ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

Дубна

E2-98-195

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HELICITY AMPLITUDES FOR THE PROCESS OF DOUBLE BREMSSTRAHLUNG IN FORWARD DIRECTION IN SMALL-ANGLE $e^-e^\pm$-SCATTERING

Submitted to «Nuclear Physics B»

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1998

1 Introduction

The problem of a complete description of jet–like processes becomes important because colliders and detectors are now able to work with polarized particles. Besides, jets formed by 2 and 3 particles and described in QED reactions of the type $e\bar{e} \to (e\gamma\gamma)\bar{e}$, $e\bar{e} \to (ee\bar{e})\bar{e}$ provide a realistic model for hadronic jets and that is, in particular, a motivation of the present paper. Processes of the type $2 \to 4$ for peripheral kinematics, such as

$$e\bar{e} \to (e\gamma)(\bar{e}\gamma), e\bar{e} \to (e\bar{a}a)\bar{e}, \gamma\gamma \to a\bar{a}, b\bar{b}$$

were in details considered in a series of papers [1, 2, 3]. Here we study the double bremsstrahlung (DB) in one direction. In the unpolarized case this process was investigated in papers [4, 5] where the expressions for differential and total cross sections in leading logarithmical approximation were obtained. In this work helicity amplitudes of the DB are calculated. Electrons are supposed to be ultrarelativistic (having energy $E \gg m$) and the terms, which give small (of order m^2/E^2 compared to those of order unity) contributions to the total cross section are systematically omitted. The process

$$e^-(p_1) + e^\pm(p_2) \to e^-(p_1') + \gamma(k_1) + \gamma(k_2) + e^\pm(p_2')$$

of two real photons emission in $e^-e^\pm$ collisions is described in Born approximation by as much as 32 Feynman diagrams (FD). 16 scattering–type FD are relevant in the kinematics of forward scattering at high energies. Indeed the main (non–decreasing with energy) contributions to the total cross section arise from FD which have the spin 1 (photon) state as an intermediate of the crossing channel [1]. Since we are concerned to the case when both photons are emitted along electron (p_1) direction, only 6 FD out of 32 correspond to our problem.

2 Determination of the amplitudes

Extracting the factor $e^4 = (4\pi\alpha)^2$ we can represent the matrix element in the form (for definiteness the Möller scattering process is considered, see Fig.1):

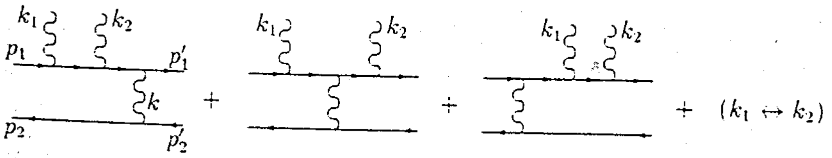

Figure 1: Feynman diagrams for double bremsstrahlung in $e\bar{e}, ee$ scattering

$$M^{ee\to(e\gamma\gamma)e} = \frac{g_{\rho\rho_1}}{k^2}\bar{u}_2'\gamma_\rho u_2 \cdot \bar{u}_1' O_{\rho_1\mu_1\mu_2} u_1 e^{\mu_1}(k_1) e^{\mu_2}(k_2), \quad (1)$$
$$k = p_2' - p_2 = p_1 - p_1' - k_1 - k_2.$$

In calculations the well–known Gribov's decomposition of the metric tensor entering the virtual photon Green function is used

$$g_{\rho\rho_1} = g^\perp_{\rho\rho_1} + \frac{2}{s}(\tilde{p}_\rho p'_{\rho_1} + \tilde{p}_{\rho_1} p'_\rho) \approx \frac{2}{s}\tilde{p}_\rho p'_{\rho_1} \quad (2)$$
$$\tilde{p} = p_1 - \frac{m^2}{s}p_2, \qquad p' = p_2 - \frac{m^2}{s}p_1.$$

Here $g^\perp_{\rho\rho_1}$ means the metric tensor with non–zero components in transverse to the beam axes direction, $\tilde{p}$ and p' are almost light–like vectors with components $\tilde{p} = \varepsilon(1,1,0,0)$, $p' = \varepsilon(1,-1,0,0)$ (we work in the center-of-mass (cms) reference frame where $\boldsymbol{p}_1 + \boldsymbol{p}_2 = 0$).

Contributions to the cross section arising from the dropped terms in the above formula are suppressed by the factor of $m^2/s \ll 1$. And subsequently we will systematically omit all the quantities of such order.

Sudakov's decomposition of the 4–vectors is very convenient in analysis. Let us write down these expressions for photons momenta:

$$k_{1,2} = \alpha_{1,2}p' + x_{1,2}\tilde{p} + k_{1,2\perp} \qquad k = \alpha p' + \beta\tilde{p} + k_\perp, \quad (3)$$
$$p'k_\perp = \tilde{p}k_\perp = 0, \qquad \tilde{p}^2 = p'^2 \approx 0, \qquad k^2 = s\alpha\beta - \bar{k}^2, \qquad s = 2p_1p_2 \gg m^2.$$

The on–shell conditions for incoming and outgoing particles give:

$$s\alpha_{1,2} = \frac{\bar{k}^2_{1,2}}{x_{1,2}}, \qquad s(\alpha + \alpha_1 + \alpha_2) = -\frac{1}{\Delta}[m^2(1-\Delta) + (\bar{k} + \bar{k}_1 + \bar{k}_2)^2], \quad (4)$$

where $\bar{k}, \bar{k}_1, \bar{k}_2$ are the two–dimensional euclidean vectors perpendicular to the beam (p_1) axes. The quantities $x_{1,2}$ and $\Delta = 1 - x_1 - x_2$ can be interpreted as the energy fractions of the emitted photons and the scattered electron

$$x_i = \frac{k_{i0}}{\varepsilon}, \qquad \Delta = \frac{p'_{10}}{\varepsilon}$$

and supposed to be $x_i \sim \Delta \sim 1$. After excluding "nonphysical" components α_i, α the kinematical invariants take the following form:

$$-(p_1 - k_1)^2 + m^2 = a_1 = \frac{1}{x_1}(m^2x_1^2 + \bar{k}_1^2),$$
$$-(p_1 - k_2)^2 + m^2 = a_2 = \frac{1}{x_2}(m^2x_2^2 + \bar{k}_2^2),$$

$$(p_1' + k_2)^2 - m^2 = a_2' = \frac{1}{x_2\Delta}(m^2 x_2^2 + \vec{r}_2^2),$$
$$(p_1' + k_1)^2 - m^2 = a_1' = \frac{1}{x_1\Delta}(m^2 x_1^2 + \vec{r}_1^2), \tag{5}$$
$$-(p_1' + k)^2 + m^2 = d = \frac{1}{x_1 x_2}\Big[x_1 x_2(1-\Delta)m^2 + x_2(1-x_2)\bar{k}_1^2 + x_1(1-x_1)\bar{k}_2^2 + 2x_1 x_2 \bar{k}_1 \bar{k}_2\Big]$$
$$(p_1 - k)^2 - m^2 = d' = \frac{1}{x_1 x_2 \Delta}\Big[x_1 x_2(1-\Delta)m^2 + x_2(1-x_2)(k_1 + k x_1)^2 + x_1(1-x_1)(\bar{k}_2 + \bar{k}x_2)^2 + 2x_1 x_2(\bar{k}_1 + \bar{k}x_1)(\bar{k}_2 + \bar{k}x_2)\Big]$$
$$\vec{r}_1 = x_1(\bar{k}_2 + \bar{k}) + (1-x_2)\bar{k}_1, \qquad \vec{r}_2 = x_2(\bar{k}_1 + \bar{k}) + (1-x_1)\bar{k}_2.$$

In kinematics of a jet moving close to $\boldsymbol{p}_1$ direction euclidean 2–vectors $\bar{k}_i, \bar{k}$ have typical magnitude of a few electron masses.

Using the on-shell condition for the spectator (positron in Bhabha and electron in Möller scattering)

$$p_2'^2 = (p_2 + k)^2 = m^2, \qquad s\alpha\beta - \bar{k}^2 + s\beta + m^2\alpha = 0$$

we put the momentum transfer squared (the 4–momentum of virtual photon k) in the form:

$$k^2 = s\alpha\beta - \bar{k}^2 = -\frac{1}{1+\alpha}\left(\bar{k}^2 + m^2\alpha^2\right), \qquad s\alpha = \frac{d}{\Delta}. \tag{6}$$

From the above it is clearly seen the physical meaning of the "nonphysical" Sudakov's parameter α. Namely, it relates to the invariant mass squared of a jet generated by the scattered electron and two accompanied photons:

$$(p_1 - k)^2 = (p_1' + k_1 + k_2)^2 = m^2 - \bar{k}^2 - s\alpha = s_1. \tag{7}$$

Let us introduce helicity states of fermions and photons in jet kinematics [4]:

$$u^\sigma(p) = \begin{pmatrix} \sqrt{\varepsilon_+} \\ \sigma\sqrt{\varepsilon_-} \end{pmatrix} w^\sigma, \quad w^{+1} = \begin{pmatrix} e^{-\mathrm{i}\phi/2} \\ \frac{\theta}{2}e^{\mathrm{i}\phi/2} \end{pmatrix}, \quad w^{-1} = \begin{pmatrix} -\frac{\theta}{2}e^{-\mathrm{i}\phi/2} \\ e^{\mathrm{i}\phi/2} \end{pmatrix}, \tag{8}$$
$$\sigma = 2\lambda = \pm 1, \quad \theta \ll 1, \quad \varepsilon_\pm = \varepsilon \pm m, \quad \sum_\lambda u^\lambda(p)\bar{u}^\lambda(p) = \hat{p} + m,$$

where σ denotes helicity states of fermions.

Gauge freedom in choosing photon polarization vectors permits to pick up them in such a way that they won't have the Sudakov's projection on $\tilde{p}$:

$$e^\eta_\mu(k) = \alpha^\eta_e p' + e^\eta_\perp, \qquad e^\eta_\perp = \frac{1}{\sqrt{2}}(e_x + \mathrm{i}\eta e_y) \equiv \bar{e}^\eta. \tag{9}$$

Using Lorentz condition $e(k)k = 0$ we find:

$$s\alpha_e(k_i) = \frac{2\bar{e}^\eta \bar{k}_i}{x_i} = \frac{2}{\sqrt{2}x_i}(\delta_{\eta,1}\chi_i + \delta_{\eta,-1}\chi_i^*), \qquad \chi_i = k_{ix} + \mathrm{i}k_{iy}. \tag{10}$$

We further exploit the freedom in choosing the general phase factor by putting $\phi_1 = 0$. Then (2, 8, 9) give for the matrix element

$$M^{\eta_1\eta_2}_{\sigma_1\sigma_1',\sigma_2\sigma_2'} = \frac{2}{sk^2}\bar{u}_2'^{\sigma_2'}\tilde{p}u_2^{\sigma_2} \cdot \bar{u}_1'^{\sigma_1'} O_{\eta\mu_1\mu_2} u_1^{\sigma_1} \cdot p_\eta' \cdot e^{\eta_1}_{\mu_1}(k_1) e^{\eta_2}_{\mu_2}(k_2)$$
$$= \frac{2s}{k^2}\delta_{\sigma_2\sigma_2'}\frac{1}{s}\mathcal{M}^{\eta_2\eta_1}_{\sigma_1\sigma_1'}. \tag{11}$$

Let us concentrate on $\mathcal{M}^{\eta_2\eta_1}_{\sigma_1\sigma_1'}$ and choose $\sigma_1 = +1$ bearing in mind that the case $\sigma_1 = -1$ can be obtained by applying the parity conservation relation $\mathcal{M}^{\eta_2\eta_1}_{\sigma_1\sigma_1'} = \mathcal{M}^{-\eta_2,-\eta_1}_{-\sigma_1,-\sigma_1'}$. One will see that only 7 non–zero helicity amplitudes exist. In the above formula the quantity $p_\eta' O_{\eta\mu_1\mu_2} e^{\eta_1}_{\mu_1} e^{\eta_2}_{\mu_2}$ can be represented by FD, depicted in Fig.1 and the corresponding amplitudes read:

$$\mathcal{M} = (1 + \mathcal{P}_{12})Q, \qquad Q = \mathcal{M}_1 + \mathcal{M}_2 + \mathcal{M}_3 \tag{12}$$
$$\mathcal{M}_1 = \frac{1}{a_1 d}\bar{u}_1'\hat{p}'(p_1 - k_1 - k_2 + m)e_2(p_1 - k_1 + m)e_1 u_1,$$
$$\mathcal{M}_2 = -\frac{1}{a_1 a_2'}\bar{u}_1' e_2(p_1 - k_1 - k + m)p'(p_1 - k_1 + m)e_1 u_1,$$
$$\mathcal{M}_3 = \frac{1}{d' a_2'}\bar{u}_1' e_2(p_1 - k_1 - k + m)e_1(p_1 - k + m)p' u_1,$$

where the permutation operator $\mathcal{P}_{12}$ is defined as

$$\mathcal{P}_{12} f(k_1, x_1, \eta_1; k_2, x_2, \eta_2) = f(k_2, x_2, \eta_2; k_1, x_1, \eta_1).$$

We note at this point that the quantity $\bar{u}_1' Q_{\eta_1\eta_2} u_1$ is gauge invariant regarding virtual photon gauge transformations. This means that it tends to zero when the limit $k_\perp \to 0$ is taken. Only $(1 + \mathcal{P}_{12})\bar{u}_1' Q_{\eta_1\eta_2} u_1$ is gauge invariant regarding all three photons gauge transformations. The latter property can be conceived from the gauge condition

$$k^\mu \bar{u}_1' O_\mu u_1 = (\alpha p' + k_\perp)^\mu \bar{u}_1' O_\mu u_1 = 0 \Longrightarrow p_\mu' \bar{u}_1' Q_\mu u_1 = -\frac{k_\perp^\mu}{\alpha}\bar{u}_1' Q_\mu u_1. \tag{13}$$

We now convince that the property (13) ($Q \sim k_\perp$) can be inferred from (12) explicitly. Rearrange for this aim $\mathcal{M}_i$ in (13), using the Dirac equation, dropping helicity state indices, and cast it down to the following form:

$$\mathcal{M}_1 = \frac{s\Delta}{a_1 d}\bar{u}_1' e_2(p_1 - k_1 + m)e_1 u_1 + \frac{1}{a_1 d}\bar{u}_1' p' k e_2(p_1 - k_1 + m)e_1 u_1,$$

$$
\begin{aligned}
\mathcal{M}_2 &= -\frac{s(1-x_1)}{a_1 a_2'}\bar{u}_1' e_2 (p_1 - k_1 + m) e_1 u_1 - \frac{1}{a_2'}\bar{u}_1' e_2 p' e_1 u_1 + \\
&\qquad \frac{1}{a_1 a_2'}\bar{u}_1' e_2 k p' (p_1 - k_1 + m) e_1 u_1, \\
\mathcal{M}_3 &= \frac{s}{a_2' d'}\bar{u}_1' e_2 (p_1 - k_1 + m) e_1 u_1 - \frac{s}{a_2' d'}\bar{u}_1' e_2 k e_1 u_1 - \\
&\qquad \frac{1}{a_2' d'}\bar{u}_1' e_2 (p_1' + k_2 + m) e_1 k p' u_1.
\end{aligned}
\tag{14}
$$

We see from the above that the last terms in the formulae for $\mathcal{M}_1, \mathcal{M}_2, \mathcal{M}_3$ are explicitly proportional to $k_\perp$ because

$$
\hat{p}'\hat{k} = \hat{p}'\hat{k}_\perp. \tag{15}
$$

The sum of the first three terms also proportional to $k_\perp$ since (see (5))

$$
A_1 \equiv \frac{\Delta}{a_1 d'} - \frac{1-x_1}{a_1 a_2'} + \frac{1}{a_2' d'}, \qquad A_1|_{k_\perp \to 0} = 0. \tag{16}
$$

Let us consider the sum of the second terms in the expressions for $\mathcal{M}_2, \mathcal{M}_3$ in (14). Using the relations (5)) one gets:

$$
-\frac{\hat{p}'}{a_2'} - \frac{s(\alpha\hat{p}' + \hat{k}_\perp)}{a_2' d'} = -\frac{s\hat{k}_\perp}{a_2' d'} + \frac{\hat{p}'\vec{k}^2}{a_2' d'}. \tag{17}
$$

From (15, 16, 17) one can convinced that the gauge condition

$$
\mathcal{M}_1 + \mathcal{M}_2 + \mathcal{M}_3|_{k_\perp \to 0} = 0
$$

is obviously fulfilled. For further consideration it will be convenient to present $\sum \mathcal{M}_i$ as a sum of terms explicitly proportional to $k_\perp$ (hereinafter we drop hats):

$$
\begin{aligned}
\sum \mathcal{M}_i = \bar{u}_1' \Big\{ & A_1 s e_2 (p_1 - k_1 + m) e_1 + \frac{1}{a_1 d} p' k_\perp e_2 (p_1 - k_1 + m) e_1 - \frac{\vec{k}^2}{a_2' d'} e_2 e_1 p' - \\
& \frac{s}{a_2' d'} e_2 k_\perp e_1 + \frac{1}{a_1 a_2'} e_2 k_\perp p' (p_1 - k_1 + m) e_1 - \frac{1}{a_2' d'} e_2 (p_1' + k_2 + m) e_1 k_\perp p' \Big\} u_1.
\end{aligned}
\tag{18}
$$

Here and in what follows we omit the terms in rhs, containing in the numerator m^2, but, of course, keep those proportional to m. Subsequent calculations involve determination of sundry bilinear spinor combinations, which are given in the appendix. There, using helicity states of fermion and photons, all the necessary ingredients entering (18) explicitly defined. Performing calculations we arrive to the result which we represent in the form:

$$
\begin{aligned}
\mathcal{M}^{++}_{++} &= s\sqrt{\Delta}\psi(1 + \mathcal{P}_{12}) \Big\{ A_1 \left(\frac{2(1-x_1)}{x_1 x_2}\chi_1\chi_2 + \frac{2}{x_1}\chi_1^2 \right) - \frac{2(1-x_1)\chi\chi_1}{x_1(a_1 a_2')} \\
&\qquad + \frac{2\chi(\rho_2 x_1 + \chi_1 x_2)}{x_1 x_2 (a_2' d')} \Big\}, \\
\mathcal{M}^{--}_{++} &= s\sqrt{\Delta}\psi(1 + \mathcal{P}_{12}) \Big\{ A_1 \left(\frac{2}{x_1 x_2}\chi_1^*\chi_2^* + \frac{2}{x_1\Delta}\chi_1^* q^* \right) - \frac{2\chi^*\chi_1^*}{x_1(a_1 d)} \\
&\qquad + \frac{2}{a_2' d'} \left(\frac{\chi_2^*\chi^*}{x_2} + \frac{\chi^* q^*}{\Delta} \right) \Big\}, \\
\mathcal{M}^{\pm\mp}_{++} &= \mathcal{M}^{\pm\mp}_{++}(\chi_1, x_1; \chi_2, x_2) + \mathcal{M}^{\mp\pm}_{++}(\chi_2, x_2; \chi_1, x_1), \\
\mathcal{M}^{+-}_{++}(\chi_1, x_1; \chi_2, x_2) &= s\sqrt{\Delta}\psi \Big\{ A_1 \frac{2}{x_1 x_2}\chi_1^*\chi_2 + \frac{2\chi\chi^*}{a_2' d'} \\
&\qquad - \frac{2\chi_1^*\chi}{x_1(a_1 a_2')} + \frac{2}{a_2' d'} \frac{\chi_2\chi^* x_1 + \chi_1^*\chi x_2}{x_1 x_2} \Big\}, \\
\mathcal{M}^{-+}_{++}(\chi_1, x_1; \chi_2, x_2) &= s(1-x_1)\sqrt{\Delta}\psi \Big\{ A_1 \left(\frac{2}{x_1 x_2}\chi_1\chi_2^* + \frac{2}{x_1}\frac{\chi_1 q^*}{\Delta} \right) \\
&\qquad - \frac{2\chi_1\chi^*}{x_1(a_1 d)} + \frac{2}{x_2\Delta}\frac{\chi\rho_2^*}{a_2' d'} \Big\}, \\
\mathcal{M}^{--}_{+-} &= 2ms\sqrt{\Delta}\psi^*(1 + \mathcal{P}_{12}) \Big\{ A_1 \left[\chi_1^* \left(1 - \frac{1-\Delta}{x_1\Delta} \right) - \frac{x_1}{x_2}\chi_2^* \right] \\
&\qquad - \frac{1-\Delta}{\Delta}\frac{\chi^*}{a_2' d'} + \frac{x_1\chi^*}{a_1 a_2'} \Big\}, \\
\mathcal{M}^{+-}_{+-} &= \mathcal{M}^{+-}_{+-}(\chi_1, x_1; \chi_2, x_2) + \mathcal{M}^{-+}_{+-}(\chi_2, x_2; \chi_1, x_1), \\
\mathcal{M}^{-+}_{+-} &= \mathcal{M}^{-+}_{+-}(\chi_1, x_1; \chi_2, x_2) + \mathcal{M}^{+-}_{+-}(\chi_2, x_2; \chi_1, x_1), \\
\mathcal{M}^{+-}_{+-}(\chi_1, x_1; \chi_2, x_2) &= 2ms\sqrt{\Delta}\psi^* \Big\{ -A_1 \left(\frac{x_1}{x_2}\chi_2 + \frac{x_1}{\Delta}q \right) + \frac{x_1\chi}{a_1 d} \Big\}, \\
\mathcal{M}^{-+}_{+-}(\chi_1, x_1; \chi_2, x_2) &= 2ms\sqrt{\Delta}\psi^* \Big\{ -A_1 \frac{x_2}{x_1\Delta}\chi_1 - \frac{x_2}{\Delta}\frac{\chi}{a_2' d'} \Big\}, \quad \mathcal{M}^{++}_{+-} = 0,
\end{aligned}
\tag{19}
$$

$$
\psi = e^{\mathrm{i}\phi_1'/2}, \quad \psi^2 = -\frac{q}{\varepsilon\theta_1'\Delta}, \quad q = \chi + \chi_1 + \chi_2 \quad \rho_i = r_{ix} + \mathrm{i}r_{iy}, \quad i = 1, 2. \tag{20}
$$

Note that in (19) the explicit Bose symmetry between two photons present:

$$
\begin{aligned}
\mathcal{M}^{\pm\pm}_{+\pm}(\chi_1, x_1; \chi_2, x_2) &= \mathcal{M}^{\pm\pm}_{+\pm}(\chi_2, x_2; \chi_1, x_1), \\
\mathcal{M}^{\pm\mp}_{+\pm}(\chi_1, x_1; \chi_2, x_2) &= \mathcal{M}^{\mp\pm}_{+\pm}(\chi_2, x_2; \chi_1, x_1).
\end{aligned}
$$

3 Checking the results

Useful check of the results obtained is the limiting case when one of the photons becomes soft. For definiteness let us suppose that the second photon does. Then we have:

$$
\begin{aligned}
\mathcal{M}^{\eta_2\eta_1}_{\sigma\sigma_1} &= s\frac{\sqrt{2}}{x_2}\left[\delta_{\eta_2,1}\left(-\frac{\chi_2}{a_2}+\frac{\rho_2}{a_2'}\right)+\delta_{\eta_2,-1}\left(-\frac{\chi_2^*}{a_2}+\frac{\rho_2^*}{a_2'}\right)\right]\cdot \\
&\cdot\sqrt{2(1-x_1)}\left(-\frac{x_1}{DD'}\right)\left\{\psi(1-x_1)\left[(D-D')\frac{\chi_1}{x_1}-D'\chi\right]\delta_{\eta_1,1}\delta_{\sigma_1',1}\right. \\
&\left.-m(D-D')x_1\psi^*\delta_{\eta_1,-1}\delta_{\sigma_1',-1}+\psi\left[(D-D')\frac{\chi_1^*}{x_1}-D'\chi^*\right]\delta_{\eta_1,-1}\delta_{\sigma_1',1}\right\} \qquad (21) \\
D' &= m^2x_1^2+\vec{k}_1^2, \qquad D=m^2x_1^2+\vec{r}_1^2
\end{aligned}
$$

We see that the **amplitude factorizes** in this limit. The sum of squares of the modules in the first multiplier agree with the known accompanied radiation factor

$$
-\left(\frac{p_1}{p_1k_2}-\frac{p_1'}{p_1'k_2}\right)^2, \qquad (22)
$$

whereas for the second multiplier that sum looks

$$
\frac{4x_1^2(1-x_1)}{D^2D'^2}\left[\vec{k}^2DD'(1+(1-x_1)^2)-2m^2(1-x_1)(D-D')^2\right] \qquad (23)
$$

which agree with the summed over spin states matrix element squared of a single bremsstrahlung process [2].

Another verification comes from the identity that must have to be satisfied:

$$
\begin{aligned}
2\Big\{|\mathcal{M}^{++}_{++}|^2+|\mathcal{M}^{--}_{++}|^2+|\mathcal{M}^{+-}_{++}|^2+|\mathcal{M}^{-+}_{++}|^2+|\mathcal{M}^{+-}_{+-}|^2+|\mathcal{M}^{-+}_{+-}|^2+|\mathcal{M}^{--}_{+-}|^2\Big\} &= \\
\mathrm{Tr}(p_1'+m)O_{\mu_1\mu_2}(p_1+m)\tilde{O}_{\mu_1\mu_2} \qquad (24)
\end{aligned}
$$

We convinced in its validity by direct computations.

4 Conclusions

This work is a sequel of a series of papers where the helicity analysis of the processes $\gamma\gamma\to e\bar{e}\mu\bar{\mu}, e\bar{e}e\bar{e}$ [3], $e\bar{e}\to(e\gamma)(\bar{e}\gamma)$ [6], $e\bar{e}\to e\bar{e}\mu\bar{\mu}, e\bar{e}e\bar{e}$ [4] in Born approximation in the kinematics of a jet moving forward was performed. This specific kinematical condition provides the main contribution to the total cross section at high energies. We hope that the results obtained can be useful at the colliding beam facilities for the purposes of monitoring and calibration of polarized beams. Besides they may provide an essential background in the developed physical programs aiming to measure asymmetries of different types.

Appendix

Here we give the explicit expressions of the bilinear spinor structures entering matrix element. We use standard representation of the Dirac matrices:

$$
\hat{p}'=\varepsilon\begin{pmatrix}1 & \sigma_z\\ -\sigma_z & -1\end{pmatrix}; \qquad \gamma_0=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}, \qquad \gamma=\begin{pmatrix}0 & \boldsymbol{\sigma}\\ -\boldsymbol{\sigma} & 0\end{pmatrix},
$$
$$
\hat{a}_\perp=\begin{pmatrix}0 & -\boldsymbol{a\sigma}\\ \boldsymbol{a\sigma} & 0\end{pmatrix} \qquad (25)
$$

In what follows $\bar{u}_1'$ and u_1 stand for the Dirac bispinors $\bar{u}^{\sigma_1'}(p_1'), u^{\sigma_1}(p_1)$ (afterwards indices σ_1', σ_1 are dropped). We take throughout the paper

$$
\sigma_1=1, \quad \delta_{\sigma_1',\pm1}=\delta_\pm, \quad \delta_{\eta_2,\pm1}=\delta_{2\pm}, \quad \delta_{\eta_1,\pm1}=\delta_{1\pm}\ .
$$

Then the relevant bilinear combinations read (see (8,9, 10,20)):

$$
\begin{aligned}
\bar{u}_1'p'u_1 &= s\sqrt{\Delta}\delta_+\psi, \\
\bar{u}_1'e_2e_1p'u_1 &= -2s\sqrt{\Delta}\delta_{2+}\delta_{1-}\delta_+\psi, \\
\bar{u}_1'e_{2\perp}u_1 &= -\sqrt{2\Delta}\delta_{2-}\left[k'\delta_++m\frac{1-\Delta}{\Delta}\delta_-\right]\psi^*, \quad k'\equiv\varepsilon\theta_1', \\
\bar{u}_1'e_{2\perp}e_{1\perp}u_1 &= -2\sqrt{\Delta}\delta_{2+}\delta_{1-}\left[-k'\delta_-+m\frac{1+\Delta}{\Delta}\delta_+\right]\psi, \\
\bar{u}_1'e_{2\perp}e_{1\perp}k_{1\perp}p'u_1 &= 2\sqrt{\Delta}\delta_{2-}\delta_{1+}\left[k'\delta_++m\frac{1-\Delta}{\Delta}\delta_-\right]\chi_1\psi^*, \\
\bar{u}_1'e_2p'u_1 &= s\sqrt{2\Delta}\delta_{2-}\delta_-\psi^*, \\
\bar{u}_1'e_{2\perp}k_{1\perp}p'u_1 &= -s\sqrt{2\Delta}\delta_+\delta_{2+}\chi_1\psi, \\
\bar{u}_1'p'k_\perp e_{2\perp}e_{1\perp}u_1 &= 2s\sqrt{\Delta}\delta_{2+}\delta_{1-}\delta_-\chi\psi^*, \\
\bar{u}_1'p'e_{2\perp}e_{1\perp}k_{1\perp}u_1 &= 2s\sqrt{\Delta}\delta_{2-}\delta_{1+}\delta_-\chi_1\psi^*, \\
\bar{u}_1'e_{2\perp}k_\perp e_{1\perp}u_1 &= 2\sqrt{\Delta}\delta_{2-}\delta_{1-}\left[k'\delta_++m\frac{1-\Delta}{\Delta}\delta_-\right]\chi^*\psi^*, \qquad (26) \\
\bar{u}_1'p'k_\perp e_{1\perp}u_1 &= -s\sqrt{2\Delta}\delta_+\delta_{1-}\chi^*\psi, \\
\bar{u}_1'p'e_{2\perp}k_\perp e_{1\perp}u_1 &= 2s\sqrt{\Delta}\delta_-\delta_{2-}\delta_{1-}\chi^*\psi^*, \\
\bar{u}_1'p'e_{2\perp}k_\perp e_{1\perp}k_{1\perp}u_1 &= 2s\sqrt{\Delta}\delta_+\delta_{2+}\delta_{1+}\chi\chi_1\psi, \\
\bar{u}_1'e_{2\perp}e_{1\perp}k_\perp p'u_1 &= -2s\sqrt{\Delta}\delta_-\delta_{2-}\delta_{1+}\chi\psi^*, \\
\bar{u}_1'r_{2\perp}e_{2\perp}e_{1\perp}k_\perp p'u_1 &= 2s\sqrt{\Delta}\delta_+\delta_{2-}\delta_{1+}\rho_2^*k\psi.
\end{aligned}
$$

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