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CONNECTION BETWEEN THE INTERFERENCE CORRELATIONS OF IDENTICAL PARTICLES IN HEAVY ION PHYSICS AND ASTROPHYSICS: ANALOGY AND DIFFERENCE

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1. About 45 years have elapsed after the publication of the classical work of Hanbury-Brown and Twiss [1], in which the method of determining the angular sizes of stars by the way of studying the intensity correlations at the registration of two optical photons was elaborated. Six years later Goldhaber et al. [2] showed that in the framework of the statistical model the effect of symmetrization of wave functions of identical pions leads to the narrowing of the distribution of angles between their flight directions, depending on the radius R of the generation region. In the seventies, in the cycle of papers by Kopylov and Podgoretsky [3—8] the deep analogy between pair correlations of identical particles, produced in nuclear and hadron collisions, and pair correlations of photons, emitted by optical sources, in particular, by stars, was established for the first time. That became the base for the creation of the general correlation method in high energy physics, overstepping the narrow bounds of the stationary statistical model [2] and allowing one to measure not only the space sizes of the generation region of identical particles but also the shape of this region and the duration of the generation processes [8].

During the last 25 years the method of correlations of identical particles with small relative velocities has got wide propagation in the practical activity of many laboratories of the world, and it has been formed as the new perspective trend in nuclear physics and particle physics, including hundreds of experimental and theoretical works.

Both the methods (in high energy physics, as well as in astronomy) are based on the correlation properties of identical particles. However, in the recent years the very important analogy between them has obscured the difference of principle (even opposition!) in the arrangement and performance of the corresponding experiments, indicated clearly in the paper [7].

2. Really, in high energy physics (heavy ion physics) we deal with the momentum-energy correlations. The model of one-particle sources is used. It is assumed that the emission sources have a sufficiently broad momentum spectrum. The sizes of sources themselves and the distances between them are by many orders smaller than the uncertainties in the space localization of the detectors; the times of the source decays are very small in comparison with the time characteristics of detectors. The momenta of two identical pions in the final state are measured and fixed. We will not dwell on details here, restricting ourselves by the description of the structure of principle of these correlations.

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The basic formula of the pion interferometry, which should be averaged over the space-time distribution of two sources at the calculation of the correlation function, independently of concrete processes, has the form

$$W=\frac{1}{2}\,|\langle p_1|x_1\rangle\langle p_2|x_2\rangle+\langle p_2|x_1\rangle\langle p_1|x_2\rangle^2=$$

$$=\frac{1}{2}\,|\exp(-ip_1x_1)\exp(-ip_2x_2)+\exp(-ip_1x_2)\exp(-ip_2x_1)|^2= \qquad (1)$$

$$=1+\cos((p_1-p_2)(x_1-x_2)).$$

Formula (1) describes the probability of the event that two noninteracting spinless identical particles, emitted at the definite space points $\mathbf{x}_1$ and $\mathbf{x}_2$ at the definite time moments t_1 and t_2, are registered by detectors in the states with the definite 4-momenta $p_2=\{E_1, \mathbf{p}_1\}$ and $p_2=\{E_2, \mathbf{p}_2\}$. This relation corresponds to the Dirac transformation from the space-time representation to the momentum-energy one. In so doing, $x_1=\{t_1, \mathbf{x}_1\}$ and $x_2=\{t_2, \mathbf{x}_2\}$ mean the 4-coordinates of sources, or, more exactly, their centres (taking into account the non-zero dimensions of the sources themselves).

By definition, the argument of cosine in Eq.(l) is the following:

$$(p_1-p_2)(x_1-x_2)=(\mathbf{p}_1-\mathbf{p}_2)(\mathbf{x}_1-\mathbf{x}_2)-(E_1-E_2)(t_1-t_2). \qquad (2)$$

Due to the equality $(p_1-p_2)(p_1+p_2)=0$, the difference of energies is:

$$E_1-E_2=(\mathbf{p}_1-\mathbf{p}_2)\mathbf{u}, \qquad (3)$$

where $\mathbf{u}=\dfrac{(\mathbf{p}_1+\mathbf{p}_2)}{E_1+E_2}$ is the velocity of the pair of the identical pions. It should be stressed that here we use the unit system in which $\hbar=c=1$.

3. In the framework of the model of one-particle sources, the double inclusive cross-section of the production of two identical pions with small relative momenta in the collisions of heavy ions has the structure:

$$\frac{d^6\sigma}{d^3\mathbf{p}_1d^3\mathbf{p}_2}=R(\mathbf{p}_1, \mathbf{p}_2)\frac{d^6\sigma^{(0)}}{d^3\mathbf{p}_1d^3\mathbf{p}_2}, \qquad (4)$$

$$R(\mathbf{p}_1, \mathbf{p}_2)=1+\langle\cos((p_1-p_2)(x_1-x_2))\rangle= \qquad (5)$$

$$=1+\int W_p(x_1, x_2)\cos(q(x_1-x_2))d^4x_1d^4x_2.$$

Here $\dfrac{d^6\sigma^{(0)}}{d^3\mathbf{p}_1d^3\mathbf{p}_2}$ is the «backgound» cross-section of the pion producton without the quantum statistics effect (pion identity) and the final state interaction effect, too, $W_p(x_1, x_2)$ is the distribution of 4-coordinates of two sources, normalized by unity, which depends on the 4-momentum sum $P=p_1+p_2$, in general, and does not depend practically on the small 4-momentum difference $q=p_1-p_2$ [9,10]. Pair correlations of pions, containing the information on space-time parameters of the multiple generation region, are described by the expression

$$C_p(q)=R(\mathbf{p}_1, \mathbf{p}_2)-1=\langle\cos(q(x_1-x_2))\rangle. \qquad (6)$$

It is essential that the correlation function $C_p(q)$ tends to zero at sufficiently large values of the momentum difference $|\mathbf{q}|=|\mathbf{p}_1-\mathbf{p}_2|>>1/r$ and the energy difference $|q_0|=|E_1-E_2|>>1/\tau$, where r is the characteristic radius of the emission region, τ is the characteristic duration of the particle production process. It should be noted that often one names the ratio $R(\mathbf{p}_1, \mathbf{p}_2)$ itself of the true two-particle spectrum to the «background» one as the correlation function. Concerning the background, in heavy ion collisions we can neglect kinematic constraints and most of the dynamic correlations, and construct the reference two-particle distribution as the product of one-particle spectra by mixing the particles from different events.

The final state interaction is neglected in Eqs. (1), (5), and (6). This approximation is sufficiently good for pions. In general, the final state interaction also influences the character of pair correlations at small relative velocities. It should be emphasized that in the case of narrow pair correlations of non-identical particles only the final state interaction effect plays role [9—11]. We will not discuss this question in detail here.

4. In the case of the emission of unpolarized identical particles with the non-zero spin j the correlations decrease. Neglecting the final state interaction, one should write for the ratio $R(\mathbf{p}_1, \mathbf{p}_2)$ of the true two-particle spectrum to the «background» one the following expression [9,10,12]:

$$R(\mathbf{p}_1, \mathbf{p}_2)=1+\frac{(-1)^{2j}}{2j+1}\langle\cos(q(x_1-x_2))\rangle, \qquad (7)$$

which coincides with Eq. (5) at $j=0$. Let us note that the factor $N(j)=2j+1$ in the denominator of Eq. (7) means the number of spin states of a particle with the spin j; the sign factor $(-1)^{2j}$ is equal to $(+1)$ for bosons and to (-1) for fermions.

Formula (7) is related to the identical particles with the non-zero mass. When the unpolarized photons are emitted, one should introduce in Eq.(7), instead of the factor $\frac{(-1)^{2j}}{2j+1}$, the factor

$$K = \frac{1}{4}(1+\cos^2\theta), \tag{8}$$

in which θ is the angle between the directions of photon flights, taking into account the transversality of the electromagnetic field of radiation [13]. Really, the angle θ is very small at small values of the momentum difference, so that $K = 1/2$. In so doing, the correlation function of two photons with small relative momenta is described by the formula

$$C_p(q) = \frac{1}{2}\langle \cos(q(x_1 - x_2))\rangle \tag{9}$$

differing from Eq.(6) by the factor $1/2$. This factor corresponds to die presence of two possible polarization states of a photon (although the photon has the spin 1, the photon helicity can take only two values (+ 1) and (− 1) due to the zero mass of photon).

5. We discussed above the momentum-energy correlations of pions and photons in nuclear and particle physics (heavy ion physics). In optics and astronomy we deal with the space-time correlations of photons [7]. In this situation the sizes of the emission region, the distances between sources and the duration of the emission process are by many orders larger than the corresponding characteristics of detectors. Let us denote the distance between two sources as $r_{12} = |\mathbf{x}_1 - \mathbf{x}_2|$, the distance between two detectors as $r_{34} = |\mathbf{x}_3 - \mathbf{x}_4|$ and the distance between the observation region and the emission region as L. The space-time correlations are observed when the conditions

$$r_{34} << r_{12} << L \tag{10}$$

are satisfied (i.e., the emission region is very large, and it is situated very far from the observation region). Then, the amplitude of the registration of two identical particles (concretely, two photons with the same polarizations) by two detectors is proportional to

$$A = \frac{1}{\sqrt{2}}(\langle x_3|p_1\rangle\langle x_2|p_2\rangle + \langle x_3|p_2\rangle\langle x_4|p_1\rangle) = \tag{11}$$

$$= \frac{1}{\sqrt{2}}(\exp(-ip_1x_3)\exp(-ip_2x_4) + \exp(-ip_1x_4)\exp(-ip_2x_3)),$$

and it corresponds to the Dirac transformation from the momentum-energy representation to the space-time one. In accordance with this, the equation

$$W = |A|^2 = 1 + \cos((p_1 - p_2)(x_3 - x_4)) =$$

$$= 1 + \cos[(\mathbf{p}_1 - \mathbf{p}_2)(\mathbf{x}_3 - \mathbf{x}_4) - (E_1 - E_2)(t_3 - t_4)] \tag{12}$$

describes the probability of the event that two photons, emitted in the states with the definite 4-momenta $p_1 = \{E_1, \mathbf{p}_1\}$ and $p_2 = \{E_2, \mathbf{p}_2\}$, will be registered by two detectors at the definite space points $\mathbf{x}_3$ and $\mathbf{x}_4$ at the definite time moments t_3 and t_4. In so doing, $x_3 = \{t_3, \mathbf{x}_3\}$ and $x_4 = \{t_4, \mathbf{x}_4\}$ in Eqs. (11) and (12) are already the 4-coordinates of detectors, but not sources: the detectors and the sources are mutually replaced as compared with the situation in high energy physics [7].

6. When the unpolarized photons with close momenta are emitted, it should be written instead of Eq. (12), passing to the usual unit system:

$$W = 1 + \frac{1}{2}\cos[(\mathbf{k}_1 - \mathbf{k}_2)(\mathbf{x}_3 - \mathbf{x}_4) - (\omega_1 - \omega_2(t_3 - t_4)], \tag{13}$$

where $\omega_1 = E_1/\hbar$ and $\omega_2 = E_2/\hbar$ are the emission frequencies, $\mathbf{k}_1 = \mathbf{p}_1/\hbar$ and $\mathbf{k}_2 = \mathbf{p}_2/\hbar$ are the wave vectors. Let $\mathbf{n}_1$ and $\mathbf{n}_2$ be the unit vectors along the directions from two sources to the observation region, which can be considered here formally as a point, in accordance with the condition (10). Then

$$\mathbf{k}_1 = \frac{\omega_1}{c}\mathbf{n}_1, \quad \mathbf{k}_2 = \frac{\omega_2}{c}\mathbf{n}_2. \tag{14}$$

Taking into account the small values of the angular distance between sources and the relatively small values of the frequency difference, the approximate relation

$$\mathbf{k}_1 - \mathbf{k}_2 = (\omega_1 - \omega_2)\mathbf{l} + \frac{\omega}{c}(\theta_1 - \theta_2) \tag{15}$$

is valid. Here $\omega = \frac{\omega_1 + \omega_2}{2}$, $\mathbf{l}$ is the unit vector along the direction from the centre of the source group (concretely, the star centre) to the observation region, θ_1 and θ_2 are the transverse vectors, which are perpendicular to the vector $\mathbf{l}$, their moduli $|\theta_2|$ and $|\theta_2|$ are the angles between the vectors $\mathbf{l}$ and $\mathbf{n}_1$, $\mathbf{l}$ and $\mathbf{n}_2$, respectively (i.e., the angular distance between the centre of the star and the sources on its disk). Using Eq. (15), we can rewrite the formula (13) for the probability of coincidences (double counts) in the form:

$$W = 1 + \frac{1}{2}\cos\left[(\omega_1 - \omega_2)\left(r_{34} - \frac{r_{34}^{(\parallel)}}{c}\right) - \frac{\omega}{c}(\theta_1 - \theta_2)\,\mathbf{r}_{34}^{(\perp)}\right], \tag{16}$$

where $r_{34}=r_3-r_4$ is the difference of the time moments of registration, $r_{34}^{(\|)}=\mathbf{r}_{34}\mathbf{l}$ is the longitudinal component of the vector $\mathbf{r}_{34}=\mathbf{x}_3-\mathbf{x}_4$, and $\mathbf{r}_{34}^{(\perp)}$ is the transverse component of the same vector with respect to the direction of l. Performing the integration of the expression (16) over the star disk with the radius $r=L\theta_0$ and the frequency interval $(\omega_0-\Delta\omega/2, \omega_0+\Delta\omega/2)$ at the fixed parameters of the observation region, we find that the number of double counts is proportional to

$$N_{34}=1+\frac{1}{2}\left(\frac{2J_1(\rho)}{\rho}\frac{\sin\delta}{\delta}\right)^2, \tag{17}$$

where J_1 is the first-order Bessel function,

$$\rho=\frac{\omega_0}{c}\,|\mathbf{r}_{34}^{(\perp)}|\,\theta_0,\quad \delta=\frac{1}{2}\Delta\omega\left(r_{34}^{(\|)}/c-t_{34}\right). \tag{18}$$

We see that the space-time correlations of intensities in two counters of photons are determined directly by the distance between the observation points and by the difference of the registration time moments. Just these parameters are measured and fixed experimentally. The information on angular sizes of the source group (concretely, star) appears only due to the dependence of the intensity correlations on the difference of transverse momenta of photons emitted by different regions of the star surface. The frequency ω_0 can be measured independently by the methods of spectral analysis.

Thus, the comparison of correlation methods in high energy physics and astronomy shows that the use of the term «HBT-interferometry» in nuclear and particle physics (heavy ion physics) seems to be not quite adequate.

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