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TOWARDS $N=2$ SUSY HOMOGENEOUS QUANTUM COSMOLOGY; EINSTEIN—YANG—MILLS SYSTEMS

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I. INTRODUCTION

In order to quantize a pure bosonic system one can apply supersymmetry as a mighty tool for dealing with the problems of a quantum theory [1] - [5]. The quantization can be done in two ways. The first one is to embed the system in a four dimensional supersymmetric field theory and then reduce it to one dimension [2] - [3], [6], or alternatively, consider the desired Lagrangian as a bosonic part of a supersymmetric sigma-model after the dimensional reduction [7] - [8]. These two approaches are not equivalent in general and the results can be different. The second method i.e., the method of supersymmetric Quantum Mechanics seems more convenient for our purposes and we shall follow it hereafter.

In the spatially homogeneous cosmological models the only dynamical variable is time t, other (spatial) coordinates can be integrated out from the action. Therefore, one can simply consider the corresponding mechanical system and then try to make a supersymmetric sigma-model extension. The case of pure gravity and gravity with scalar fields was investigated recently by Graham and Bene in the framework of $N = 2$ $SUSY$ Quantum Mechanics. However, the construction of quantum Hamiltonian, proposed there, is occurred to be Hermitean not self-adjoint for the case of indefinite signature of the metric in the minisuperspace. In this paper we use another construction of the corresponding Hamiltonian, which in accordance with general lines of quantization, is Hermitean self-adjoint for any type of signature of the metric in minisuperspace. The obtained quantum states coincide with those found in [7] - [8] only in null fermion and filled fermion sectors, while in other fermion sectors they exist only if the manifold, determined by the minisuperspace metric has corresponding nontrivial cohomologies.

We apply developed $N = 2$ $SUSY$ sigma-model technique for the quantization of $SU(2)$ Einstein-Yang-Mills system in homogeneous axially-symmetric $Bianchi - I, II, VIII, IX$, $Kantowski - Sachs$ (KS) and closed $Friedman - Robertson - Walker$ (FRW) cosmological models. Since the work by Bartnik and McKinnon [9] where an infinite set of regular particle-like $SU(2)$ non-Abelian EYM configurations was obtained, the further interest to EYM system is caused by an unexpected properties of their classical solutions. In particular, it has been shown, that non-Abelian EYM black holes violate naive "no-hair" conjecture in an external region [10], as well as they demonstrate rather unusual internal structure [11] - [12] with the generic space-time singularity being an infinitely oscillating, but not of a Mixmaster type. The metric in space-time region under an event horizon of spherically symmetric black hole is equivalent to homogeneous cosmological $Kantowski - Sachs$ metric and this correspondence allows to apply the methods, developed in the quantum cosmology for study of black hole singularities. Classical EYM solutions in different $(Bianchi)$ cosmologies are still not investigated so far, except axially-symmetric $Bianchi - I$ model, where the chaotic behavior of the metric, inspired by chaos in YM equations of motion has been observed [13] - [14]. In all classical EYM systems mentioned above, the nonlinear nature of source YM field produces nontrivial space-time configurations mainly in strong field regions i.e., near black hole or cosmological space-time singularities, where pure classical description of space-time should be replaced by a quantum field theory and our present work is one step towards this goal.

We show that all considered EYM models, containing initially purely bosonic (gravitational and YM) degrees of freedom, admit $N = 2$ supersymmetrization in the framework of $N = 2$ $SUSY$ sigma-model. The inclusion of non-Abelian gauge fields to pure gravitational systems produces additional parts in superpotentials, which as we shall see below, are equal to Yang-Mills Chern-Simons terms. The connection between the superpotential and the "winding number" in some supersymmetric Yang-Mills field theories and sigma-models was discussed earlier [2] - [3]. However, direct generalization to the EYM supersymmetric sigma-models is not straightforward, since the expression for the space-time metric, which in turn determines the form of the ansatz for Yang-Mills field, can be arbitrary. Therefore the fact, that $N = 2$ supersymmetric sigma-model based on the axially-symmetric homogeneous EYM systems respects this result is quite nontrivial.

The paper is organized as follows: in Section **II** we discuss formal aspects of $N = 2$ $SUSY$ sigma-models, starting with the superfield approach; in Section **III** the desired embedding of EYM systems into $N = 2$ $SUSY$ sigma-model is described and explicit expressions for superpotentials are given; the quantization and $SUSY$ breaking by YM instantons are discussed in Section **IV**.

II. $N = 2$ $SUSY$ QUANTUM MECHANICS

Let us first recall some main features of $N = 2$ supersymmetric Quantum Mechanics, developed mainly in [1] - [5]. We shall follow the superfield approach, since it is more geometrical, rather than the component one, and obtained component form of the corresponding Lagrangian is obviously invariant under the desired $SUSY$ transformations. Consider the superspace, spanned by the coordinates $(t, \theta, \bar{\theta})$, where t is time, while θ and it's conjugate $\bar{\theta}$ are nilpotent Grassman variables. The $N = 2$ supersymmetry transformations in the superspace with the complex odd parameter ϵ have the following form

$$\begin{aligned} \delta t &= i\epsilon\bar{\theta} + i\bar{\epsilon}\theta \\ \delta\theta &= \epsilon \quad \delta\bar{\theta} = \bar{\epsilon}, \end{aligned} \tag{1}$$

which are generated by the linear differential operators:

$$\Omega = \frac{\partial}{\partial\bar{\theta}} + i\theta\frac{\partial}{\partial t}, \quad \text{and} \quad \bar{\Omega} = \frac{\partial}{\partial\theta} + i\bar{\theta}\frac{\partial}{\partial t}. \tag{2}$$

Now one can introduce the main object of the theory – the real vector superfield Φ^i

$$\Phi^i = q^i + \bar{\theta}\xi^i - \theta\bar{\xi}^i + \bar{\theta}\theta F^i, \tag{3}$$

where q^i stands for all bosonic degrees of freedom of the system, ξ^i and $\bar{\xi}^i$ are their fermionic superpartners and F^i is an auxiliary bosonic field. Since the superfield Φ^i transforms under the supersymmetry transformations as

$$\delta\Phi^i = (\bar{\epsilon}\Omega + \epsilon\bar{\Omega})\Phi^i, \tag{4}$$

the most general supersymmetric Lagrangian can be obtained in terms of supercovariant derivatives

$$D = \frac{\partial}{\partial\theta} - i\bar{\theta}\frac{\partial}{\partial t}, \quad \text{and} \quad \bar{D} = \frac{\partial}{\partial\bar{\theta}} - i\theta\frac{\partial}{\partial t}, \tag{5}$$

which anticommute with Ω and $\bar{\Omega}$; the resulting Lagrangian

$$L = \int d\theta d\bar{\theta}(-\frac{1}{2}g_{ij}(D\Phi^i)(\bar{D}\Phi^i) + W), \tag{6}$$

is invariant under supersymmetry transformations (4) by the construction and it corresponds to the one-dimensional $N = 2$ supersymmetric sigma-model, characterized by the metric g_{ij} $(i, j = 1, ..., n)$ of the "target" manifold $M(g_{ij})$ and the superpotential W, both being a functions of the superfield Φ^i.

Note, that the Lagrangian (6) is *self – adjoint* for any signature of the metric g_{ij}. This fact is especially important for considering of homogeneous systems coupled with gravity, since in these cases the manifold M described by the metric g_{ij} is not Riemannian.

After integration over the Grassman variables and elimination of an auxiliary field F^i, one gets a more familiar component form of the Lagrangian

$$\begin{aligned} L = &\frac{1}{2}g_{ij}(q)\dot{q}^i\dot{q}^j + ig_{ij}(q)\bar{\xi}^i(\dot{\xi}^j + \Gamma^j_{kl}\dot{q}^k\xi^l) \\ &+\frac{1}{2}R_{ijkl}\bar{\xi}^i\xi^j\bar{\xi}^k\xi^l - \frac{1}{2}g^{ij}(q)\partial_i W\partial_j W - \partial_i\partial_j W\bar{\xi}^i\xi^j, \end{aligned} \tag{7}$$

where R_{ijkl} and Γ^i_{jk} are the Riemann curvature and Christoffel connection, corresponding to the metric g_{ij}. The supersymmetry transformations can be also written in the component form

$$\begin{aligned} \delta q^i &= \bar{\epsilon}\xi^i - \epsilon\bar{\xi}^i, \\ \delta\xi^i &= \epsilon(-i\dot{q}^i + \Gamma^i_{jk}\bar{\xi}^j\xi^k - \partial^i W), \\ \delta\bar{\xi}^i &= \bar{\epsilon}(i\dot{q}^i + \Gamma^i_{jk}\bar{\xi}^j\xi^k - \partial^i W), \end{aligned} \tag{8}$$

which allow to find the conserved supercharges using the standard Noether theorem technique:

$$\begin{aligned} Q &= \xi^i(g_{ij}\dot{q}^j + i\partial_i W) \\ \bar{Q} &= \bar{\xi}^i(g_{ij}\dot{q}^j - i\partial_i W) \end{aligned} \tag{9}$$

Following the general lines of quantization of the system with bosonic and fermionic degrees of freedom [15], introduce the canonical Poisson brackets

$$\{q^i, P_{q^j}\} = \delta^i_j, \quad \{\xi^i, P_{\xi^j}\} = -\delta^i_j, \quad \{\bar{\xi}^i, P_{\bar{\xi}^j}\} = -\delta^i_j, \tag{10}$$

where P_{q^j}, P_{ξ^j}, and $P_{\bar{\xi}^j}$ are momenta, conjugate to q^i, ξ^i and $\bar{\xi}^i$. After finding their explicit form

$$P_{q^i} = g_{ij}\dot{q}^i + i\Gamma_{j;ik}\bar{\xi}^j\xi^k, \tag{11}$$

$$P_{\xi^i} = -ig_{ij}\bar{\xi}^j, \quad P_{\bar{\xi}^i} = 0. \tag{12}$$

one can conclude from (12), that the system possesses the second-class fermionic constraints

$$\chi_{\xi^i} = P_{\xi^i} + ig_{ij}\bar{\xi}^j, \quad \text{and} \quad \chi_{\bar{\xi}^i} = P_{\bar{\xi}^i}, \tag{13}$$

since

$$\{\chi_{\xi^i}, \chi_{\bar{\xi}^j}\} = -ig_{ij}. \tag{14}$$

Therefore, the quantization has to be done using the Dirac brackets, defined for any two functions V_a and V_b as

$$\{V_a, V_b\}_D = \{V_a, V_b\} - \{V_a, \chi_c\}\frac{1}{\{\chi_c, \chi_d\}}\{\chi_d, V_b\}. \tag{15}$$

Using (15), one can easily find non-vanishing Dirac brackets between bosonic and fermionic degrees of freedom

$$\{q^i, P_{q^j}\}_D = \delta^i_j, \quad \{\xi^i, \bar{\xi}^j\}_D = -ig^{ij}. \tag{16}$$

Then, after replacing Dirac brackets with graded commutator

$$\{,\}_D \to i[,]_\pm, \tag{17}$$

one obtains the following (anti)commutation relations:

$$[q^i, P_{q^j}]_- = i\delta^i_j, \quad [\xi^i, \bar{\xi}^j]_+ = g^{ij}. \tag{18}$$

To make a quantum expressions for supercharges (9) it is convenient to introduce the projected fermionic operators $\bar{\xi}^a = e^a_\mu \bar{\xi}^\mu$ and $\xi^a = e^a_\mu \xi^\mu$ where e^i_a is inverse to the tetrad e^a_i ($e^a_i e^i_b = \delta^a_b$), related to the metric g_{ij} of the "target" manifold M and to the metric of it's tangent space η_{ab} in the usual way $e^a_i e^b_j \eta_{ab} = g_{ij}$.

However, the explicit form of the supercharges depends on the choice of operator ordering and therefore is ambiguous. We take it as in [3]

$$\begin{aligned} Q &= \xi^a e^i_a(P_i + i\omega_{iab}\bar{\xi}^a\xi^b + i\partial_i W) \\ \bar{Q} &= \bar{\xi}^a e^i_a(P_i + i\omega_{iab}\bar{\xi}^a\xi^b - i\partial_i W), \end{aligned} \tag{19}$$

where ω_{iab} is the corresponding spin connection.

In what follows, we shall consider the systems subject to the classical Hamiltonian constraint

$$H_0 = \frac{1}{2}g^{ij}P_iP_j + \frac{1}{2}g^{ij}(q)\partial_i W\partial_j W = 0, \tag{20}$$

which in the quantum case should be replaced by the condition on the quantum state $|\rho\rangle$

$$H|\rho\rangle = 0, \tag{21}$$

with the Hamiltonian

$$H = \frac{1}{2}[Q, \bar{Q}]_+, \tag{22}$$

giving H_0 in the classical limit i.e., when all fermionic fields are set equal to zero.

The important point is that the operators (9) are nilpotent and mutually Hermitean adjoint with respect to the measure $\sqrt{|-g|}d^n q$ and therefore, the energy operator H is *self − adjoint* for any signature of the metric g_{ij}. Now the Lagrangian (7) is self-adjoint by the construction, since we use the real superfields and hence the complex Noether charges and their Quantum Mechanical expressions are Hermitean adjoint to each other.

Obviously, now one can consider two first order differential equations on the wave function

$$\bar{Q}|\rho\rangle = 0, \quad and \quad Q|\rho\rangle = 0, \tag{23}$$

and therefore linearize the operator equation (21); the existence of normalizable solutions of the system (23) means in turn, that supersymmetry is unbroken Quantum Mechanically.

In order to solve the system consider the Fock space spanned by the fermionic creation and annihilation operators $\bar{\xi}^a$ and ξ^a respectively with $[\xi^a, \bar{\xi}^b]_+ = \eta^{ab}$. The general state in this Fock space is obtained in terms of the series expansion

$$\begin{aligned} |\rho\rangle &= F(q)|0\rangle + ... + \frac{1}{n!}\bar{\xi}^{a_1}...\bar{\xi}^{a_n}F_{a_1...a_n}(q)|0\rangle \\ &= F(q)|0\rangle + ... + \frac{1}{n!}\bar{\xi}^{i_1}...\bar{\xi}^{i_n}F_{i_1...i_n}(q)|0\rangle, \end{aligned} \tag{24}$$

where the coefficients in expansions of this series are p - forms defined on the manifold $M(g_{ij})$, and their number due to the nilpotency of fermionic creation operators is finite. Since the fermion number operator $N = \bar{\xi}^a\xi_a$ commutes with the Hamiltonian H and

$$[N, Q]_- = -Q, \quad [N, \bar{Q}]_- = \bar{Q}, \tag{25}$$

one can consider the states characterized by the different fermion numbers separately. Now the solution in empty and filled fermion sectors is simply expressed in terms of the superpotential W as follows:

$$|\rho_0\rangle = \text{const} * e^{-W}|0\rangle, \tag{26}$$

$$|\rho_n\rangle = \text{const} * \frac{1}{n!}\bar{\xi}^{a_1}...\bar{\xi}^{a_n}\epsilon_{a_1...a_n}e^{+W}|0\rangle. \tag{27}$$

In order to investigate the solutions in other fermion sectors, let us first recall [2], that in the case of vanishing superpotential operators $\bar{Q}_0$ and Q_0 (supercharges with $W = 0$) act on the p - forms F as an exterior and co-exterior derivatives respectively. So, the solution of equation $\bar{Q}_0|\rho\rangle = 0$ cannot be written as

$$|\rho_p\rangle = \bar{Q}_0|\sigma_{p-1}\rangle, \tag{28}$$

only if the corresponding p-th cohomology group $H^p(M)$ of the manifold $M(g_{ij})$ is nontrivial. Before generalizing this result to the systems with non-zero superpotential W, first note that

$$\bar{Q} = e^{-W}\bar{Q}_0 e^{W}, \quad \text{and} \quad Q = e^{W} Q_0 e^{-W}. \tag{29}$$

Now, using (28) and (29) one can prove that the general solution in p - fermion sectors $(p = 1, ..., n-1)$ of the first equation in (23) for the case of trivial cohomology group $H^p(M)$ is

$$|\rho_p\rangle = \bar{Q}|\sigma_{p-1}\rangle. \tag{30}$$

However, because Q and $\bar{Q}$ are Hermitean adjoint to each other, the second equation in (23) indicates, that this state has zero norm and consequently is unphysical. Therefore the possible existence of supersymmetric ground-states i.e. the solutions of zero-energy Shrödinger-type equation (21) is directly related with the topology of the considered manifold $M(g_{ij})$, since all states except those in purely bosonic and filled fermion sectors can be excluded even without solving the system (23), if the topology of the manifold $M(g_{ij})$ is trivial.

For purely bosonic systems with nonvanishing potential energy the described $N = 2$ supersymmetrization turns out to be the simplest possible one and it can be applied for canonical quantization of any appropriate homogeneous cosmological model coupled with matter. After the choice of operator ordering in the supercharges equation (21) in null fermion sector corresponds to the Wheeler-De Witt equation for the considered Einstein-matter system and it's solution (26) is then easily obtained in terms of superpotential W, since $SUSY$ allows to linearize the Quantum Hamiltonian equation.

III. $N = 2$ SUPERSYMMETRIZATION OF $SU(2)$ EINSTEIN-YANG-MILLS COSMOLOGICAL MODELS

Now we are in a position to make the $N = 2$ supersymmetric extension of homogeneous axially-symmetric $SU(2)$ Einstein-Yang-Mills systems given by the action

$$S = \int d^4x\sqrt{|-G|}(R - \frac{1}{2}F^A_{\mu\nu}F^{A\mu\nu}). \tag{31}$$

We restrict ourselves to a subclass of homogeneous space-times which admit representation in a form of an unconstrained Hamiltonian system for a corresponding classical coupled system of equations, i.e., we consider axially-symmetric $Bianchi - I, II,$ $VIII, IX$ (axially-symmetric $Bianchi - VII$ is equivalent to $Bianchi - I$), $Kantowski - Sachs$ and closed $Friedmann - Robertson - Walker$ cosmological models.

The general diagonal *Bianchi* -type axially-symmetric space-times are parametrized by two independent functions of a cosmological time $b_1(t)$ and $b_3(t)$

$$ds^2 = -dt^2 + b_1^2(t)[(\omega^1)^2 + (\omega^2)^2] + b_3^2(t)(\omega^3)^2, \tag{32}$$

where ω^i are basis left-invariant one-forms ($d\omega^i = \frac{1}{2}C^i_{jk}\omega^j \wedge \omega^k$) for the spatially homogeneous three metrics, depending on 3 spatial (not necessarily Cartezian) coordinates x, y, z:

$Bianchi - I$

$$\omega^1 = dx, \quad \omega^2 = dy, \quad \omega^3 = dz. \tag{33}$$

$Bianchi - II$

$$\omega^1 = dz, \quad \omega^2 = dx, \quad \omega^3 = dy - xdz. \tag{34}$$

$Bianchi - VIII$

$$\begin{aligned} \omega^1 &= dx + (1 + x^2)dy + (x - y - x^2y)dz, \\ \omega^2 &= dx + (-1 + x^2)dy + (x + y - x^2y)dz, \\ \omega^3 &= 2xdy + (1 - 2xy)dz, \end{aligned} \tag{35}$$

$Bianchi - IX$

$$\begin{aligned} \omega^1 &= \sin z dx - \cos z \sin x dy, \\ \omega^2 &= \cos z dx + \sin z \sin x dy; \\ \omega^3 &= \cos x dy + dz. \end{aligned} \tag{36}$$

As it was shown by Darian and Kunzle [13], the general ansatz for $SU(2)$ Yang-Mills field, compatible with the symmetries of axially-symmetric *Bianchi* -type cosmological models is also expressed in terms of two independent real valued functions $\alpha(t)$ and $\gamma(t)$ of a cosmological time only and has the form

$$A = \alpha(t)(\omega^1\tau_1 + \omega^2\tau_2) + \gamma(t)\omega^3\tau_3, \tag{37}$$

where τ_i are $SU(2)$ group generators, normalized as $[\tau_i, \tau_j] = \epsilon_{ijk}\tau_k$.

$Kantowski - Sachs$ space-time

$$ds^2 = -dt^2 + b_3^2(t)dr^2 + b_1^2(t)d\theta^2 + b_1^2(t)(\sin\theta)^2 d\phi^2, \tag{38}$$

does not belong to *Bianchi* classification and admits an additional spherical symmetry, so $SU(2)$ YM ansatz has the different form, originated from Witten ansatz for static spherically symmetric case after mutual replacement $r \to t$, $t \to r$:

$$A_0 = 0, \; A_r = \gamma(t)L_1, \; A_\theta = -L_3 + \alpha(t)L_2, \; A_\phi = \sin\theta(L_2 + \alpha(t)L_3), \tag{39}$$

where $L_1 = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$, $L_2 = (\cos\theta\cos\phi, \cos\theta\sin\phi, -\sin\theta)$, $L_3 = (-\sin\phi, \cos\phi, 0)$ are spherical projections of $SU(2)$ generators.

We also consider closed $Friedmann - Robertson - Walker$ model separately, because it's general YM ansatz [16] ($SU(2)$ YM field on S^3) is not obtained from $Bianchi - IX$ after setting $\alpha(t) = \gamma(t)$ in (37). Closed FRW model with the interval

$$ds^2 = -dt^2 + b^2(t)(d\chi^2 + \sin^2\chi(d\theta^2 + \sin^2\theta d\phi^2)), \tag{40}$$

(χ, θ and ϕ are angles on S^3) admits the following representation for $SU(2)$ YM field, expressed in terms of single real-valued function $\alpha(t)$:

$$A_0 = 0 \quad A_j = \frac{1}{2}(\alpha(t) + 1)U\partial_j U^{-1}, \tag{41}$$

$$U = \exp(i\chi(\sin\theta(\sigma_1\cos\phi + \sigma_2\sin\phi) + \sigma_3\cos\theta); \qquad j = 1, 2, 3; \tag{42}$$

Inserting these ansatzes into the action and integrating over all variables except t one obtains the one-dimensional Lagrangian

$$L_0 = \frac{1}{2} g_{ij}(q)\dot{q}^i \dot{q}^j - V(q) = K - V, \tag{43}$$

here $g_{ij}(q)$ is the metric in the extended minisuperspace i.e., in the configuration space of spatially homogeneous axially-symmetric three-metrics coupled with the corresponding $SU(2)$ Yang-Mills fields.

Let us consider the functions $q^i = (b_1, b_3, \alpha, \gamma)$ as a bosonic components of the superfield (3). One can introduce the same number of fermionic fields ($\bar{\xi}^i$ and ξ^i) and therefore make $N = 2$ supersymmetrization of the Lagrangian L_0 if, and only if, the potential $V(q)$ admits the expression via a function $W(q)$, called superpotential:

$$V(q) = \frac{1}{2} g^{ij}(q) \frac{\partial W(q)}{\partial q^i} \frac{\partial W(q)}{\partial q^j}. \tag{44}$$

In this case $N = 2$ $SUSY$ Lagrangian (7) and the corresponding Hamiltonian, obtained after usual Legendre transformation are *self – adjoint* for any signature of the metric g_{ij} in the extended minisuperspace.

The kinetic terms for all *Bianchi* models and *Kantowksi – Sachs* are the same:

$$K = -\dot{b}_1^2 b_3 - 2\dot{b}_1 \dot{b}_3 b_1 + \dot{\alpha}^2 b_3 + \dot{\gamma}^2 \frac{b_1^2}{2b_3}, \tag{45}$$

and the only difference between them is due to the potential terms. Using the expression for the metric on the extended "minisuperspace"

$$g_{b_1 b_1} = -2b_3, \quad g_{b_1 b_3} = -2b_1, \quad g_{\alpha\alpha} = 2b_3, \quad g_{\gamma\gamma} = \frac{b_1^2}{b_3}, \tag{46}$$

and the explicit form of the potentials, we have found some superpotentials as a solutions of (44), hence making $N = 2$ $SUSY$ extension of the given Einstein-Yang-Mills systems. The results are collected in the **Table I.**

One should note that the obtained superpotentials W in all these cases turn out to be the direct sums of pure gravitational W_{gr} (first listed in [8] in terms of Misner variables) and Yang-Mills parts W_{YM}. This fact is quite interesting and does not follow *a priori* from the general expectations, since in the sigma-model approach considered above, gravitational and Yang-Mills variables in the Lagrangian L_0 are not separated. Moreover, it seems that the YM field is a unique one, which being coupled with gravity can allow the corresponding superpotential to be in the form of direct sum. It follows from the statement, that the superpotential is also the least Euclidean action - solution of the Euclidean Hamilton-Jacobi equation of the considered system. One can reconstruct from the superpotential the corresponding Euclidean solutions, those which give the main contribution to the wave function in a quasiclassical approach. So, the gravitational part of the superpotential W_{gr} determines the Euclidean gravitational background configurations which should not be changed if a matter field is added. It is possible, only if a matter configurations do

TABLE I.

	Lagrangian L_0	*Superpotential* $W = W_{gr} + \mathbf{W_{YM}}$
BI	$K - [\frac{1}{b_3}\alpha^2\gamma^2 + \frac{b_3}{2b_1^2}\alpha^4];$	$0 + \alpha^2\gamma;$
BII	$K - [\frac{1}{4}\frac{b_3^3}{b_1^2} + \frac{1}{b_3}\alpha^2\gamma^2 + \frac{1}{2}\frac{b_3}{b_1^2}(\alpha^2 + \gamma)^2];$	$\frac{1}{2}b_3^2 + (\alpha^2\gamma + \frac{1}{2}\gamma^2);$
BVIII	$K - [\frac{1}{4}\frac{b_3^3}{b_1^2} + b_3 + \frac{1}{b_3}\alpha^2\gamma^2 + \frac{1}{2}\frac{b_3}{b_1^2}(\alpha^2 - \gamma)^2];$	$\frac{1}{2}(2b_1^2 - b_3^2) + (\alpha^2\gamma - \frac{1}{2}\gamma^2);$
BIX	$K - [\frac{1}{4}\frac{b_3^3}{b_1^2} - b_3 + \frac{1}{b_3}\alpha^2(\gamma - 1)^2 + \frac{1}{2}\frac{b_3}{b_1^2}(\alpha^2 - \gamma)^2];$	$\frac{1}{2}(2b_1^2 + b_3^2) + (\alpha^2(\gamma - 1) - \frac{1}{2}\gamma^2);$ *or* $\frac{1}{2}(b_3^2 - 4b_1 b_3) + (\alpha^2(\gamma - 1) - \frac{1}{2}\gamma^2);$
KS	$K - [\frac{1}{b_3}\alpha^2\gamma^2 - b_3 + \frac{1}{2}\frac{b_3}{b_1^2}(\alpha^2 - 1)^2];$	$2b_1 b_3 + \gamma(\alpha^2 - 1);$
FRW	$-\frac{3}{2}b\dot{b}^2 + \frac{1}{2}b\dot{\alpha}^2 + \frac{3}{2}b - \frac{1}{2}\frac{(1-\alpha^2)^2}{b};$	$\frac{3}{2}b^2 + (\frac{1}{3}\alpha^3 - \alpha);$

not contribute to the energy-momentum tensor. Yang-Mills part of superpotential W_{YM} just provides such possibility since it produces self-dual YM instantons with the energy-momentum tensor identically vanished. We discuss this point in more details in next Section.

Note that full superpotential $W = W_{gr} + W_{YM}$ does not exist as a solution of (44) if we cancel one of the relevant YM function α or γ; there are no nontrivial self-dual solutions of YM equations of motion with one of YM functions canceled and W_{YM} ceases to exist in this case. The question about other solutions of equation (44) which are not a direct sums of gravitational and YM parts is still opened, however, it seems unlikely that such solutions can be obtained in a closed analytical form.

On the other hand, the one more crucial observation can be done, that for all considered models Yang-Mills part of the superpotential coincides with the corresponding Chern-Simons functional, calculated on a 3 -dimensional slice $t = const$. Indeed, it can be checked the YM Chern-Simons terms

$$W_{YM} = \frac{1}{2}\int d^3x \sqrt{|-G|}\epsilon^{0\lambda\mu\nu}(A_\lambda^a \partial_\mu A_\nu^a + \frac{1}{3} f^{abc} A_\lambda^a A_\mu^b A_\nu^c), \tag{47}$$

turn out to be a solutions of Euclidean Hamilton-Jacobi equation and therefore play a role of the Yang-Mills part of the superpotential. Such coincidence of YM Chern-Simons terms (47) with YM superpotentials (44) in framework of the one-dimensional sigma-model describing YM field coupled with gravity, seems to be very surprising. Definitely, this statement is not true in a general case of an arbitrary space-time and takes place for the suggested models as a consequence of the symmetries of the space-time metrics and corresponding YM ansatzes. Note, that there exist no similar expressions for the W_{gr} part of the superpotential in terms of a functional of gravitational variables except $Bianchi-IX$ model with a nonzero cosmological constant, where Chern-Simons functional in terms of Ashtekar's variables [17] is also an exact solution of Ashtekar-Hamilton-Jacobi equation [18].

So, we have shown, that the considered homogeneous axially-symmetric EYM systems admit $N = 2$ supersymmetric sigma-model extension with the superpotentials given explicitly in the **Table I** and this gives us the suitable background for the quantization.

IV. THE QUANTIZATION AND *SUSY* BREAKING BY YM INSTANTONS

A.

As it can be seen from the supersymmetry transformations (8), in order to prevent $N = 2$ $SUSY$ breaking at the classical level, the classical pure bosonic configurations must satisfy the properties

$$\dot{q}^i(t) = 0, \quad \text{and} \quad \partial^i W(q_i(t)) = 0, \tag{48}$$

along with the classical Hamiltonian constraint (20). Such classical configurations really exist in an usual field theory in a flat space-time, and the simplest well-known example is a scalar rest particle ($\dot{q}^i = 0$) on a bottom of a potential with $V(q^i) = 0$.

In contrary with such examples, dealing with unconstrained homogeneous systems with gravity included, any nontrivial classical solution of Einstein (or Einstein coupled with a matter) equations never has all momenta vanished $\dot{q}^i(t) \neq 0$. These systems satisfy (20) due to the dynamical balance between the kinetic and potential terms with both positive and negative signs.

Hence, any homogeneous Einstein (or Einstein-matter) system, being embedded into $N = 2$ supersymmetric sigma-model never has solutions of equations of motion with unbroken supersymmetry, i.e., supersymmetry is always spontaneously broken at the "tree level".

Let us see what happens in the Quantum Mechanical approach. In the Einstein-Yang-Mills systems considered above the number of bosonic functions q^i is four, which is also the fermion number of the filled fermion sector. Therefore we shall consider the solutions of zero energy Schrödinger-type equation (21) in these empty and filled fermion sectors.

The superpotential $W(q)$ is always defined up to the sign, since it is the "square root" of the bosonic potential $V(q)$. Both signs are physically acceptable and correspond to the solutions in empty (26) and filled (27) fermion sectors when finding the supersymmetric wave functions. The normalizability of bosonic wave function for "positive" superpotential means in turn the normalizability of filled fermionic wave function for the "negative" superpotential and vice versa. We define the norm of the physical state as $\pm \int \sqrt{|-g|}\langle\rho||\rho\rangle d^4q$ in order

to avoid the problem of the negative norm in four fermion sector, caused by the time-like component of the fermionic field. The plus sign in the definition of the norm corresponds to $+W(q)$ while the minus sign has to be taken for $-W(q)$.

Let us accept for definiteness the positive sign of the superpotential. First consider the pure gravitational systems, when α and γ functions along with their fermionic partners are set equal to zero. As it was stated above, the supersymmetry is spontaneously broken for any nontrivial solutions of Einstein equations. Quantum Mechanically the supersymmetry is restored for $Bianchi\ - I, II, IX_{(1)}$, $Kantowski - Sachs$ and FRW models since the solution of (21), $|\rho_0^{gr}\rangle = \text{const} * e^{-W_{gr}}|0\rangle$ in null fermion sector are normalizable:

$$\int_0^{+\infty} db_1 \int_0^{+\infty} db_3 \sqrt{|-g|}e^{-2W_{gr}} < \infty. \tag{49}$$

Therefore we are facing the interesting situation, where unlike to the ordinary supersymmetric Quantum Mechanics, the supersymmetry being spontaneously broken at the "tree level" is then restored Quantum Mechanically.

The only exceptions are the second (in the **Table I**) superpotential for $Bianchi - IX_{(2)}$ and $Bianchi - VIII$ where the supersymmetry remains broken at the quantum level as well, since their norm (49) diverges at the upper limit.

The further inclusion of Yang-Mills field spontaneously breaks the supersymmetry again, because, as one can see from the **Table I**, Yang-Mills part of the superpotential W_{YM} for all considered models, being the corresponding Chern-Simons term, is the odd function of α and γ; consequently, YM parts of wave functions $|\rho_0^{ym}\rangle = \text{const} * e^{\pm W_{YM}}|0\rangle$ both in null and filled fermion sectors are not normalizable:

$$\int_{-\infty}^{+\infty} d\alpha \int_{-\infty}^{+\infty} d\gamma \sqrt{|-g|}e^{\pm 2W_{YM}} \to \infty. \tag{50}$$

In order to find possible supersymmetric wave functions in one, two and three fermion sectors, one has to investigate the topology of the extended minisuperspace. The simplest way of doing that is going to the Misner parametrization [19] of the space-time metric (32):

$$ds^2 = -N^2(t)dt^2 + \frac{1}{6}e^{2A(t)+2B(t)}[(\omega^1)^2 + (\omega^2)^2] + \frac{1}{6}e^{2A(t)-4B(t)}(\omega^3)^2. \tag{51}$$

In terms of Misner variables the metric in the extended minisuperspace (46) has the simple diagonal form

$$g_{AA} = -1, \quad g_{BB} = 1, \quad g_{\alpha\alpha} = 2e^{-2A-2B}, \quad g_{\gamma\gamma} = e^{-2A+4B}, \tag{52}$$

which shows that the topology of the extended minisuperspace is equivalent to the Minkowski one with all cohomologies trivial $H^p(M(g_{ij})) = 0$, $p = 1, 2, 3$ and, in accordance with the discussion of Section **II**, no physical states in one, two and three fermion sectors exist since they have zero norm. Similarly, there are no physical states except the ones in null and filled fermion sectors in a considered pure gravitational systems.

B.

Let us discuss in more details the mechanism of the spontaneous supersymmetry breaking in null fermion sector when YM field is added to a pure gravitational system (such as $Bianchi - I, II, IX_{(1)}$, KS and FRW) which is Quantum Mechanically supersymmetric, since it admits normalizable zero energy solution of Wheeler-De Witt equation (21). This mechanism is occurred to be quite similar to the one considered in [1], [20] - [22] where the $SUSY$ breaking by instanton configurations has been discussed.

Indeed, as it was already mentioned, the superpotential $W(q)$ (if exists), is one of the solutions of Euclidean Hamilton-Jacobi equation and represents a "least" Euclidean action of field configurations, giving the main quasiclassical contribution into the wave function and providing the $SUSY$ breaking after inclusion of the Yang-Mills field. Explicit form of superpotential allows to reconstruct such classical configurations by solving the first order system:

$$g_{ij}\dot{q}^j = -\frac{\partial(W_{gr} + W_{YM})}{\partial q^i}. \qquad (53)$$

For pure gravitational degrees of freedom these equations are equivalent to the (anti)self-duality gravitational equations $R_{\mu\nu\lambda\sigma} = \pm\tilde{R}_{\mu\nu\lambda\sigma}$ while $W_{YM}(q)$ part of the superpotential in (53) gives rise to the (anti)self-dual Yang-Mills equations $F^a_{\mu\nu} = \pm\tilde{F}^a_{\mu\nu}$ on a given gravitational background determined by the W_{gr}.

Then, (anti)self-dual Yang-Mills instantons in our systems can be interpreted as a tunneling solutions (with the nonvanishing Euclidean action) between topologically distinct vacua. In this case YM instanton contribution provides the $SUSY$ breakdown due to the energy shift from the initial zero to some positive level and this fact is expressed in nonnormalizability of YM part of zero energy wave function $|\rho_0^{YM}\rangle = \text{const} * e^{-W_{YM}}|0\rangle$.

As the illustration of these statements, let us consider Euclidean configurations in $Bianchi - IX$ and $Kantowski - Sachs$ EYM systems.

BIX. The solutions of Hamilton-Jacobi equations (53), which correspond to the gravitational part of both possible superpotentials $W_{gr(BIX_{(1)})} = \frac{1}{2}(2b_1^2 + b_3^2)$ and $W_{gr(BIX_{(2)})} = \frac{1}{2}(b_3^2 - 4b_1b_3)$ have been discussed by Gibbons and Pope [23]. For our purposes we would like to mention some of them using the slightly different notations.

One of the solutions of equations (53) with the normalizable superpotential $W_{gr(BIX_{(1)})}$ is occurred to be the (anti)self-dual Eguchi-Hanson [24] metric which has the form

$$ds^2 = f^2dr^2 + \frac{r^2}{4}((\omega^1)^2 + (\omega^2)^2) + \frac{r^2}{4}f^{-2}(\omega^3)^2, \qquad (54)$$

with

$$f^2 = \left(1 - \left(\frac{a}{r}\right)^4\right)^{-1}, \qquad (55)$$

and ω^i is determined by (36). In order to bring this metric to the form (32), one should introduce the "Euclidean time" r as

$$dt = \left(1 - \left(\frac{a}{r}\right)^4\right)^{-\frac{1}{2}} dr. \qquad (56)$$

Eguchi-Hanson metric has vanishing Euclidean action $S^{gr}_{EH} = 0$, which is completely determined by it's surface contribution [25], since the volume contribution is canceled identically ($R = 0$ "on shell") for EYM systems.

Inserting the expression for the metric functions into the Hamilton-Jacobi equations for Yang-Mills part of superpotential $W_{YM(BIX)} = -\alpha^2(\gamma - 1) + \frac{1}{2}\gamma^2$ and differentiating with respect to the introduced variable r one obtains the system

$$\dot{\alpha} = \frac{2}{r}f^2(\alpha\gamma - \alpha), \qquad (57)$$

$$\dot{\gamma} = \frac{2}{r}(\alpha^2 - \gamma), \qquad (58)$$

which is self-duality YM equations on Eguchi-Hanson background solved by the family of instanton solutions [26]

$$\alpha = \frac{a_1 \sinh(\rho)}{\sinh(a_1(\rho + a_2))}, \quad \gamma = a_1 \tanh(\rho)\coth(a_1(\rho + a_2)), \quad \frac{r^2}{a^2} = \coth(\rho), \qquad (59)$$

with the action $S^{YM}_{EH} = 8\pi^2\frac{a_1^2 - 1}{2}$ for $a_1 > 1$, $a_2 = 0$, and $S^{YM}_{EH} = 8\pi^2\frac{a_1^2}{2}$ for $a_1 > 1, 0 < a_2 < \infty$, where a_1 and a_2 are the constants of integration.

The extremal Euclidean configurations, produced by the nonnormalizable superpotential $W_{gr(BIX_{(2)})}$ are self-dual Taub-NUT gravitational instantons with the nonvanishing action [27]; similarly, YM part of the superpotential gives rise to the self-dual YM instantons [28] on a Taub-NUT background.

KS. For EYM system in Kantowski-Sachs space-time with $W_{gr(KS)} = 2b_1b_3$ the gravitational degrees of freedom b_1 and b_3 obey to the following self-duality equations

$$\dot{b}_1b_3 + \dot{b}_3b_1 = b_3, \qquad (60)$$

$$\dot{b}_1 = 1, \qquad (61)$$

satisfied by

$$b_1 = t, \quad \text{and} \quad b_3 = 1, \qquad (62)$$

which is nothing more than flat Euclidean R^4 space-time metric with r and t interchanged. From the Yang-Mills part of (53) with $W_{YM(KS)} = -\gamma(\alpha^2 - 1)$ one obtains the usual YM (anti)self-duality equations in R^4, written in the "polar" coordinates

$$\dot{\alpha} = \alpha\gamma, \qquad (63)$$

$$\dot{\gamma}t^2 = \alpha^2 - 1, \qquad (64)$$

with well-known family of YM instanton solutions, having the topological charge $k = 1$ [29]

$$\gamma = \dot{\psi}, \quad and \quad \alpha = e^{\psi}\dot{g}, \qquad (65)$$

where

$$\psi = -\ln(\frac{1-g^2}{2t}), \quad g = (\frac{a_1 - t}{a_1 + t})(\frac{a_2 - t}{a_2 + t}). \tag{66}$$

Note, that the dimension of the moduli space $\mathcal{M}$ of $SU(2)$ Yang-Mills instantons with a topological charge k on a given Riemannian $4-D$ manifold $\overline{\mathbf{M}}$ (which has first Betti number c_1 and the dimension c_2^- of maximal submanifold in cohomologies $H^2(\overline{\mathbf{M}}, R)$ where the corresponding intersection form is negatively defined) is [30]

$$\dim(\mathcal{M}_{SU(2)}) = 8k - 3(1 - c_1 + c_2^-), \tag{67}$$

and in a simplest $Kantowski-Sachs$ case with $\overline{\mathbf{M}} = R^4$ ($k=1$, $c_1 = c_2^- = 0$) is equal to five. In the framework of our approach, since we quantize the system reduced to one-dimension, only some of these instantons are taken into account. In fact, we deal with the subclass of all possible YM instantons, originated from the chosen ansatzes, which share the space-time symmetries in Lorentsian sector. However, their contribution breaks the supersymmetry fatally in conformity with the general expectations, as it should take place in a full $4-D$ quantum theory.

To summarize, it is shown that the spontaneous supersymmetry breaking which takes place if the Yang-Mills field is added to pure gravity is caused in a quasiclassical approach by YM instanton contribution to the wave function. This contribution, in accordance with general expectations, provides the energy shift ΔE from a zero level. To estimate this energy shift for EYM systems an instanton calculation technique can be used, which also should give the possibility to find the lowest level normalizable wave function $|\rho^1\rangle$, $H|\rho_1^{EYM}\rangle = \Delta E|\rho_1^{EYM}\rangle$ for the considered models. This work is in a progress now.

V. CONCLUSIONS

We would like to conclude with the following remarks. $N=2$ $SUSY$ Quantum Mechanical sigma-model approach allows to obtain the conserved supercharges being Hermitean adjoint to each other, along with the self-adjoint expressions for Hamiltonian and Lagrangian for any signature of a sigma-model metric. This gives the possibility to use the supersymmetry as a tool for quantization of various homogeneous systems coupled with gravity if they can be embedded into the considered $N=2$ $SUSY$ sigma-model. The desired embedding has been done for coupled $SU(2)$ EYM systems in some cosmological models which admit explicit expressions for the superpotentials being direct sum of gravitational and Yang-Mills parts. After the quantization the only nontrivial zero energy wave functions in null and filled fermion sectors turn out to have diverging norm and this fact indicates spontaneous breaking of supersymmetry, caused by YM instantons.

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Донец Е.Е., Тентюков М.Н., Цулая М.М. E2-98-214
Об $N=2$ SUSY однородной квантовой космологии;
системы Эйнштейна—Янга—Миллса

Обсуждается применение $N=2$ суперсимметричной квантовой механики для квантования пространственно однородных систем, связанных с гравитацией. Основываясь на суперполевом формализме в $N=2$ суперсимметричной сигма-модели, получены эрмитово сопряженные выражения для квантового гамильтониана и лагранжиана при произвольной сигнатуре метрики в мини-суперпространстве. Развитый формализм далее применен к связанным $SU(2)$ системам Эйнштейна—Янга—Миллса (ЭЯМ) в аксиально симметричных Бианки-I,II,VIII,IX космологических моделях, а также в моделях Кантовского—Сакса и Фридмана—Робертсона—Уокера. Показано, что все эти модели допускают суперсимметризацию в рамках $N=2$ сигма-модели и получены соответствующим выражением для суперпотенциалов, которые оказываются прямой суммой гравитационной и янг-миллсовых частей. Янг-миллсова часть суперпотенциала при этом в точности равна члену Черна—Саймонса. Спонтанное нарушение суперсимметрии, вызванное янг-миллсовыми инстантонами в ЭЯМ системах, обсуждаются на нескольких физически содержательных примерах.

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Donets E.E., Tentyukov M.N., Tsulaia M.M. E2-98-214
Towards $N=2$ SUSY Homogeneous Quantum Cosmology;
Einstein—Yang—Mills Systems

The application of $N=2$ supersymmetric Quantum Mechanics for the quantization of homogeneous systems coupled with gravity is discussed. Starting with the superfield formulation of $N=2$ SUSY sigma-model. Hermitean self-adjoint expressions for quantum Hamiltonians and Lagrangians for any signature of a sigma-model metric are obtained. This approach is then applied to coupled $SU(2)$ Einstein—Yang—Mills (EYM) systems in axially-symmetric Bianchi-I,II,VIII,IX, Kantowski—Sachs and closed Friedmann—Robertson—Walker cosmological models. It is shown, that all these models admit the embedding into $N=2$ SUSY sigma-model with the explicit expressions for superpotentials, being direct sums of gravitational and Yang—Mills (YM) parts. In addition, YM parts of superpotentials exactly coincide with the corresponding Chern—Simons terms. The spontaneous SUSY breaking, caused by YM instantons in EYM systems is discussed in a number of examples.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

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