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NEUTRON STAR IN PRESENCE OF TORSION-DILATON FIELD

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1 Introduction

In recent years the interest in scalar-tensor theories of gravity has been renewed. One reason for this is the important role which these theories play in the understanding of inflantionary epoch. On the other hand the scalar-tensor gravitation (the so called "dilaton gravity") arises naturally from the low-energy limit of the super-string theory [1].

The predictions of scalar-tensor theories may differ drastically from these of general relativity. For example such a phenomenon – "spontaneous scalarization" was recently discovered by Damour and Esposito-Farese as a non-perturbative strong field effect in a massive neutron star [2].

Many theories of gravity with propagating torsion involving a scalar field have been proposed in the last decades, too. In such theories contrary to the usual Einstein-Cartan gravity [3]-[5], there are long-range torsion mediated interactions. Carrol and Field [6] have examined some observational consequences of propagating torsion in a wide class of models involving a scalar field. They conclude that for reasonable models the torsion could be detected experimentally.

Recently a new interesting model with propagating torsion was proposed by Saa [7]-[11]. This model involves a non-minimally coupled scalar field as a potential of the torsion of space-time. As one can see Saa's model is very close to the dilaton gravity.

In the present article we investigate both analytically and numerically a neutron star in the Saa's model and compare obtained results with these in the general relativity. We also discuss new predictions of the theory under consideration.

The paper is organized as follows. In section 2 we consider briefly Saa's model. In section 3 we give the necessary information for the vacuum solutions of the field equations. The equations determining static equilibrium solutions for a neutron star are discussed in section 4. Numerical results for the neutron star are discussed in section 5.

2 The model with torsion-dilaton field

Here we give a brief description of Saa's model. For more details one can see [7]-[9], [12].

Consider four-dimensional Einstein-Cartan manifold $\mathcal{M}^{(1,3)}$, i.e. four-dimensional manifold equipped with metric $g_{\alpha\beta}$ and affine connection $\Gamma_{\alpha\beta}{}^{\gamma}$ with torsion tensor $S_{\alpha\beta}{}^{\gamma}$.

The main idea of articles [7]–[9] is to make the volume form d^4Vol compatible with the affine connection on the Einstein-Cartan manifold $\mathcal{M}^{(1,3)}$

via the compatibility condition:

$$\pounds_v \left(d^4Vol\right) = (\nabla_\mu v^\mu) d^4Vol, \tag{1}$$

where $\pounds_v$ is the Lee derivative along an arbitrary vector field v and ∇_μ is the covariant derivative with respect to the affine connection. It turns out that compatibility condition (1) is fulfilled if and only if the torsion vector

$$\mathcal{S}_\alpha = \tfrac{2}{3} S_{\alpha\mu}{}^\mu$$

is potential, i.e. if there exists a potential Θ, such that

$$S_\alpha = \nabla_\alpha \Theta \equiv \partial_\alpha \Theta. \tag{2}$$

In this case Saa's condition (1) implies the form

$$d^4Vol = f(x)d^4x = e^{-3\Theta}\sqrt{|g|}d^4x \tag{3}$$

of the volume element in Einstein-Cartan manifold. As it was pointed out in [12] compatibility condition (1) leads to covariantly constant scalar density $f = e^{-3\theta}\sqrt{|g|}$ with respect to the transposed connection $(\Gamma^T)_{\alpha\beta}{}^\gamma = \Gamma_{\beta\alpha}{}^\gamma$, not with respect to the usual connection $\Gamma_{\alpha\beta}{}^\gamma$. Therefore the Einstein-Cartan manifold for which compatibility condition (1) is fulfilled was called transposed-equi-affine and the corresponding theory of gravity – transposed-equi-affine theory of gravity (TEATG).

The most important mathematical consequence of the condition (1) which leads to new equations of gravity is the generalized Gauss' formula:

$$\int_{\mathcal{M}} d^4Vol (\nabla_\mu v^\mu) = \int_{\partial\mathcal{M}} d^3\Sigma_\mu v^\mu. \tag{4}$$

The natural choice of the lagrangian density for gravity is:

$$\mathcal{L}_G = -\tfrac{c}{2\kappa} R = -\tfrac{c}{2\kappa}\left(\overset{\{\}}{R} + 6\nabla_\mu S^\mu + 12 S_\mu S^\mu - \tilde{K}_{\mu\nu\lambda}\tilde{K}^{\mu\nu\lambda}\right), \tag{5}$$

c being the velocity of light, $\kappa = 8\pi c^{-2} G$ being the Einstein constant, G being the Newton constant. Here $R = g^{\alpha\beta}R_{\alpha\beta}$ is the scalar curvature with respect to the affine connection, $\tilde{K}_{\mu\nu\lambda} = K_{\mu\nu\lambda} + 2g_{\mu[\nu}S_{\lambda]}$ is the traceless part of the contorsion: $\tilde{K}^\mu{}_{\mu\nu} = \tilde{K}^\mu{}_{\nu\mu} \equiv 0$, and $\overset{\{\}}{R}$ is the scalar curvature with respect to the Levi-Chevita connection.

The traceless part of the torsion doesn't vanish only if spin-non-zero matter presents. In the present article we consider only spinless matter (as we know from Einstein-Cartan theory of gravity, the effects due to the spin become essential at density over $10^{57} g/cm^3$ [3] which is too far from the physics in

the stars). Therefore we put $\tilde{K}_{\alpha\beta\gamma} \equiv 0$ and obtain a semi-symmetric affine connection:

$$S_{\alpha\beta}{}^\gamma = S_{[\alpha}\delta^\gamma_{\beta]}. \tag{6}$$

In this case we have:

$$\mathcal{L}_G = -\tfrac{c}{2\kappa} R = -\tfrac{c}{2\kappa}\left(\overset{\{\}}{R} + 6\nabla_\mu S^\mu + 12 S_\mu S^\mu\right). \tag{7}$$

Denoting the lagrangian density for the matter by $\mathcal{L}_M$ and using the volume element (3) we write down the action of gravity and matter in the form:

$$\mathcal{A} = \mathcal{A}_G + \mathcal{A}_M = -\tfrac{c}{2\kappa}\int d^4Vol\, R + \tfrac{1}{2c}\int d^4Vol \mathcal{L}_M. \tag{8}$$

Due to the new Gauss' formula (4) the term $6\nabla_\mu S^\mu$ in the lagrangian (7) gives a surface term in the action integral (8) and doesn't contribute to the equations of motion. Hence, these equations may be derived from the modified action:

$$\tilde{\mathcal{A}} = -\tfrac{c}{2\kappa}\int d^4Vol \left(\overset{\{\}}{R} + 12 S^\mu S_\mu\right) + \tfrac{1}{2c}\int d^4Vol\, \mathcal{L}_M. \tag{9}$$

This action is very close to the one of the dilatonic gravity arising from low-energy limit of the superstring theory. Two essential differences between our case and the dilatonic one are: 1) the matter action includes the dilaton-like term $e^{-3\theta}$ which arises in a natural way, as a part of the volume element of space-time, and 2) the sign before the term $12S^\mu S_\mu$. Following the above described reasons we call the field Θ, which originates from the space-time torsion and plays the role of the dilaton field in Saa's model, "a torsion-dilaton field".

Taking variations with respect to the metric $g_{\alpha\beta}$ and torsion-dilaton field Θ, and using the generalized Gauss' formula, we obtain the following equations of motion for the geometrical fields g and θ:

$$\begin{aligned} G_{\mu\nu} + \nabla_\mu\nabla_\nu\Theta - g_{\mu\nu}\Box\Theta &= \tfrac{\kappa}{c^2} T_{\mu\nu}, \\ \Box\Theta &= \tfrac{\kappa}{c^2}\left(\mathcal{L}_M - \tfrac{1}{3}\frac{\delta\mathcal{L}_M}{\delta\Theta}\right) - \tfrac{1}{2}R. \end{aligned} \tag{10}$$

Here $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}$ is the Einstein tensor for the affine connection, its trace is $G = g^{\mu\nu}G_{\mu\nu} = -R$; $T_{\mu\nu} = \delta\mathcal{L}_M/\delta g^{\mu\nu}$ is the symmetric energy-momentum tensor of the matter ; its trace is $T = g^{\mu\nu}T_{\mu\nu}$ and $\nabla_\sigma S^\sigma = g^{\mu\nu}\nabla_\mu\nabla_\nu\Theta = \Box\Theta$. From the first equation of the system (10) it follows that:

$$R = \tfrac{2\kappa}{c^2}\left(3\mathcal{L}_M - \frac{\delta\mathcal{L}_M}{\delta\Theta} + T\right). \tag{11}$$

Then combining this result with the second equation of the system (10) we obtain:

$$\nabla_\sigma S^\sigma = \Box\Theta = -\,\tfrac{2\kappa}{c^2}\left(\mathcal{L}_M - \tfrac{1}{3}\frac{\delta\mathcal{L}_M}{\delta\Theta} + \tfrac{1}{2}T\right). \tag{12}$$

The equation (12) shows that under proper boundary conditions, and in the presence only of spinless matter, the torsion-dilaton field Θ is completely determined by the matter distribution. Further on, as a basic system we will use the system:

$$\begin{aligned} G_{\mu\nu} + \nabla_\mu\nabla_\nu\Theta - g_{\mu\nu}\Box\Theta &= \tfrac{\kappa}{c^2}T_{\mu\nu}, \\ \nabla_\sigma S^\sigma = \Box\Theta &= -\,\tfrac{2\kappa}{c^2}\left(\mathcal{L}_M - \tfrac{1}{3}\frac{\delta\mathcal{L}_M}{\delta\Theta} + \tfrac{1}{2}T\right). \end{aligned} \tag{13}$$

From this system one can derive (using Bianchi identity) the differential consequence:

$$\nabla_\sigma T^\sigma_\alpha + T^\sigma_\alpha S_\sigma = \tfrac{c^2}{2\kappa}RS_\alpha \tag{14}$$

which is a generalization of the well-known conservation law $\overset{\{\}}{\nabla}_\sigma T^\sigma_\alpha = 0$ in general relativity.

To have a complete set of dynamical equations one has to add to the above relations the equations of motion of the very matter. For the purpose of the present article we need to consider only a perfect fluid. Its theory was recently described in [12]. Here we give the basic results.

The continuity condition describing the conservation of the fluid matter can be written in the form:

$$\int_{\partial\Delta^{(1,3)}} d^3\Sigma_\alpha\, n(x)u^\alpha(x) = 0, \tag{15}$$

where $u^\alpha(x)$ is the fluid four-velocity, normalized by the relation $g_{\alpha\beta}u^\alpha u^\beta = 1$, $n(x)$ is properly defined a fluid density, $d^3\Sigma_\alpha$ is a proper three dimensional surface element depending on the choice of the volume element via the Gauss' formula, and $\Delta^{(1,3)}$ is an arbitrary domain.

Considering the volume element (3) as an universal one we must use it in the continuity condition, too. Therefore according to the generalized Gauss' formula we can rewrite relation (15) in the form of a continuity equation of autoparallel type:

$$\nabla_\alpha(n(x)u^\alpha(x)) = 0. \tag{16}$$

We take the lagrangian of the fluid with internal pressure p in the usual form:

$$\mathcal{L}_\mu = -\varepsilon = -nc^2 - n\Pi, \tag{17}$$

where Π is the elastic potential energy of the fluid ; $d\Pi = -pd(\frac{1}{n})$ and the symbol " d " means that the corresponding differential form isn't exact. Taking into account the relation $\mathcal{L}_\mu - \frac{1}{3}\frac{\delta\mathcal{L}_\mu}{\delta\Theta} = p$ it's not difficult to obtain the equations of motion for geometrical fields $g_{\alpha\beta}$ and Θ in presence of perfect fluid:

$$\begin{aligned} G_{\mu\nu} + (\nabla_\mu\nabla_\nu - g_{\mu\nu}\Box)\Theta &= \tfrac{\kappa}{c^2}\Big((\varepsilon + p)u^\mu u^\nu - p\,g^{\mu\nu}\Big), \\ \Box\Theta &= -\,\tfrac{\kappa}{c^2}(\varepsilon - p); \end{aligned} \tag{18}$$

In addition one can show that:

$$\begin{aligned} \nabla_\sigma T^\sigma_\alpha &= (\varepsilon + p)\,(\delta^\sigma_\alpha - u^\sigma u_\alpha)\,S_\sigma, \qquad \text{or} \\ \overset{\{\}}{\nabla}_\sigma T^\sigma_\alpha &= 3(\varepsilon + p)\,u^\sigma u_\alpha S_\sigma. \end{aligned} \tag{19}$$

Making use of (19) and of the continuity condition (16) one can obtain the equations of motion of the perfect fluid (just as in general relativity):

$$(\varepsilon + p)u^\beta\overset{\{\}}{\nabla}_\beta u_\alpha = \left(\delta^\beta_\alpha - u_\alpha u^\beta\right)\overset{\{\}}{\nabla}_\beta p. \tag{20}$$

The equations (19) are equations of a geodesic type. In particular, considering dust matter ($p \equiv 0$) we have:

$$u^\beta\overset{\{\}}{\nabla}_\beta u_\alpha = 0, \tag{21}$$

i.e. we can conclude (just as in general relativity) that a test particle in the theory under consideration will move on a geodesic line. We will need this conclusion in the next sections. For more details concerning the relativistic perfect fluid in the theory under consideration we refer to [12].

3 Spherically symmetric vacuum solution

The asymptotic flat, static and spherically symmetric general solutions of the vacuum geometrical field equations (10) are known [13], [14]. In Schwarzshild's coordinates they are described as a two parameter - $\{K, a\}$ family of solutions[1]:

$$g_{00} = e^\nu,$$

$$r = \tfrac{1}{2}ae^{\frac{(3K-1)\nu}{2}}\sinh^{-1}\left(\tfrac{\rho\nu}{2}\right),$$

[1] We use asymptotic conditions $\nu \to 0, \Theta \to 0$ at $r \to \infty$ without loss of generality.

$$g_{11} = e^{\lambda} = \left(\tfrac{1+\delta}{2} e^{\frac{-\rho\nu}{2}} + \tfrac{1-\delta}{2} e^{\frac{\rho\nu}{2}} \right)^{-2},$$

$$\rho = \sqrt{3\left(K - \tfrac{1}{2}\right)^2 + \tfrac{1}{4}},$$

$$\delta = \tfrac{3K-1}{\rho}$$

$$\xi = \tfrac{1}{2}\nu' = \tfrac{1}{2}\,\tfrac{a}{\rho}\,\tfrac{1}{r^2}\, e^{\frac{(3K-1)\nu}{2}} e^{\frac{\lambda}{2}}$$

$$S_r = \Theta' = K\xi. \tag{22}$$

Here and further on the prime denotes a differentiation with respect to the variable r. All quantities in formulae (22) are represented as functions of the variable ν. This is the most convenient form of the vacuum solutions.

The parameter K presents the ratio of the torsion force (as defined in [12]) and the gravitational one: $K = S_r/(\frac{1}{2}\nu')$. In the case when $K = 0$ we have the usual torsionless Schwarzschild's solution and $a \equiv r_g$ is the standard gravitational radius r_g.

In the model under consideration the value of the fundamental parameter of the theory K (which is constant in vacuum) is not an independent integration constant. Instead, we shall show that it is determined by the total mass of the star, or by its radius and depends on the matter distribution, on the equation of state of the star's matter, and so on via the solution of the full system of equation of the star's state.

The parameter a is positive ($a > 0$), and may take arbitrary values. It is related to the total mass of the star, too.

The asymptotic behaviour of the solution is:

$$g_{00} \sim 1 - \frac{2MG}{c^2 r}, \tag{23}$$

$$g_{11} \sim 1 + \frac{2G(M - M_\theta)}{c^2 r}, \tag{24}$$

where $M = \frac{1}{2}\frac{c^2}{G}\frac{a}{\rho}$ describes the asymptotic dependence of g_{00} on the variable r, and the mass $M_\theta = 3\lim_{r\to\infty} r^2 S_r$ describes the asymptotic dependence of the torsion-dilaton potential Θ on r (or the asymptotic of S_r). The mass M_θ may be considered as a source of torsion-dilaton force and is analogous to the "scalar mass" introduced in [15]. As one can see the scalar mass M_θ depends on K, and may be less or greater than the mass M.

In the model under consideration, a test particle moves along geodesic lines. Therefore, the keplerian-like mass measured by a test particle in the asymptotic region of space-time will be the mass $M = \lim_{r\to\infty} \frac{1}{2} r^2 \nu'$. As we will see in the

next section, the mass M is positively defined and it's natural to consider it as a total mass-energy of the geometric-fields-complex $\{g, \Theta\}$.

The appearance of two masses in the asymptotic solution is related to the violation of the strong equivalence principle (the weak equivalence principle is not violated). In our case the ratio M_θ to M is just $3K$ and depends on the ratio K of the torsion-dilaton force to gravitational one in vacuum.

4 The Basic equations for a star

4.1 General considerations

Here we will discuss some general properties of the system of equations of the star without specifying the matter's equation of state.

The system of the equation (18) for geometrical fields g and Θ can be rewritten in the form:

$$\begin{aligned} \overset{\{\}}{G}{}^\nu_\mu + 3\Sigma^\nu_\mu &= \tfrac{\kappa}{c^2} T^\nu_\mu, \\ \overset{\{\}}{\nabla}_\sigma S^\sigma - 3S_\sigma S^\sigma &= -\tfrac{\kappa}{c^2}(\epsilon - p), \\ T^\nu_\mu &= (\epsilon + p)u^\nu u_\mu - p g^\nu_\mu, \end{aligned} \tag{25}$$

where $\overset{\{\}}{\nabla}$ is the covariant derivative with respect to the Levi-Chevita connection, $\overset{\{\}}{G}{}^\nu_\mu$ is the corresponding Einstein's tensor and

$$\Sigma^\nu_\mu = \overset{\{\}}{\nabla}_\mu S^\nu + S_\mu S^\nu - g^\nu_\mu \overset{\{\}}{\nabla}_\sigma S^\sigma + g^\nu_\mu S_\sigma S^\sigma. \tag{26}$$

In this paper we restrict ourselves with consideration of the static and spherically symmetric case. Hence, the metric has the form

$$ds^2 = e^{\nu(r)}(c\,dt)^2 - e^{\lambda(r)}dr^2 - r^2(d\theta^2 + \sin^2(\theta)d\varphi^2) \tag{27}$$

in which the functions $\nu = \nu(r)$, $\lambda = \lambda(r)$ depend only on the Schwarzshild's radial coordinate r and the torsion-dilaton field Θ depends only on r, too.

In this case we obtain the following equations for the functions ν, λ, Θ, and p:

$$\begin{aligned} -e^{-\lambda}\left(\tfrac{1}{r} - \tfrac{\lambda'}{r}\right) + \tfrac{1}{r^2} &= \tfrac{\kappa}{c^2}\varepsilon - 3\Sigma^0_0, \\ -e^{-\lambda}\left(\tfrac{\nu'}{r} + \tfrac{1}{r^2}\right) + \tfrac{1}{r^2} &= -\tfrac{\kappa}{c^2}p - 3\Sigma^1_1, \\ -\tfrac{1}{2}e^{-\lambda}\left(\nu'' + \tfrac{1}{2}\nu'^2 + \tfrac{\nu'-\lambda'}{r} - \tfrac{\nu'\lambda'}{2}\right) &= -\tfrac{\kappa}{c^2}p - 3\Sigma^2_2, \\ e^{-\lambda}\left(S_r' + \left(\tfrac{\nu'-\lambda'}{2} + \tfrac{2}{r}\right)S_r - 3{S_r}^2\right) &= \tfrac{\kappa}{c^2}(\varepsilon - p), \\ p' &= -\tfrac{1}{2}(\varepsilon + p)\nu', \\ p &= p(\varepsilon). \end{aligned} \tag{28}$$

Here $p = p(\varepsilon)$ is the matter's equation of state and correspondingly:

$$\begin{aligned}
\Sigma^0_0 &= \left(S_r' - \tfrac{1}{2}\lambda' S_r + \tfrac{2}{r}S_r - S_r{}^2\right) e^{-\lambda},\\
\Sigma^1_1 &= \left(\tfrac{1}{2}\nu' S_r + \tfrac{2}{r}S_r - 2S_r{}^2\right) e^{-\lambda},\\
\Sigma^2_2 &= \left(S_r' + \tfrac{1}{2}\nu' S_r - \tfrac{1}{2}\lambda' + \tfrac{1}{r}S_r - S_r{}^2\right) e^{-\lambda}.
\end{aligned} \tag{29}$$

The second equation of the system (28) (i.e. $\overset{\{\}}{G}{}^1{}_1 + 3\Sigma^1_1 = \frac{\kappa}{c^2} T^1_1$) may be considered as a constraint creating a relation between S_r, ν' and e^λ, namely:

$$e^\lambda = \frac{1 + r\nu' - 6rS_r - \frac{3}{2}r^2\nu' S_r + 6r^2 S_r{}^2}{1 + \frac{\kappa}{c^2} p r^2}. \tag{30}$$

Using this relation we can put our system (28) in a normal form:

$$\begin{aligned}
\nu' &= 2\xi,\\
\Theta' &= S_r,
\end{aligned}$$

$$\begin{aligned}
\xi' &= -\tfrac{\xi}{r} + \left(\tfrac{2\kappa}{c^2}\varepsilon - \tfrac{\xi}{r} - \tfrac{\kappa}{c^2}(\varepsilon - p) r\xi\right) e^\lambda,\\
S_r' &= -\tfrac{S_r}{r} + \left(\tfrac{\kappa}{c^2}(\varepsilon - p) - \tfrac{S_r}{r} - \tfrac{\kappa}{c^2}(\varepsilon - p) r S_r\right) e^\lambda,\\
p' &= -(\varepsilon + p)\xi,\\
p &= p(\varepsilon),
\end{aligned}$$

$$e^\lambda = \frac{1 + 2r\xi - 6rS_r - 3r^2\xi S_r + 6r^2 S_r{}^2}{1 + \frac{\kappa}{c^2} p r^2}. \tag{31}$$

It's seen that the first two equations are separated, and the rest generate a subsystem independent of them.

The equations (31) must be solved with proper initial and boundary conditions. From a physical point of view the solutions regular at the center are the most interesting. The regularity means that there exists a local lorentzian system in neighborhood of the center, i.e. $e^{\lambda(0)} = 1$ and the pressure is finite at $r = 0$. Hence, we have $lim_{r\to 0}\, r\xi(r) = 0$, otherwise, as it may be seen from equation for p, the pressure will have at least logarithmic singularity at the center. On the other hand the condition $e^{\lambda(0)} = 1$ requires that $lim_{r\to 0}\, rS_r(r) = 0$, too. The expansion of the equations for ξ and S_r around the center is:

$$\begin{aligned}
\xi' &= -\frac{2\xi}{r},\\
S_r' &= -\frac{2S_r}{r}.
\end{aligned} \tag{32}$$

Hence, the behaviour of ξ and S_r around $r = 0$ is $\frac{constant}{r^2}$. To fulfill the above restrictions at $r \to 0$ we must put $constant = 0$. Hence, we obtain $\xi(0) = 0$, $S_r(0) = 0$. As a final result these considerations imply the following initial conditions:

$$\xi(0) = 0, \quad S_r(0) = 0, \quad p(0) = p_c, \quad (\varepsilon(0) = \varepsilon_c). \tag{33}$$

At the star surface $r = R$ we have to match interior solution with the exterior (vacuum) solution. We will consider the model of the star without surface tension, hence $p(R) = 0$. Then the matter distribution must be continuous at the surface of the star and one can show that ξ and S_r must be continuous at $r = R$. Obviously ν and Θ must be continuous at the star surface, too. Using matching conditions:

$$\begin{aligned}
\xi(R) &= \xi^{ext}(R),\\
S_r(R) &= S_r{}^{ext}(R),
\end{aligned} \tag{34}$$

we can obtain the vacuum solutions parameters K and a as functions of ε_c, i.e.

$$K = K(\varepsilon_c), \quad a = a(\varepsilon_c). \tag{35}$$

For arbitrary values $\nu_c = \nu(0)$ and $\Theta_c = \Theta(0)$, ν and Θ will not fulfill the matching conditions:

$$\begin{aligned}
\nu(R) &= \nu^{ext}(R),\\
\Theta(R) &= \Theta^{ext}(R).
\end{aligned} \tag{36}$$

Therefore, the separated equations $\nu' = 2\xi$ and $\Theta' = S_r$ must be solved under proper initial conditions in the following form:

$$\begin{aligned}
\nu_c &= \nu^{ext}(R) - \int_0^R 2\xi(r)\, dr,\\
\Theta_c &= \Theta^{ext}(R) - \int_0^R \Theta(r)\, dr.
\end{aligned} \tag{37}$$

As a result we obtain all parameters K, a, ν_c, Θ_c, R as functions only of the central density ε_c. Hence, the whole geometry of the space-time in vacuum and in the star is completely determined by the matter which carries *only* the same properties described by mass, matter density, pressure, equation of state and so on which are familiar from the general relativity. A very important feature of the model under consideration is that we are not forced to assign to the matter new properties, charges, or something else. Nevertheless we have a new geometric field (the torsion-dilaton field Θ) the physical problem is well defined by the usual physical properties of the matter.

Let's go back to the subsystem:

$$\xi' = -\,\frac{\xi}{r} + \left(\frac{2\kappa}{c^2}\varepsilon - \frac{\xi}{r} - \frac{\kappa}{c^2}(\varepsilon - p)r\xi\right)e^{\lambda},$$
$$S_r' = -\,\frac{S_r}{r} + \left(\frac{\kappa}{c^2}(\varepsilon - p) - \frac{S_r}{r} - \frac{\kappa}{c^2}(\varepsilon - p)rS_r\right)e^{\lambda},$$
$$p' = -(\varepsilon + p)\xi,$$
$$p = p(\varepsilon),$$

$$e^{\lambda} = \frac{1 + 2r\xi - 6rS_r - 3r^2\xi S_r + 6r^2 {S_r}^2}{1 + \frac{\kappa}{c^2}pr^2}. \qquad (38)$$

We can't define a local gravitational mass in the form $m_{GR}(r) = \frac{c^2}{2G}r(1 - e^{-\lambda})$, as in general relativity because in our case $m_{GR}(r)$ is in general not positively defined (see for example the vacuum solution). As we have noted in the previous section the full mass felt by a test particle is $\lim_{r\to\infty} r^2\xi(r)$ Hence, in the theory under consideration we define the local mass as $m(r) = \frac{c^2}{G}r^2\xi(r)$. Similarly, we can define a local scalar mass $m_\theta(r) = 3\frac{c^2}{G}r^2S_r(r)$. Because of the initial conditions we have $m(0) = 0$ and $m_\theta(0) = 0$. Now the system (38) can be rewritten in terms of the masses $m(r)$ and $m_\theta(r)$:

$$m' = \left(1 - \left(1 + \frac{\kappa}{c^2}(\varepsilon - p)r^2\right)e^{\lambda}\right)\frac{m}{r} + \frac{16\pi}{c^2}r^2\varepsilon e^{\lambda},$$
$$m_\theta' = \left(1 - \left(1 + \frac{\kappa}{c^2}(\varepsilon - p)r^2\right)e^{\lambda}\right)\frac{m_\theta}{r} + \frac{24\pi}{c^2}r^2(\varepsilon - p)e^{\lambda},$$
$$p' = -\,\frac{G}{c^2}(\varepsilon + p)\,\frac{m(r)}{r^2}$$
$$p = p(\varepsilon),$$

$$e^{\lambda} = \frac{1 + \frac{2G}{c^2 r}(m - m_\theta) - \frac{G^2}{c^4 r^2}m_\theta(m - \frac{2}{3}m_\theta)}{1 + \frac{\kappa}{c^2}pr^2}. \qquad (39)$$

This system is a generalization of Tolman-Oppenheimer-Volkoff's one for a star in general relativity [16]. Using the first and the second equation, it's not difficult to show that $m(r)$ and $m_\theta(r)$ are positively defined. Indeed, taking into account regularity at the center we obtain:

$$m = e^{A(r)}\int_0^r e^{-A(r)}\,\frac{16\pi}{c^2}\varepsilon r^2 dr,$$
$$m_\theta = e^{A(r)}\int_0^r e^{-A(r)}\,\frac{24\pi}{c^2}(\varepsilon - p)r^2 dr, \qquad (40)$$

where

$$A(r) = \int_0^r \frac{1 - \left(1 + \frac{\kappa}{c^2}(\varepsilon - p)r^2\right)e^{\lambda}}{r}dr.$$

In the same way from the above equations we have:

$$(m - m_\theta)' = \left(1 - \left(1 + \frac{\kappa}{c^2}(\varepsilon - p)r^2\right)e^{\lambda}\right)\frac{m - m_\theta}{r} - \frac{8\pi}{c^2}r^2(\varepsilon - 3p)e^{\lambda}. \qquad (41)$$

Solving this equation with an initial condition $(m - m_\theta)(0) = 0$ we obtain :

$$m - m_\theta = -e^{A(r)}\int_0^r e^{-A(r)}(\varepsilon - 3p)dr. \qquad (42)$$

Hence, it's seen that $m - m_\theta \leq 0$ inside and outside the matter if $\varepsilon - 3p \geq 0$. In other words we obtain for $k(r) = \frac{1}{3}\frac{m_\theta(r)}{m(r)}$ that $k \geq \frac{1}{3}$ and k takes a value $\frac{1}{3}$ when the matter is ultrarelativistic ($\varepsilon = 3p$). The parameter k takes its maximum value $\frac{1}{2}$ in the case of nonrelativistic matter ($\varepsilon \gg p$). If we assume following Zel'dovich [18], [19] that $\varepsilon < 3p$ may happen, then in general the vacuum value of k which is just $K = k(R)$ may change its sign passing through the zero at $\varepsilon = p$. For realistic equations of state we obtain $K \in [\frac{1}{3}, \frac{1}{2}]$.

For completeness we will give an expression which is a generalization of the well-known Tolman's formula [21]. From equations (25) we have

$$\overset{\{\}}{R}{}^0_0 + 3\overset{\{\}}{\nabla}_0 S^0 = \frac{\kappa}{c^2}\left(T^0_0 - \frac{1}{2}T\right) - \frac{3}{2}\Box\Theta. \qquad (43)$$

In the static and spherically symmetric case, one may show that the following relation is fulfilled:

$$\overset{\{\}}{R}{}^0_0 + 3\overset{\{\}}{\nabla}_0 S^0 = \frac{1}{e^{-3\Theta}\sqrt{|g|}}\partial_\alpha\left(\sqrt{|g|}e^{-3\Theta}g^{0\beta}\overset{\{\}}{\Gamma}{}_{0\beta}{}^{\alpha}\right), \qquad (44)$$

where $\overset{\{\}}{\Gamma}{}_{\alpha\beta}{}^{\gamma} = \left\{{\gamma \atop \alpha\beta}\right\}$ are Christoffel symbols. Hence, we obtain

$$\int_{\mathcal{M}}\left(\overset{\{\}}{R}{}^0_0 + 3\overset{\{\}}{\nabla}_0 S^0\right)e^{-3\Theta}\sqrt{|g|}d^3x = \oint_{\Sigma_\infty} g^{0\beta}\overset{\{\}}{\Gamma}{}_{0\beta}{}^{\alpha}e^{-3\Theta}\sqrt{|g|}d\Sigma_\alpha = 4\pi\,\frac{MG}{c^2}. \qquad (45)$$

Therefore for the total mass we can write down

$$M = \frac{c^2}{4\pi G}\int_{\mathcal{M}}\left(\overset{\{\}}{R}{}^0_0 + 3\overset{\{\}}{\nabla}_0 S^0\right)e^{-3\Theta}\sqrt{|g|}d^3x =$$
$$\frac{1}{c^2}\int_{\mathcal{M}}(2T^0_0 - T)e^{-3\Theta}\sqrt{|g|}d^3x - \frac{c^2}{4\pi G}\,\frac{3}{2}\int_{\mathcal{M}}\Box\Theta e^{-3\Theta}\sqrt{|g|}d^3x. \qquad (46)$$

Taking into account that

$$\int_{\mathcal{M}}\Box\Theta e^{-3\Theta}\sqrt{|g|}d^3x = -\,\frac{4\pi}{3}\,\frac{M_\theta c^2}{G}$$

and $2T^0_0 - T = \varepsilon + 3p$ we obtain:

$$M = \frac{1}{c^2}\int_{\mathcal{M}}(\varepsilon + 3p)e^{-3\Theta}\sqrt{|g|}d^3x + \frac{1}{2}M_\theta. \qquad (47)$$

On the other hand, taking into account that

$$M_\theta = 3KM = \frac{3}{c^2}\int_{\mathcal{M}}(\varepsilon - p)e^{-3\Theta}\sqrt{|g|}d^3x$$

one can rewrite the above formula in the form

$$
\begin{aligned}
M &= \frac{1}{1-\frac{3}{2}K}\,\frac{1}{c^2}\int_{\mathcal{M}}(\varepsilon+3p)e^{-3\Theta}\sqrt{|g|}d^3x \\
&= \frac{4}{c^2}\int_{\mathcal{M}}\varepsilon e^{-3\Theta}\sqrt{|g|}d^3x = \frac{16\pi}{c^2}\int_0^R \varepsilon e^{\frac{(\lambda+\nu)}{2}-3\Theta}r^2dr.
\end{aligned}
\tag{48}
$$

4.2 Neutron star model

First we consider non-interacting neutron gas at zero temperature [17], [22]. The energy density and the pressure in a proper normalization are given by:

$$
\varepsilon = \frac{m_N^4c^5}{3\pi^2h^3}g(\mu),
$$
$$
p = \frac{m_N^4c^5}{3\pi^2h^3}f(\mu),
$$

where

$$
g(\mu) = \frac{1}{8}\left(8\mu\sqrt{\mu+\mu^2}-\sqrt{\mu+\mu^2}(2\mu-3)-3ln(\sqrt{\mu}+\sqrt{1+\mu})\right),
$$
$$
f(\mu) = \frac{1}{8}\left(\sqrt{\mu+\mu^2}(2\mu-3)+3ln(\sqrt{\mu}+\sqrt{1+\mu})\right), \tag{49}
$$

$\mu = (\frac{q_{Fermi}}{m_N c})^2$, q_{Fermi} being the Fermi's momentum, m_N being the neutron mass.

We are interested in the difference between the predictions of the theory under consideration and of the general relativity. For this purpose the equation of state for a non-interacting neutron gas is sufficient.

As a more realistic equation of state we consider the analytical approximation (according to Zel'dovich and Novikov [19]) of Tsuruta-Cameron's equation of state [20]. In this case the interaction between the nucleons is taken into account in a simple approximation and the pressure is given by:

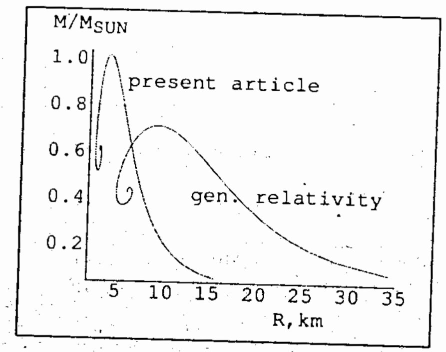
$$
p = \varepsilon + \frac{\rho_0c^2}{2} - \frac{\rho_0c^2}{2}\left(1+\frac{4\varepsilon}{\rho_0c^2}\right)^{1/2} \tag{50}
$$

where $\rho_0 = 5*10^{15}g/cm^3$.

It turns out that these two examples present typical results which qualitatively agree with the results for the other equations of state of star's matter.

4.3 Numerical results and discussions

We have solved the system of equations (31) coupled with the state equations (49) and (50) numerically using the method due to Runge-Kutta-Merson with automatic error control. The results are shown in the corresponding figures.



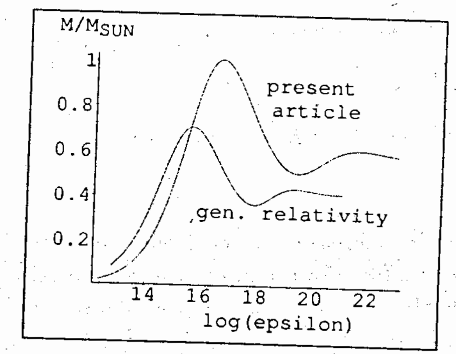


Figure 1: a) $M-R$ dependence. b) $M-log(\varepsilon_c)$ dependence.

First we concentrate our attention on the case of non-interacting neutron gas. In Fig. 1a) the M - R dependence is represented. It's seen that the M - R curve in our case is fairly similar to the one of general relativity, but there are significant differences, too. The maximum mass M_{max} in our case is $\approx 1M_{\odot}$, while in general relativity the Oppenheimer-Volkoff's mass is $M_{OV} = 0.7M_{\odot}$. The radius corresponding to the mass $M_{\odot}$ is $R = 4.2km$, while in the case of general relativity $R = 9.6km$. If we look at Fig. 1b) where the dependence of M on the central density ε_c is shown, we note that M_{max} lies at $\varepsilon_c \approx 4.5*10^{16}g/cm^3$, while M_{OV} lies at $\varepsilon_c \approx 5*10^{15}g/cm^3$ in general relativity. The average density in our case is about 10 times greater than the one in general relativity. Hence, in the model under consideration the neutron star is more compact and has a mass about 1.5 - times greater than M_{OV}.

At the Fig. 2a) the dependence $k(r)$ is shown inside the star (for central density $7.5*10^{15}g/cm^3$). In accordance to the general considerations k increases from the center of the star to the surface, where k takes a value $K = k(R) \approx 0.45 - 0.46$, which is close to 0.5. The dependence $K(\varepsilon_c)$ of K on the star central density ε_c is shown in Fig. 2b). It's seen that K decreases when

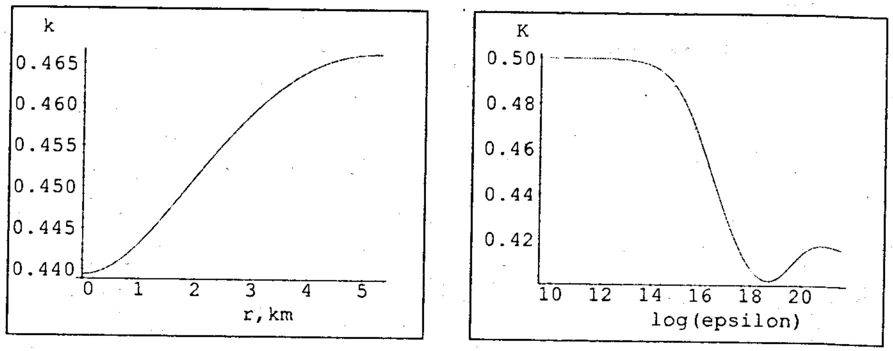


Figure 2: a) $K - r$ dependence. b) $K - ln(\varepsilon_c)$ dependence.

density increases, which is similar to the previous case. So, the ratio of the torsion force to the gravitational one takes its minimum value at the center of the star and is the greatest at the surface.

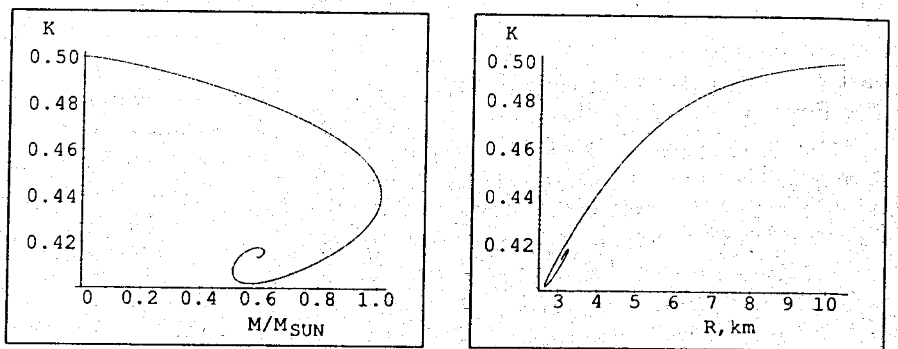


Figure 3: a) $K - M$ dependence. b) $K - R$ dependence.

As it may be seen from Fig. 3a) expressing the dependence $K(M)$, the torsion-urged effects are relatively strongest in the case of small masses – with increasing of the star mass (up to the point where the star loses its stability) K decreases. It's seen from Fig. 3b), where the dependence of K on the star radius R is shown, that in the area of stability K decreases when R decreases too – the more compact stars are, the smaller K they have.

Fig. 4a) presents the dependencies $\theta(r)$ and $\nu(r)$ inside the star. One may see that $\frac{1}{2}\nu - 3\Theta < 0$ everywhere. The dependencies $m(r)$ and $m_\theta(r)$ are shown in Fig. 4b) for central density $7.5 * 10^{15}$.

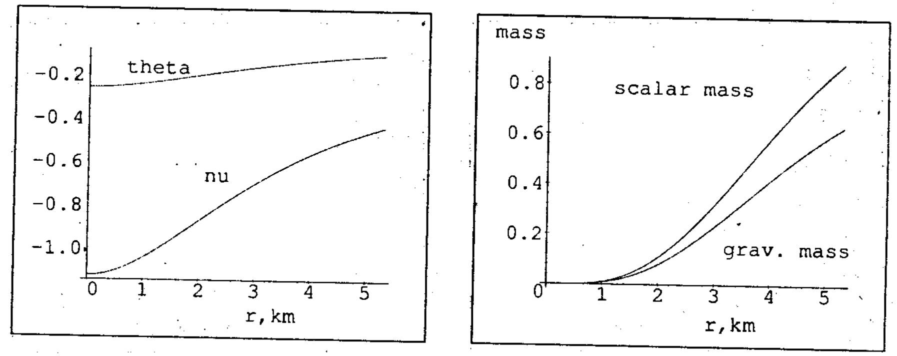


Figure 4: a) $\Theta, \nu - r$ dependence. b) $m, m_\theta - r$ dependence.

The following figures illustrate the case of Tsuruta-Cameron equation of state (TCES).

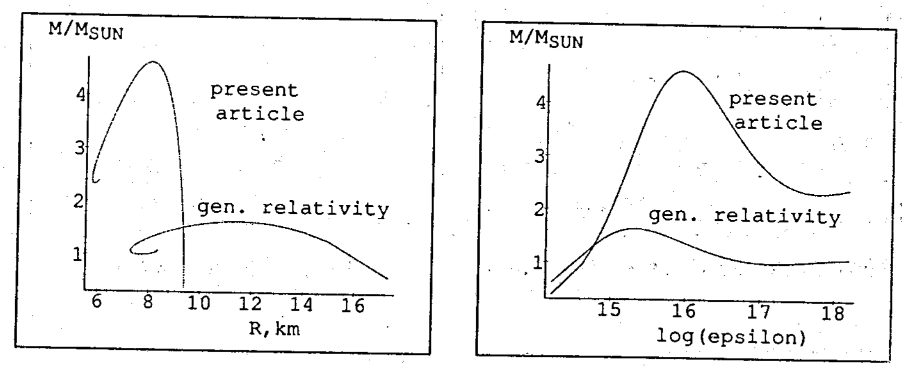


Figure 5: TCES. a) $M - R$ dependence . b) $M - ln(\varepsilon_c)$ dependence.

We see from Fig. 5a) that the maximum mass in this case is about $4.5M_\odot$ and the corresponding radius is about $7.5km$ - the same quantities in general relativity are correspondingly $\approx 1.6M_\odot$ and $\approx 11.5km$. Hence, the interaction between the nucleons leads to an increase in the maximum mass, as in general relativity.

Note the differences between the Fig. 3b) and Fig. 4b) (for the case of non-interacting neutron gas), and the corresponding Fig. 7b) and Fig. 8b) (for the case of Tsuruta-Cameron equation of state). There one can see the strong dependence of some results in the Saa's model of gravity with propagating torsion on the equation of state of star's matter.

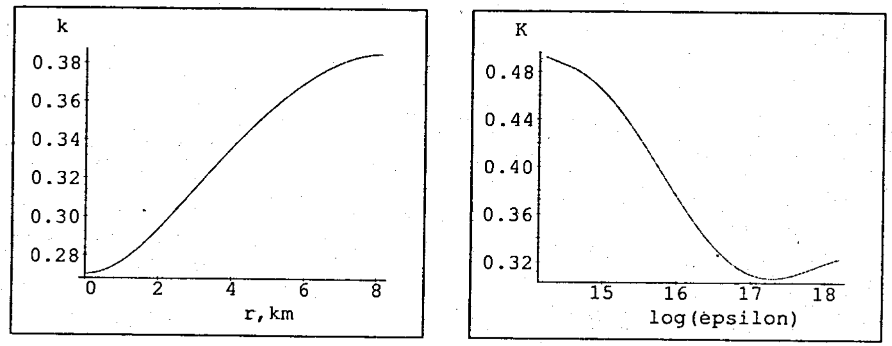


Figure 6: TCES. a) $k - r$ dependence. b) $M - log(\varepsilon_c)$ dependence.

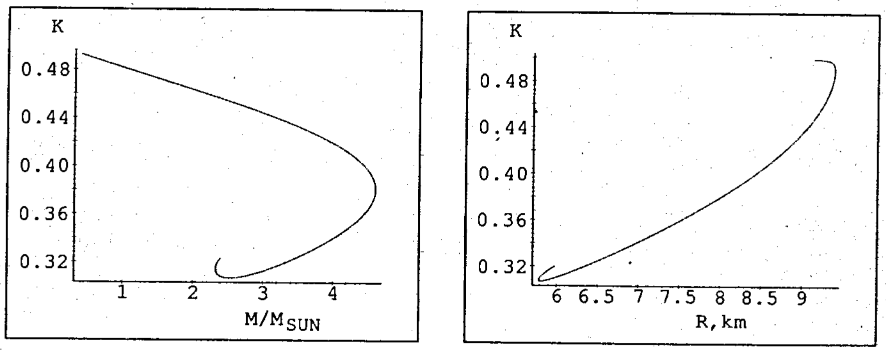


Figure 7: TCES. a) $K - M$ dependence. b) $K - R$ dependence.

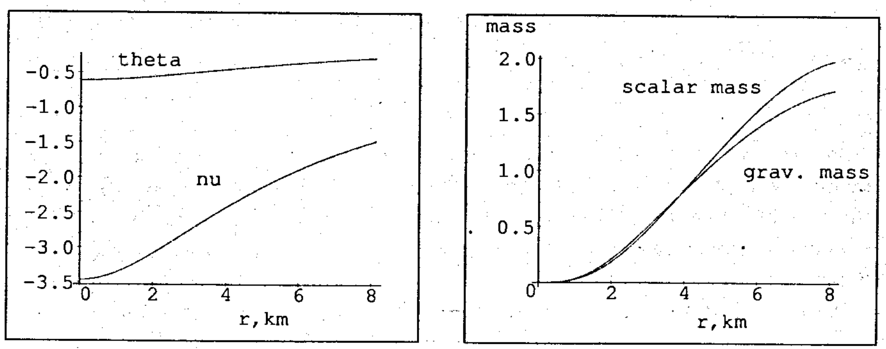


Figure 8: TCES. a) $\Theta, \nu - r$ dependence. b) $m, m_\theta - r$ dependence.

We have also examined the Harrison-Wheeler's equation of state [16]. As in general relativity the numerical results are very close to these for the non-interacting neutron gas. For example the maximum mass is $\approx 1M_\odot$ and the corresponding radius is $3.8km$.

Other equations of state (of politropic type) have been examined, too. The corresponding maximum mass of a neutron star reaches a value about $6 - 6.5M_\odot$.

4.4 Summary

In this article we have examined the basic spherically symmetric stationary state of stars in the Saa's model of gravity with propagating torsion.

In the model under investigation there is no need to consider unknown charges creating the torsion-dilaton field. Its source is the very spinless matter. The whole geometry of the space-time (including metric and torsion) is determined by the familiar properties of this matter.

The parameters of the vacuum solution are determined only by the spinless matter without adopting an existence of new properties, too. In contrast to the corresponding models in the general relativity here we have two parameters K and a of the vacuum solutions. The values of these parameters depend on the mass distribution in the star which is related with the equation of state of the star's matter. For a fixed equation of state both parameters become functions only of the total mass, but these functions are not the same for the different equations of state. The first parameter K being the ratio of the magnitude of torsion-dilaton force and of the magnitude of gravitational force for realistic equations of matter state takes values in the interval $[\frac{1}{3}, \frac{1}{2}]$, depending on the total mass. The second one – a is analogous to the gravitational radius in general relativity and takes positive values depending on the value of the parameter K and on the value of the total mass of the star.

To be specific in the present article we restrict our attention to the model of neutron stars. Numerical results and analytical considerations show that the space-time torsion may have a significant role in their structure. The new torsion force decreases the role of the gravity in the star configuration and may lead to an increasing of the maximum neutron star mass up to $5 - 6M_\odot$. If real, these properties of the model may change essentially the interpretation of the observations in astrophysics.

The consistence of the present model of stars, as far as of the whole Saa's model of gravity with propagating torsion with the reality is still an open problem. The results of the present article may have not only independent value, but are necessary for reaching the solution of this critical problem. The corresponding results will be presented elsewhere.

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Бояджиев Т., Физиев П., Язаджиев С. E2-98-218
Нейтронная звезда в присутствии поля кручения-дилятации

Развита общая теория звезд в модели Саа гравитации с распространяющимся кручением и изучено основное стационарное состояние нейтронной звезды. Наши численные результаты показывают, что сила кручения уменьшает роль гравитации в строении звезд и увеличивает максимально возможную массу нейтронной звезды вплоть до $5-6M_{\odot}$, в зависимости от уравнения состояния звезды.

Работа выполнена в Лаборатории теоретической физики им. Н.Н.Боголюбова ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна, 1998

Boyadjiev T., Fiziev P., Yazadjiev S. E2-98-218
Neutron Star in Presence of Torsion-Dilaton Field

We develop the general theory of stars in Saa's model of gravity with propagating torsion and study the basic stationary state of neutron star. Our numerical results show that the torsion force decreases the role of the gravity in the star configuration and increases the maximum of the neutron star mass up to $5-6M_{\odot}$ depending on the equation of state of matter.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna, 1998