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Kh.M.Beshtoev

THE RESONANCE BREMSSTRAHLUNG  
OF A FAST CHARGED PARTICLE IN A MEDIUM

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# 1 Introduction

In our previous work [1] we discussed the problem of a charged relativistic particle passing through a medium. It was shown that at the velocity of the particle  $v \cong c'$  ( $c' = \frac{c}{n}$ ) the Cherenkov radiation differs from zero. This radiation is rather weak and at velocity  $v$  when  $v < c'$ , this radiation disappears. There was also a simple mechanism proposed to avoid singularity in the electrodynamics when  $v = c'$ . The medium is supposed to behave as a collective and the characteristics of this medium are described by a dielectric permittivity  $\epsilon$ . It is clear that  $\epsilon$  is a function of frequency  $\epsilon = \epsilon(\omega)$  and at high frequencies when  $\epsilon \rightarrow 1$  the medium loses the collective property and the charged particle begins to interact with nuclei and electrons of the medium.

In this work we study the resonance bremsstrahlung of a charged particle passing through the medium at velocities  $v$ ,  $v \cong c'$  and higher. It is clear that this charged particle interacts with the medium as a collective. The property of this collective is determined by a dielectric permittivity  $\epsilon = \epsilon_1 + i\epsilon_2$  ( $\epsilon_1 = n^2$ ,  $\epsilon = \epsilon(\omega)$ ).

It is interesting to remark, that after discovering of a radiation which differs from the luminescent radiation, Vaviliv S.I. [2] came to a conclusion that, the more probable reason of the new radiation is bremsstrahlung of the Kompton electrons knocked out by gamma-rays from atoms of the liquid.

At first, we discuss bremsstrahlung of a fast charged particle when it takes part in individual electromagnetic interactions, then the resonance bremsstrahlung of a fast charged particle in the medium.

## 2 The Bremsstrahlung of a Fast Charged Particle

The full energy  $\epsilon$  of bremsstrahlung of a fast charged particle is determined by the following equation [3]:

$$\epsilon = \int_{-\infty}^{\infty} J dt, \quad (1)$$

$$J = \frac{2e^2}{3m^2c^3} \frac{[\vec{E} + \frac{1}{c}[\vec{v}\vec{H}]]^2 - \frac{1}{c^2}(\vec{E}\vec{v})^2}{1 - \frac{v^2}{c^2}}, \quad (2)$$

where  $\vec{E}$ ,  $\vec{H}$  are tensivity of external electric and magnetic fields;  $m$ ,  $e$  are mass and charge of the particle.

If the external field is divided into two components, parallel and transversal to velocity  $\vec{v}$ , then for component  $J_{\parallel}$  we obtain

$$J_{\parallel} = \frac{2e^2}{3m^2c^3} \vec{E}_{\parallel}^2, \quad (3)$$

and for the transversal component  $J_{\perp}$ , we obtain

$$J_{\perp} = \frac{2e^2}{3m^2c^3} \frac{\vec{E}_{\perp}^2}{1 - \frac{v^2}{c^2}}. \quad (4)$$

The angle distribution of bremsstrahlung energy is determined by the following equation:

$$I = \int \frac{dI}{d\Omega} d\Omega, \quad d\Omega = \sin\theta d\theta d\varphi \quad (5)$$

$$\frac{dI}{d\Omega} = \frac{c^2}{4\pi c^3} \left( \frac{2(\vec{n}\vec{w})(\vec{v}\vec{w})}{c(1 - \frac{v\vec{n}}{c})^5} + \frac{\vec{w}^2}{(1 - \frac{v\vec{n}}{c})^4} - \frac{(1 - \frac{v^2}{c^2})(\vec{n}\vec{w})^2}{(1 - \frac{v\vec{n}}{c})^6} \right), \quad (6)$$

where  $\vec{n}$  is the direction of the bremsstrahlung.

If the velocity  $\vec{v}$  and acceleration  $\vec{w}$  are parallel, then for  $I_{\parallel}$  we obtain the following equation:

$$\frac{dI_{\parallel}}{d\Omega} = \frac{e^2}{4\pi c^3} \frac{w^2 \sin^2\theta}{(1 - \frac{v}{c} \cos\theta)^5}, \quad (7)$$

if  $\vec{v}$  and  $\vec{w}$  are transversal, then for  $\frac{dI_{\perp}}{d\Omega}$  we obtain the following equation:

$$\frac{dI_{\perp}}{d\Omega} = \frac{e^2}{4\pi c^3} \left( \frac{1}{(1 - \frac{v}{c} \cos\theta)^4} - \frac{(1 - \frac{v^2}{c^2}) \sin^2\theta \cos^2\varphi}{(1 - \frac{v}{c} \cos\theta)^6} \right). \quad (8)$$

In the angle distribution of the bremsstrahlung of the fast charged particle, as we can see from (1)-(8), there is a peak in forward distribution and width of this peak is determined by the following equation:

$$\theta \sim \sqrt{1 - \frac{v^2}{c^2}} \quad (9)$$

Now we go on discussing bremsstrahlung of a fast charged particle in a medium.

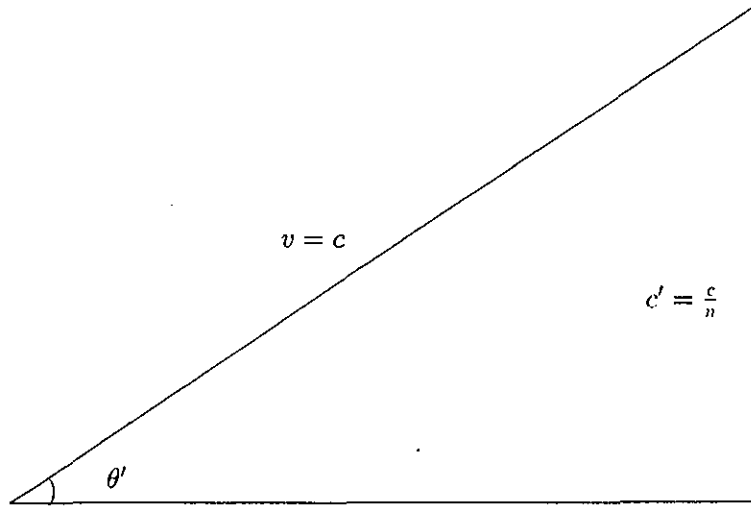
### 3 The Resonance Bremsstrahlung of a Fast Particle in a Medium

We study bremsstrahlung of a fast charged particle in a medium with velocities  $v \geq \frac{c}{n}$ . And we are interested in the bremsstrahlung in the region of frequencies  $\omega$ , when  $n(\omega) > 1$ , i.e. optics and X-ray regions. Actually, it is the region of frequencies where the charged particle interacts with the medium collectively but not individually. Then the following substitution should be done in equations (1)-(8):

$$c \rightarrow c' = \frac{c}{n}, \quad (10)$$

where  $n^2 = \text{Re}\epsilon$  ( $\epsilon$  is dielectric permittivity). Moreover, in denominators of (1)-(8) (see [2]) there is factor  $\epsilon$  reduced with the same factor  $\epsilon$  which appears at the substitution (10).

Besides, since the velocity of the light in the medium is  $c' = \frac{c}{n}$ , therefore the field around the particle will spread out in this medium with the velocity  $c'$ , (which is less than  $c$ ) and the field will remain behind this relativistic particle. The triangle of the velocities of this particle in the medium has the following form:



$$\sin \theta' = \frac{c}{vn} = \frac{1}{\beta n},$$

$$\theta' = \frac{\pi}{2} - \theta.$$

So, we can see that the field around the relativistic particle ( $v > c$ ) has the form of a thin surface film of a rotation cone with the slope angle  $\theta'$  to the direction of the moving particle, and  $\theta'$  is determined by the value  $\sin \theta' = \frac{1}{\beta n}$ . Then, the larger the refraction coefficient  $n$  is, the smaller the scope angle of the surface of the rotation cone is obtained (see work [1]). It is clear that the bremsstrahlung must go mainly in the transversal direction to the surface of the rotation cone.

So, the equations (1)-(8) transform in:

$$\varepsilon = \int_{-\infty}^{\infty} J dt = \int_{-\infty}^{\infty} (J_{\parallel} + J_{\perp}) dt, \quad (11)$$

where  $J_{\parallel}$  is:

$$J_{\parallel} = \frac{2e^2}{3m^2c^3} \vec{E}_{\parallel}^2, \quad (12)$$

and  $J_{\perp}$  is:

$$J_{\perp} = \frac{2e^2}{3m^2c^3} \frac{\vec{E}_{\perp}^2}{1 - \epsilon \frac{v^2}{c^2}} \quad (13)$$

From equation (13) we see that singularity in the medium at the charged particle velocity  $v^2 = \frac{c^2}{\epsilon}$  appears there. Since  $\epsilon$  is a function of  $\omega$  ( $\epsilon = \epsilon(\omega)$ ), then the singularity takes place at different velocities

$$v^2(\omega) = \frac{c^2}{\epsilon(\omega)}. \quad (14)$$

At frequencies  $\omega$  ( $\omega \rightarrow \infty$ ) when  $\epsilon(\omega) \rightarrow 1$ , the charged particle begins to interact with the medium individually but not as with a collective, then the bremsstrahlung becomes individual.

If  $\epsilon$  in (10) is a complex value  $\epsilon(\omega) = \epsilon_1(\omega) + i\epsilon_2(\omega)$ , then (13) transforms into the following equation:

$$J_{\perp} = \frac{2e^2}{3m^2c^3} \frac{\vec{E}_{\perp}^2 [(1 - \epsilon_1 \frac{v^2}{c^2}) + i\epsilon_2 \frac{v^2}{c^2}]}{(1 - \epsilon_1 \frac{v^2}{c^2})^2 + (\epsilon_2 \frac{v^2}{c^2})^2} \quad (15)$$

This equation has a typical resonance form (see [4]) with the point of the resonance at  $\epsilon \frac{v^2}{c^2} = 1$  and with width  $\Gamma$

$$\frac{\Gamma}{2} = \epsilon_2 \frac{v^2}{c^2}. \quad (16)$$

So, the charged particle bremsstrahlung in the medium depends on its velocity as:

$$J_{\perp} \sim \frac{1}{1 - \epsilon \frac{v^2}{c^2}}, \quad (17)$$

and at  $\epsilon_1 \frac{v^2}{c^2} \sim 1$ , it has resonance. The equation for bremsstrahlung when  $\epsilon$  is a complex value, is determined by equations (12) and (13) where  $J \rightarrow |J|$ .

Another equation for the bremsstrahlung full energy of the charged particle in the medium is given in chapter 4.

Now we consider the angle distribution of the charged particle bremsstrahlung in the medium.

If we take into account that  $c \rightarrow c' = \frac{c}{\epsilon}$  in the medium, then equations (7), (8) are transformed in the following equations:

$$\frac{dI_{\parallel}}{d\Omega} = \frac{e^2}{4\pi c^3} \frac{w^2 \sin^2 \theta}{(1 - \sqrt{\epsilon} \frac{v}{c} \cos \theta)^5}, \quad (18)$$

$$\frac{dI_{\perp}}{d\Omega} = \frac{e^2 w^2}{4\pi c^3} \left( \frac{1}{(1 - \sqrt{\epsilon} \frac{v}{c} \cos \theta)^4} - \frac{(1 - \frac{v^2}{c^2}) \sin^2 \theta \cos^2 \varphi}{(1 - \sqrt{\epsilon} \frac{v}{c} \cos \theta)^6} \right). \quad (19)$$

From equations (18), (19) we see, that their denominators are equal to zero (i.e. (18), (19) have singularity) at:

$$\frac{\sqrt{\epsilon} v}{c} \cos \theta = 1 \quad \text{or} \quad \cos \theta = \frac{c}{\sqrt{\epsilon} v}, \quad (20)$$

i.e. in the region of frequencies  $\omega$ , where  $\epsilon(\omega) > 1$ , there is the bremsstrahlung in direction  $\theta$ , determined by equation (20). This bremsstrahlung has typical resonance form:

$$\frac{dI_{\parallel}}{d\Omega} = \frac{e^2}{4\pi c^3} w^2 \sin^2 \theta \frac{[(1 - \sqrt{\epsilon_1} \beta \cos \theta) + i \sqrt{\epsilon_2} \beta \cos \theta]^5}{[(1 - \epsilon_1 \beta^2 \cos^2 \theta)^2 + \epsilon_2 \beta^2 \cos^2 \theta]^5}, \quad (18')$$

$$\frac{dI_{\perp}}{d\Omega} = \frac{e^2 w^2}{4\pi c^3} \left( \frac{[(1 - \sqrt{\epsilon_1} \beta \cos \theta) + i \sqrt{\epsilon_2} \beta \cos \theta]^4}{[(1 - \epsilon_1 \beta^2 \cos^2 \theta)^2 + \epsilon_2 \beta^2 \cos^2 \theta]^4} - \right.$$

$$\left. \frac{e^2 w^2}{4\pi c^3} \left( (1 - \frac{v^2}{c^2}) \sin^2 \theta \cos^2 \varphi \frac{[(1 - \sqrt{\epsilon_1} \beta \cos \theta) + i \sqrt{\epsilon_2} \beta \cos \theta]^6}{[(1 - \epsilon_1 \beta^2 \cos^2 \theta)^2 + \epsilon_2 \beta^2 \cos^2 \theta]^6} \right) \right). \quad (19')$$

The equations for the resonance bremsstrahlung of the charged particle in the medium can be obtained from (18') and (19') by using the following substitution  $\frac{dI_{\parallel}}{d\Omega} \rightarrow \left| \frac{dI_{\parallel}}{d\Omega} \right|$ ,  $\frac{dI_{\perp}}{d\Omega} \rightarrow \left| \frac{dI_{\perp}}{d\Omega} \right|$ .

The integrated on  $\varphi$  values of  $\left| \frac{dI_{\parallel}}{d\Omega} \right|$ ,  $\left| \frac{dI_{\perp}}{d\Omega} \right|$  are:

$$\frac{d\epsilon_{\parallel}}{dt} = \int \left| \frac{dI_{\parallel}}{d\Omega} \right| d\varphi, \quad \frac{d\epsilon_{\parallel}}{dx} = \frac{1}{v} \frac{d\epsilon_{\parallel}}{dt}, \quad (21)$$

$$\frac{d\epsilon_{\perp}}{dt} = \int \left| \frac{dI_{\perp}}{d\Omega} \right| d\varphi, \quad \frac{d\epsilon_{\perp}}{dx} = \frac{1}{v} \frac{d\epsilon_{\perp}}{dt}, \quad (22)$$

where  $x = vt$ .

At the point of the resonance from (20), (21) we get the following equations ( $\cos \theta = \frac{1}{\sqrt{\epsilon_1} \beta}$ ,  $w = \omega \beta$ ):

$$\frac{d\epsilon_{\parallel}}{dx} = \frac{e^2}{2c^2} \omega^2 \beta \sin^2 \theta \left( \frac{\epsilon_1}{\epsilon_2} \right)^{\frac{5}{2}}, \quad (23)$$

$$\frac{d\epsilon_{\perp}}{dx} = \frac{e^2}{2c^2} \omega^2 \beta \left( \frac{\epsilon_1}{\epsilon_2} \right)^2 + \frac{e^2}{8\pi c^2} \omega^2 \beta^3 \sin^2 \theta \epsilon_2 \left( \frac{\epsilon_1}{\epsilon_2} \right)^3. \quad (24)$$

So, in dependence on the velocity of the charged particle, the resonance bremsstrahlung in the medium goes in different directions  $\theta$  (see (20)), and at the velocity  $v = \frac{c}{\sqrt{\epsilon_1}}$ , the resonance bremsstrahlung goes in the forward direction. When  $v < \frac{c}{\sqrt{\epsilon_1}}$ , the resonance bremsstrahlung disappears. It is clear that since  $\epsilon$  depends on  $\omega$ , then the velocity values  $v$  also depend on  $\omega$ . From (18) and (19) we also see that the direction of the resonance bremsstrahlung coincides with the direction of the Cherenkov radiation [5].

Let us discuss another approach for obtaining the bremsstrahlung of the fast particle in the medium.

#### 4 Full Bremsstrahlung of a Fast Charged Particle in a Medium

The power of bremsstrahlung  $P$  of a charged particle can be presented in the following form [3] (i.e. the lost power of the charged particle through transverse acceleration):

$$P = \frac{2e^2}{3m^2 c^3} \gamma^5 \omega^2 |\vec{p}|^2, \quad (25)$$

where  $\omega$  is frequency.

Since  $\vec{p} = cm\vec{\beta}\gamma$ , then equation (25) is transformed into the following equation:

$$P = \frac{2e^2}{3c^3} \gamma^4 \omega^2 \beta^2, \quad (26)$$

where  $\gamma = \frac{1}{\sqrt{1-\beta^2}}$ .

In a medium with dielectric permittivity  $\epsilon$ , equation (26) transforms into the following equation:

$$P = \frac{2e^2}{3c} \omega^2 \beta^2 \frac{1}{(1 - \epsilon \beta^2)^2}. \quad (27)$$

Equation (27) has singularity at  $\beta^2 = \frac{1}{\epsilon}$ . This singularity can be removed using the method proposed in [1], if we take into account that  $\epsilon = \epsilon_1 + i\epsilon_2$ ,

$$\epsilon_1 = n^2, \epsilon = \epsilon(\omega), \epsilon_1 = \epsilon_1(\omega), \epsilon_2 = \epsilon_2(\omega). \quad (28)$$

Then  $1 - \epsilon \beta^2 = (1 - \epsilon_1 \beta^2) - \epsilon_2 \beta^2$  and  $P \rightarrow |P|$

$$|P| = \frac{2e^2}{3c} \omega^2 \beta^2 \frac{1}{(1 - \epsilon_1 \beta^2)^2 + (\epsilon_2 \beta^2)^2}. \quad (29)$$

So, equation (29) is the equation for the bremsstrahlung of the charged particle with velocity  $\beta = \frac{v}{c}$  in the medium and this bremsstrahlung has a resonance character at  $\beta^2 = \frac{1}{\epsilon_1}$ . As we can see from (27), this bremsstrahlung goes in the forward direction (the bremsstrahlung at other angles is given by equations (18)-(19)).

At velocity  $\beta = \frac{1}{\sqrt{\epsilon_1}}$  (in the point of the resonance) the density of radiation  $|P|$  of the charged particle in the medium is given by the following equation:

$$|P| = \frac{2e^2 \omega^2 \epsilon_1(\omega)}{3c \epsilon_2^2(\omega)}, \quad (30)$$

or

$$\frac{d\epsilon}{dx} = \frac{2e^2 \omega^2 \epsilon_1(\omega)}{3c^2 \beta \epsilon_2^2(\omega)}, \quad (31)$$

and it is inversely proportional to the coefficient  $\epsilon_2$  in square. The equation (31) defines the losses of the charged particle in the medium per unit of the length without taking into account the absorption part (we suppose that  $\epsilon_2 \ll 1$ ), which must be proportional to  $\exp(-\epsilon_2)$ .

Equations (23), (24) and (31) are the same equations which are necessary to be compared with the following standard equation for the Cherenkov radiation:

$$\frac{d\epsilon}{dx} = \frac{e^2}{2c^2} \omega^2 \left(1 - \frac{1}{\epsilon_1 \beta^2}\right). \quad (32)$$

The expression for  $\epsilon$  is given in [3], [6].

## 5 Conclusion

The bremsstrahlung of the fast charged particle in the medium with dielectric permittivity  $\epsilon$  at velocities  $v \geq \frac{c}{n}$  ( $Re\epsilon = n^2$ ) was considered. The bremsstrahlung has singularity at  $\beta = \frac{1}{n \cos \theta}$  ( $\beta = \frac{v}{c}$ ,  $\theta$  is an angle of the bremsstrahlung). And this bremsstrahlung was interpreted as resonance bremsstrahlung with the width characterized by  $Im\epsilon = \epsilon_2$ , and the less  $\epsilon_2$  is, the higher the peak of this resonance rises. The angle distribution of the bremsstrahlung is determined by  $\cos \theta = \frac{1}{n\beta}$  and this angle coincides with the angle of the Cherenkov radiation. At  $\beta = \frac{1}{n}$  this resonance bremsstrahlung goes in the forward direction. The resonance bremsstrahlung depends on frequency  $\omega$  ( $\epsilon \equiv \epsilon(\omega)$ ).

Evidently, an accurate experiment is needed to study the radiation of the fast charged particle in the medium at velocities  $\beta \geq \frac{1}{n}$ .

It is necessary to remark that in work [7] was reported about the noticeable radiation in the forward direction in optics region, which cannot be the Cherenkov radiation, since the Cherenkov radiation in the forward direction must be equal to zero. This radiation was interpreted by A.A. Tyapkin [8] as a coherent sum of radiation of the charged particle in the medium at its velocity  $v$  equal to the velocity of light in the medium.

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