $H_d(z)$ which depends on the scaling variable z as presented in Fig. 4. The result shows that the description in terms of z-representation coincides with good accuracy with the scaling function for proton-proton collisions. We have checked the sensitivity of such universal behaviour with respect to the ratio of fractal dimensions α. The situation is depicted by two dashed lines in the figure. We can see that, in the case of deuteron, the value of $\alpha = 2$ is distinguished with regard to the z scaling universality. The obtained results confirm z-scaling in pd collisions at high energies. The dependence of the function $H_{Be}(z)$ on z for the process $p + Be \to \pi^{\pm} + X$ at the incident proton momenta $p =$ 200, 300, and 400 GeV/c is presented in Fig. 5. Similar behaviour for heavier nuclei as titanium (A=48) and tungsten (A=184) are shown in Figs. 6(a) and 6(b). The functions $H_A(z)$ for the both nuclei exhibit energy independence in contrast to the behaviour of $Ed^3\sigma/dq^3$ as a function of $m_\perp$. As we can see from Fig. 7, the energy evolution with respect to the $m_\perp$ variable is manifested especially in the hard part of the spectrum. Thus, besides the energy independence, the A-universality of the scaling functions is found. The obtained results give us strong argument to use z-scaling formalism for the analysis of experimental data on inclusive cross sections in pA interactions. We would like to remember that the scaling in the proposed form is valid for pp collisions in the energy range $\sqrt{s} > 20$ GeV [30]. It is reasonable to assume that similar restriction exists for proton-nucleus interactions and that full asymptotic regime will be achieved for corresponding center-of-mass energies.

In order to construct the scaling function $H_A(z)$ for particle production in pA interactions, it is necessary to know the values of the average multiplicity density $\rho_A(s,\eta)$ of secondaries produced in pA collisions. The values enter to the scheme in Eq. (20). At present there are no experimental data on ρ_A for pA collisions at high enough energies ($\sqrt{s} > 20$ GeV). Therefore, we have used the Monte Carlo code HIJING [34, 35] to determine the energy dependence of the multiplicity densities of charged particles for different nuclei - Al, Ti, W, and Au (Figure 3.). The atomic numbers of the nuclei change from 27 to 197. The results of simulations of $\rho_A(s)$ obtained in the central region of the corresponding NN interactions are denoted by points in Fig. 3. The densities of charged particles were averaged over the impact parameters b $(0 < b < 10fm)$. The obtained values can be parametrized by the formula

$$\rho_A(s) \simeq 0.67 \cdot A^{0.18} \cdot s^{0.105}, \quad A \geq 2. \tag{40}$$

In the case of pp or $\bar{p}p$ collisions, the fit $\rho(s) = 0.74 s^{0.105}$ used in Refs. [36, 37] is shown in the same figure. The comparison of the multiplicity densities for pp and pA

collisions indicates similar energy dependence for both cases. It can be a consequence of the Pomeron trajectory with Pomeron intercept $\Delta = \alpha_P - 1 \simeq 0.105$.

4 Results and discussion

We would like to present some qualitative picture, the substantial elements of which are the basic characteristics of the underlying parton subprocess (2) in terms of the scaling proposed. Generally it is based on the scheme suggested in Ref. [30] for pp or $\bar{p}p$ collisions. In the scenario, we focus our study to the regime of local parton interactions of incident hadrons and nuclei. It manifests itself in the production of particles with high $q_\perp$ at high energies. In this regime the parton distribution functions of incoming objects are separated and, therefore, the scaling function $H(z)$ reflects the fragmentation process of produced partons into the observable hadrons.

The cross section of hadron interactions at the level of single parton-parton scattering (4) is governed by the minimal energy $\hat{s}^{1/2}_{min}$ of colliding constituents. Thus, for the high energy regime of elementary parton interactions, the expression for the differential cross section can be written in the form

$$E\frac{d^3\sigma}{dq^3} \sim s^{-2}_{min}(x_1,x_2) \cdot G_{A_1}(x_{1min}) \cdot G_{A_2}(x_{2min}) \cdot D^h(z_q). \tag{41}$$

Here x_{1min} and x_{2min} satisfy the condition $s(x_{1min}, x_{2min}) = s_{min}(x_1,x_2)$. The relation reflects quantitative measure of the proportionality to the elementary parton cross section which is the number of partons expressed by structure functions. It allows to introduce the concept of parton structure function of nuclei $G(x_i)$ [38]. The fragmentation of the secondary produced parton is described by the fragmentation function $D^h(z_q)$.

The approach based on the principles of self-similarity, locality and fractality is different. Fractal character in the initial state regards the parton composition of hadrons and nuclei and reveals itself with more resolution at high energies. Leading by these principles, we construct the variable z according to Eq. (27). The cross section can be expressed in therms of z as follows

$$E\frac{d^3\sigma}{dq^3} \sim g(\lambda_1,\lambda_2) \cdot H(z). \tag{42}$$

The scaling properties of the function $H(z)$ and comparison of the Eqs. (41) and (42) give us some arguments to write the relation

$$H(z) \sim D^h(z_q). \tag{43}$$

It reflects universality of hadronization mechanism with the variable z considered as a hadronization parameter.

Really, z can be interpreted in terms of parton-parton collision with the subsequent formation of a string stretched by the leading quark out of which the inclusive hadron is formed. The energy of the colliding constituents $s^{1/2}_x$ is just the energy of the string which connects the two objects in the final state of the subprocess (2). The string evolves further and splits into pieces. The resultant number of the string pieces is proportional to number or density of the final hadrons measured in experiment. As known from various experimental and theoretical studies concerning the multiple production, the produced multiplicity is proportional to the excitation of transverse degrees of freedom. Therefore, the string transverse energy is a measure of multiplicity. Such ideas allow us to interpret the ratio

$$\sqrt{s_h} \equiv \hat{E}_{\perp}^{kin}/\rho(s) \tag{44}$$

as a quantity proportional to the transverse energy of a string piece $\sqrt{s_h}$, which does not split already, but during the hadronization converts into the observed hadron. The process of string splitting is self-similar in the sense that the leading piece of the string forgets the string history and its hadronization does not depend on the number and behaviour of other pieces. We consider that the factor Ω in the definition of z reflects fractal structure of the colliding objects and represents degree of "softness" of the initial partons participating the elementary interaction. Maximal softness corresponds to the maximal tension of the generated string what is expressed by the condition (9). Then we write following relation

$$\sqrt{s_h} = \Omega \cdot z. \tag{45}$$

So, in the inclusive hadron production, we consider the variable z as a quantity proportional to the length of the elementary string, or to the formation length, on which the inclusive hadron is formed from its QCD ancestor.

The complementary interpretation of the physical meaning of the variable z is based on ideas concerning fractality in high energy collisions. The fractal objects are usually characterized by power law dependence of their fractal measures [31]. The fractal measure, considered in our case, is given by all possible configurations of elementary interactions that lead to the production of the inclusive particle. It has the following form

$$\Omega(x_1, x_2) \sim (1-x_1)^{\delta_1}(1-x_2)^{\delta_2}.$$

The formula expresses the factorization of the fractal measure with respect to the fractal measures of colliding objects. Both are described by power law dependence in the space of fractions $\{x_1, x_2\}$. The single measure reflects number of constituent configurations in the colliding object taking part in production of the inclusive particle. The measure is characterized by the fractal dimension δ. Fractal dimensions can be different for various colliding objects. Results of our analysis show that the fractal dimension of nucleus δ_A is related to the nucleon fractal dimension δ_N by the following simple form

$$\delta_A = A \cdot \delta_N. \tag{46}$$

The relation reflects the additivity of fractal dimensions. In the framework of the fractal picture, the number of initial configurations is maximized according to Eq. (8), and the variable

$$z = \hat{E}_{\perp}^{kin}/(\Omega \cdot \rho(s))$$

can be interpreted as the energy of elementary constituent collision per one initial configuration and per one produced particle. This energy is the energy of elementary string piece and is proportional to its formation length according to the transformations (22) - (24).

The outlined picture has restricted range of application in the low energy region, where manifestation of the self-similar mechanism of particle production becomes complicated. In the experiments with nuclei at low energies one has usually deal with the problem of rescattering mechanism. The effect can be, however, neglected at high energies for high transverse momenta of secondary particles. In this region, we consider unlikely large contribution to the inclusive cross section due to rescatterings of particles in the surrounded target. This statement is supported by Figs. 4, 5, and 6. The A-dependence of experimental data used in our analysis does not violate the general features of the z-scaling construction. Moreover, the observed form of the scaling function $H(z)$ for pp and pA collisions is practically the same.

The situation in nucleus-nucleus systems may be different. The high $q_\perp$ enhancement in AA relative to pA interactions was discussed, in particular, in Ref. [26]. It corresponds to the larger inverse slopes of cross section dependence on a transverse momentum relative to the proton-nucleus collisions. In the case of AA interactions the excitation of nuclear medium is realized in an extended volume which can significantly influence particle production mechanism. We consider that the dependence of $H_{AA}(z)$ on z for hadron and QCD phases can be quantitatively distinguished. Possible violations of the scaling, especially in the region of high transverse momenta, could be very interesting. Here one can expect a manifestation of the transition of nuclear matter to parton phase especially in AA collision. The transition corresponds to the joining of partons from different nucleons of nuclei known as cumulative process [18, 19, 20]. The corresponding regime of particle production is kinematically forbidden in nucleon-nucleon collisions. The higher stage of cumulation corresponds the larger values of the variable z and can manifest itself more prominent just in the high momentum tail of the spectrum. As a result the enhance of the scaling function $H_{AA}(z)$ for the nucleus-nucleus interactions in comparison with the scaling found in pp and pA collisions may be expected. We suggest, therefore, that the comparison of the z-scaling for pp and pA collisions with data on AA interactions can give valuable information regarding exotic physical phenomena such as quark-gluon plasma formation and others.

5 Conclusions

Inclusive particle production in pA collisions at high energies in terms of the z-scaling is considered. The scaling function $H_A(z)$ is expressed via the invariant inclusive cross section $Ed^3\sigma/dq^3$ and is normalized to the multiplicity density of particles produced in pA collisions. The definition of the scaling variable z includes fractal properties of the colliding objects. The dynamical ingredient of the scaling relates to the energy dependent scale which is the average multiplicity density of charged particles produced in the central pseudorapidity region in the corresponding NN interaction.

The observed A-dependence of available experimental data for different nuclei (D, Be, Ti, and W) does not violate the general features of the z-scaling. It was shown that the fractal dimensions of nuclei are expressed via the fractal dimension of nucleon $\delta_A = A \cdot \delta_N$. Our analysis confirms the energy independence of the scaling functions $H_A(z)$. Thus, the scaling function $H(z)$ demonstrates main features of the hadronization process in terms of the formation length z.

The z-scaling found in pp and pA collisions reflects general properties of the particle production mechanism such as self-similarity, locality, scale-relativity and fractality and can serve as an effective tool in searching for new physical phenomena in future experiments planed at RHIC (BNL) and LHC (CERN).

6 Appendix A

In this Appendix we present some properties of the momentum fractions x_1 and x_2. The relation between the variables follows from the minimum recoil mass hypothesis in the elementary constituent interaction. The second restriction which we lay upon the variables, is the requirement of maximal tension of the string formed in the final state of the binary subprocess (2). String tension coefficient Ω was chosen in the form (8) relating fractal properties of colliding objects with the character of the phenomenological string. Such construction leads to specific structure of the variables x_1 and x_2 according to which the notation (2) can be rewritten to the symbolic form

$$(\lambda_1+\chi_1)+(\lambda_2+\chi_2) \rightarrow (\lambda_1+\lambda_2)+(\chi_1+\chi_2). \tag{A1}$$

The relation should be understand in the way that only parts of the interacting partons underlay the production of the inclusive particle while the other parts are responsible for the creations of its recoil. The variables have the following form

$$x_1 = \lambda_1+\chi_1, \qquad x_2 = \lambda_2+\chi_2, \tag{A2}$$

with

$$\lambda_1 \rightarrow \frac{\sqrt{m_\perp^2+q_z^2}-q_z}{\sqrt{s}}, \qquad \lambda_2 \rightarrow \frac{\sqrt{m_\perp^2+q_z^2}+q_z}{\sqrt{s}}, \tag{A3}$$

and

$$\chi_1 = \sqrt{\mu_1^2+\omega_1^2}-\omega_1 \qquad \chi_2 = \sqrt{\mu_2^2+\omega_2^2}+\omega_2. \tag{A4}$$

Relations (A3) represent an approximation of Eq. (7). It is reasonable in the considered kinematical range provided that emission angle of the inclusive particle is not too small. From Eqs. (12) and (13) it follows

$$\mu_1\mu_2 = \lambda_1\lambda_2+\lambda_0, \tag{A5}$$

$$\frac{\omega_i}{\omega_j} = \frac{\mu_i^2}{\lambda_1\lambda_2+\lambda_0}, \qquad i \neq j, \tag{A6}$$

$$\frac{\omega_1}{\mu_1} = \frac{\omega_2}{\mu_2}. \tag{A7}$$

Next, we present the energy decomposition of the elementary parton-parton subprocess. Using Eqs. (A4) - (A7), it can be shown that

$$\chi_1\chi_2 = \mu_1\mu_2 = \lambda_1\lambda_2+\lambda_0 \tag{A8}$$

and

$$x_1x_2 = (\sqrt{\lambda_1\lambda_2}+\sqrt{\lambda_1\lambda_2+\lambda_0})^2+(\sqrt{\lambda_1\chi_2}-\sqrt{\lambda_2\chi_1})^2. \tag{A9}$$

The relation reflects separation of the transverse and longitudinal degrees of freedom. The corresponding decomposition of the energy takes the following form

$$\hat{s}_x = \hat{s}_\lambda+\hat{s}_\chi+2\hat{s}_{cor}+\hat{s}_\parallel. \tag{A10}$$

The first and the second term are squares of the energy terms given by Eq. (29). They represent the parts of the transverse energy of the string which correspond to the inclusive particle and its recoil, respectively. As Eq. (A10) is quadratic, the third term,

$$\hat{s}_{cor} = (\sqrt{\lambda_1\chi_1}P_1+\sqrt{\lambda_2\chi_2}P_2)^2, \tag{A11}$$

accounts for correlation between the two parts. Neglecting masses M_ix_i in the expression (28), we get the relation for the total transverse energy

$$\hat{E}_\perp = \hat{s}_\lambda^{1/2}+\hat{s}_\chi^{1/2} \sim (\sqrt{\lambda_1\lambda_2}+\sqrt{\lambda_1\lambda_2+\lambda_0})\sqrt{s_A} \tag{A12}$$

and

$$\hat{E}_\perp \sim \sqrt{q_\perp^2+m_1^2}+\sqrt{q_\perp^2+m_1^2+\lambda_0 s_A} \sim \sqrt{q_\perp^2+m_1^2}+\sqrt{q_\perp^2+m_2^2}. \tag{A13}$$

The last term in Eq. (A10),

$$\hat{s}_\parallel = (\sqrt{\lambda_1\chi_2}-\sqrt{\lambda_2\chi_1})^2(2P_1P_2), \tag{A14}$$

is square of the momentum of the elementary string representing its longitudinal motion. According to this construction we divide the full phase space into two asymmetric hemispheres corresponding to the proton and nucleus parts, respectively. In such a way the energy of the elementary parton-parton subprocess can be divided to the energy released to the transverse direction and the energy flowing longitudinally with respect to boundary between the two hemispheres. The boundary is given by the equation

$$\frac{\chi_1}{\chi_2} = \frac{\lambda_1}{\lambda_2}. \tag{A15}$$

For the sake of simplicity, let us consider the case of $\lambda_0 \sim 0$ when illustrating the relation. Using Eq. (A8) and the definition of χ_i, we can write Eq. (A15) in the form

$$\lambda_2 = \frac{\lambda_1}{\alpha-2\lambda_1(\alpha-1)}. \tag{A16}$$

The variables λ_i can be expressed in terms of the relative momenta $Q = q/q_{max}$,

$$\lambda_1 = \frac{Q+Q_z}{2}, \qquad \lambda_2 = \frac{Q-Q_z}{2}, \tag{A17}$$

which is good approximation for the particles with small masses. Substituting the expressions into Eq. (A16), we get

$$Q - \frac{Q_z}{v} = Q_\perp^2. \tag{A18}$$

The v is the velocity of the recoil object and is given by Eq. (35). Since the right hand side of the last relation is non-negative, there exist a limiting angle $\theta(\alpha)$ which is determined by the relation

$$\cos\theta = v \tag{A19}$$

for any given resolution α. The angle defines the cone separating the proton and the nucleus hemispheres in the region of small relative transverse momenta $Q_\perp$. The partition in the $(Q_\perp, Q_\parallel)$ plane is depicted in Fig. 8 for the values of $\theta = 44.4^0$ and $\theta = 16.4^0$. The numbers correspond to the illustrative examples for $\alpha = 6$ and $\alpha = 48$, respectively. As bulk of the produced particles at high energies relates to the small values of $Q_\perp$, the angle given by Eq. (A19) form the boundary between the proton and nucleus hemispheres for most of the secondaries. When approaching the full phase space limit, the boundary deviates from the cone and bends towards 90^0 as shown by thick lines in Fig. 8. The deflection is the effect of finite size of the phase space. The dashed and dashed-dot curves show the hemisphere boundaries for given values of α. The curve for $\alpha = 1$ represents proton-proton (or generally equal mass $M_1 = M_2$) collisions. In the case of the proton interactions with heavy nucleus, the boundary tends to the value of $\lambda_1 = 1/2$ which corresponds the asymptotic value $\alpha = \infty$. The parts of the hemispheres which do not depend on the resolution α are given by the relations

$$\lambda_1 \le \lambda_2 \quad => \quad \lambda_1 \le \chi_1; \quad \chi_2 \le \lambda_2. \tag{A20}$$

$$\lambda_1 \ge 1/2 \quad => \quad \lambda_1 \ge \chi_1; \quad \chi_2 \ge \lambda_2. \tag{A21}$$

Finally, we will examine the limiting case when the resolution α tends to infinity. We consider the following situation

$$\delta_1 \to 0, \qquad \delta_2 \equiv \delta. \tag{A22}$$

First, let us note that for given λ_1 and λ_2 the x_1 (x_2) is increasing (decreasing) function of α, or equivalently

$$\mu_1 \frac{\partial \mu_1}{\partial \alpha} \ge \chi_1 \frac{\partial \omega_1}{\partial \alpha}, \qquad \mu_2 \frac{\partial \mu_2}{\partial \alpha} \le -\chi_2 \frac{\partial \omega_2}{\partial \alpha}. \tag{A23}$$

Using Eq. (10) - (13) and performing the limit $\alpha \to \infty$, it can be shown that

$$x_1 \to 1, \qquad x_2 \to x = \frac{\lambda_2 + \lambda_0}{1 - \lambda_1}, \tag{A24}$$

and

$$\Omega \to m(1-x)^\delta. \tag{A25}$$

With the neglect of masses, x turns into the Bjorken variable

$$x = \frac{\lambda_2 + \lambda_0}{1 - \lambda_1} \sim -\frac{\frac{1}{2}(P_1 - q)^2}{(P_1P_2) - (P_2q)} = -\frac{Q^2}{2P_2Q}, \tag{A26}$$

where Q is the transferred momentum between P_1 and q. The corresponding expressions for the derivatives of z are given in the Appendix B.

7 Appendix B

The invariant differential cross section for the production of inclusive particle is normalized as

$$\int E\frac{d\sigma}{dq^3} d\eta d^2 q_\perp = \sigma_{inel} < N >, \tag{B1}$$

where σ_{inel} is the inelastic cross section and $< N >$ is the average multiplicity. The inclusive cross section can be expressed in terms of the variables λ_1 and λ_2 in the way

$$E\frac{d\sigma}{dq^3} = \frac{1}{2\pi} \frac{\sqrt{(P_1P_2)^2 - M_1^2 M_2^2}}{[(P_1P_2) - M_1M_2]^2} \frac{d^2\sigma}{d\lambda_1 d\lambda_2}. \tag{B2}$$

In the region of high energies, the relation can be written in the approximate form

$$E\frac{d\sigma}{dq^3} = \frac{1}{\pi s A} \frac{d^2\sigma}{d\lambda_1 d\lambda_2}, \tag{B3}$$

where s is square of the center-of-mass energy for the corresponding NN collision and A is the nucleus mass number. We suppose that the inclusive cross section is given by solution (17) as a function of a single variable $z = z(\lambda_1, \lambda_2)$. In Eq.(18), we have chosen y as another independent combination of λ_1 and λ_2. Relation between the y and the NN center-of-mass (pseudo)rapidity η can be written as

$$y = \eta - \frac{1}{2}\ln A \tag{B4}$$

in a high energy approximation. Using the variables, we get the normalization

$$\int \frac{d^2\sigma}{d\lambda_1 d\lambda_2} d\lambda_1 d\lambda_2 = \int \frac{d^2\sigma}{dzdy} dzdy = \sigma_{inel} \int \rho_A(\eta)\psi(z) d\eta dy = \sigma_{inel} < N >, \tag{B5}$$

where $\rho_A(\eta) \equiv d < N > /dy$ is the average multiplicity density of particles produced in pA collisions. The function $\psi(z)$ is normalized to unity

$$\int_{z_{min}}^{\infty} \psi(z) dz = 1. \tag{B6}$$

According to the choice of Eq. (28), we have $z_{min} = 0$. The ψ can be expressed in terms of the inclusive cross section

$$\psi(z) = \frac{\pi s A}{\rho_A \sigma_{inel}} J^{-1} E\frac{d\sigma}{dq^3}, \tag{B7}$$

where

$$J = \frac{\partial y}{\partial \lambda_1}\frac{\partial z}{\partial \lambda_2} - \frac{\partial y}{\partial \lambda_2}\frac{\partial z}{\partial \lambda_1}. \tag{B8}$$

Next, we present the expressions for the partial derivatives of z. As the variable has the form given by Eqs. (27), (28), (29), and (8), one gets

$$\begin{aligned}\Omega\rho(s)\frac{\partial z}{\partial \lambda_1} &= [\lambda_1 M_1^2 + \lambda_2(P_1P_2)]\hat{s}_\lambda^{-1/2} \\ &+ \left[\chi_1\frac{\partial \chi_1}{\partial \lambda_1}M_1^2 + \chi_2\frac{\partial \chi_2}{\partial \lambda_1}M_2^2 + \lambda_2(P_1P_2)\right]\hat{s}_\chi^{-1/2} \\ &+ \left(1+\frac{\partial \chi_1}{\partial \lambda_1}\right)\left(\frac{\delta_1\hat{E}_\perp^{kin}}{1-x_1} - M_1\right) + \frac{\partial \chi_2}{\partial \lambda_1}\left(\frac{\delta_2\hat{E}_\perp^{kin}}{1-x_2} - M_2\right),\end{aligned} \tag{B9}$$

$$\begin{aligned}\Omega\rho(s)\frac{\partial z}{\partial \lambda_2} &= [\lambda_2 M_2^2 + \lambda_1(P_1P_2)]\hat{s}_\lambda^{-1/2} \\ &+ \left[\chi_1\frac{\partial \chi_1}{\partial \lambda_2}M_1^2 + \chi_2\frac{\partial \chi_2}{\partial \lambda_2}M_2^2 + \lambda_1(P_1P_2)\right]\hat{s}_\chi^{-1/2} \\ &+ \frac{\partial \chi_1}{\partial \lambda_2}\left(\frac{\delta_1\hat{E}_\perp^{kin}}{1-x_1} - M_1\right) + \left(1+\frac{\partial \chi_2}{\partial \lambda_2}\right)\left(\frac{\delta_2\hat{E}_\perp^{kin}}{1-x_2} - M_2\right).\end{aligned} \tag{B10}$$

The derivatives of χ_i satisfy the relations

$$\frac{\partial \chi_1}{\partial \lambda_j} = (\chi_1+\omega_1)^{-1}\left(\mu_1\frac{\partial \mu_1}{\partial \lambda_j} - \chi_1\frac{\partial \omega_1}{\partial \lambda_j}\right), \quad \frac{\partial \chi_2}{\partial \lambda_j} = (\chi_2-\omega_2)^{-1}\left(\mu_2\frac{\partial \mu_2}{\partial \lambda_j} + \chi_2\frac{\partial \omega_2}{\partial \lambda_j}\right). \tag{B11}$$

The derivatives of ω_i are given in terms of λ_j as follows

$$\begin{aligned}\frac{\partial \omega_1}{\partial \lambda_1} &= \frac{(\alpha-1)\lambda_2}{2(1-\lambda_2)}, & \frac{\partial \omega_1}{\partial \lambda_2} &= \frac{(\alpha-1)(\lambda_1+\lambda_0)}{2(1-\lambda_2)^2}, \\ \frac{\partial \omega_2}{\partial \lambda_1} &= \frac{(\alpha-1)(\lambda_2+\lambda_0)}{2\alpha(1-\lambda_1)^2}, & \frac{\partial \omega_2}{\partial \lambda_2} &= \frac{(\alpha-1)\lambda_1}{2\alpha(1-\lambda_1)}.\end{aligned} \tag{B12}$$

For the derivatives of μ_i it can be written

$$\begin{aligned}\frac{\partial \mu_1}{\partial \lambda_1} &= \frac{\alpha(\lambda_2-2\lambda_1\lambda_2-\lambda_0)}{2\mu_1(1-\lambda_2)}, & \frac{\partial \mu_1}{\partial \lambda_2} &= \frac{\alpha(\lambda_1+\lambda_0)(1-\lambda_1)}{2\mu_1(1-\lambda_2)^2}, \\ \frac{\partial \mu_2}{\partial \lambda_1} &= \frac{(\lambda_2+\lambda_0)(1-\lambda_2)}{2\alpha\mu_2(1-\lambda_1)^2}, & \frac{\partial \mu_2}{\partial \lambda_2} &= \frac{\lambda_1-2\lambda_1\lambda_2-\lambda_0}{2\alpha\mu_2(1-\lambda_1)}.\end{aligned} \tag{B13}$$

We can neglect mass terms in the central region of collision for high $q_\perp$ and write

$$z = \frac{2\sqrt{\lambda_1\lambda_2}\sqrt{s_A}}{\Omega\cdot\rho(s)}, \qquad \lambda_0 \sim 0, \tag{B14}$$

in the considered energy region. In this case the derivatives of variable z simplify and the factor (21) can be written as follows

$$g(\lambda_1,\lambda_2) = \frac{4}{\Omega^2\rho^2(s)}\left[1+\delta_1\frac{\lambda_1+\chi_1 f_1}{(1-x_1)} + \delta_2\frac{\lambda_2+\chi_2 f_2}{(1-x_2)}\right], \tag{B15}$$

where

$$f_1 = \frac{(2-\lambda_2)\chi_1 - \alpha\lambda_1\chi_2}{2(\chi_1+\omega_1)(1-\lambda_2)}, \qquad f_2 = \frac{(2-\lambda_1)\chi_2 - \alpha^{-1}\lambda_2\chi_1}{2(\chi_2-\omega_2)(1-\lambda_1)}. \tag{B16}$$

The formulae for the derivatives of z can be expressed in a more closed form when considering the case of the infinite resolution. Using Eqs. (A24) and (A25) we have

$$\begin{aligned}\Omega\rho(s)\frac{\partial z}{\partial \lambda_1} &= \frac{x}{1-\lambda_1}\left(\frac{\delta\hat{E}_\perp^{kin}}{1-x} - M_2\right) + [\lambda_1 M_1^2 + \lambda_2(P_1P_2)]\hat{s}_\lambda^{-1/2} \\ &+ \left[(\lambda_1-1)M_1^2 + \frac{(\lambda_1\lambda_2+\lambda_0)}{(1-\lambda_1)^2}xM_2^2 + \lambda_2(P_1P_2)\right]\hat{s}_\chi^{-1/2},\end{aligned} \tag{B17}$$

$$\begin{aligned}\Omega\rho(s)\frac{\partial z}{\partial \lambda_2} &= \frac{1}{1-\lambda_1}\left(\frac{\delta\hat{E}_\perp^{kin}}{1-x_2} - M_2\right) + [\lambda_2 M_2^2 + \lambda_1(P_1P_2)]\hat{s}_\lambda^{-1/2} \\ &+ \left[\frac{(\lambda_1\lambda_2+\lambda_0)}{(1-\lambda_1)^2}\lambda_1 M_2^2 + \lambda_1(P_1P_2)\right]\hat{s}_\chi^{-1/2}.\end{aligned} \tag{B18}$$

In the approximation of Eq. (B14), the factor g becomes

$$g(\lambda_1,\lambda_2) = \frac{4}{\Omega^2\rho^2(s)}\left(1+\frac{\delta x}{(1-\lambda_1)(1-x)}\right). \tag{B19}$$

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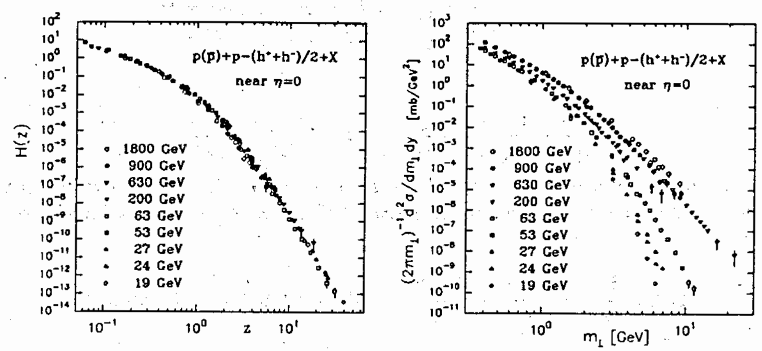
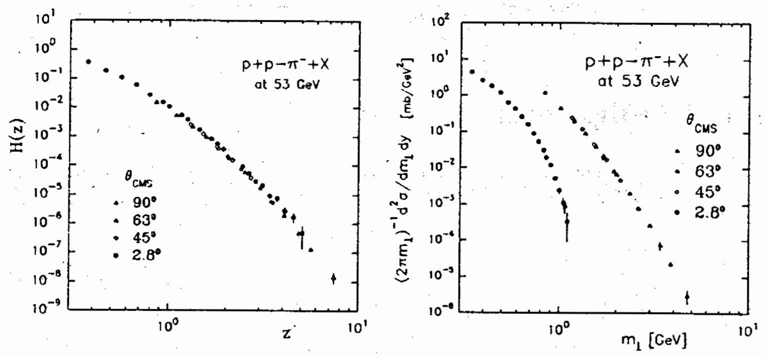


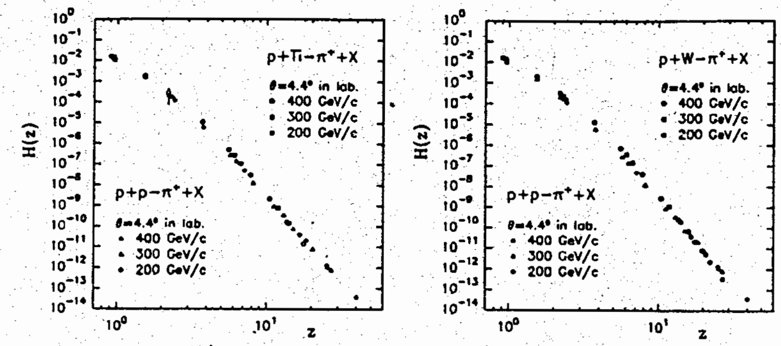
a) b)

Figure 1. (a) Scaling function $H_p(z)$ for the charged hadrons in central region produced in pp or $\bar{p}p$ interactions at $\sqrt{s} = 19 - 1800$ GeV. Detection angle θ is 90^0 c.m.s except the data at $\sqrt{s} = 63$ GeV, where $\theta = 50^0$; (b) The corresponding inclusive differential cross sections as functions of the transverse mass $m_\perp$. Experimental data are taken from Refs. [39, 40, 41, 42, 33].



a) b)

Figure 2. (a) Scaling function $H_p(z)$ for π^--meson production in the central and fragmentation regions at $\sqrt{s} = 53$ GeV; (b) The corresponding inclusive differential cross sections as functions of $m_\perp$. Experimental data are taken from Refs. [41, 43].



a) b)

Figure 6. The scaling functions (a) $H_p(z)$, $H_{Ti}(z)$ and (b) $H_p(z)$, $H_W(z)$ for π^+-meson production. Experimental data are taken from Ref. [33].

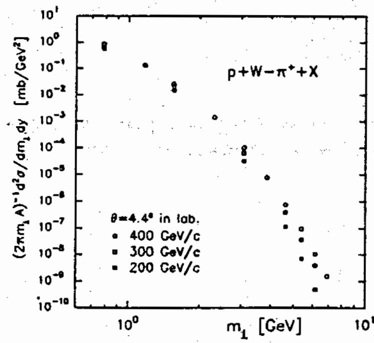


Figure 7. The normalized inclusive differential cross sections for π^+-meson production in pW interactions for various incoming proton momenta as a function of the transverse mass $m_\perp$. Experimental data are taken from Ref. [33].

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Z-скейлинг в протон-ядерных взаимодействиях
при высоких энергиях

Исследуется новый z-скейлинг в инклюзивном рождении частиц в pA-взаимодействиях. Концепция z-скейлинга базируется на фундаментальных физических принципах, таких как принципы подобия, локальности, масштабной относительности и фрактальности, отражающие общие особенности взаимодействия частиц. Скейлинговая функция $H_A(z)$ выражается через инвариантное сечение $Ed^3\sigma/dq^3$ и плотность распределения частиц $dN/d\eta$ при $\eta = 0$, определенной в системе центра масс NN. Исследуется зависимость $H_A(z)$ от скейлинговой переменной z, энергии в системе центра масс $\sqrt{s}$ и атомного номера A. Показано, что экспериментальные данные по сечениям рождения $\pi^{\pm}$-мезонов в pA-взаимодействиях подтверждают скейлинговые свойства $H_A(z)$. Полученные результаты представляют интерес для будущих экспериментов, планируемых на ядерных коллайдерах RHIC (BNL) и LHC (CERN), по поиску новых физических явлений в pp-, pA- и AA-взаимодействиях.

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Zborovsky I. et al. E2-98-250

Z-Scaling in Proton-Nucleus Collisions at High Energies

New scaling, z-scaling, in the inclusive particle production in pA collisions is studied. The concept of z-scaling is based on the fundamental principles such as self-similarity, locality, scale relativity and fractality reflecting the general features of particle interactions. The scaling function $H_A(z)$ is expressed via the invariant cross section $Ed^3\sigma/dq^3$ and the average multiplicity density $dN/d\eta$ of particles produced at pseudorapidity $\eta = 0$ in the corresponding nucleon-nucleon interaction. The dependence of $H_A(z)$ on scaling variable z, the center-of-mass energy $\sqrt{s}$ and the atomic number A is investigated. It is shown that the available experimental data on cross section in pA collisions confirm the scaling properties of the function $H_A(z)$. The obtained results can be of interest for future experiments at RHIC (BNL) and LHC (CERN) in searching for new physical phenomena, in pp, pA, and AA collisions.

The investigation has been performed at the Laboratory of High Energies, JINR.

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