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VIOLATION OF THE FACTORIZATION THEOREM IN LARGE-ANGLE RADIATIVE BHABHA SCATTERING

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1 Introduction

Large–angle Bhabha scattering process (LABS) has an important role in e^+e^- colliding beam physics [1]. First, it is traditionally used for calibration, because it has large cross section and can be recognized easily. Second, it might provide an essential background in a study of quarkonia physics. The result obtained below can be used also to construct Monte–Carlo event generators for Bhabha scattering processes.

In our previous papers we considered the following contributions to the large angle Bhabha cross section: pair production (virtual, soft [2] and hard [3]) and two hard photons [4]. This paper is devoted to the calculation of radiative corrections (RC) to a single hard photon emission process. We consider the kinematics essentially of the type $2 \to 3$, when all possible scalar products of 4–momenta of external particles are large compared to the electron mass squared.

Considering virtual corrections, we separate gauge–invariant sets of Feynman diagrams (FD). Loop corrections, connected with emission and absorption of virtual photons by the same fermionic line, are called as *Glass–type* (G) ones. The case, when loop includes exchange by two virtual photons between different fermionic lines, are dubbed Box–type (B) of FD. The third class includes the vertex function and vacuum polarization contributions ($\Gamma\Pi$–type). We see explicitly that all terms, which contain square of *large logarithms* $\ln(s/m^2)$, as well as those, which contain the infrared singularity parameter (fictitious photon mass λ), are cancelled out in the total sum, where the emission of an additional soft photon is considered also.

We note here, that the part of general result which is connected with the scattering type diagrams (see Fig.1(1,5)) was used to describe radiative deep inelastic scattering process (DIS) with RC taken into account in the paper [5] (we labeled it as the Compton tensor with heavy photon). Similar set of FD can be used to describe the annihilation channel [3].

The problem of virtual RC calculations at the 1–loop level is rather cumbersome for the process

$$e^+(p_2) \; + \; e^-(p_1) \; \longrightarrow \; e^+(p_2') \; + \; e^-(p_1') \; + \; \gamma(k_1). \qquad (1)$$

Namely, if at the Born level we need to consider 8 FD, then at the 1–loop level we have as much as 72 FD. Besides, performing loop momentum integration, we introduce scalar, vector and tensor integrals up to 3 rank with 2,3,4, and 5 denominators (a set of relevant integrals is given in Appendices A,B). A high degree of topological symmetries of FD for cross section can be exploited to calculate the matrix element squared. Using it, we can restrict ourselves to the consideration of interferences of the Born level amplitudes (Fig.1(1-4)) with those, which contain 1–loop integrals (Fig.1(5-16)). Our calculation is simplified since we omit the electron mass m in evaluation of the corresponding traces due to kinematical region under consideration:

$$s \sim s_1 \sim -t_1 \sim -t \sim -u \sim -u_1 \sim \chi_{1,2} \sim \chi'_{1,2} \gg m^2, \quad (2)$$
$$s = 2p_1p_2\,, \quad t = -2p_2p_2'\,, \quad u = -2p_1p_2' \quad s_1 = 2p_1'p_2'\,,$$
$$t_1 = -2p_1p_1'\,, \quad u_1 = -2p_2p_1'\,, \quad \chi_{1,2} = 2k_1p_{1,2}\,, \quad \chi'_{1,2} = 2k_1p'_{1,2}\,,$$
$$s + s_1 + t + t_1 + u + u_1 = 0, \quad s + t + u = \chi_1',$$
$$s_1 + t + u_1 = -\chi_1, \quad t + \chi_1 = t_1 + \chi_1'. \quad (3)$$

We found that some kind of *local factorization* took place both for the G– and B–type FD: the leading logarithmic contribution to the summed over spin states matrix element squared arising from an interference of one of the four FD at the Born level (Fig.1(1-4)) with some 1-loop corrected FD (Fig.1(5-16)) turns out to be proportional to an interference of the corresponding amplitudes at the Born level. The latter has the form:

$$\begin{aligned}
E_0 &= (4\pi\alpha)^{-3}\sum|M_1|^2 = -\frac{16}{t^2}\frac{1}{4}\mathrm{Tr}(\hat{p}_1' O_{11'}\hat{p}_1\tilde{O}_{11'})\cdot\frac{1}{4}\mathrm{Tr}(\hat{p}_2\gamma_\sigma\hat{p}_2'\gamma_\rho) \\
&= -\frac{16}{t\chi_1\chi_1'}(u^2+u_1^2+s^2+s_1^2), \qquad (4)\\
O_0 &= (4\pi\alpha)^{-3}\sum M_1M_2^* = \frac{8}{tt_1}\left(\frac{s}{\chi_1\chi_2}+\frac{s_1}{\chi_1'\chi_2'}+\frac{u}{\chi_1\chi_2'}+\frac{u_1}{\chi_2\chi_1'}\right)\\
&\times\ (u^2+u_1^2+s^2+s_1^2),\\
I_0 &= (4\pi\alpha)^{-3}\sum M_1(M_3^*+M_4^*) = -(1+\hat{Z})\frac{4}{ts_1}\Big\{-\frac{4u_1\chi_2'}{\chi_1}+\frac{4u(s_1+t_1)(s+t)}{\chi_2\chi_1'}\\
&- \frac{2}{\chi_1\chi_2}[2suu_1+(u+u_1)(uu_1+ss_1-tt_1)]+\frac{2}{\chi_1\chi_1'}[2t_1uu_1+(u+u_1)(uu_1\\
&+\ tt_1-ss_1)]\Big\}, \quad O_{11'} = \gamma_\rho\frac{\hat{p}_1'+\hat{k}_1}{\chi_1'}\gamma_\mu-\gamma_\mu\frac{\hat{p}_1-\hat{k}_1}{\chi_1}\gamma_\rho, \quad \tilde{O}_{11'} = O_{11'}(\rho\leftrightarrow\mu),
\end{aligned}$$

where $\hat{Z}$-operator acts as follows:

$$\hat{Z} = \left|\begin{matrix} p_1 \longleftrightarrow p_1' & s \longleftrightarrow s_1 \\ p_2 \longleftrightarrow p_2' & u \longleftrightarrow u_1 \\ k_1 \to -k_1 & t, t_1 \to t, t_1 \end{matrix}\right|.$$

One can be convinced, that the total summed over spin states matrix element squared can be obtained using symmetry properties, which are realized by means of the permutation operations:

$$\begin{aligned}
\sum|M|^2 &= (4\pi\alpha)^3 F, \qquad (5)\\
F &= (1+\hat{P}+\hat{Q}+\hat{R})\Phi = 16\,\frac{ss_1(s^2+s_1^2)+tt_1(t^2+t_1^2)+uu_1(u^2+u_1^2)}{ss_1tt_1}\\
&\times\ \left(\frac{s}{\chi_1\chi_2}+\frac{s_1}{\chi_1'\chi_2'}-\frac{t}{\chi_2\chi_2'}-\frac{t_1}{\chi_1\chi_1'}+\frac{u}{\chi_1\chi_2'}+\frac{u_1}{\chi_2\chi_1'}\right), \quad \Phi = E_0+O_0-I_0.
\end{aligned}$$

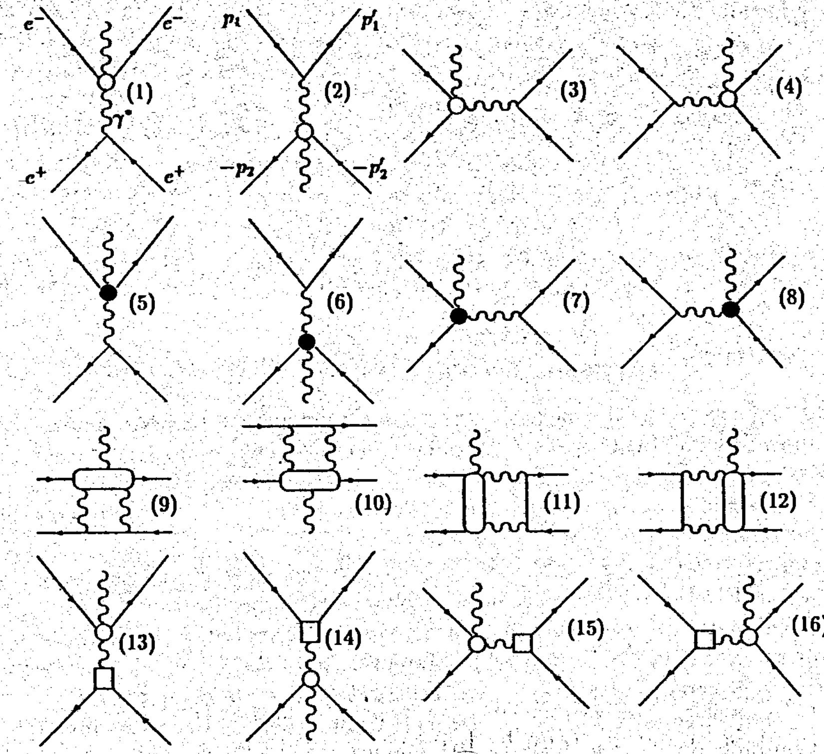

Fig. 1: G– and B–type Feynman diagrams for radiative Bhabha scattering.

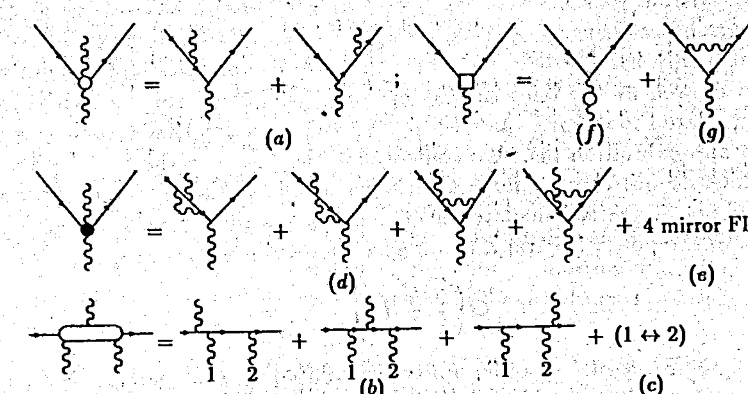

Fig. 2: Content of the notation for Fig. 1.

Explicit form of $\hat{P}, \hat{Q}, \hat{R}$ operators reads

$$\hat{P} = \left| \begin{array}{ll} p_1 \longleftrightarrow -p_2', & s \longleftrightarrow s_1 \\ p_2 \longleftrightarrow -p_1' & t \longleftrightarrow t_1 \\ k_1 \to k_1 & u, u_1 \to u, u_1 \end{array} \right|, \quad \hat{Q} = \left| \begin{array}{ll} p_2 \longleftrightarrow -p_1' & s \longleftrightarrow t_1 \\ p_2' \to p_2' & s_1 \longleftrightarrow t \\ p_1, k_1 \to p_1, k_1 & u, u_1 \to u, u_1 \end{array} \right|,$$

$$\hat{R} = \left| \begin{array}{ll} p_1 \longleftrightarrow -p_2' & s \longleftrightarrow t \\ p_1' \to p_1' & s_1 \longleftrightarrow t_1 \\ p_2, k_1 \to p_2, k_1 & u, u_1 \to u, u_1 \end{array} \right|. \tag{6}$$

The differential cross section at the Born level in the case of large–angle kinematics (2) was found in Ref. [6] by F.A. Berends et al.,

$$d\sigma_0(p_1, p_2) = \frac{\alpha^3}{32 s \pi^2} F \frac{d^3 p_1' d^3 p_2' d^3 k_1}{\varepsilon_1' \varepsilon_2' \omega_1} \delta^{(4)}(p_1 + p_2 - p_1' - p_2' - k_1), \tag{7}$$

where ε_1, ε_2, and ω_1 are the energies of outgoing fermions and photon, respectively. The collinear kinematical regions (real photon is emitted along the direction of one of charged particles) corresponding to the case when one of invariants χ_i, χ_i' of order m^2 gives the main contribution to the total cross section. They require separate investigation, and will be considered elsewhere.

Our paper is organized as follows. In Sect. 2 we consider the contribution due to the set of FD Fig.1(5-8) called here as *glasses* (G–type diagrams). Using the crossing symmetry we construct the whole G–type contribution from the gauge invariant set of FD drawn in Fig.1(5). Moreover, only the set of FD depicted in Fig.2(d) can be considered in practical calculations due to an additional mirror symmetry of diagrams Fig.2(d,e). So, we start from a check of the gauge invariance of the Compton tensor described by FD Fig.2(d,e) with all fermions and one of the photons on mass shell. In Sect. 3 we consider the contribution of amplitudes containing vertex functions and polarization operator of virtual photon shown in Fig.1(13-15) and Fig.2(f,d). In Sect. 4 we take into account the contribution of FD with two virtual photon exchange drawn in Fig.1(9-12), called here as *boxes* (B–type diagrams). Again, using the crossing symmetry of FD, we show how to use in calculations only FD Fig.1(9). We show that the terms containing the infrared singularities as well as the ones containing *large logarithms* can be written in a simple form, related to definite contributions to the radiative Bhabha cross section in the Born approximation (4). We control also the terms in the matrix element squared, which do not contain *large logarithms* and are infrared finite. Thus our consideration permits us to calculate the cross section in the kinematical region (2), in principle, to the power accuracy, i.e., neglecting terms of the order

$$O\left(\frac{\alpha}{\pi}\frac{m^2}{s}L_s^2\right) \tag{8}$$

as compared to the terms of order unity, calculated in this paper. Note that the terms in (8) are less than 10^{-4} for a typical moderate high-energy colliders

(DAΦNE, VEPP-2M, BEPS). Unfortunately the form of the non–leading terms is too complicated to be presented analytically therefore we have performed only numerical estimation of them. In Sect. 5 we consider emission of an additional soft photon in our process of radiative Bhabha process. In conclusion we note that the expression for the total correction taking into account virtual and real soft photons emission in the leading logarithmic approximation has a very elegant and handy form, though does not look as one would expect in the approach based on renormalization group ideas. Besides analytic expressions, we give also its numerical values along with the non–leading terms for a few points in typical experimental conditions.

2 Contribution of G–type diagrams

We start now by explicitly checking the gauge invariance of the tensor

$$\bar{u}(p_1') R^{\sigma\mu}_{1,1'} u(p_1). \tag{9}$$

This check was done indirectly in the paper [5], where Compton tensor with heavy photon was written in terms of explicitly gauge–invariant tensor structures. We use the following expression:

$$R^{\sigma\mu}_{1,1'} = R^{\chi_1} + R^{\chi_1'}. \tag{10}$$

$$R^{\chi_1} = A_2 \gamma_\sigma \hat{k}_1 \gamma_\mu + \int \frac{d^4 k}{i\pi^2} \left\{ \frac{\gamma_\lambda (\hat{p}_1' - \hat{k}) \gamma_\sigma (\hat{p}_1 - \hat{k}_1 - \hat{k}) \gamma_\lambda (\hat{p}_1 - \hat{k}_1) \gamma_\mu}{-\chi_1 (0)(2)(q)} \right.$$
$$\left. + \frac{\gamma_\lambda (\hat{p}_1' - \hat{k}) \gamma_\sigma (\hat{p}_1 - \hat{k}_1 - \hat{k}) \gamma_\mu (\hat{p}_1 - \hat{k}) \gamma_\lambda}{(0)(1)(2)(q)} \right\}, \tag{11}$$

where

$$(0) = k^2 - \lambda^2, \quad (2) = (p_1' - k)^2 - m^2, \quad (1) = (p_1 - k)^2 - m^2,$$
$$(q) = (p_1 - k_1 - k)^2 - m^2, \quad A_2 = \frac{2}{\chi_1}\left(L_{\chi_1} - \frac{1}{2}\right), \quad L_{\chi_1} = \ln\frac{\chi_1}{m^2}. \tag{12}$$

The quantity R^{χ_1} corresponds to FD depicted in Fig.2(d), while $R^{\chi_1'}$ corresponds to FD drawn in Fig.2(e). The first term in the right hand side of Eq.(11) corresponds to the first two FD of Fig.2(d) under conditions (2). One can convince himself, that the gauge invariance condition $R^{\sigma\mu}_{1,1'} k_\mu = 0$ is fulfilled even in the matrix form in the pure algebraic way. As for gauge invariance condition regarding the heavy photon Lorentz index it provides some check of loop momentum integrals given in the Appendix A:

$$\bar{u}(p_1') R^{\sigma\mu}_{1,1'} u(p_1) q_\sigma e_\mu(k_1) = A k_1^\mu e_\mu(k_1), \qquad A = -2\frac{L_{\chi_1} - 2}{\chi_1} - 6\frac{L_{\chi_1'} - 1}{\chi_1'}. \tag{13}$$

So, the gauge invariance is fulfilled due to Lorentz condition for the on-shell photon $e(k_1)k_1 = 0$. As we stated above the use of crossing symmetries of amplitudes permits us to consider only R^{χ_1}. For interference of amplitudes at the Born level (see Fig.1(1-4) and Fig.1(5-8)) we obtain in terms of the replacement operators:

$$(\Delta|M|^2)_G = 2^5\alpha^4\pi^2(1+\hat{P}+\hat{Q}+\hat{R})(1+\hat{Z})[E_{15}^{\chi_1}+O_{25}^{\chi_1}-I_{35}^{\chi_1}-I_{45}^{\chi_1}]. \qquad (14)$$

with

$$\begin{aligned}
E_{15}^{\chi_1} &= \frac{16}{t^2}\frac{1}{4}\mathrm{Tr}(\hat{p}_1' R^{\chi_1}\hat{p}_1 O_{11'})\cdot\frac{1}{4}\mathrm{Tr}(\hat{p}_2\gamma_\rho\hat{p}_2'\gamma_\sigma),\\
O_{25}^{\chi_1} &= \frac{16}{tt_1}\frac{1}{4}\mathrm{Tr}(\hat{p}_1' R^{\chi_1}\hat{p}_1\gamma_\rho)\cdot\frac{1}{4}\mathrm{Tr}(\hat{p}_2\gamma_\sigma\hat{p}_2' O_{22'}), \qquad (15)\\
I_{35}^{\chi_1} &= \frac{4}{ts_1}\frac{1}{4}\mathrm{Tr}(\hat{p}_1' R^{\chi_1}\hat{p}_1 O_{12}\hat{p}_2\gamma_\sigma\hat{p}_2'\gamma_\rho), \quad I_{45}^{\chi_1} = \frac{4}{ts}\frac{1}{4}\mathrm{Tr}(\hat{p}_1' R^{\chi_1}\hat{p}_1\gamma_\rho\hat{p}_2\gamma_\sigma\hat{p}_2' O_{1'2'}),\\
O_{11'} &= \gamma_\rho\frac{\hat{p}_1'+\hat{k}_1}{\chi_1'}\gamma_\mu - \gamma_\mu\frac{\hat{p}_1-\hat{k}_1}{\chi_1}\gamma_\rho, \quad O_{22'} = \gamma_\mu\frac{-\hat{p}_2'-\hat{k}_1}{\chi_2'}\gamma_\rho - \gamma_\rho\frac{-\hat{p}_2+\hat{k}_1}{\chi_2}\gamma_\mu,\\
O_{12} &= -\gamma_\mu\frac{\hat{p}_1-\hat{k}_1}{\chi_1}\gamma_\rho - \gamma_\rho\frac{-\hat{p}_2+\hat{k}_1}{\chi_2}\gamma_\mu, \quad O_{1'2'} = \gamma_\rho\frac{\hat{p}_1'+\hat{k}_1}{\chi_1'}\gamma_\mu + \gamma_\mu\frac{-\hat{p}_2'-\hat{k}_1}{\chi_2'}\gamma_\rho.
\end{aligned}$$

In logarithmic approximation the G-type amplitudes contribution to the cross section has the form:

$$d\sigma_G = \frac{d\sigma_0}{F}\frac{\alpha}{\pi}(1+\hat{P}+\hat{Q}+\hat{R})\Phi\left[-\frac{1}{2}L_{t_1}^2+\frac{3}{2}L_{t_1}+2L_{t_1}\ln\frac{\lambda}{m}\right], \qquad L_{t_1} = \ln\frac{-t_1}{m^2}. \quad (16)$$

3 Vacuum polarization and vertex insertion contributions

Let us examine a set of ΓΠ-type FD. A contribution of Dirac formfactor of fermions and vacuum polarization (see Fig.3) can be parametrized as $(1+\Gamma_t)/(1-\Pi_t)$, while a contributions of Pauli formfactor are proportional to fermion mass and omitted here. Then we obtain

$$d\sigma_{\Gamma\Pi} = \frac{d\sigma_0}{F}\frac{\alpha}{\pi}2(1+\hat{P}+\hat{Q}+\hat{R})(\Gamma_t+\Pi_t)\Phi, \qquad (17)$$

where

$$\begin{aligned}
\Gamma_t &= \frac{\alpha}{\pi}\left\{\left(\ln\frac{m}{\lambda}-1\right)(1-L_t)-\frac{1}{4}L_t-\frac{1}{4}L_t^2+\frac{1}{2}\zeta_2\right\}, \qquad (18)\\
\Pi_t &= \frac{\alpha}{\pi}\left(\frac{1}{3}L_t-\frac{5}{9}\right), \qquad L_t = \ln\frac{-t}{m^2}.
\end{aligned}$$

In realistic computations the vacuum polarization caused by hadrons and muons can be taken into account in a very simple fashion (see [7]), that is just by adding it to Π_t.

4 Contribution of the B-type set of Feynman diagrams

The procedure resembling the one used in the previous section being applied to the B-type set of FD (Fig.1(9-12a)) permits us to use in practical calculations only part of 1-loop diagrams, namely three of those in the scattering channel with uncrossed exchanged photon legs:

$$(\Delta|M|^2)_B = 2^5\alpha^4\pi^2\mathrm{Re}(1+\hat{P}+\hat{Q}+\hat{R})[(1-\hat{P}_{22'})I_{19}^{\chi_1}+(1+\hat{P}_{22'})I_{29}^{\chi_1}-I], \quad (19)$$

where

$$\hat{P}_{22'} = \left|\begin{matrix} p_2 \longleftrightarrow -p_2' & s \longleftrightarrow u\\ p_1 \longleftrightarrow p_1 & s_1 \longleftrightarrow u_1\\ p_1', k_1 \to p_1', k_1 & t, t_1 \to t, t_1\end{matrix}\right|, \qquad (20)$$

and

$$\begin{aligned}
I_{19}^{\chi_1} &= \int\frac{d^4k}{i\pi^2}\frac{1}{(0)(q)((p_2+k)^2-m^2)}\frac{16}{t}\frac{1}{4}\mathrm{Tr}(\hat{p}_1' B^{\chi_1}\hat{p}_1 O_{11'})\\
&\times \frac{1}{4}\mathrm{Tr}\,(\hat{p}_2\gamma_\sigma(-\hat{p}_2-\hat{k})\gamma_\lambda\hat{p}_2'\gamma_\rho),\\
I_{29}^{\chi_1} &= \int\frac{d^4k}{i\pi^2}\frac{1}{(0)(q)((p_2+k)^2-m^2)}\frac{16}{t_1}\frac{1}{4}\mathrm{Tr}(\hat{p}_1' B^{\chi_1}\hat{p}_1\gamma_\rho)\\
&\times \frac{1}{4}\mathrm{Tr}\,(\hat{p}_2\gamma_\sigma(-\hat{p}_2-\hat{k})\gamma_\lambda\hat{p}_2' O_{22'}),\\
I &= \int\frac{d^4k}{i\pi^2}\frac{1}{(0)(q)}\left\{\frac{4}{s_1}\frac{1}{4}\mathrm{Tr}(\hat{p}_2'\gamma_\rho\hat{p}_1' B^{\chi_1}\hat{p}_1 O^{12}\hat{p}_2(\hat{A}+\hat{B})\right. \qquad (21)\\
&+ \left.\frac{4}{s_1}\frac{1}{4}\mathrm{Tr}(\hat{p}_2' O_{1'2'}\hat{p}_1 B^{\chi_1}\hat{p}_1\gamma_\rho\hat{p}_2(\hat{A}+\hat{B}))\right\},\\
&\hat{A} = \frac{\gamma_\sigma(-\hat{p}_2-\hat{k})\gamma_\lambda}{(p_2+k)^2-m^2}, \quad \hat{B} = \frac{\gamma_\lambda(-\hat{p}_2'+\hat{k})\gamma_\sigma}{(-p_2'+k)^2-m^2}.
\end{aligned}$$

Here

$$\begin{aligned}
B^{\chi_1} &= \frac{\gamma_\lambda(\hat{p}_1-\hat{k}_1-\hat{k})\gamma_\sigma(\hat{p}_1-\hat{k}_1)\gamma_\mu}{-\chi_1(d)} + \frac{\gamma_\lambda(\hat{p}_1-\hat{k}_1-\hat{k})\gamma_\mu(\hat{p}_1-\hat{k})\gamma_\sigma}{(d)(1)} \qquad (22)\\
&+ \frac{\gamma_\mu(\hat{p}_1'+\hat{k}_1)\gamma_\lambda(\hat{p}_1-\hat{k})\gamma_\sigma}{\chi_1'(1)}, \qquad (q) = (p_2-p_2'+k)^2-\lambda^2,\\
(d) &= (p_1-k_1-k)^2-m^2, \qquad (1) = (p_1-k)^2-m^2, \qquad (0) = k^2-\lambda^2.
\end{aligned}$$

The result of analytical evaluations shows the absence of both double logarithmic ($\sim L_s^2$) and infrared logarithmic ($\sim \ln(\lambda/m)L$) terms in box contribution. In spite of explicitly seen proportionality of separate contributions to the structures E_0, O_0, I_0, the total expression reveals to be somewhat tangled though having factorized form in each gauge invariant subset of diagrams. We parametrize the correction coming from the B–type FD as follows:

$$d\sigma_B = d\sigma_0 \frac{\alpha}{\pi} L_s \Delta_B, \qquad \Delta_B = 2\ln\frac{ss_1}{uu_1} + \frac{2}{F}(\Phi_Q + \Phi_R)\ln\frac{tt_1}{ss_1}. \qquad (23)$$

The total virtual correction to the cross section has the form:

$$\begin{aligned} d\sigma^{virt} &= d\sigma_G + d\sigma_{\Gamma\Pi} + d\sigma_B \\ &= \frac{\alpha}{\pi}\left[-L_s^2 + L_s\left(\frac{11}{3} + 4\ln\frac{\lambda}{m} + \Delta_G + \Delta_{\Gamma\Pi} + \Delta_B\right) + \mathcal{O}(1)\right], \qquad (24) \\ \Delta_G + \Delta_{\Gamma\Pi} &= \frac{1}{F}\left(\Phi\ln\frac{s^2}{tt_1} + \Phi_R\ln\frac{t^2}{ss_1} + \Phi_Q\ln\frac{t_1^2}{ss_1} + \Phi_P\ln\frac{s_1^2}{tt_1}\right), \end{aligned}$$

where $\Phi_P = \hat{P}\Phi$, $\Phi_Q = \hat{Q}\Phi$, and $\Phi_R = \hat{R}\Phi$.

5 Contribution from additional soft photon emission

Consider now the process of radiative Bhabha scattering accompanied by emission of the additional soft photon in the center–of–mass system reference frame. Soft we imply that its energy does not exceed some small, compared to the energy of the initial beams ε, quantity $\Delta\varepsilon$. The corresponding cross section has the form:

$$\begin{aligned} d\sigma^{soft} &= d\sigma_0 \cdot \delta^{soft}, \\ \delta^{soft} &= -\frac{4\pi\alpha}{16\pi^3}\int\frac{d^3k_2}{\omega_2}\left(-\frac{p_1}{p_1k_2} + \frac{p_1'}{p_1'k_2} + \frac{p_2}{p_2k_2} - \frac{p_2'}{p_2'k_2}\right)^2\Bigg|_{\omega_2<\Delta\varepsilon}. \qquad (25) \end{aligned}$$

The soft photon energy does not exceed $\Delta\varepsilon \ll \varepsilon_1 = \varepsilon_2 \equiv \varepsilon \sim \varepsilon_1' \sim \varepsilon_2'$. In order to calculate the right hand side of Eq.(25), we use the master formula [8]:

$$-\frac{4\pi\alpha}{16\pi^3}\int\frac{d^3k}{\omega}\frac{(q_i)^2}{(q_ik)^2}\Bigg|_{\omega<\Delta\varepsilon} = -\frac{\alpha}{\pi}\ln\left(\frac{\Delta\varepsilon\cdot m}{\lambda\cdot\varepsilon_i}\right), \quad \omega = \sqrt{k^2+\lambda^2}, \qquad (26)$$

$$\frac{4\pi\alpha}{16\pi^3}\int\frac{d^3k}{\omega}\frac{2q_1q_2}{(kq_1)(kq_2)}\Bigg|_{\omega<\Delta\varepsilon} = \frac{\alpha}{\pi}\left[L_q\ln\left(\frac{m^2(\Delta\varepsilon)^2}{\lambda^2\varepsilon_1\varepsilon_2}\right) + \frac{1}{2}L_q^2 - \frac{1}{2}\ln^2\left(\frac{\varepsilon_1}{\varepsilon_2}\right) - \frac{\pi^2}{3} + \mathrm{Li}_2\left(\cos^2\frac{\theta}{2}\right)\right]. \qquad (27)$$

Here we used the following notation:

$$L_q = \ln\frac{-q^2}{m^2}, \quad q_1^2 = q_2^2 = m^2, \quad -q^2 = -(q_1 - q_2)^2 \gg m^2, \qquad (28)$$
$$q_{1,2} = (\varepsilon_{1,2}, \mathbf{q}_{1,2}), \qquad \theta = \widehat{\mathbf{q}_1\mathbf{q}_2},$$

where ε_1, ε_2, θ are the energies and angle between 3–momenta $\mathbf{q}_1$, $\mathbf{q}_2$, respectively. λ is the fictitious photon mass (all is defined in the center–of–mass system).

Below we put the concrete contribution of each possible term in the right hand side of Eq.(25):

$$\begin{aligned} \frac{\pi}{\alpha}\delta^{soft} &= -\Delta_1 - \Delta_2 - \Delta_1' - \Delta_2' + \Delta_{12} + \Delta_{1'2'} + \Delta_{11'} + \Delta_{22'} - \Delta_{12'} - \Delta_{1'2}. \\ \Delta_1 &= \Delta_2 = \ln\frac{\Delta\varepsilon\cdot m}{\varepsilon\lambda}; \quad \Delta_1' = \ln\frac{\Delta\varepsilon\cdot m}{\varepsilon_1'\lambda}, \quad \Delta_2' = \ln\frac{\Delta\varepsilon\cdot m}{\varepsilon_2'\lambda}. \\ \Delta_{12} &= 2L_s\ln\frac{\Delta\varepsilon\cdot m}{\varepsilon\lambda} + \frac{1}{2}L_s^2 - \frac{\pi^2}{3}. \\ \Delta_{1'2'} &= L_{s_1}\ln\left(\frac{(\Delta\varepsilon\cdot m)^2}{\varepsilon_1'\varepsilon_2'\lambda^2}\right) + \frac{1}{2}L_{s_1}^2 - \frac{1}{2}\ln^2\left(\frac{\varepsilon_1'}{\varepsilon_2'}\right) - \frac{\pi^2}{3} + \mathrm{Li}_2\left(\cos^2\frac{\theta_{1'2'}}{2}\right). \\ \Delta_{11'} &= L_{t_1}\ln\left(\frac{(\Delta\varepsilon\cdot m)^2}{\varepsilon_1'\varepsilon\lambda^2}\right) + \frac{1}{2}L_{t_1}^2 - \frac{1}{2}\ln^2\left(\frac{\varepsilon_1'}{\varepsilon}\right) - \frac{\pi^2}{3} + \mathrm{Li}_2\left(\cos^2\frac{\theta_{1'}}{2}\right). \\ \Delta_{22'} &= L_t\ln\left(\frac{(\Delta\varepsilon\cdot m)^2}{\varepsilon\varepsilon_2'\lambda^2}\right) + \frac{1}{2}L_t^2 - \frac{1}{2}\ln^2\left(\frac{\varepsilon_2'}{\varepsilon}\right) - \frac{\pi^2}{3} + \mathrm{Li}_2\left(\sin^2\frac{\theta_{2'}}{2}\right). \\ \Delta_{1'2} &= L_{u_1}\ln\left(\frac{(\Delta\varepsilon\cdot m)^2}{\varepsilon\varepsilon_1'\lambda^2}\right) + \frac{1}{2}L_{u_1}^2 - \frac{1}{2}\ln^2\left(\frac{\varepsilon_1'}{\varepsilon}\right) - \frac{\pi^2}{3} + \mathrm{Li}_2\left(\sin^2\frac{\theta_{1'}}{2}\right). \\ \Delta_{12'} &= L_u\ln\left(\frac{(\Delta\varepsilon\cdot m)^2}{\varepsilon\varepsilon_2'\lambda^2}\right) + \frac{1}{2}L_u^2 - \frac{1}{2}\ln^2\left(\frac{\varepsilon}{\varepsilon_2'}\right) - \frac{\pi^2}{3} + \mathrm{Li}_2\left(\cos^2\frac{\theta_{2'}}{2}\right). \qquad (29) \end{aligned}$$

$$L_u = \ln\frac{-u}{m^2}, \quad L_{u_1} = \ln\frac{-u_1}{m^2}, \quad \mathrm{Li}_2(z) \equiv -\int_0^z\frac{dx}{x}\ln(1-x). \qquad (30)$$

where ε_1', ε_2' – are the center–of–mass energies of the scattered electron and positron, respectively; $\theta_{1'}$, $\theta_{2'}$ are their scattering angles (measured from the initial electron momentum direction); $\theta_{1'2'}$ is the angle between the scattered electron and positron momenta.

Extracting large logarithms, we obtain

$$\delta^{soft} = \frac{\alpha}{\pi}\left\{4(L_s - 1)\ln\frac{m\Delta\varepsilon}{\lambda\varepsilon} + L_s^2 + L_s\ln\frac{tt_1}{uu_1} + L_s\ln\frac{1 - c_{1'2'}}{2} + \mathcal{O}(1)\right\}. \qquad (31)$$
$$c_{1'2'} = \cos\theta_{1'2'}.$$

It can be written in another form, using the experimentally measurable quantities

N	y_1	y_2	c_1	c_2	Δ_L	Δ
1	0.36	0.89	−0.70	−0.10	10.70	−24.53
2	0.59	0.66	0.29	−0.06	4.86	−11.41
3	0.67	0.67	0.50	0.30	5.82	−35.58
4	0.68	0.65	0.60	−0.50	4.10	−10.45

Table 1: Numerical estimations for Δ_L and Δ versus y_1, y_2, c_1, c_2

— energy fractions of the scattered leptons and the scattering angles:

$$y_i = \frac{\varepsilon_i'}{\varepsilon}, \quad c_i = \cos\theta_i', \quad \frac{1}{2}(1 - c_{1'2'}) = \frac{y_1 + y_2 - 1}{y_1 y_2}, \quad -\frac{t}{s} = y_2\frac{1+c_2}{2},$$
$$-\frac{u}{s} = y_2\frac{1-c_2}{2}, \; -\frac{t_1}{s} = y_1\frac{1-c_1}{2}, \; \frac{s_1}{s} = y_1 + y_2 - 1, \; -\frac{u_1}{s} = y_1\frac{1+c_1}{2}. \tag{32}$$

6 Conclusions

The double logarithmic terms of the type L_s^2 and those, proportional to $L_s \ln(\lambda/m)$ cancel out in total sum with the corresponding terms from the soft–photon contribution (31). Omitting vacuum polarization, we arrive to the result which in the logarithmic approximation has the form:

$$\begin{aligned} \mathrm{d}\sigma^{\text{soft+virt}} &= \mathrm{d}\sigma_0\frac{\alpha}{\pi}\Big[L_s\Big(4\ln\frac{\Delta\varepsilon}{\varepsilon} + \Delta_L\Big) + \Delta(y_1, y_2, c_1, c_2)\Big], \\ \Delta_L &= 3 + \ln\frac{(1-c_1)(1-c_2)}{(1+c_1)(1+c_2)} + \ln\frac{y_1+y_2-1}{y_1y_2} + \frac{1}{F}\Big[\Phi\ln\frac{s^2}{tt_1} + \Phi_P\ln\frac{s_1^2}{tt_1} \\ &\quad + \Phi_Q\ln\frac{t_1^2}{ss_1} + \Phi_R\ln\frac{t^2}{ss_1}\Big] + 2\ln\frac{ss_1}{uu_1} + \frac{2}{F}(\Phi_Q + \Phi_R)\ln\frac{tt_1}{ss_1}. \end{aligned} \tag{33}$$

The function $\Delta(y_1, y_2, c_1, c_2)$ is quite complicated. To compare it with Δ_L, we give their numerical values (omitting vacuum polarization) for a certain set of points from physical regions (34) and $y_1 + y_2 > 1$, $D > 0$ (see Table 1). Considering the kinematics typical for large–angle inelastic Bhabha scattering we put the lowest order contribution that was obtained earlier in [9] and the radiative corrections computed in this work.

After performing loop integration and shifting logarithms ($L_i = L_s + L_{is}$) one can see that the terms containing infrared singularities and double–logarithmic terms $\sim L_s^2$, are associated with the factor, equal to the corresponding Born contribution. This is valid for all types of contributions.

The phase volume

$$\mathrm{d}\Gamma = \frac{\mathrm{d}^3p_1'\mathrm{d}^3p_2'\mathrm{d}^3k_1}{\varepsilon_1'\varepsilon_2'\omega_1}\delta^{(4)}(p_1 + p_2 - p_1' - p_2' - k_1)$$

can be transformed in different ways [9]. We introduce variables (see Eq. (32))

$$y_i = \frac{\varepsilon_i'}{\varepsilon}, \quad c_i = \cos\theta_i', \quad \theta_i' = \widehat{\mathbf{p}_1, \mathbf{p}_i'}, \quad 0 < y_i < 1, \quad -1 < c_{1,2} < 1, \tag{34}$$

which parametrize the kinematics of the outgoing particles (these do not include common degree of freedom — the rotation around the beam axes). Then, the phase volume takes the form:

$$\mathrm{d}\Gamma = \frac{\pi s}{2\sqrt{D(y_1, y_2, c_1, c_2)}}\mathrm{d}y_1\mathrm{d}y_2\mathrm{d}c_1\mathrm{d}c_2\Theta(y_1 + y_2 - 1)\Theta(D(y_1, y_2, c_1, c_2)), \tag{35}$$

$$D(y_1, y_2, c_1, c_2) = \rho^2 - c_1^2 - c_2^2 - 2c_{1'2'}c_1c_2, \quad \rho^2 = 2(1 - c_{1'2'})\frac{(1-y_1)(1-y_2)}{y_1y_2}.$$

The allowed region of integration is the triangle in y_1, y_2 plane and the interior of the ellipse $D > 0$ in c_1, c_2 plane.

Let us discuss now the relation of our result to the renormalization group approach. The dependence on $\Delta\varepsilon/\varepsilon$ in (33) will disappear when one takes into account the two hard photons emission. The leading contribution arises from the kinematics when the second hard photon is emitted close to the direction of motion of one of the incoming or outgoing particles. It reads:

$$\begin{aligned} \mathrm{d}\sigma^{\text{hard}} &= \frac{\alpha}{2\pi}L_s\Big[\frac{1+z^2}{1-z}\Big(\mathrm{d}\sigma_0(zp_1, p_2, p_1', p_2') + \mathrm{d}\sigma_0(p_1, zp_2, p_1', p_2')\Big)\mathrm{d}z \\ &\quad + \frac{1+z_1^2}{1-z_1}\mathrm{d}\sigma_0\Big(p_1, p_2, \frac{p_1'}{z_1}, p_2'\Big)\mathrm{d}z_1 + \frac{1+z_2^2}{1-z_2}\mathrm{d}\sigma_0\Big(p_1, p_2, p_1', \frac{p_2'}{z_2}\Big)\mathrm{d}z_2\Big], \\ &\quad z = 1 - x_2, \quad z_i = \frac{y_i}{y_i + x_2}, \quad x_2 = \frac{\omega_2}{\varepsilon}. \end{aligned} \tag{36}$$

The energy fraction of the additional photon varies within the limits $\Delta\varepsilon/\varepsilon < x_2 = \omega_2/\varepsilon < 1$. This formula agrees with the Drell–Yan form of radiative Bhabha scattering (with switched off vacuum polarization)

$$\mathrm{d}\sigma(p_1, p_2, p_1', p_2') = \int \mathrm{d}x_1\mathrm{d}x_2\mathcal{D}(x_1)\mathcal{D}(x_2)\mathrm{d}\sigma_0\Big(x_1p_1, x_2p_2, \frac{p_1'}{z_1}, \frac{p_2'}{z_2}\Big)\mathcal{D}(z_1)\mathcal{D}(z_2)\mathrm{d}z_1\mathrm{d}z_2, \tag{37}$$

where the non–singlet structure functions $\mathcal{D}$ [10] are

$$\begin{aligned} \mathcal{D}(z) &= \delta(1-z) + \frac{\alpha}{2\pi}L\mathcal{P}^{(1)}(z) + \Big(\frac{\alpha}{2\pi}L\Big)^2\frac{1}{2!}\mathcal{P}^{(2)}(z) + \ldots, \\ \mathcal{P}^{(1)}(z) &= \lim_{\Delta\to 0}\Big[\frac{1+z^2}{1-z}\Theta(1-z-\Delta) + \delta(1-z)\Big(2\ln\Delta + \frac{3}{2}\Big)\Big]. \end{aligned} \tag{38}$$

In our calculations we see explicitly a factorization of the terms containing double logarithmic contributions and infrared single logarithmic ones, which arise from

G– and ΓΠ–type FD. To be precise, the corresponding contributions to the cross section have a structure of the Born cross section (7). But we faced with that the above claim fails to be true for the terms containing single logarithms. Hence, the Drell–Yan form (37) is not valid in this case, and the factorization theorem is broken down, because the mass singularities (large logarithms) do not factorize before the Born structure. That is because of plenty of different–type amplitudes and kinematical variables, which describe our process. The reason for the violation of a naive usage of factorization in the Drell–Yan form has presumably the same origin with that found in paper [11], where the authors claimed, that it is necessary to study independently the renormalization group behaviour of leading logarithms before different amplitudes of the same process. Note, that in the case of the $e\mu \to e\mu\gamma$ reaction, which can be easily extracted from our results, the factorization does take place. We see also from (33) that the factorization will take place if all the logarithmic terms become equal, i.e., $\ln(s_1/m^2) = \ln(s/m^2) = \ldots$. The source for the violation of the factorization theorem, we found, might have a relation to some of the ones, have been found in other problems [12].

Numerical estimation (see Table 1) for Φ–factory energy range ($\sqrt{s} \simeq 1$ GeV) shows that the contributions of the non–leading terms coming from virtual and soft real photons emission could reach 35%. The process of additional hard photon emission will also contribute to Δ_L and Δ. To get an explicit form of that correction, one has to take into account a definite experimental setup.

Obviously, an analogous phenomenon of the factorization theorem violation takes place in QCD in processes like $q\bar{q} \to q\bar{q}g$ and $q\bar{q} \to q\bar{q}\gamma$. A consistent investigation of the latter processes, with taking into account the phenomenon found, can give a certain correction to predictions for large–angle jet production and direct hard photon emission at proton–antiproton colliders.

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Appendix A

Loop integrals for G–type Feynman diagrams

In this and the following appendices we used partially the results of our previous work [13] and refer to it for further details. After this digression let us turn to the

problem. Two types of FD require different approaches. For the set of FD, labeled as *glasses* (G), only three independent external momenta are relevant due to the conservation law: $p_1 + q = p_1' + k_1$. Choosing p_1, p_1', q we use the notation:

$$
\begin{aligned}
J_{ijk} &= \int \frac{\mathrm{d}^4k}{\mathrm{i}\pi^2} \frac{1}{(i)(j)(k)}, \qquad J_{012q} = \int \frac{\mathrm{d}^4k}{\mathrm{i}\pi^2} \frac{1}{(0)(1)(2)(q)}, \\
J^{\mu}_{ijk} &= \int \frac{\mathrm{d}^4k}{\mathrm{i}\pi^2} \frac{k^{\mu}}{(i)(j)(k)} = a_{ijk}p_1^{\mu} + b_{ijk}p_1'^{\mu} + c_{ijk}q^{\mu}, \\
J^{\mu\nu}_{ij\ldots} &= \int \frac{\mathrm{d}^4k}{\mathrm{i}\pi^2} \frac{k^{\mu}k^{\nu}}{ij\ldots} = g^T_{ij\ldots}g^{\mu\nu} + a^T_{ij\ldots}p_1^{\mu}p_1^{\nu} + b^T_{ij\ldots}p_1'^{\mu}p_1'^{\nu} + c^T_{ij\ldots}q^{\mu}q^{\nu} \\
&+ \alpha^T_{ij\ldots}(p_1p_1')^{\mu\nu} + \beta^T_{ij\ldots}(p_1q)^{\mu\nu} + \gamma^T_{ij\ldots}(p_1'q)^{\mu\nu}, \\
J^{\mu\nu\lambda}_{012q} &= \int \frac{\mathrm{d}^4k}{\mathrm{i}\pi^2} \frac{k^{\mu}k^{\nu}k^{\lambda}}{(0)(1)(2)(q)} = K_{g1}(gp_1)^{\mu\nu\lambda} + K_{g2}(gp_1')^{\mu\nu\lambda} + K_{gq}(gq)^{\mu\nu\lambda} \\
&+ K_{111}p_1^{\mu}p_1^{\nu}p_1^{\lambda} + K_{222}p_1'^{\mu}p_1'^{\nu}p_1'^{\lambda} + K_{qqq}q^{\mu}q^{\nu}q^{\lambda} + K_{112}(p_1^2p_1')^{\mu\nu\lambda} \\
&+ K_{122}(p_1p_1'^2)^{\mu\nu\lambda} + K_{11q}(p_1^2q)^{\mu\nu\lambda} + K_{1qq}(p_1q^2)^{\mu\nu\lambda} + K_{22q}(p_1'^2q)^{\mu\nu\lambda} \\
&+ K_{2qq}(p_1'q^2)^{\mu\nu\lambda} + K_{12q}(p_1p_1'q)^{\mu\nu\lambda},
\end{aligned} \tag{A.1}
$$

where the inverse propagators are

$$
\begin{aligned}
&(0) = k^2 - \lambda^2, \ (1) = (p_1 - k)^2 - m^2, \\
&(2) = (p_1' - k)^2 - m^2, \ (q) = (p_1' - q - k)^2 - m^2.
\end{aligned} \tag{A.2}
$$

λ is a fictitious photon mass. The symmetrized tensor structures are defined as follows:

$$
\begin{aligned}
(pq)^{\mu\nu} &= p^{\mu}q^{\nu} + p^{\nu}q^{\mu}, \qquad (p^2q)^{\mu\nu\lambda} = p^{\mu}p^{\nu}q^{\lambda} + p^{\mu}p^{\lambda}q^{\nu} + p^{\nu}p^{\lambda}q^{\mu}, \\
(gp)^{\mu\nu\rho} &= g^{\mu\nu}p^{\rho} + g^{\mu\rho}p^{\nu} + g^{\rho\nu}p^{\mu}, \\
(pqr)^{\mu\nu\lambda} &= p^{\mu}q^{\nu}r^{\lambda} + p^{\mu}q^{\lambda}r^{\nu} + p^{\nu}q^{\mu}r^{\lambda} + p^{\nu}q^{\lambda}r^{\mu} + p^{\lambda}q^{\mu}r^{\nu} + p^{\lambda}q^{\nu}r^{\mu}.
\end{aligned}
$$

The vector and tensor integrals can be calculated by multiplying both sides of the expression (A.2) by vectors p_1^{μ}, $p_1'^{\mu}$ and q^{μ}. Then one has to use the relations

$$
2p_1k = (0) - (1), \quad 2k_1k = (q) - (1) + \chi_1, \quad 2p_1'k = (0) - (2). \tag{A.3}
$$

and compare the coefficients before vector components on both sides.

Considering the vector and tensor integrals with three denominators, we use ultra violet divergent integrals with two denominators. Using the Feynman trick to join denominators, they can be expressed as

$$
\begin{aligned}
&\int \frac{\mathrm{d}^4k}{\mathrm{i}\pi^2} \frac{1}{[(k-b)^2 - d]^2} = \ln\frac{\Lambda^2}{d} - 1, \\
&\int \frac{\mathrm{d}^4k}{\mathrm{i}\pi^2} \frac{k^{\mu}}{[(k-b)^2 - d]^2} = b^{\mu}\left(\ln\frac{\Lambda^2}{d} - \frac{3}{2}\right).
\end{aligned} \tag{A.4}
$$

We put here the complete list of these integrals (in approximation Eq.(2)):

$$
\begin{aligned}
J_{01} &= L_\Lambda + 1, \quad J_{1q} = L_\Lambda - 1, \quad J_{2q} = L_\Lambda - L_t + 1,\\
J_{0q} &= L_\Lambda - L_{\chi_1} + 1, \quad J_{12} = L_\Lambda - L_{t_1} + 1, \quad J_{02} = L_\Lambda + 1,\\
J^\mu_{01} &= \frac{1}{2} p_1^\mu \Big(L_\Lambda - \frac{1}{2} \Big), \quad J^\mu_{1q} = (p_1^\mu - \frac{1}{2} k_1^\mu) \Big(L_\Lambda - \frac{3}{2} \Big),\\
J^\mu_{2q} &= \frac{1}{2} (p_1^\mu - k_1^\mu + p_1'^\mu) \Big(L_\Lambda - L_t + \frac{1}{2} \Big), \quad J^\mu_{0q} = (p_1^\mu - k_1^\mu) \Big(\frac{1}{2} L_\Lambda - \frac{1}{2} L_{\chi_1} + \frac{1}{4} \Big),\\
J^\mu_{12} &= (p_1^\mu + p_1'^\mu) \Big(\frac{1}{2} L_\Lambda - \frac{1}{2} L_{t_1} + \frac{1}{4} \Big), \quad J^\mu_{02} = p_1'^\mu \Big(\frac{1}{2} L_\Lambda - \frac{1}{4} \Big).
\end{aligned}
\tag{A.5}
$$

where

$$
L_q = L_t = \ln \frac{-t}{m^2}, \quad L_{\chi_1} = \ln \frac{\chi_1}{m^2}, \quad L_{\chi_1'} = \ln \frac{\chi_1'}{m^2} - i\pi, \quad L_\Lambda = \ln \frac{\Lambda^2}{m^2}.
$$

The scalar integrals with three denominators read

$$
\begin{aligned}
J_{012} &= \frac{1}{2t_1} \Big[-2 L_\lambda L_{t_1} + L_{t_1}^2 - \frac{\pi^2}{3} \Big], \qquad J_{12q} = \frac{1}{2(\chi_1' - \chi_1)} (L_t^2 - L_{t_1}^2),\\
J_{02q} &= \frac{1}{t + \chi_1} \Big[L_t (L_t - L_{\chi_1}) + \frac{1}{2} (L_t - L_{\chi_1})^2 + 2 \mathrm{Li}_2 \Big(1 + \frac{\chi_1}{t} \Big) \Big],\\
J_{01q} &= -\frac{1}{2\chi_1} L_{\chi_1}^2 - \frac{\pi^2}{3\chi_1}, \quad \mathrm{Li}_2(z) \equiv - \int_0^z \frac{dx}{x} \ln(1-x), \quad L_\lambda = \ln \frac{\lambda^2}{m^2}.
\end{aligned}
\tag{A.6}
$$

The coefficients for vector integrals with three denominators are

$$
\begin{aligned}
a_{012} &= b_{012} = \frac{1}{t_1} L_{t_1}, \quad c_{012} = 0,\\
a_{01q} &= J_{01q} + \frac{2}{\chi_1} (L_{\chi_1} - 1), \quad b_{01q} = -c_{01q} = \frac{1}{\chi_1} (-L_{\chi_1} + 2),\\
a_{02q} &= 0, \quad b_{02q} = \frac{\chi_1}{\chi_1 + t} J_{02q} + \frac{2 t L_t}{(\chi_1 + t)^2} + \frac{(\chi_1 - t) L_{\chi_1}}{(\chi_1 + t)^2}, \quad c_{02q} = \frac{L_{\chi_1} - L_t}{\chi_1 + t},\\
a_{12q} &= \frac{t}{t - t_1} J_{12q} + \frac{(t + t_1) L_{t_1} - 2 t L_t}{(t - t_1)^2} + \frac{2}{t - t_1}, \quad b_{12q} = J_{12q} - a_{12q},\\
c_{12q} &= \frac{t_1}{t - t_1} J_{12q} + \frac{-(t + t_1) L_t + 2 t_1 L_{t_1}}{(t - t_1)^2} + \frac{2}{t - t_1},
\end{aligned}
\tag{A.7}
$$

The tensor integrals for G-type FD (see Eq.(A.1)) have the following form:

$$
\begin{aligned}
g^T_{012} &= \frac{1}{4} (L_\Lambda - L_{t_1}) + \frac{3}{8}, \quad a^T_{012} = b^T_{012} = \frac{1}{2t_1} (L_{t_1} - 1), \quad \alpha^T_{012} = \frac{1}{2t_1},\\
c^T_{012} &= \beta^T_{012} = \gamma^T_{012} = 0,
\end{aligned}
\tag{A.8}
$$

$$
\begin{aligned}
g^T_{01q} &= \frac{1}{4} (L_\Lambda - L_{\chi_1}) + \frac{3}{8}, \quad a^T_{01q} = J_{01q} + \frac{3}{\chi_1} L_{\chi_1} - \frac{9}{2\chi_1},\\
b^T_{01q} &= c^T_{01q} = -\gamma^T_{01q} = -\frac{1}{2\chi_1} (L_{\chi_1} - 2), \quad \beta^T_{01q} = -\alpha^T_{01q} = \frac{1}{2\chi_1} (L_{\chi_1} - 3),
\end{aligned}
$$

$$
\begin{aligned}
g^T_{02q} &= \frac{1}{4} L_\Lambda - \frac{\chi_1}{4(t + \chi_1)} L_{\chi_1} - \frac{t}{4(t + \chi_1)} L_t + \frac{3}{8}\\
b^T_{02q} &= \frac{3\chi_1^2 - 4t\chi_1 - t^2}{2(t + \chi_1)^3} L_{\chi_1} + \frac{t(t + 4\chi_1)}{(t + \chi_1)^3} L_t + \frac{t - \chi_1}{2(t + \chi_1)^2} + \frac{\chi_1^2}{(t + \chi_1)^2} J_{02q},\\
c^T_{02q} &= \frac{L_t - L_{\chi_1}}{2(t + \chi_1)}, \quad \gamma^T_{02q} = \frac{t + 2\chi_1}{2(t + \chi_1)^2} (L_{\chi_1} - L_t) - \frac{1}{2(t + \chi_1)},\\
a^T_{02q} &= \alpha^T_{02q} = \beta^T_{02q} = 0,
\end{aligned}
$$

$$
\begin{aligned}
g^T_{12q} &= \frac{1}{4} L_\Lambda + \frac{t_1 L_{t_1} - t L_t}{4(t - t_1)} + \frac{3}{8},\\
a^T_{12q} &= \frac{3t^2 + 4tt_1 - t_1^2}{2(t - t_1)^3} L_{t_1} - \frac{3t^2}{(t - t_1)^3} L_t + \frac{4t - t_1}{(t - t_1)^2} + \frac{t^2}{(t - t_1)^2} J_{12q},\\
b^T_{12q} &= \frac{-t^2 + 4tt_1 + 3t_1^2}{2(t - t_1)^3} L_{t_1} + \frac{t(t - 4t_1)}{(t - t_1)^3} L_t + \frac{3t_1}{(t - t_1)^2} + \frac{t_1^2}{(t - t_1)^2} J_{12q},\\
c^T_{12q} &= \frac{3t_1^2}{(t - t_1)^3} L_{t_1} + \frac{t^2 - 4tt_1 - 3t_1^2}{2(t - t_1)^3} L_t + \frac{4t_1 - t}{(t - t_1)^2} + \frac{t_1^2}{(t - t_1)^2} J_{12q},\\
\alpha^T_{12q} &= -\frac{t^2 + 4tt_1 + t_1^2}{2(t - t_1)^3} L_{t_1} + \frac{t(t + 2t_1)}{(t - t_1)^3} L_t - \frac{2t + t_1}{(t - t_1)^2} - \frac{tt_1}{(t - t_1)^2} J_{12q},\\
\beta^T_{12q} &= \frac{t_1(5t + t_1)}{2(t - t_1)^3} L_{t_1} - \frac{t(t + 5t_1)}{2(t - t_1)^3} L_t + \frac{3(t + t_1)}{2(t - t_1)^2} + \frac{tt_1}{(t - t_1)^2} J_{12q},\\
\gamma^T_{12q} &= -\frac{t_1(t + 5t_1)}{2(t - t_1)^3} L_{t_1} + \frac{-t^2 + 5tt_1 + 2t_1^2}{2(t - t_1)^3} L_t + \frac{t - 7t_1}{2(t - t_1)^2} - \frac{t_1^2}{(t - t_1)^2} J_{12q}.
\end{aligned}
\tag{A.9}
$$

Four-denominator scalar integral reads:

$$
J_{012q} = -\frac{1}{t_1 \chi_1} \Big[-L_\lambda L_{t_1} + 2 L_{t_1} L_{\chi_1} - L_t^2 - 2 \mathrm{Li}_2 \Big(1 - \frac{t}{t_1} \Big) - \frac{\pi^2}{6} \Big].
\tag{A.10}
$$

Vector 4-denominator integrals are:

$$
\begin{aligned}
a_{012q} &= \frac{1}{d} \Big[-(t\chi_1' + t_1\chi_1) J_{12q} + (t + \chi_1)^2 J_{02q} - \chi_1 (\chi_1' - t_1) J_{01q} - t_1 (t + \chi_1) Y \Big],\\
b_{012q} &= \frac{1}{d} \Big[(t_1\chi_1' + t\chi_1) J_{12q} - (tt_1 + \chi_1'\chi_1) J_{02q} + \chi_1 (\chi_1 - t_1) J_{01q} + t_1 (t_1 - \chi_1) Y \Big],\\
c_{012q} &= \frac{1}{d} \Big[-t_1 (\chi_1' + \chi_1) J_{12q} + t_1 (t + \chi_1) J_{02q} + \chi_1 t_1 J_{01q} - t_1^2 Y \Big].
\end{aligned}
\tag{A.11}
$$

$$Y = J_{012} + \chi_1 J_{012q}, d = -2t_1\chi_1\chi_1'. \tag{A.12}$$

2-rank 4-denominator tensors are:

$$
\begin{aligned}
g^T_{012q} &= \frac{1}{2}(J_{12q} - \chi_1 c_{012q}),\\
a^T_{012q} &= \frac{1}{d}\Big[(t+\chi_1)^2(J_{12q} - \chi_1 c_{012q}) - (\chi_1 t_1 + \chi_1' t)a_{12q} + \chi_1(t_1 - \chi_1')a_{01q}\\
&- t_1(t+\chi_1)(a_{012} + \chi_1 a_{012q})\Big],\\
b^T_{012q} &= \frac{1}{d}\Big[(t_1-\chi_1)^2(J_{12q} - \chi_1 c_{012q}) + (\chi_1' t_1 + \chi_1 t)b_{12q} + \chi_1(\chi_1 - t_1)b_{01q}\\
&- (t_1 t + \chi_1\chi_1')b_{02q} + t_1(t_1 - \chi_1)(a_{012} + \chi_1 b_{012q})\Big],\\
\gamma^T_{012q} &= \frac{1}{d}\Big[-t_1(t_1-\chi_1)(J_{12q} - 2\chi_1 c_{012q}) + (\chi_1' t_1 + \chi_1 t)c_{12q} - (tt_1 + \chi_1\chi_1')c_{02q}\\
&+ \chi_1(t_1 - \chi_1)b_{01q}\Big],\\
\alpha^T_{012q} &= \frac{1}{d}\Big[-(tt_1 + \chi_1\chi_1')(J_{12q} - \chi_1 c_{012q}) + (\chi_1' t_1 + \chi_1 t)a_{12q} - \chi_1(\chi_1 - t_1)a_{01q}\\
&+ t_1(t_1 - \chi_1)(a_{012} + \chi_1 a_{012q})\Big],\\
\beta^T_{012q} &= \frac{1}{d}\Big[t_1(t_1+\chi_1')(J_{12q} - 2\chi_1 c_{012q}) - (\chi_1 t_1 + \chi_1' t)c_{12q} + (\chi_1 + t)^2 c_{02q}\\
&+ \chi_1(\chi_1' - t_1)b_{01q}\Big],\\
c^T_{012q} &= \frac{1}{t}\Big[J_{12q} - 4g^T_{012q} + t_1\alpha^T_{012q} + (\chi_1' - t_1)\beta^T_{012q} + t\gamma^T_{012q}\Big].
\end{aligned} \tag{A.13}
$$

We put now the coefficients of 3-rank tensor structures:

$$
\begin{aligned}
K_{1g} &= \frac{1}{d}[-(t+\chi_1)^2 A_1 - t_1(t+\chi_1)A_8 + (tt_1 + \chi_1\chi_1')A_{18}],\\
K_{2g} &= \frac{1}{d}[(tt_1 + \chi_1\chi_1')A_1 + t_1(t_1-\chi_1)A_8 - (t_1-\chi_1)^2 A_{18}],\\
K_{qg} &= \frac{1}{d}[-t_1(t+\chi_1)A_1 - t_1^2 A_8 + t_1(t_1-\chi_1)A_{18}],\\
K_{111} &= \frac{1}{d}[-(t+\chi_1)^2 A_2 - t_1(t+\chi_1)A_9 + (tt_1 + \chi_1\chi_1')A_{19}],\\
K_{112} &= \frac{1}{d}[(tt_1 + \chi_1\chi_1')A_2 + t_1(t_1-\chi_1)A_9 - (t_1-\chi_1)^2 A_{19}],\\
K_{11q} &= \frac{1}{d}[-t_1(t+\chi_1)A_2 - t_1^2 A_9 + t_1(t_1-\chi_1)A_{19}],\\
K_{12q} &= \frac{1}{t+\chi_1}[t_1 K_{112} + \alpha^T_{12q} - \alpha^T_{01q} - 2K_{1g}],
\end{aligned}
$$

$$
\begin{aligned}
K_{1qq} &= \frac{1}{t+\chi_1}[t_1 K_{11q} + \beta^T_{12q} - \beta^T_{01q}],\\
K_{qqq} &= \frac{1}{t+\chi_1}[t_1 K_{1qq} + c^T_{12q} - c^T_{01q}],\\
K_{122} &= -\frac{1}{t_1}[(t_1-\chi_1)K_{12q} + \alpha^T_{12q} - \alpha^T_{01q} - 2K_{2g}],\\
K_{2qq} &= -\frac{1}{t_1}[(t_1-\chi_1)K_{qqq} + c^T_{12q} - c^T_{02q}],\\
K_{22q} &= -\frac{1}{t_1}[(t_1-\chi_1)K_{2qq} + \gamma^T_{12q} - \gamma^T_{02q}],\\
K_{222} &= -\frac{1}{t_1}[(t_1-\chi_1)K_{22q} + b^T_{12q} - b^T_{02q}],
\end{aligned} \tag{A.14}
$$

where

$$
\begin{aligned}
A_1 &= g^T_{12q} - g^T_{02q}, \quad A_{18} = g^T_{012} - g^T_{01q} + \chi_1 g^T_{012q}, \quad A_8 = g^T_{12q} - g^T_{01q},\\
A_2 &= a^T_{12q} - 4K_{1g}, \quad A_{19} = a^T_{012} - a^T_{01q} + \chi_1 a^T_{012q}, \quad A_9 = a^T_{12q} - a^T_{01q}.
\end{aligned} \tag{A.15}
$$

We give below some checking equations for coefficients before tensor structures of G-type integrals. The complete checking system can be obtained by contraction of general tensor expansion with relevant vectors, simplifying the numerators of the integrand and using a set of vector integrals given above. Additional check can be inferred by contraction with metric tensor. In this case the scalar integrals should be used. The complete set of 10 equations for the 2-rank tensor and 24 equations for the 3-rank 4-denominator tensor integrals for the G type was convinced to be fulfilled. For definiteness we give four equations of such a type, obtained by contraction with metric tensor. They are:

$$
\begin{aligned}
4g^T_{012q} + tc^T_{012q} - t_1\alpha^T_{012q} + (\chi_1 - t_1)\beta^T_{012q} + (t+\chi_1)\gamma^T_{012q} &= J_{12q},\\
6K_{1g} - t_1 K_{112} + (\chi_1 - t_1)K_{11q} + tK_{1qq} + (t+\chi_1)K_{12q} &= a_{12q},\\
6K_{2g} - t_1 K_{122} + (\chi_1 + t)K_{22q} + tK_{2qq} + (\chi_1 - t_1)K_{12q} &= b_{12q},\\
6K_{qg} + tK_{qqq} + (\chi_1 - t_1)K_{1qq} + (t+\chi_1)K_{2qq} - t_1 K_{12q} &= c_{12q}.
\end{aligned} \tag{A.16}
$$

Another indirect check is the absence of infrared divergence containing terms in all the vector and tensor integrals.

Appendix B

Loop integrals for B–type Feynman diagrams

We use here the following set of denominators:

$$
\begin{aligned}
(1) &= (p_1 - k)^2 - m^2, \quad (2) = (p_1 - k_1 - k)^2 - m^2, \quad (3) = (p_2 + k)^2 - m^2,\\
(4) &= (p_1 - k_1 - p_1' - k)^2 - \lambda^2, \quad (5) = k^2 - \lambda^2.
\end{aligned} \tag{B.1}
$$

4-momentum conservation law we use reads $p_1 + p_2 = p_1' + p_2' + k_1$. Scalar products of the loop momentum k with the external 4-vectors can be expressed in terms of the denominators:

$$
\begin{aligned}
&2p_1k = (5) - (1), \quad 2p_2k = (3) - (5), \quad 2p_1'k = (4) - (2) - t - \chi_1, \\
&2k_1k = (2) - (1) + \chi_1, \quad 2p_2'k = (3) - (4) + t.
\end{aligned} \qquad (\mathrm{B.2})
$$

Using these relations one can consider only one type of integrals with 5 denominators, namely the scalar one. Using the elegant technique developed in the paper of Van–Neerven and Vermasseren [14] it can be expressed in the form:

$$
\begin{aligned}
J_{12345} &= -\frac{1}{D}[D_1J_{2345} + D_2J_{1345} + D_3J_{1245} + D_4J_{1235} + D_5J_{1234}], \quad D = 2ss_1t\chi_1\chi_1', \\
D_1 &= s_1t[-t(s-s_1) - s\chi_1 - s_1\chi_1' - \chi_1\chi_1'], \\
D_2 &= st[t(s-s_1) + s\chi_1 + s_1\chi_1' - \chi_1\chi_1'], \\
D_3 &= \chi_1\chi_1'[-t(s+s_1) - s\chi_1 + s_1\chi_1' + \chi_1\chi_1'], \\
D_4 &= s\chi_1[t(s-s_1) + s\chi_1 - s_1\chi_1' - \chi_1\chi_1'], \\
D_1 &= s_1\chi_1'[t(s-s_1) - s\chi_1 + s_1\chi_1' + \chi_1\chi_1'].
\end{aligned} \qquad (\mathrm{B.3})
$$

It is interesting to note that the method described above to calculate the coefficients of the tensor structures cannot be applied to the tensor integrals with 5 denominators given above. Some additional information is needed to close the system of algebraic equations.

We mention a trick which permits to obtain additional equations for vector and tensor integrals whose denominators do not contain the term $k^2 - \lambda^2$. It consists in shifting a loop momentum. Thus, for J^μ_{1234} we have

$$
\int \frac{\mathrm{d}^4k}{\mathrm{i}\pi^2}\frac{k}{(1)(2)(3)(4)}\bigg|_{k=p_1-\tilde{k}} = \int \frac{\mathrm{d}^4\tilde{k}}{\mathrm{i}\pi^2}\frac{(p_1-\tilde{k})}{(\tilde{1})(\tilde{2})(\tilde{3})(\tilde{4})} = p_1J_{1234} + \tilde{a}(p_1+p_2) + \tilde{c}k_1 + \tilde{d}p_1',
$$

$$
(\tilde{1}) = \tilde{k}^2 - m^2, \ (\tilde{2}) = (\tilde{k}-k_1)^2 - m^2, \ (\tilde{3}) = (p_1+p_2+\tilde{k})^2 - m^2, \ (\tilde{4}) = (\tilde{k}-p_1'-k_1)^2.
$$

The comparison of right hand side of this equation with the standard expansion

$$
J^\mu_{1234} = (ap_1 + bp_2 + ck_1 + dp_1')^\mu_{1234}
$$

leads to the new relation:

$$
a_{1234} = J_{1234} = b_{1234}.
$$

Analogous useful relations can be obtained for tensor integrals as well. We put below the relevant scalar, vector and tensor integrals with 3 and 4 denominators from (B.1) and introduce the parametrization:

$$
\begin{aligned}
J_{ij\ldots} &= \int\frac{\mathrm{d}^4k}{\mathrm{i}\pi^2}\frac{1}{(i)(j)\ldots}, \quad J^\mu_{ij\ldots} = \int\frac{\mathrm{d}^4k}{\mathrm{i}\pi^2}\frac{k^\mu}{(i)(j)\ldots} = (a_{ij\ldots}p_1 + b_{ij\ldots}p_2 \\
&+ c_{ij\ldots}k_1 + d_{ij\ldots}p_1')^\mu, \\
J^{\mu\nu}_{ij\ldots} &= \int\frac{\mathrm{d}^4k}{\mathrm{i}\pi^2}\frac{k^\mu k^\nu}{(i)(j)\ldots} = (g^Tg + a^Tp_1p_1 + b^Tp_2p_2 + c^Tk_1k_1 + d^Tp_1'p_1' \\
&+ \alpha^T(p_1p_2) + \beta^T(p_1k_1) + \gamma^T(p_1p_1') + \rho^T(p_1'p_2) + \sigma^T(k_1p_2) + \tau^T(p_1'k_1))^{\mu\nu}_{ij\ldots}.
\end{aligned} \qquad (\mathrm{B.4})
$$

Vector 3-denominator integrals are:

$$
a_{245} = -c_{245} = J_{245} + \frac{L_{\chi_1} - L_t}{t+\chi_1}, \quad b_{245} = 0,
$$

$$
d_{245} = -\frac{\chi_1}{t+\chi_1}J_{245} - \frac{2\chi_1L_{\chi_1}}{(t+\chi_1)^2} + \frac{(\chi_1-t_1)L_t}{(t+\chi_1)^2},
$$

$$
a_{145} = -\frac{t}{\chi_1-t_1}J_{145} + \frac{2\chi_1'L_{\chi_1'}}{(t_1-\chi_1)^2} - \frac{t+\chi_1'}{(\chi_1-t_1)^2}L_t,
$$

$$
b_{145} = 0, \ c_{145} = d_{145} = \frac{L_t - L_{\chi_1'}}{\chi_1' - t},
$$

$$
a_{345} = -c_{345} = -d_{345} = \frac{L_t}{t}, \ b_{345} = -J_{345} + \frac{2L_t}{t},
$$

$$
a_{125} = J_{125} + \frac{L_{\chi_1}}{\chi_1}, \ b_{125} = d_{125} = 0, \ c_{125} = \frac{L_{\chi_1} - 2}{\chi_1},
$$

$$
a_{235} = -c_{235} = \frac{L_{s_1} - L_{\chi_1}}{s - \chi_2}, \ d_{235} = 0,
$$

$$
b_{235} = -\frac{\chi_1}{s-\chi_2}J_{235} - \frac{2\chi_1L_{\chi_1}}{(s-\chi_2)^2} + \frac{\chi_1 - s_1}{(s-\chi_2)^2}L_{s_1},
$$

$$
a_{135} = -b_{135} = \frac{L_s}{s}, \ c_{135} = d_{135} = 0,
$$

$$
a_{234} = -c_{234} = J_{234} - \frac{L_{s_1}}{s_1}, \ b_{234} = -\frac{L_{s_1}}{s_1}, \ d_{234} = -J_{234} + \frac{2L_{s_1}}{s_1},
$$

$$
a_{123} = J_{123} + b_{123}, \ b_{123} = \frac{L_{s_1} - L_s}{s - s_1}, \ d_{123} = 0,
$$

$$
c_{123} = -\frac{s}{s-s_1}J_{123} - \frac{2}{s-s_1} + \frac{2sL_s}{(s-s_1)^2} - \frac{(s+s_1)L_{s_1}}{(s-s_1)^2},
$$

$$
a_{124} = J_{124}, \ b_{124} = 0, \ c_{124} = -J_{124} + \frac{L_{\chi_1'} - 2}{\chi_1'}, \ d_{124} = -\frac{L_{\chi_1'}}{\chi_1'},
$$

$$a_{134} = \frac{s}{s-\chi_1'} J_{134} + \frac{2\chi_1' L_{\chi_1'} - (s+\chi_1')L_s}{(s-\chi_1')^2}, \quad b_{134} = a_{134} - J_{134},$$

$$c_{134} = d_{134} = -\frac{s}{s-\chi_1'} J_{134} + \frac{-(\chi_1'+s)L_{\chi_1'} + 2sL_s}{(s-\chi_1')^2}. \tag{B.5}$$

Vector integrals with 4 denominators read:

$$\begin{aligned}
a_{1245} &= \frac{\Delta_{3a}}{\Delta_3}, \quad b_{1245} = 0, \quad c_{1245} = \frac{\Delta_{3c}}{\Delta_3}, \\
d_{1245} &= \frac{\Delta_{3d}}{\Delta_3}, \quad \Delta_3 = 2t_1\chi_1\chi_1', \\
\Delta_{3a} &= \chi_1'[\chi_1(2t_1+\chi_1')J_{1245} + \chi_1' J_{124} - \chi_1 J_{125} - (t+\chi_1)J_{245} + (t_1+\chi_1)J_{145}], \\
\Delta_{3c} &= t_1[-\chi_1\chi_1' J_{1245} + \chi_1' J_{124} + \chi_1 J_{125} - (t+\chi_1)J_{245} + (t-\chi_1')J_{145}], \\
\Delta_{3d} &= \chi_1[-\chi_1\chi_1' J_{1245} - \chi_1' J_{124} + \chi_1 J_{125} + (\chi_1'-t_1)J_{245} + (t-\chi_1')J_{145}].
\end{aligned} \tag{B.6}$$

$$\begin{aligned}
a_{1235} &= \frac{\Delta_{4a}}{\Delta_4}, \quad b_{1235} = \frac{\Delta_{4b}}{\Delta_4}, \quad c_{1235} = \frac{\Delta_{4c}}{\Delta_4}, \\
d_{1235} &= 0, \quad \Delta_4 = 2s\chi_1\chi_2, \\
\Delta_{4a} &= \chi_2[s\chi_1 J_{1235} - (s-s_1)J_{123} - (s-\chi_2)J_{235} + \chi_1 J_{125} + sJ_{135}], \\
\Delta_{4b} &= \chi_1[s\chi_1 J_{1235} + (s-s_1)J_{123} - (s+\chi_2)J_{235} - \chi_1 J_{125} + sJ_{135}], \\
\Delta_{4c} &= s[-s\chi_1 J_{1235} + (\chi_2-\chi_1)J_{123} + (s-\chi_2)J_{235} + \chi_1 J_{125} - sJ_{135}].
\end{aligned} \tag{B.7}$$

$$\begin{aligned}
a_{1345} &= \frac{\Delta_{2a}}{\Delta_2}, \quad b_{1345} = \frac{\Delta_{2b}}{\Delta_2}, \quad c_{1345} = d_{1345} = \frac{\Delta_{2c}}{\Delta_2}, \quad \Delta_2 = 2stu, \\
\Delta_{2a} &= -st(s+t)J_{1345} + t(s+t)J_{345} + s(s+t)J_{135} + (ut - s\chi_1')J_{145} \\
&\qquad + (us - t\chi_1')J_{134}, \\
\Delta_{2b} &= -st(s+u)J_{1345} + t(s-u)J_{345} + s(s+u)J_{135} - (s+u)^2 J_{145} \\
&\qquad + (u\chi_1' - st)J_{134}, \\
\Delta_{2c} &= s[stJ_{1345} - tJ_{345} - sJ_{135} + (s+u)J_{145} + (t-u)J_{134}].
\end{aligned} \tag{B.8}$$

$$\begin{aligned}
a_{2345} &= -c_{2345} = \frac{\Delta_{1a}}{\Delta_1}, \quad b_{2345} = \frac{\Delta_{1b}}{\Delta_1}, \quad d_{2345} = \frac{\Delta_{1d}}{\Delta_1}, \\
\Delta_1 &= -2s_1u_1t, \\
\Delta_{1a} &= -s_1u_1tJ_{2345} - u_1(t+\chi_1)J_{245} - u_1s_1J_{234} + u_1(s-\chi_2)J_{235} + tu_1J_{345}, \\
\Delta_{1b} &= -s_1t(t+\chi_1)J_{2345} + (t+\chi_1)^2 J_{245} + s_1(t+\chi_1)J_{234} + (u_1\chi_1 + s_1t)J_{235} \\
&\qquad + t(u_1-s_1)J_{345}, \\
\Delta_{1c} &= -s_1t(s-\chi_2)J_{2345} + (u_1\chi_1 + s_1t)J_{245} + s_1(u_1-t)J_{234} + (s-\chi_2)^2 J_{235} \\
&\qquad + t(s-\chi_2)J_{345}.
\end{aligned} \tag{B.9}$$

$$\begin{aligned}
a_{1234} &= J_{1234} + \frac{\Delta_{5b}}{\Delta_5}, \quad b_{1234} = \frac{\Delta_{5b}}{\Delta_5}, \quad c_{1234} = -J_{1234} - \frac{\Delta_{5b}}{\Delta_5} + \frac{\Delta_{5c}}{\Delta_5}, \\
d_{1234} &= -J_{1234} + \frac{\Delta_{5a}}{\Delta_5} - \frac{\Delta_{5b}}{\Delta_5}, \quad \Delta_5 = 2s_1\chi_1'\chi_2', \quad \chi_2' = s - s_1 - \chi_1', \\
\Delta_{5a} &= \chi_2'[-(s-s_1)J_{123} + (s-\chi_1')J_{134} + \chi_1' J_{124} - s_1J_{234} + s_1\chi_1' J_{1234}], \\
\Delta_{5b} &= \chi_1'[(s-s_1)J_{123} + (2s_1 - s + \chi_1')J_{134} - \chi_1' J_{124} - s_1J_{234} + s_1\chi_1' J_{1234}], \\
\Delta_{5c} &= s_1[(\chi_2'-\chi_1')J_{123} - (s-\chi_1')J_{134} + \chi_1' J_{124} + s_1J_{234} - s_1\chi_1' J_{1234}].
\end{aligned} \tag{B.10}$$

We put now the tensor coefficients for B-type integrals with 4 denominators.

$$g^T_{1245} = \frac{1}{2}[2J_{124} - a_{124} - \chi_1 c_{1245} + (t+\chi_1)d_{1245}].$$

$$\begin{aligned}
a^T_{1245} &= \frac{1}{t_1\chi_1}[\chi_1'(-J_{124} + a_{124} - c_{145}) + t_1a_{145} - (t+\chi_1)a_{245} \\
&+ t_1\chi_1a_{1245} - \chi_1'(t+\chi_1)d_{1245}],
\end{aligned}$$

$$c^T_{1245} = \frac{1}{\chi_1\chi_1'}[t_1(-J_{124} + a_{124}) + \chi_1c_{125} + (t_1-\chi_1)c_{145} - \chi_1\chi_1'c_{1245}].$$

$$d^T_{1245} = \frac{1}{t_1\chi_1'}[\chi_1(-J_{124} + a_{124} - a_{245}) + (t_1-\chi_1)c_{145} - t_1d_{245} - \chi_1\chi_1'd_{1245}].$$

$$\beta^T_{1245} = \frac{1}{\chi_1}[-J_{124} + a_{124} + c_{145} + \chi_1c_{1245}].$$

$$\gamma^T_{1245} = \frac{1}{t_1}[J_{124} - a_{124} + a_{245} + c_{145} + (t+\chi_1)d_{1245}].$$

$$\tau^T_{1245} = \frac{1}{\chi_1'}[-J_{124} + a_{245} + \chi_1c_{1245} - (t+\chi_1)d_{1245}].$$

$$b^T_{1245} = \alpha^T_{1245} = \rho^T_{1245} = \sigma^T_{1245} = 0. \tag{B.11}$$

As a check one can use the result of contraction by the metric tensor:

$$4g^T_{1245} + \chi_1\beta^T_{1245} - t_1\gamma^T_{1245} + \chi_1'\tau^T_{1245} = J_{124}. \tag{B.12}$$

$$g^T_{1235} = \frac{1}{2}[2J_{123} - a_{123} + b_{123} - \chi_1c_{1235}].$$

$$a^T_{1235} = \frac{1}{s\chi_1}[\chi_2J_{123} - (\chi_1+\chi_2)a_{123} + \chi_1a_{125}) - \chi_1\chi_2c_{1235}].$$

$$b^T_{1235} = \frac{1}{s\chi_2}[\chi_1(J_{123} - a_{235}) + (\chi_1 + \chi_2)b_{123} - \chi_2 b_{235} - \chi_1^2 c_{1235}]\,,$$

$$c^T_{1235} = \frac{1}{\chi_1\chi_2}[s(J_{123} + b_{123}) - (s - \chi_2)a_{235} + \chi_2 c_{123} - s\chi_1 c_{1235}]\,,$$

$$\alpha^T_{1235} = \frac{1}{s}[-J_{123} + a_{123} - a_{235} - b_{123}],$$

$$\beta^T_{1235} = \frac{1}{\chi_1}[-J_{123} + a_{123} + \chi_1 c_{1235}],$$

$$\sigma^T_{1235} = \frac{1}{\chi_2}[-J_{123} + a_{235} - b_{123} + \chi_1 c_{1235}],$$

$$d^T_{1235} = \gamma^T_{1235} = \rho^T_{1235} = \tau^T_{1235} = 0\,. \qquad \text{(B.13)}$$

One of the checking relations here has the form

$$4g^T_{1235} + s\alpha^T_{1235} + \chi_1\beta^T_{1235} + \chi_2\sigma^T_{1235} = J_{123}. \qquad \text{(B.14)}$$

$$\begin{aligned}
g^T_{1345} &= \frac{1}{2}[J_{134} + tc_{1345}],\\
a^T_{1345} &= \frac{1}{st(\chi_1' - s - t)}[(s+t)^2 J_{134} + t(\chi_1' - s - t)a_{145} - (s(s+t) + t\chi_1')a_{134}\\
&+ \chi_1'(s+t)(c_{145} - c_{134}) + t(s+t)^2 c_{1345}],\\
b^T_{1345} &= \frac{1}{s}[b_{134} - b_{345} - (\chi_1' - t)\rho^T_{1345}],\\
c^T_{1345} &= d^T_{1345} = \tau^T_{1345} = \frac{1}{t(\chi_1' - s - t)}[(\chi_1' - t)(c_{145} - c_{134}) - s(b_{134} - tc_{1345})],\\
\alpha^T_{1345} &= \frac{1}{st(\chi_1' - s - t)}[-t(\chi_1' - s - t)a_{345} + \chi_1'(\chi_1' - t)(c_{145} - c_{134})\\
&- s\chi_1'(a_{134} - J_{134}) + st\chi_1' c_{1345}],\\
\beta^T_{1345} &= \gamma^T_{1345} = \frac{1}{t(\chi_1' - s - t)}[(s+t)(b_{134} - tc_{1345}) - \chi_1'(c_{145} - c_{134})],\\
\rho^T_{1345} &= \sigma^T_{1345} = \frac{1}{st(\chi_1' - s - t)}[-(\chi_1' - t)^2 c_{145} + t(\chi_1' - s - t)a_{345}\\
&+ (\chi_1'(\chi_1' - t) - st)c_{134} + s(\chi_1' - t)b_{134} - st(\chi_1' - t)c_{1345}]. \qquad \text{(B.15)}
\end{aligned}$$

The relation of the same type for the above coefficients reads:

$$4g^T_{1345} + \chi_1' c^T_{1345} + s\alpha^T_{1345} + (\chi_1 - t_1)\beta^T_{1345} + (\chi_2 - u_1)\sigma^T_{1345} = J_{134}. \qquad \text{(B.16)}$$

$$\begin{aligned}
g^T_{2345} &= \frac{1}{2}[J_{234} + \chi_1 a_{2345} + (t + \chi_1)d_{1345}],\\
a^T_{2345} &= c^T_{2345} = -\beta^T_{2345} = \frac{1}{s_1 t}[-ta_{345} - (s_1 + \chi_1)a_{235} + s_1 t a_{2345}],\\
b^T_{2345} &= \frac{1}{s_1 t(\chi_1 + s_1 + t)}[s_1 t(b_{235} - b_{345}) - \chi_1(\chi_1 + t)a_{235} - t(t + \chi_1)a_{345}\\
&- s_1 t(\chi_1 + t)b_{2345}],\\
d^T_{2345} &= \frac{1}{\chi_1 + s_1 + t}\Big[d_{245} - d_{234} - \frac{\chi_1 + s_1}{s_1 t(\chi_1 + s_1 + t)}\Big(s_1 t(a_{245} - a_{234})\\
&+ t(\chi_1 + s_1)a_{345} + (\chi_1 + s_1)^2 a_{235} - s_1 t(\chi_1 + s_1)a_{2345}\Big)\Big];\\
\alpha^T_{2345} &= -\sigma^T_{2345} = \frac{1}{s_1 t}[-\chi_1 a_{235} - ta_{345}],\\
\gamma^T_{2345} &= -\tau^T_{2345} = \frac{1}{s_1 t(\chi_1 + s_1 + t)}[s_1 t(a_{245} - a_{234}) + t(\chi_1 + s_1)a_{345}\\
&+ (\chi_1 + s_1)^2 a_{235} - s_1 t(\chi_1 + s_1)a_{2345}]\\
\rho^T_{2345} &= \frac{1}{s_1 t(\chi_1 + s_1 + t)}[-s_1 t a_{234} + \chi_1(\chi_1 + s_1)a_{235} + t(\chi_1 + s_1)a_{345}\\
&- s_1 t\chi_1 a_{2345} - s_1 t(\chi_1 + t)d_{2345}]. \qquad \text{(B.17)}
\end{aligned}$$

The above coefficients have to satisfy the relation,

$$4g^T_{2345} - \chi_1 a^T_{2345} + (s - \chi_2)\alpha^T_{2345} - (t + \chi_1)\gamma^T_{2345} - u_1\rho^T_{2345} = J_{234}.$$

$$g^T_{1234} = \frac{1}{2}\left[J_{123} - \chi_1'\frac{\Delta^{(3)}}{\Delta}\right],$$

$$a^T_{1234} = 2\frac{\Delta^{(2)}}{\Delta} + J_{1234} + \tilde{b}_{1234}\,,$$

$$b^T_{1234} = \tilde{b}_{1234}\,,$$

$$c^T_{1234} = 2\frac{\Delta^{(2)}}{\Delta} - 2\frac{\Delta^{(3)}}{\Delta} + J_{1234} + \tilde{b}_{1234} + \tilde{c}_{1234} - 2\tilde{\gamma}_{1234}\,,$$

$$d^T_{1234} = 2\frac{\Delta^{(2)}}{\Delta} - 2\frac{\Delta^{(1)}}{\Delta} + J_{1234} + \tilde{b}_{1234} + \tilde{a}_{1234} - 2\tilde{\alpha}_{1234}\,,$$

$$\alpha^T_{1234} = \frac{\Delta^{(2)}}{\Delta} + \tilde{b}_{1234}\,,$$

$$\beta^T_{1234} = \frac{\Delta^{(3)}}{\Delta} - 2\frac{\Delta^{(2)}}{\Delta} - J_{1234} - \tilde{b}_{1234} + \tilde{\gamma}_{1234}$$

$$\gamma^T_{1234} = \frac{\Delta^{(1)}}{\Delta} - 2\frac{\Delta^{(2)}}{\Delta} - J_{1234} - \tilde{b}_{1234} + \tilde{\alpha}_{1234}$$

$$\rho^T_{1234} = -\frac{\Delta^{(2)}}{\Delta} - \tilde{b}_{1234} + \tilde{\alpha}_{1234}\,,$$

$$\sigma^T_{1234} = -\frac{\Delta^{(2)}}{\Delta} - \tilde{b}_{1234} + \tilde{\gamma}_{1234}\,,$$

$$\tau^T_{1234} = 2\frac{\Delta^{(2)}}{\Delta} - \frac{\Delta^{(1)}}{\Delta} - \frac{\Delta^{(3)}}{\Delta} + J_{1234} + \tilde{b}_{1234} + \tilde{\beta}_{1234} - \tilde{\alpha}_{1234} - \tilde{\gamma}_{1234}, \tag{B.18}$$

where the quantities with the sign $\tilde{}$ are defined as follows:

$$\tilde{a}_{1234} = \frac{1}{s\chi'_1}(L_s - L_{s_1} - L_{\chi'_1})\,,$$

$$\tilde{b}_{1234} = \frac{1}{\chi'_2}\left[\chi'_1\frac{\Delta^{(2)}}{\Delta} + \frac{\chi'_1}{s-\chi'_1}J_{134} + \frac{L_{s1}}{s_1} - \frac{L_s}{s-\chi'_1} + \frac{\chi'_1(s_1-\chi'_2)}{s_1(s-\chi'_1)^2}(L_{\chi'_1} - L_s)\right]\,,$$

$$\tilde{c}_{1234} = \frac{1}{\chi'_2}\left[\frac{s_1^2}{\chi'_2}\frac{\Delta^{(2)}}{\Delta} - \frac{s_1}{\chi'_2}J_{124} + (\frac{s_1}{\chi'_2} + \frac{s_1}{s-s_1})J_{123} + \frac{2-L_s}{s-s_1}\right.$$
$$\left. - \frac{2-L_{\chi'_1}}{\chi'_1} - \frac{2s_1}{(s-s_1)^2}(L_s - L_{s_1})\right]\,,$$

$$\tilde{\alpha}_{1234} = \frac{L_{\chi'_1} - L_s}{s_1(s-\chi'_1)}\,, \qquad \tilde{\beta}_{1234} = \frac{\Delta^{(3)}}{\Delta} - \frac{L_s - L_{s_1}}{\chi'_1(s-s_1)}\,,$$

$$\tilde{\gamma}_{1234} = \frac{1}{\chi'_2}\left[\chi'_1\frac{\Delta^{(3)}}{\Delta} - J_{123} + \frac{L_s - L_{s_1}}{s-s_1} + \frac{L_s - L_{\chi'_1}}{s-\chi'_1}\right]. \tag{B.19}$$

One of the checking relations takes the form

$$2g^T_{1234} + \chi_1\beta^T_{1234} + \chi_2\sigma^T_{1234} + \chi'_1\tau^T_{1234} = a_{134} - a_{234} + \chi_1 a_{1234}.$$

At the end of this Appendix we give the table of scalar integrals with two, three and four denominators. We imply the real part everywhere and the ultraviolet asymptotic is assumed as well.

$$\begin{aligned}
&J_{12} = -1 + L_\Lambda\,, \qquad J_{13} = 1 + L_\Lambda - L_s\,,\\
&J_{14} = 1 + L_\Lambda - L_{\chi'_1}\,, \qquad J_{15} = J_{24} = J_{34} = J_{35} = L_\Lambda + 1\,,\\
&J_{23} = 1 + L_\Lambda - L_{s_1}\,, \qquad J_{25} = 1 + L_\Lambda - L_{\chi_1}\,,\\
&J_{45} = 1 + L_\Lambda - L_t,
\end{aligned} \tag{B.20}$$

where

$$\begin{aligned}
&L_\Lambda = \ln\frac{\Lambda^2}{m^2}, \quad L_s = \ln\frac{s}{m^2}, \quad L_\lambda = \ln\frac{\lambda^2}{m^2},\\
&L_{s_1} = \ln\frac{s_1}{m^2}, \quad L_{\chi'_1} = \ln\frac{\chi'_1}{m^2}, \quad L_{\chi_1} = \ln\frac{\chi_1}{m^2}, \quad L_t = \ln\frac{-t}{m^2}.
\end{aligned} \tag{B.21}$$

3-denominator scalar integrals are

$$\begin{aligned}
J_{123} &= \frac{1}{2(s-s_1)}(L_s^2 - L_{s_1}^2), \quad J_{345} = \frac{1}{t}\left[\frac{1}{2}L_t^2 + \frac{2\pi^2}{3}\right],\\
J_{124} &= \frac{1}{\chi'_1}\left[\frac{1}{2}L^2_{\chi'_1} - \frac{\pi^2}{6}\right], \quad J_{125} = \frac{1}{\chi_1}\left[-\frac{1}{2}L^2_{\chi_1} - \frac{\pi^2}{3}\right],\\
J_{134} &= \frac{1}{s-\chi'_1}\left[\frac{3}{2}L_s^2 + \frac{1}{2}L^2_{\chi'_1} - 2L_sL_{\chi'_1} + 2\mathrm{Li}_2\left(1 - \frac{\chi'_1}{s}\right)\right],\\
J_{235} &= \frac{1}{s_1+\chi_1}\left[\frac{3}{2}L^2_{s_1} + \frac{1}{2}L^2_{\chi_1} - 2L_{s_1}L_{\chi_1} + 2\mathrm{Li}_2\left(1 + \frac{\chi_1}{s_1}\right) - \frac{3\pi^2}{2}\right],\\
J_{135} &= \frac{1}{s}\left[\frac{1}{2}L_s^2 - L_sL_\lambda - \frac{2\pi^2}{3}\right], \quad J_{234} = \frac{1}{s_1}\left[\frac{1}{2}L^2_{s_1} - L_{s_1}L_\lambda - \frac{2\pi^2}{3}\right],\\
J_{245} &= \frac{1}{t+\chi_1}\left[\frac{1}{2}L_t^2 - \frac{1}{2}L^2_{\chi_1} + 2\mathrm{Li}_2\left(1 + \frac{\chi_1}{t}\right)\right],\\
J_{145} &= \frac{1}{-t+\chi'_1}\left[\frac{1}{2}L^2_{\chi'_1} - \frac{1}{2}L_t^2 - \frac{\pi^2}{2} - 2\mathrm{Li}_2\left(1 - \frac{\chi'_1}{t}\right)\right].
\end{aligned} \tag{B.22}$$

4-denominator scalar integrals read:

$$\begin{aligned}
J_{1245} &= \frac{1}{\chi_1\chi'_1}\left[-L^2_{\chi_1} - L^2_{\chi'_1} - L_t^2 - 2L_{\chi_1}L_{\chi'_1} + 2L_{\chi_1}L_t + 2L_{\chi'_1}L_t + \frac{2\pi^2}{3}\right],\\
J_{2345} &= \frac{1}{s_1t}\left[L^2_{s_1} - L_{s_1}L_\lambda - 2L_{s_1}L_{\chi_1} + 2L_{s_1}L_t - \frac{5\pi^2}{6}\right],\\
J_{1345} &= \frac{1}{st}\left[L_s^2 - L_sL_\lambda - 2L_sL_{\chi'_1} + 2L_sL_t + 7\frac{\pi^2}{6}\right],\\
J_{1235} &= \frac{1}{s\chi_1}\left[L^2_{s_1} + L_sL_\lambda - 2L_sL_{\chi_1} + 2\mathrm{Li}_2\left(1 - \frac{s_1}{s}\right) - 5\frac{\pi^2}{6}\right],\\
J_{1234} &= \frac{1}{s_1\chi'_1}\left[-L_s^2 - L_{s_1}L_\lambda + 2L_{s_1}L_{\chi'_1} - 2\mathrm{Li}_2\left(1 - \frac{s}{s_1}\right) - 7\frac{\pi^2}{6}\right].
\end{aligned} \tag{B.23}$$

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