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THE TOPOLOGICAL WU–YANG MONOPOLE

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1 Introduction

The Wu–Yang monopole has been introduced in paper [1] in the conceptual analysis of the global aspects of gauge fields and gauge symmetry. It is an infinite–energy stationary solution of the classical equations of Yang–Mills theory with a singularity at the origin. Recently, Faddeev and Niemi have treated this monopole as a knot–like vortex of the type of the condensate in a superfluid liquid [2]. In the present paper, in the context of such a treatment we construct a generalization of the Wu–Yang monopole with a nontrivial topological "charge", which reflects the superfluid rigid dynamics of the gauge field system as a whole [3, 4]. In QCD, this topological "charge" leads to an additional mass for the η_0–meson which fixes the value of the finite density of the knot energy.

2 The Topological Wu–Yang Monopole

The Wu–Yang (WY) monopole [1]

$$A_0^a = 0, \qquad A_i^a = b_i^a = \frac{1}{g}\epsilon_{iak}\frac{n_k}{r} \tag{1}$$

$$n_k = \frac{x_k}{r}, \qquad r = \sqrt{x_1^2 + x_2^2 + x_3^2}$$

is a stationary solution of the YM classical equations

$$\mathcal{D}_\mu^{ab} G_{\mu\nu}^b = 0 \Longrightarrow \begin{cases} [\mathcal{D}_i^2(A)]^{ab} A_0^b = \mathcal{D}_j^{ab}(A)\partial_0 A_j^b \\ [\mathcal{D}_0(A)]^{ab} G_{0j}^b = \mathcal{D}_i^{ab}(A) G_{ij}^b \end{cases}, \tag{2}$$

where $\mathcal{D}_\mu^{ab}(A) = \delta^{ab}\partial_\mu + g\epsilon^{acb}A_\mu^c$, $G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g\epsilon^{abc}A_\mu^b A_\nu^c$. Following [1], we consider this solution between two spheres with radii ε and R, $\varepsilon < r < R$, $\varepsilon \ll R$, so in a finite space volume $V(\varepsilon, R)$, in order to exclude the singularity at the origin, $r = 0$. The knot interpretation of the WY monopole supposes that the density of its energy (which coincides with the density of its action)

$$< \alpha_s G^2 > = \lim_{R\to\infty,\varepsilon\to 0} \alpha_s \frac{\int_V d^3x G_{ij}^2(b)}{V} \equiv \lim_{R\to\infty,\varepsilon\to 0} 2(V\varepsilon)^{-1} \quad (\alpha_s = \frac{g^2}{4\pi}) \tag{3}$$

is finite, and can be considered as a parameter of the theory. Eqs.(2) contain time derivatives and their solutions depend on the initial data. From this point of view, the WY monopole represents some initial data (i.e. an initial "position"). At the initial "position" $A_i^a \equiv b_i^a$, eqs.(2) have the form

$$[\mathcal{D}_i^2(A)]^{ab} A_0^b = 0, \qquad [\mathcal{D}_0(A)]^{ab}\mathcal{D}_i^{bc}(A) A_0^c = 0. \tag{4}$$

The result we present here is based on the existence of a nontrivial solution to these equations in the considered region with a volume $V(\varepsilon, R)$

$$A_0^a(t, x) = N(t)\Phi^a(x) \tag{5}$$

$$\ddot{N} = 0 \qquad \Phi^a = 2\pi n^a f(r)/g, \qquad f(r) = 1 - \varepsilon/r$$

with boundary conditions $f(\varepsilon) = 0$, $f(R \to \infty) = 1$.

The variable $N(t)$ can be interpreted as a zero mode of the Gauss constraint, which describes the superfluid rigid motion of the YM field as a whole [3, 4]. This global motion is like the one of a free particle of mass M

$$L(N) = \frac{M\dot{N}^2}{2} \tag{6}$$

$$M = \int_V d^3x[\mathcal{D}_i^{ab}\Phi^b]^2 = \int_V d^3x\frac{1}{2}\Delta(\Phi^a)^2 = \frac{4\pi^2}{\alpha_s}\int_\varepsilon^R dr\frac{d}{dr}(r^2\frac{d}{dr}f) = \frac{4\pi^2}{\alpha_s}\varepsilon.$$

With the help of solutions (1), (5) a Dirac-like representation [5] for the physical field can be constructed

$$\hat{A}_0 = 0, \quad \hat{A}_i = e^{N\hat{\Phi}}(\hat{b}_i + \partial_i)e^{-N\hat{\Phi}}; \qquad \hat{A} = g\frac{\tau_a A^a}{2i}, \tag{7}$$

which means that in the non-Abelian theory, in addition to the perturbative harmonic excitations of the type of photons in QED (which are neglected here), a global type excitation of the gauge field as a whole is possible, characterized by some initial "position" and "velocity" in the field configuration space.

The physical field so obtained (7) is a generalization of the WY monopole with a nontrivial topological "charge"

$$X[A] = \lim_{R\to\infty,\varepsilon\to 0}\frac{g^2}{16\pi^2}\int_{V(\varepsilon,R)} d^3x\epsilon^{ijk}\left[A^a{}_i\partial_j A^a{}_k + \frac{2}{3}g\epsilon_{abc}A^a{}_i A^b{}_j A^c{}_k\right]. \tag{8}$$

In the case under consideration, this topological "charge" coincides with the zero mode of the Gauss constraint

$$X[A] = N(t). \tag{9}$$

The WY monopole and similar solutions of the classical equations are also present in the $SU(3)$ theory. To construct them, one has to choose a minimal subgroup $SU(2)$ so that the irreducible (fundamental) representation of the $SU(3)$ group will also be the irreducible (joint) one of the $SU(2)$ subgroup. For example, the role of τ_1, τ_2, τ_3 is played in the $SU(3)$ theory by $(\lambda_2, \lambda_5, \lambda_7)$:

$$\hat{b}_i = g\frac{b_i^1\lambda_2 + b_i^2\lambda_5 + b_i^3\lambda_7}{2i}, \qquad b_i^a = \frac{\epsilon^{aik}n^k}{gr}. \tag{10}$$

3 The $U(1)$-Problem in QCD

It is well known [6] that the topological variable N mixes with the nineth pseudoscalar meson (η_0) in the effective QCD action [7,8]

$$L_{eff} = \frac{M\dot{N}^2}{2} + \frac{V}{2}(\dot{\eta}_0^2 - m_0^2\eta_0^2) + C\eta_0(t)\dot{N}, \tag{11}$$

where m_0 is the spontaneous chiral symmetry breaking mass of η_0-meson and the constant C reads

$$C = \frac{N_f}{F_\pi}\sqrt{\frac{2}{3}},$$

with $F_\pi = 92Mev$ — the weak-decay coupling constant, and N_f — the number of flavors. The diagonalization of Lagrangian (11) leads to an additional mass term for the η_0-meson

$$\Delta m_0^2 = \frac{C^2}{MV} \quad \text{with} \quad M = \gamma\varepsilon,\ \gamma = \frac{4\pi^2}{\alpha_s}, \tag{12}$$

if in the infinite volume limit the quantity $MV = \gamma\varepsilon V$ remains finite with $\varepsilon \to 0$. This ensures also a finite energy density for the WY monopole (see eq.(3)) in correspondence with its interpretation as a knoted vortex [2]. Note that in this case eq.(12)

$$\Delta m_\eta{}^2 = \frac{C^2}{\gamma\varepsilon V} = \frac{N_f^2}{2F_\pi^2}\frac{\alpha_s}{6\pi^2} < \alpha_s G^2 > \tag{13}$$

reproduces the relation between the gluon condensate [9] and the ($\eta'-\eta$)-mass difference which is in agreement with the present experimental data [10].

4 Conclusion

From a physical point of view, the topological generalization of the Wu-Yang monopole means that the non-Abelian (gluon) field, despite the monopole condensate and harmonic excitations of the type of photons in QED, allows also a rigid type excitation of the gauge field system as a whole represented by the zero mode of the Gauss constraint differential operator. For this (global) dynamics, the WY monopole specifies the initial data.

The identification of this zero mode with the topological excitation which realizes the Kogut-Susskind mechanism of solution of the $U(1)$-problem in QCD does not contradict the physical interpretation of the Wu-Yang monopole as a condensate with a finite energy density. On the other hand, the effective mass of the topological excitation disappears in the infinite volume limit. This fact means that the region of validity of the quantum theory for the rigid dynamics is restricted to the finite space volume, as for a superfluid liquid [3, 2].

The superfluid dynamics of the non-Abelian fields, in contrast to the semi-classical instantons [11], is considered in Minkowski space. It can bear relation to a colour confinement mechanism due to a complete destructive interference phenomenon as a result of taking an average from the colour state amplitudes over different initial data in the gauge field configuration space [4].

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[11] V.N. Gribov has been arguing in the last years, that instantons are nonphysical solutions since they are permanently tunnelling and their zero energy belongs to the nonphysical part of the spectrum. (One of the authors (V.P.) is grateful to his widow Julia Nyiri for the permition to refer to the discussion with Vladimir Naumovich on this problem in May 1996, in Budapest.)

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Топологический монополь Ву–Янга

Мы даем обобщение монополя Ву–Янга с непрерывным топологическим зарядом. Последний может быть рассмотрен как безмассовое топологическое возмущение, которое ведет к решению проблемы дополнительной массы η_0-мезона.

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The Topological Wu–Yang Monopole

We present generalization of the Wu–Yang monopole with a continuous topological «charge». The latter can be considered as a massless topological excitation which realizes the Kogut–Susskind mechanism of solution of the $U(1)$-problem in QCD.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

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