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NJL MODEL WITHOUT $\bar{q}q$ THRESHOLDS

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1. INTRODUCTION

The NJL model has been very intensively investigated during the last 15 years [1–4] and it appears to be one of the most frequently used tools to investigate finite temperature effects in the low energy QCD [5]. In particular, it has been used in order to estimate signals from the chiral symmetry restoration transition expected to occur in relativistic heavy-ion collisions. In these applications there arises the problem of unphysical quark-antiquark pair production thresholds appearing due to the absence of quark confinement in the standard NJL model. Different methods for a solution of this problem have been proposed [6–10]. The approach via nonperturbative solution of Dyson-Schwinger Equations (DSE) of QCD has been successfully generalized to finite temperatures and densities [11] and allows a discussion of the deconfinement transition under extreme conditions met in heavy ion collisions or in the interior of neutron stars. The investigation of observable consequences of this transition within the DSE approach is in progress.

In this short note we consider an extension of the standard NJL model where the confinement is introduced by excluding the infrared domain from quark loop integrals, which has been suggested by [10]. For this purpose, we introduce besides the usual ultraviolet cut-off Λ connected with the scale, where spontaneous chiral symmetry breaking occurs, an infrared cut-off μ below which the long range effects of the quark confinement phenomenon dominate. However, we will suggest in the present paper a choice for the cut-off μ, that still preserves the low-energy theorems for scalar and pseudoscalar mesons and describes spontaneous chiral symmetry breaking while restricting the quark propagation to the domain of momenta $\mu \leq |\vec{k}| \leq \Lambda$ and thus excluding the decay of low mass mesons in $q\bar{q}$ pairs, see Fig. 1. At the end of the paper, we estimate a basic low-energy process such as the sigma meson decay. Here the self-consistent choice of the infrared cutoff $\mu = c \cdot m(T)$ ($c = const$) is suggested which is motivated from the Lattice-QCD result that chiral symmetry restoration and deconfinement at finite temperature occur at the same temperature.

2. SU(2)× SU(2) LAGRANGIAN, GAP EQUATION AND CHIRAL MASS FORMULAE

Let us consider an SU(2)×SU(2) NJL model defined by the Lagrangian

$$\mathcal{L}_q = \bar{q}(i\partial\!\!\!/ - m^0)q + \frac{G}{2}\left[(\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2\right] . \quad (1)$$

After bosonization of the 4-fermion model (1) one obtains the equivalent representation in terms of one scalar (σ) and three pseudoscalar ($\vec{\pi}$) mesons

$$\mathcal{L}_{meson} = -\frac{\tilde{\sigma}^2 + \vec{\pi}^2}{2G} - i\mathrm{Trln}\left\{1 + \frac{1}{i\partial\!\!\!/ - m}[\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi}]\right\} . \quad (2)$$

Here, the scalar fields σ and $\tilde{\sigma}$ are connected by the relation

$$-m^0 + \tilde{\sigma} = -m + \sigma , \quad (3)$$

where m^0 is the current quark mass, m is the constituent quark mass and the vacuum expectation of σ vanishes, $\langle\sigma\rangle_0 = 0$. Then, from the condition

$$\left.\frac{\delta\mathcal{L}}{\delta\sigma}\right|_{\sigma=0,\vec{\pi}=0} = 0 , \quad (4)$$

the gap equation is obtained[1])

$$\begin{aligned} m^0 &= m(1 - 8GI_1^{(\mu\,\Lambda)}(m)) \\ &= mG(\frac{1}{\bar{G}} - 8I_1^{(0\,\Lambda)}(m)) \\ &= m\frac{(1 - 8\bar{G}I_1^{(0\,\Lambda)}(m))}{(1 - 8\bar{G}I_1^{(0\,\mu)}(m))} \\ &\approx m + 2\bar{G}\langle\bar{q}q\rangle_0 , \end{aligned} \quad (5)$$

where $\langle\bar{q}q\rangle_0$ is the quark condensate (see Eq. (15)) and $I_1^{(a\,b)}(m)$ is obtained from the quadratically divergent integral

$$I_1(m) = -i\frac{N_c}{(2\pi)^4}\int\frac{d^4k}{m^2 - k^2 - i\varepsilon} \quad (6)$$

by applying a 3-dimensional UV cut-off regularization ($b = \Lambda$) and placing an IR cut-off ($a = \mu$) in order to model confinement [10]

$$I_1^{(\mu\,\Lambda)}(m) = \frac{N_c}{(2\pi)^2}\int_\mu^\Lambda dk\frac{k^2}{E(k)} = \frac{N_c m^2}{8\pi^2}\left[x\sqrt{x^2+1} - \ln(x + \sqrt{x^2+1})\right]\Big|_{\mu/m}^{\Lambda/m} , \quad (7)$$

where $E(k) = \sqrt{k^2 + m^2}$ and N_c is the number of colors.

We give also the expression for the logarithmic divergent integral $I_2(p, m)$

$$\begin{aligned} I_2(p^2, m) &= -i\frac{N_c}{(2\pi)^4}\int\frac{d^4k}{(m^2 - k^2 - i\varepsilon)(m^2 - (k-p)^2 - i\varepsilon)} \\ &= \frac{N_c}{2\pi^2}\int_\mu^\Lambda dk\frac{k^2}{E(4E^2 - p^2 + i\varepsilon)} . \end{aligned} \quad (8)$$

[1]Here $\bar{G}$ is defined by the relation $1/G - 8I_1^{(0\,\mu)} = 1/\bar{G}$ (see also eq. (12)). The IR cut-off in the I_1 integral is thereby absorbed by the renormalization of the coupling constant $\bar{G}$.

We shall consider $\mu = c \cdot m$, where $0.5 \le c \le 1$, thus connecting the IR cut-off with the quark condensate ($m \approx -2G\langle\bar{q}q\rangle_0$)[2]. Then, for the pion momentum $p^2 = M_\pi^2$ we can neglect p^2 in Eq. (8) and obtain for $I_2(0,m)$ the expression

$$I_2(0,m) = \frac{N_c}{8\pi^2}\left[\ln(x+\sqrt{x^2+1}) - (1+1/x^2)^{-1/2}\right]\Big|_{\mu/m}^{\Lambda/m} . \tag{9}$$

Now let us consider the quadratic pion part of the Lagrangian (2) in the one-quark-loop approximation (see Fig. 2) [3]

$$\mathcal{L}_\pi^{(2)} = -\frac{\vec{\pi}^2}{2}\left\{\frac{1}{\bar{G}} - 8I_1^{(0\Lambda)}(m) - 4p^2 I_2(p^2,m)\right\} . \tag{10}$$

After renormalization of the pion fields

$$\vec{\pi} = g_\pi \vec{\pi} \quad , \quad g_\pi = [4I_2(0,m)]^{-1/2} , \tag{11}$$

we obtain the following expression for the pion mass:

$$M_\pi^2 = g_\pi^2\left[\frac{1}{\bar{G}} - 8I_1^{(0\Lambda)}(m)\right] = \frac{g_\pi^2 m^0}{\bar{G}m}(1 - 8\bar{G}I_1^{(0\mu)}(m)) . \tag{12}$$

Here, we use the gap equation (5). We can see that this pion mass formula is in accordance with the Goldstone theorem since for $m^0 = 0$ the mass of the pion vanishes and it is the Goldstone boson of the (spontaneously broken) chiral symmetry.

For the σ meson mass in the one-loop approximation (see Fig. 2) we obtain

$$M_\sigma^2 = g_\sigma^2\left[\frac{1}{\bar{G}} - 8I_1^{(0\Lambda)}(m)\right] + 4m^2 , \tag{13}$$

where $g_\sigma^{-2} = 4I_2(M_\sigma^2, m)$. From Eq. (8) we can see that this constant has an imaginary part if $\mu = 0$ since $M_\sigma^2 > 4m^2$. Therefore, in the quark loop with σ legs (Fig. 2) the appearance of free quarks is possible.

In the next section, we describe a simple method that avoids unphysical quark production thresholds in quark loop diagrams.

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[2]The value of c is defined by the condition of the absence of $\bar{q}q$ thresholds in the scalar and vector meson decay channels (see below).

[3]The expression in the brackets can be written in the form $1/G + \Pi_\pi(p)$, where $\Pi_\pi(p)$ is the polarization operator of the pion.

3. QUARK CONFINEMENT BY INFRARED (IR) CUT-OFF AND DEFINITION OF THE MODEL PARAMETERS

In the standard NJL model ($\mu = 0$) the conventional input parameters, which are known from phenomenology, are the pion decay constant $f_\pi = (92.4 \pm 0.3)$ MeV [12], the pion mass $M_\pi = 140$ MeV and the quark condensate $-\langle\bar{q}q\rangle_0 = (250\text{MeV})^3$. Then, from the Goldberger-Treiman relation

$$g_\pi = \frac{m}{f_\pi} , \tag{14}$$

the relation for the quark condensate

$$\langle\bar{q}q\rangle_0 = -4mI_1^{(\mu\Lambda)}(m) , \tag{15}$$

and the pion mass formula (12), we find the model parameters Λ, G and m[4]. Representative sets of parameter values are given in Table I depending on the choice of the quark condensate value and the IR cut-off μ. The σ-meson mass is given by (see Eq. (13))

$$M_\sigma^2 = \frac{g_\sigma^2}{g_\pi^2}M_\pi^2 + 4m^2 , \tag{16}$$

and the polarization operator of the σ-meson (see Fig. 2) has not an imaginary part if the threshold condition

$$M_\sigma < 2\sqrt{m^2 + \mu^2} \tag{17}$$

is fulfilled. Here, we observe that according to Ref. [10] the introduction of an infrared cut-off on the quark momentum in loop integrals can prevent the occurrence of an unphysical threshold for the meson decay into free (deconfined) quarks. In the present work, we show that the choice $\mu/m = c \approx 0.5$ leads to a removal of poles in the polarization operators of all mesons with masses in the interval $0 \le M_{\text{meson}} \le 0.805$ GeV. On the other hand, we conserve the low-energy theorems and the spontaneous breaking of chiral symmetry (see Eqs. (5) and (12)).

Concluding this section, we give the condition for the meson masses at which the integral $I_2(M^2, m)$ has not the imaginary part (confinement condition)

$$M_{\text{meson}} < 2m\sqrt{1+c^2} . \tag{18}$$

For $c = 0.5$ and $|\langle\bar{q}q\rangle_0|^{1/3} = 250$ MeV (see Table I) we obtain $m = 360$ MeV, $m_0 = 5.3$ MeV, $\Lambda = 690$ MeV, $M_\sigma = 727$ MeV, $g_\sigma = 2.74$, $g_\pi = 3.9$. This result satisfies the condition (18) $M_{\text{meson}} < 805$ MeV. Therefore, this model is suitable for the description of the low-lying mesons $\pi, \sigma, \omega, \rho$ as bound states of quarks with prohibited decay into $\bar{q}q$ although all of these mesons except the pion are heavier than a free quark pair.

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[4]The current quark mass m^0 is defined from the gap equation (5).

4. DECAY OF THE σ- MESON

The decay $\sigma \longrightarrow 2\pi$ goes through the quark triangle diagram, see Fig. 3. This diagram also satisfies the confinement condition if we use the IR cut-off $\mu = c \cdot m$ $(0.5 \leq c \leq 1)$. The amplitude of the process $\sigma \longrightarrow 2\pi$ consists of two parts: the divergent part T_1 and the convergent part T_2

$$T_1 = 8mg_\sigma g_\pi^2 I_2(M_\sigma^2, m)\sigma\vec{\pi}^2 \tag{19}$$

$$T_2 = 8mg_\sigma g_\pi^2 \mathcal{J}(M_\sigma, M_\pi, m)\sigma\vec{\pi}^2 \ , \tag{20}$$

where

$$\mathcal{J}(M_\sigma, M_\pi, m) = -i\frac{N_c}{2(2\pi)^4}(M_\sigma^2 - 2M_\pi^2)I_3(p_1, p_2, m) \ . \tag{21}$$

With the approximation that

$$\begin{aligned} I_3(p_1,p_2,m) &= \int \frac{d^4k}{(k^2-m^2)((k+p_1)^2-m^2)((k-p_2)^2-m^2)} \\ &\approx \int \frac{d^4k}{(k^2-m^2)^3} = -i\frac{3\pi^2}{2}\int_\mu^\Lambda dk\frac{k^2}{E^5} \\ &= -i\frac{\pi^2}{2m^2}\left[(1+\frac{m^2}{\Lambda^2})^{-3/2} - (1+\frac{m^2}{\mu^2})^{-3/2}\right] \ , \end{aligned} \tag{22}$$

the decay width is obtained to be $(\mu = 0.5m)$

$$\Gamma_{\sigma\to 2\pi} = \frac{3}{2\pi}\left(\frac{m^3}{g_\sigma f_\pi^2}\right)^2 \frac{(1-0.31)^2}{M_\sigma^2}\sqrt{M_\sigma^2 - 4M_\pi^2} \approx 1 \text{ GeV} \ .$$

The σ- meson mass and the decay width are in agreement with the experimental data [12–14]

$$M_\sigma^{\rm exp} = (400 - 1200) \text{ MeV} \ , \quad \Gamma_\sigma^{\rm exp} = (600 - 1000) \text{ MeV} \ . \tag{23}$$

From this example we can conclude that the NJL model with the IR-cutoff $\mu = 0.5m$ satisfies both of the low energy theorems together with spontaneous breaking of the chiral symmetry and describes the low-energy physics of scalar and pseudoscalar mesons. Similar results can be obtained for other values of μ and $\langle\bar{q}q\rangle_0$ (see Table I).

5. DISCUSSION AND CONCLUSION

In this work it has been shown that the introduction of an IR cut-off in the NJL model removes unphysical quark production thresholds from quark loop diagrams

and in this sense confines quarks. On the other hand, the dynamical chiral symmetry breaking phenomenon as well as the basic low-energy theorems remain intact.

In Fig. 1, we have shown the meson-meson coupling as a quark loop diagram with three domains in the $x-$ space:

1) $|\vec{x}| < r \approx 1/\Lambda$, where the UV cut-off Λ is the scale of spontaneous breaking of chiral symmetry. This is the region of asymptotic freedom where the quarks have their current masses m^0.

2) $|\vec{x}| > R \approx 1/\mu$, which is the region of quark confinement where only hadronic states can propagate.

3) $r \leq |\vec{x}| \leq R$, the domain where the interactions of quarks inside the quark loops take place. Here, we have spontaneous chiral symmetry breaking and constituent quarks.

In the standard NJL model only a UV cut-off is used in order to remove the UV divergencies due to the local four-point interaction of quarks. Since IR divergencies are absent in the standard NJL model, no IR cut-off has been introduced. However, as has been shown in the present paper, the introduction of such a cut-off can remove unphysical quark production thresholds of the non-confining standard NJL model and, by a proper choice of the scale of this cut-off from the dynamical quark mass (condensate) which is melted at finite temperature, the quark deconfinement and possible observable phenomena associated with this transition due to the appearance of the (now physical) decay channels into (quasi-) free quarks can be studied. As an example, we have introduced $\mu = 0.5m$ and consider the width of the sigma-meson, which is dominated at low temperatures by the $\sigma \to 2\pi$ decay channel, which closes at temperatures $T > T_\sigma^{(\pi\pi)}$ when $M_\sigma < 2M_\pi$. For still higher temperatures $T > T_\sigma^{(\bar{q}q)}$, when $M_\sigma > 2\sqrt{m^2+\mu^2}$, the $\sigma-$ meson width is given by the decay channel into (quasi-) free quarks. We plan to fulfill these investigations in our further work.

Possible consequences of this behaviour for experimental investigations of the chiral phase transition in heavy ion collisions have been drawn from such a particular behaviour of the σ- meson by considering the annihilation of pions (quarks) in the two-photon channel [15,16] and in the dilepton channel [17] within the standard NJL model. A more systematic study of the possible observable effects associated with quark deconfinement can be performed in applying the present model to studies of, e.g., the $\pi\pi$ (and $\bar{q}q$) scattering in the ρ- meson channel which is important for explaining the low-mass dilepton enhancement observed by the CERES collaboration [18].

In conclusion we would like to point out that the present paper is only a first and simplest version of the NJL model with the IR confinement. In our next work we are going to define a less ambiguous condition for the IR cut-off parameter μ and to consider also strange and excited mesons.

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TABLES

μ/m	$\lvert\langle\bar{q}q\rangle\rvert^{1/3}$ [GeV]	m [GeV]	Λ [GeV]	m_0 [MeV]	G [GeV^{-2}]	Γ [GeV]	M_σ [GeV]
0	0.250	0.304	0.659	5.26	9.54	–	–
	0.255	0.284	0.690	4.96	8.42	–	–
	0.260	0.270	0.719	4.68	7.55	–	–
	0.265	0.259	0.748	4.42	6.84	–	–
	0.270	0.250	0.777	4.18	6.23	–	–
	0.275	0.242	0.806	3.96	5.72	–	–
	0.280	0.235	0.835	3.75	5.27	–	–
	0.285	0.229	0.864	3.56	4.87	–	–
	0.290	0.224	0.893	3.38	4.52	–	–
	0.295	0.219	0.922	3.21	4.20	–	–
	0.300	0.215	0.952	3.05	3.92	–	–
0.5	0.250	0.360	0.639	5.28	10.98	1.16	0.727
	0.255	0.315	0.676	4.97	9.15	0.73	0.638
	0.260	0.292	0.707	4.68	8.03	0.57	0.593
	0.265	0.276	0.737	4.42	7.19	0.47	0.561
	0.270	0.263	0.767	4.18	6.51	0.40	0.538
	0.275	0.253	0.796	3.96	5.94	0.36	0.518
	0.280	0.245	0.825	3.75	5.45	0.32	0.502
	0.285	0.238	0.854	3.56	5.03	0.29	0.488
	0.290	0.232	0.884	3.38	4.65	0.26	0.476
	0.295	0.226	0.913	3.21	4.32	0.24	0.466
	0.300	0.221	0.943	3.06	4.02	0.23	0.456
1	0.285	0.315	0.809	3.57	6.07	0.59	0.641
	0.290	0.291	0.840	3.39	5.46	0.47	0.594
	0.295	0.275	0.871	3.22	4.98	0.39	0.564
	0.300	0.263	0.901	3.06	4.57	0.34	0.540

TABLE I. Parameter sets for the NJL model with different values of the IR cut-off μ and the phenomenological quark condensate.

FIGURES

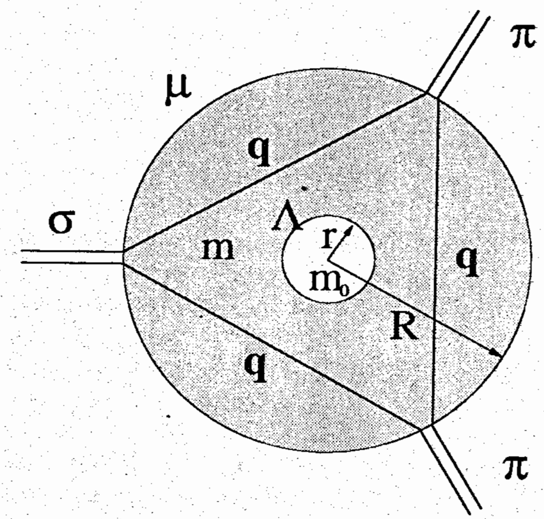

FIG. 1. Two scales relevant for the quark propagation in the low energy QCD vacuum.

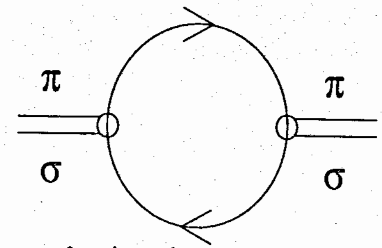

FIG. 2. One-loop diagram for the polarization operator of the pion and the sigma meson, resp.

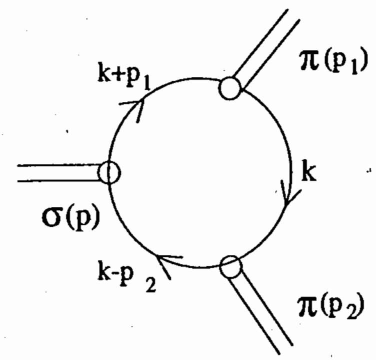

FIG. 3. Quark loop diagram for the decay $\sigma \longrightarrow 2\pi$.

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Бляшке Д. и др. E2-98-227
Модель НИЛ без $\bar{q}q$-порогов

Рассмотрена расширенная модель Намбу—Иона-Лазинио (НИЛ) для легких мезонов, в которой нефизические пороги рождения кварков исключаются путем введения инфракрасного обрезания в интегралах по импульсам. Эти интегралы возникают в модели при вычислении однопетлевых кварковых диаграмм, описывающих взаимодействия мезонов. Предлагаемая кварковая модель не нарушает низкоэнергетические теоремы. Инфракрасное обрезание определяется конституентной массой кварка (кварковым конденсатом). Даны оценки массы и ширины распада сигма-мезона.

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Blaschke D. et al. E2-98-227
NJL Model without $\bar{q}q$ Thresholds

We consider an extended Nambu—Jona-Lasinio (NJL) model for the light mesons, in which unphysical quark production thresholds are excluded by an infrared cut-off on the momentum integration within quark loop diagrams. This chiral quark model conserves the low energy theorems. The infrared cut-off is fixed by the dynamically generated quark mass (quark condensate). The sigma meson mass and its decay width are estimated in the model.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

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