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SEMI-ANALYTIC TREATMENT OF THE VAVILOV—CHERENKOV RADIATION

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1 Introduction

A specific electromagnetic radiation produced by fast electrons moving in medium was observed by P.A. Cherenkov in 1934 [1]. Tamm and Frank [2] considered the motion of a point charge in medium with a constant electric permittivity. They showed that the charge should radiate when its velocity exceeds the light velocity in medium. For frequency-independent electric permittivity the electromagnetic strengths have δ-type singularities on the surface of the so-called Cherenkov (or Mach) cone [3-6]. As a result, of the integrals involving the product of electromagnetic strengths become divergent. In particular, this is true for the total flux of electromagnetic field (EMF). To avoid this difficulty, Tamm and Frank (see, e.g., Frank's book [7]) made the Fourier transformation of EMF and integrated the energy flux up to some maximal frequency ω_0.

The goal of this treatment is to consider consequences arising from the uniform motion of a charge in a nonmagnetic medium described by the frequency-dependent one-pole electric permittivity

$$\epsilon(\omega) = 1 + \frac{\omega_L^2}{\omega_0^2 - \omega^2}. \tag{1.1}$$

This expression is a suitable extrapolation between the static case $\omega = 0,\ \epsilon(\omega) = \epsilon_0 = 1 + \omega_L^2/\omega_0^2$ and the high-frequency limit $\omega = \infty,\ \epsilon(\omega) = 1$. In the usual interpretation ω_L and ω_0 are the plasma frequency $\omega_L^2 = 4\pi N_e e^2/m$ (N_e is the number of electrons per unit volume, m is the electron mass) and some resonance frequency, resp. Quantum-mechanically, it can be associated with the energy excitation of the lowest atomic level. Our subsequent exposition does not depend on this particular interpretation of ω_L and ω_0.

Equation (1.1) is a standard parametrization describing a lot of optical phenomena [8]. It is valid when the wavelength of the EMF is much larger than the distance between particles of medium on which the light scatters. The typical atomic dimensions are of an order of $a \approx \hbar/mc\alpha$; $\alpha = e^2/\hbar c$, m is the electron mass. This gives $\lambda = c/\omega >> a$ or $\omega << mc^2\alpha/\hbar \approx 5 \cdot 10^{18} sec^{-1}$. The typical atomic frequencies are of the order $\omega_0 \approx mc^2/\hbar\alpha^2 \approx 10^{16} sec^{-1}$. As $\omega \gg \omega_0$, the physical region extends well beyond ω_0 ([9]). For $\omega >> \omega_0$, $\epsilon(\omega) \approx 1$, that is, medium oscillators have no enough time to be excited. This means that we disregard the excitation of nuclear levels and discrete structure of scatterers. According to L. Brillouin ([10], p. 20): "Also, we use the formulas of the dispersion theory in a somewhat more general way than can be justified physically. Namely, we extend these formulas to infinitesimally small wavelengths, while their derivation is justified only for wavelengths large compared with the distance between dispersing particles".

We intend to consider the effects arising from the uniform charge motion in medium with $\epsilon(\omega)$ given by (1.1). Partly, this was done by E. Fermi in 1940 [11]. He showed that a charged particle moving uniformly in medium with permittivity (1.1) should radiate at every velocity.

However, the following questions remained unanswered:

1. How the EMF strengths and energy flux are distributed in space? In particular, what is their angular distribution?
2. How do these distributions depend on the medium properties, on the charge velocity? In particular, how do these distributions differ for the charge velocity smaller and greater than the light velocity in medium?

In this consideration we restrict ourselves to the classical theory of the Vavilov-Cherenkov (VC) radiation with electric permittivity given by (1.1). It is suggested that uniform motion of a particle is maintained by some external force the origin of which is not of interest for us.

The plan of our exposition is as follows.
In section 2, we present, in a manifestly real form, the electromagnetic potentials and field strengths for a charge moving uniformly in a dielectric with $\epsilon(\omega)$ given by (1.1).
Various analytically solvable particular cases are considered in section 3. It is shown there that EMF of the charge moving in the medium with electric permittivity (1.1) should exhibit oscillations in a half-space behind the moving charge. It turns out that some critical charge velocity v_c exists which depends on the medium properties and does not depend on the frequency (despite the fact that the frequency dispersion is taken into account). Below and above v_c, the distributions of EMF radiated by a moving charge differ drastically. In section 4, we evaluate the energy losses as a function of the charge velocity for ϵ given by (1.1). This dependence shows that a charge moving in medium radiates at each velocity. In the same section, we demonstrate how the energy flux is distributed over the surface of a cylinder coaxial with the charge trajectory (this is a usual procedure in the VC effect theory). It is shown that for the charge velocity greater than v_c the main contribution to the energy flux comes from the space region where in the absence of ω dispersion the Cherenkov cone intersects the cylinder surface. It turns out that rapid oscillations of the energy flux should be observed in this region. For $v < v_c$ this space region contributes practically nothing to the energy flux. The main contribution to the energy losses comes from the space region sufficiently remote from that of mentioned above and lying behind a moving charge. These considerations support the results of experiments ([12-14]) indicating on the existence of the radiation below the Cherenkov threshold.

2 Mathematical preliminaries

Consider a point charge e uniformly moving in a non-magnetic medium with a velocity v directed along the z axis. Its charge and current densities are given by

$$\rho(\vec{r},t) = e\delta(x)\delta(y)\delta(z-vt), \quad j_z = v\rho.$$

Their Fourier transforms are

$$\rho(\vec{k},\omega) = \int \rho(\vec{r},t)\exp[i(\vec{k}\vec{r}-\omega t)]d^3\vec{r}dt = 2\pi e\delta(\omega - \vec{k}\vec{v}), \quad j_z(\vec{k},\omega) = v\rho(\vec{k},\omega) \qquad (2.1).$$

The EMF strengths of the moving charge satisfy the Maxwell equations:

$$div\vec{D} = 4\pi\rho, \quad div\vec{B} = 0, \quad curl\vec{E} = -\frac{1}{c}\dot{\vec{B}}, \quad curl\vec{H} = \frac{1}{c}\dot{\vec{D}} + \frac{4\pi}{c}\vec{j} \qquad (2.2).$$

As the medium is non-magnetic, $\vec{B} = \vec{H}$. The second and third Maxwell equations are satisfied if we put

$$\vec{H} = \vec{\nabla}\times\vec{A}, \quad \vec{E} = -\vec{\nabla}\Phi - \frac{1}{c}\dot{\vec{A}}$$

The electric field $\vec{E}$ of a moving charge induces the polarization $\vec{P}(\vec{r},t)$ which being added with $\vec{E}$ gives electric induction $\vec{D} = \vec{E} + 4\pi\vec{P}$. Usually, it is believed (see, e.g.,[15]) that the ω components of $\vec{P}$ and $\vec{E}$

$$\vec{P}_\omega = \int e^{-i\omega t}\vec{P}(\vec{r},t)dt, \quad \vec{E}_\omega = \int e^{-i\omega t}\vec{E}(\vec{r},t)dt$$

are related by the formula

$$4\pi\vec{P}_\omega = \frac{\omega_L^2}{\omega_0^2 - \omega^2 + ip\omega}\vec{E}_\omega. \qquad (2.3)$$

In the $(\vec{k},\omega)$ space the electromagnetic potentials are given by ([16])

$$\Phi(\vec{k},\omega) = \frac{4\pi}{\epsilon}\frac{\rho(\vec{k},\omega)}{k^2 - \frac{\omega^2}{c^2}\epsilon}, \quad A_z(\vec{k},\omega) = 4\pi\beta\frac{\rho(\vec{k},\omega)}{k^2 - \frac{\omega^2}{c^2}\epsilon}, \quad \beta = v/c \qquad (2.4)$$

Here $\epsilon(\omega)$ is the electric permittivity of medium. Its frequency dependence is chosen in a standard form (1.1). In the usual interpretation ω_L and ω_0 are the plasma frequency $\omega_L^2 = 4\pi N_e e^2/m$ (N_e is the number of electrons per unit volume, m is the electron mass) and some resonance frequency, resp. Quantum-mechanically, it can be associated with the energy excitation of the lowest atomic level. Our subsequent exposition does not depend on this particular interpretation of ω_L and ω_0. The static limit of $\epsilon(\omega)$ is

$$\epsilon_0 = \epsilon(\omega = 0) = 1 + \frac{\omega_L^2}{\omega_0^2}.$$

$\epsilon(\omega)$ has poles at $\omega = \pm\omega_0$. Being positive for $\omega^2 < \omega_0^2$ it jumps from $+\infty$ to $-\infty$ when one crosses the point $\omega^2 = \omega_0^2$; $\epsilon(\omega)$ has zero at $\omega^2 = \omega_3^2 = \omega_0^2 + \omega_L^2$ and tends to unity for $\omega \to \infty$. In Eq. (1.1), $\epsilon(\omega)$ is negative for $\omega_0^2 < \omega^2 < \omega_0^2 + \omega_L^2$ (Fig. 1). For the free electromagnetic wave this leads to its space damping in this ω region even for real $\epsilon(\omega)$ (see, e.g., [10,15]).
It is seen that

$$\epsilon^{-1}(\omega) = 1 - \frac{\omega_L^2}{\omega_3^2 - \omega^2}$$

has zero at $\omega^2 = \omega_0^2$ and a pole at $\omega^2 = \omega_3^2 = \omega_0^2 + \omega_L^2$.

For the EMF radiated by a moving charge the conditions for EMF damping are modified . It turns out that the damping takes place for $1 - \beta^2\epsilon > 0$. Otherwise $(1 - \beta^2\epsilon < 0)$, there is no damping.
We now define domains where $1 - \beta^2\epsilon > 0$ and $1 - \beta^2\epsilon < 0$.
For $\beta < \beta_c$ one has: $1 - \beta^2\epsilon > 0$ for $\omega^2 < \omega_c^2$ and $\omega^2 > \omega_0^2$ and $1 - \beta^2\epsilon < 0$ for $\omega_c^2 < \omega^2 < \omega_0^2$ (Fig. 2).
For $\beta > \beta_c$ one gets: $1 - \beta^2\epsilon > 0$ for $\omega^2 > \omega_0^2$ and $1 - \beta^2\epsilon < 0$ for $0 < \omega^2 < \omega_0^2$ (Fig. 3) . Here $\beta_c = \epsilon_0^{-1/2} = 1/\sqrt{1 + \omega_L^2/\omega_0^2}$, $\omega_c = \omega_0\sqrt{1 - \tilde{\epsilon}}$, $\tilde{\epsilon} = \beta^2\gamma^2/\beta_c^2\gamma_c^2$ $\gamma^2 = (1 - \beta^2)^{-1}$ $\gamma_c^2 = (1 - \beta_c^2)^{-1}$.
In what follows, β_c, despite its formal appearance and independence of ω, will play an important role for the analysis of the EMF induced by a charge moving in medium with a frequency-dependent permittivity. In the static limit $(\omega \to 0)$ it coincides with the light velocity in medium.

Strictly speaking, Eq. (1.1) is valid for media with $\epsilon_0 \approx 1$ (e.g., for gases). In what follows we apply Eq. (1.1) to the medium with $\beta_c = 0.75$, $n = \sqrt{\epsilon_0} = 1/\beta_c = 1.333$. The optical properties of this medium are close to those of water for which $n = 1.334$. For this value of the refractive index n one should use the Clausius-Mossotti (or Lorentz-Lorenz) formula ([8-10]):

$$\epsilon' = \frac{1+2\alpha(\omega)/3}{1-\alpha(\omega)/3} = 1 + \frac{\omega_L^2}{\omega_0'^2 - \omega^2}, \quad \alpha(\omega) = \frac{\omega_L^2}{\omega_0^2-\omega^2}, \quad \omega_0'^2 = \omega_0^2 - \omega_L^2/3 \qquad (2.5).$$

The conditions for EMF damping and its absence now are $1-\beta^2\epsilon' > 0$ and $1-\beta^2\epsilon' < 0$, resp. For $\omega_0^2 < \omega_L^2/3$ one always has $1-\beta^2\epsilon' > 0$, which means the absence of radiation by a uniformly moving charge .
Let now $\omega_0^2 > \omega_L^2/3$. Then, for $\beta < \beta_c'$, ($\beta_c'^2 = 1 - \omega_L^2/(\omega_0^2 + 2\omega_L^2/3)$) one has :
$1-\beta^2\epsilon' > 0$ for $\omega^2 < \omega_c'^2$ ($\omega_c'^2 = \omega_0'^2 - \beta^2\gamma^2\omega_L^2$) and for $\omega^2 > \omega_0'^2$;
$1-\beta^2\epsilon' < 0$ for $\omega_c'^2 < \omega^2 < \omega_0'^2$.
On the other hand, for $\beta > \beta_c'$:
$1-\beta^2\epsilon' > 0$ for $\omega^2 > \omega_0'^2$ and $1-\beta^2\epsilon' < 0$ for $0 < \omega^2 < \omega_0'^2$.
We see that qualitative behaviour of ϵ and ϵ' is almost the same if we identify β_c, ω_0 and ω_L with β_c', ω_0' and ω_L, resp. The sole exception is that for $\omega_0^2 < \omega_L^2/3$ there is no solution corresponding to $1-\beta^2\epsilon' < 0$. This permits us to limit ourselves to the ϵ representation in the form (1.1). All the subsequent expressions will be valid for ϵ' given by (2.5) if we change β_c, ω_0 and ω_L by β_c', ω_0' and ω_L, resp.

In what follows we use the quantity $\beta_c = 1/\sqrt{1+\omega_L^2/\omega_0^2}$ which in the static limit ($\omega \to 0$) coincides with the light velocity in medium. It is seen that β_c changes from $\beta_c = 0$ for $N >> 1$ up to $\beta_c = 1$ for $N = 0$. We refer to these limit cases as to optically dense and rarefied media, resp.

In the $\vec r, t$ representation $\Phi(\vec r, t)$ and $\vec A(\vec r, t)$ are given by

$$\Phi(\vec r, t) = \frac{e}{\pi v}\int \frac{d\omega}{\epsilon} e^{i\omega(t-z/v)} \frac{kdk}{k^2 + \frac{\omega^2}{v^2}(1-\beta^2\epsilon)} J_0(k\rho).$$

$$A_z(\vec r, t) = \frac{e}{\pi c}\int d\omega e^{i\omega(t-z/v)} \frac{kdk}{k^2 + \frac{\omega^2}{v^2}(1-\beta^2\epsilon)} J_0(k\rho). \qquad (2.6)$$

The usual way to handle with these integrals is to integrate them first over k. This can be done in a closed form [17]. The remaining integrals over ω are difficult to treat analytically. The corresponding integrands are usually interpreted as frequency distributions of EMF associated with the uniform motion of charge in medium.
In this approach, we prefer to take the above integrals first over ω. The advantage of this approach is that arising integrals can be treated analytically in various particular cases. These integration methods complement each other. The possibility to get rid of any trace of the ω dependence points out on a slightly artificial character of the ω representation (as far as we do not concern the quantum aspects of radiation). In fact, Maxwell equations (2.2) describing EMF of the uniformly moving charge can be handled without any appeal to ω representation. To prove this, we rewrite Eq. (2.3) in the $\vec r, t$ representation:

$$\vec P(t) = \frac{1}{8\pi^2}\int_{-\infty}^{\infty} G(t-t')\vec E(t')dt',$$

where

$$G(t-t') = \lim_{p\to 0+} \omega_L^2 \int_{-\infty}^{+\infty} \frac{d\omega}{\omega_0^2 - \omega^2 + ip\omega} e^{i\omega(t-t')}.$$

A direct calculation shows that $G(t-t') = 0$ for $t' > t$ and $G(t-t') = \frac{2\pi\omega_L^2}{\omega_0}\sin[\omega(t-t')]$ for $t' < t$.
Substituting $\vec P$ into the Maxwell equations (2.2) one obtains the system of integro-differential equations which depend only on the charge velocity and medium parameters.

We represent denominators entering in (2.6) in the form

$$[k^2 + \frac{\omega^2}{v^2}(1-\beta^2\epsilon)]^{-1} = \frac{v^2}{1-\beta^2}\frac{\omega^2-\omega_0^2}{(\omega^2-\omega_1^2)(\omega^2+\omega_2^2)} =$$

$$= \frac{v^2}{1-\beta^2}(\omega^2-\omega_0^2)\frac{1}{\omega_1^2+\omega_2^2}[\frac{1}{2\omega_1}(\frac{1}{\omega-\omega_1} - \frac{1}{\omega+\omega_1}) - \frac{1}{2i\omega_2}(\frac{1}{\omega-i\omega_2} - \frac{1}{\omega+i\omega_2})].$$

Here

$$\omega_3^2 = \omega_0^2 + \omega_L^2, \quad \omega_1^2 = \omega_0^2 - \Omega + (\Omega^2 - \beta^2\gamma^2\omega_0^2\omega_L^2)^{1/2}.$$

$$\omega_2^2 = -\omega_0^2 + \Omega + (\Omega^2 - \beta^2\gamma^2\omega_0^2\omega_L^2)^{1/2}, \quad \Omega = \frac{1}{2}[\omega_0^2 + \beta^2\gamma^2(k^2c^2 + \omega_L^2)].$$

Substituting these expressions into (2.6) and performing the ω integration we get for the electromagnetic potentials and field strengths

$$A_z = A_z^{(1)} + A_z^{(2)},$$

$$A_z^{(1)} = \frac{cv^2\gamma^2}{c}\int_0^\infty kdkJ_0(k\rho)F_A^{(1)}, \quad A_z^{(2)} = \frac{cv^2\gamma^2}{c}\int_0^\infty kdkJ_0(k\rho)F_A^{(2)} \qquad (2.7)$$

$$\Phi = cv\gamma^2\int_0^\infty kdkJ_0(k\rho)F_\phi - \frac{2c\omega_L^2}{v\omega_3}\sin[\omega_3(t-z/v)]\Theta(t-z/v)K_0(\rho\omega_3/v).$$

$$H_\phi = -\frac{\partial A_z}{\partial\rho} = c\beta^2c\gamma^2\int_0^\infty k^2dkJ_1(k\rho)F_A, \quad D_\rho = H_\phi/\beta,$$

$$E_\rho = -\frac{\partial\Phi}{\partial\rho} = c\gamma^2 v\int_0^\infty k^2dkJ_1(k\rho)F_\phi - \frac{2c\omega_L^2}{v^2}\sin\omega_3(t-z/v)\Theta(t-z/v)K_1(\rho\omega_3/v).$$

$$E_z = -\frac{\partial\Phi}{\partial z} - \frac{\partial A_z}{c\partial t} = c\gamma^2\int_0^\infty kdkJ_0(k\rho)[2(\beta^2 - \frac{\omega_1^2-\omega_0^2}{\omega_1^2-\omega_3^2})\frac{\omega_1^2-\omega_0^2}{\omega_1^2+\omega_2^2}\Theta(t-z/v)\cos\omega_1(t-z/v) -$$

$$-(\beta^2 - \frac{\omega_0^2+\omega_2^2}{\omega_2^2+\omega_3^2})\frac{\omega_0^2+\omega_2^2}{\omega_2^2+\omega_1^2}\cdot sign(z-vt)\exp(-\omega_2|t-z/v|)] -$$

$$-\frac{2c\omega_L^2}{v^2}\cos\omega_3(t-z/v)\Theta(t-z/v)K_0(\rho\omega_3/v),$$

$$D_z = -2c\int_0^\infty kdkJ_0(k\rho)\frac{\omega_1^2-\omega_0^2+\beta^2\gamma^2\omega_L^2}{\omega_1^2+\omega_2^2}\Theta(t-z/v)\cos\omega_1(t-z/v) -$$

$$-e\int_0^\infty kdkJ_0(k\rho)\frac{\omega_2^2+\omega_0^2-\beta^2\gamma^2\omega_L^2}{\omega_1^2+\omega_2^2}\exp(-\omega_2|t-z/v|)\cdot sign(t-z/v).$$

Here we put: $F_A = F_A^{(1)} + F_A^{(2)}$,

$$F_A^{(1)} = -\frac{\omega_1^2-\omega_0^2}{\omega_1^2+\omega_2^2}\frac{2}{\omega_1}\Theta(t-z/v)\sin\omega_1(t-z/v),\quad F_A^{(2)} = \frac{1}{\omega_2}\frac{\omega_2^2+\omega_0^2}{\omega_1^2+\omega_2^2}\exp(-\omega_2|t-z/v|),$$

$$F_\phi = F_\phi^{(1)} + F_\phi^{(2)},\quad F_\phi^{(1)} = -\frac{(\omega_1^2-\omega_0^2)^2}{(\omega_1^2+\omega_2^2)(\omega_1^2-\omega_3^2)}\Theta(t-z/v)\frac{2}{\omega_1}\sin\omega_1(t-z/v),$$

$$F_\phi^{(2)} = \frac{(\omega_0^2+\omega_2^2)^2}{\omega_2(\omega_1^2+\omega_2^2)(\omega_3^2+\omega_2^2)}\exp(-\omega_2|t-z/v|). \tag{2.8}$$

The separation of F_A and F_ϕ into two parts is justified physically. We see that $F_A^{(1)}$, $F_\phi^{(1)}$ and $F_A^{(2)}$, $F_\phi^{(2)}$ describe correspondingly the radiation field and EMF carried by a uniformly moving charge. They originate from the ω poles lying in non-damping and damping regions, resp.

When evaluating electromagnetic potentials and field strengths we have taken into account that $\epsilon(\omega)$ given by (1.1) is a limiting expression (as $p \to 0$) of

$$\epsilon(\omega) = 1 + \frac{\omega_L^2}{\omega_0^2-\omega^2+ip\omega}$$

having a pole in the upper ω half-plane (for the Fourier transform chosen in the form (2.4)). This in turn results in an infinitely small positive imaginary part in ω_1 and in factor 2 in the first terms in F_A and F_ϕ. The position of poles of $\epsilon(\omega)$ in the upper complex ω half-plane is needed to satisfy the causality condition. Sometimes in physical literature [15] it is stated that the causality condition is fulfilled if the poles of $\epsilon(\omega)$ lie in the lower ω half-plane. This is due to a different definition of the Fourier transforms corresponding to different signs of ω of the exponentials occurring in (2.6).

It is seen that Φ, E_ρ, and E_z are singular on the motion axis behind the moving charge. These singularities are due to the modified Bessel functions K outside the integrals in (2.7). For a fixed observation point z on the cylinder surface these singularities as functions of time oscillate with the frequency $\omega_3 = \omega_0/\beta_c$. For the fixed observation time t these singularities as functions of the observation point z oscillate with the frequency $\omega_0/\beta_c v$. As electric induction $\vec D$ is not singular on the motion axis, the electric polarization $\vec P = (\vec D - \vec E)/4\pi$ has the same singularity as $\vec E$. As to the magnetic field $\vec H$, it tends to zero on the motion axis:

$$H_\phi \to \frac{e\omega_L^2\omega_0}{c^3}\Theta(t-z/v)\sin[\omega_0(t-z/v)]\rho K_0(\rho\omega_0/c)\quad \text{for}\quad \rho\to 0.$$

3 Particular cases

Consider the limiting cases. In most cases we consider of the magnetic vector potential (and, rarely, of the electric potential). The behaviour of EMF strengths is restored by the differentiation of potentials.

1) Let $v \to 0$. Then,
$\omega_1 \to \omega_0$, $\omega_2 \to v\gamma k$, $A_z \to 0$, and

$$\Phi \to \frac{e\gamma}{1+\omega_L^2/\omega_0^2}\int_0^\infty dkJ_0(k\rho)\exp(-\beta\gamma kc|t-z/v|) = \frac{e}{\epsilon_0}\frac{1}{[z^2+\rho^2]^{1/2}}. \tag{3.1}$$

i.e., we obtain the field of a charge to be at rest in the medium. It turns out that only the second term in F_ϕ contributes to Φ.

2) Let $\omega_L \to 0$. This corresponds to the zero electron density, on which the moving charge exhibits scattering. Then,
$\epsilon \to 1$, $\beta_c \to 1$, $\omega_1 \to 0$, $\omega_2 \to \gamma kv$,

$$A_z \to e\beta\gamma\int_0^\infty dkJ_0(k\rho)\exp(-k\gamma|z-vt|) = \frac{e\beta}{[(z-vt)^2+\rho^2/\gamma^2]^{1/2}},$$

$$\Phi \to \frac{e}{[(z-vt)^2+\rho^2/\gamma^2]^{1/2}}, \tag{3.2}$$

i.e., we obtain the field of a charge moving in vacuum. Again, only second terms in F_ϕ and F_A contribute to Φ and A_z, resp.

3) Let $\omega_L \to \infty$. This corresponds to an optically dense medium. Then,

$$\omega_1^2 \to \frac{\omega_0^2}{\omega_L^2}k^2c^2,\quad \omega_2^2 \to \beta^2\gamma^2(\omega_L^2+k^2c^2) - \omega_0^2 + \frac{\omega_0^2}{\omega_L^2}k^2c^2,$$

$$F_A^{(1)} \to \frac{2\omega_0\omega_L}{\beta^2\gamma^2(\omega_L^2+k^2c^2)kc}\Theta(t-z/v)\sin\frac{\omega_0 kc(t-z/v)}{\omega_L},$$

$$F_A^{(2)} \to \frac{e}{\beta\gamma\sqrt{\omega_L^2+k^2c^2}}\exp(-\beta\gamma\sqrt{\omega_L^2+k^2c^2}|t-z/v|),$$

$A_z^{(2)}$ can be evaluated in a closed form:

$$A_z^{(2)} \to \frac{e\beta}{R}\exp(-\gamma\omega_L R/c),\quad R = [(z-vt)^2 + \beta_c^2\gamma_c^2\rho^2/\gamma^2]^{1/2}. \tag{3.3}$$

while the analytic form of $A_z^{(1)}$ is available only for $\rho \ge \omega_0 c(t-z/v)/\omega_L$:

$$A_z^{(1)} \to \frac{2e\omega_0}{c}\Theta(t-z/v)\sinh[\omega_0(t-z/v)]K_0(\omega_L\rho/c). \tag{3.4}$$

(it is seen that $A_z^{(1)}$ decreases exponentially when ρ grows and increases exponentially with rising $t-z/v$) and on the motion axis:

$$A_z^{(1)} = \frac{e\omega_0}{c}\Theta(t-z/v)[\exp(-\omega_0(t-z/v))E_i(\omega_0(t-z/v)) - \exp(\omega_0(t-z/v))E_i(-\omega_0(t-z/v))].$$

Here $E_i(x)$ is an integral exponent. For small and large values of $\omega_0(t-z/v)$ this gives:

$$A_z^{(1)} \approx -2\frac{e\omega_0}{c}\Theta(t-z/v)\sin(\omega_0(t-z/v))[C+\ln(\omega_0(t-z/v))]\quad \text{for}\quad \omega_0(t-z/v) << 1,$$

$$A_z^{(1)} \approx \frac{2e}{c(t-z/v)} \quad \text{for} \quad \omega_0(t-z/v) >> 1,$$

C is the Euler constant. Thus, damped oscillations of EMF should be observed on the motion axis behind the charge.

4) Let $\omega_0 \to \infty$, i.e., the resonance level lies very high. Then,

$$\omega_1^2 \to \omega_0^2 - \beta^2\gamma^2\omega_L^2, \quad \omega_2^2 \to \beta^2\gamma^2k^2c^2,$$

$$F_A^{(1)} \to \frac{\omega_L^2\beta^2\gamma^2}{\omega_0^2 - \omega_L^2\beta^2\gamma^2 + \beta^2\gamma^2k^2c^2}\frac{2}{\sqrt{\omega_0^2 - \beta^2\gamma^2\omega_L^2}}\Theta(t-z/v)\sin[\sqrt{\omega_0^2 - \omega_L^2\beta^2\gamma^2}(t-z/v),$$

$$F_A^{(2)} \to \frac{1}{\beta\gamma kc}\exp(-\beta\gamma kc|t-z/v|,$$

$$A_z^{(1)} \to \frac{2e\omega_L^2\beta^2\gamma^2}{c\omega_0}\Theta(t-z/v)\sin[\omega_0(t-z/v)]K_0(\frac{\rho\omega_0}{\beta\gamma c}), \tag{3.5}$$

$$A_z^{(2)} \to \frac{e\beta}{[(z-vt)^2+\rho^2/\gamma^2]^{1/2}}. \tag{3.6}$$

We see that a compete VP consists of VP $A_z^{(2)}$ describing the charge motion in vacuum and oscillating perturbation $A_z^{(1)}$ on the axis of the charge motion.

5) Let $\omega_0 \to 0$, i.e., the resonance level lies very low. Then,

$$\omega_1^2 \to \omega_0^2\frac{k^2c^2}{k^2c^2+\omega_L^2}, \quad \omega_2^2 \to \beta^2\gamma^2(k^2c^2+\omega_L^2) - \frac{\omega_L^2\omega_0^2}{k^2c^2+\omega_L^2},$$

$$F_A^{(1)} \approx \frac{2\omega_0^2\omega_L^2}{\beta^2\gamma^2ck}\frac{1}{(k^2c^2+\omega_L^2)^{3/2}}\Theta(t-z/v)\sin[\frac{\omega_0kc(t-z/v)}{\sqrt{k^2c^2+\omega_L^2}}],$$

$$F_A^{(2)} \approx \frac{1}{\beta\gamma}\frac{1}{\sqrt{k^2c^2+\omega_L^2}}\exp[-\beta\gamma\sqrt{k^2c^2+\omega_L^2}(t-z/v)],$$

$$A_z^{(2)} \approx \frac{e\beta}{R}\exp(-\gamma\omega_L R/c), \quad R = [(vt-z)^2 + \beta_c^2\gamma_c^2\rho^2/\gamma^2]^{1/2}. \tag{3.7}$$

We succeed to evaluate $A_z^{(1)}$ in a closed form in two cases. For $\omega_0\rho/c << 1$ one gets

$$A_z^{(1)} \approx 2e\Theta(t-z/v)\frac{1-\cos\omega_0(t-z/v)}{c(t-z/v)}, \tag{3.8}$$

On the other hand, for $\omega_0(t-z/v) << 1$

$$A_z^{(1)} \approx \frac{e\omega_0^2\omega_L}{c^3}\Theta(t-z/v)\rho c(t-z/v)K_1(\rho\omega_L/c).$$

There are VP oscillations in the half-space behind the moving charge decreasing exponentially with the rise of ρ.

6) Let $\omega_0 \to \infty$, $\omega_L \to \infty$, but ω_L/ω_0 and, therefore, β_c are finite. Then, one gets

$$\omega_1^2 \to \omega_0^2(1-\tilde{\epsilon}) + x^2\omega_0^2\frac{\tilde{\epsilon}}{1-\tilde{\epsilon}}, \quad \omega_2^2 = x^2\omega_0^2\frac{1}{1-\tilde{\epsilon}}.$$

$$A_z \to \frac{2ec\beta^2\gamma^2\tilde{\epsilon}}{\omega_0(1-\tilde{\epsilon})^{3/2}}\frac{\delta(\rho)}{\rho}\sin[\sqrt{1-\tilde{\epsilon}}\omega_0(t-z/v)] + \frac{e\beta}{[(z-vt)^2+\rho^2(1-\beta^2\tilde{\epsilon}_0)]^{1/2}},$$

$$\Phi \to \frac{2ec\beta^3\gamma^2\tilde{\epsilon}}{\omega_0(1-\tilde{\epsilon})^{3/2}}\frac{\delta(\rho)}{\rho}\sin[\sqrt{1-\tilde{\epsilon}}\omega_0(t-z/v)] + \frac{e\beta}{[(z-vt)^2+\rho^2(1-\beta^2\tilde{\epsilon}_0)]^{1/2}}$$

for $\beta < \beta_c$. Here $x = \beta\gamma kc/\omega_0$, $\tilde{\epsilon} = \beta^2\gamma^2/\beta_c^2\gamma_c^2$. For $\beta > \beta_c$ one has:

$$\omega_1^2 = \frac{\omega_0^2x^2}{\tilde{\epsilon}-1}, \quad \omega_2^2 = \omega_0^2(\tilde{\epsilon}-1) + x^2\omega_0^2\frac{\tilde{\epsilon}}{\tilde{\epsilon}-1},$$

$$A_z \to \frac{ec\beta^2\gamma^2\tilde{\epsilon}}{\omega_0(\tilde{\epsilon}-1)^{3/2}}\frac{\delta(\rho)}{\rho}\exp[-\sqrt{\tilde{\epsilon}-1}\omega_0(t-z/v)] + \frac{2e\beta}{[(z-vt)^2-\rho^2(\beta^2\tilde{\epsilon}_0-1)]^{1/2}},$$

$$\Phi \to \frac{ec\beta^3\gamma^2\tilde{\epsilon}}{\omega_0(\tilde{\epsilon}-1)^{3/2}}\frac{\delta(\rho)}{\rho}\exp[-\sqrt{\tilde{\epsilon}-1}\omega_0(t-z/v)] + \frac{2e\beta}{[(z-vt)^2-\rho^2(\beta^2\tilde{\epsilon}_0-1)]^{1/2}}$$

The origin of the first and second terms in A_z and Φ is due to the second and first terms in F_A and F_ϕ, resp.

Thus, one obtains VP of a charge moving in medium with a constant electric permittivity $\tilde{\epsilon} = \tilde{\epsilon}_0$ and the singular VP on the motion axis.

7) Let the dimensionless quantity $\tilde{\epsilon} = \beta^2\gamma^2/\beta_c^2\gamma_c^2 >> 1$. Then,

$$\omega_1^2 = \frac{\omega_0^2x_c^2}{1+x_c^2}, \quad \omega_2^2 = \tilde{\epsilon}(1+x_c^2) - \frac{1}{1+x_c^2}, \quad x_c = \beta_c\gamma_c kc/\omega_0.$$

$$F_A = F_A^{(1)} + F_A^{(2)},$$

$$F_A^{(1)} = \frac{2}{\omega_0\tilde{\epsilon}x_c(1+x_c^2)^{3/2}}\Theta(t-z/v)\sin[\omega_0(t-z/v)\frac{x_c}{\sqrt{1+x_c^2}}].$$

$$F_A^{(2)} = \frac{1}{\omega_0\sqrt{\tilde{\epsilon}}}\frac{1}{\sqrt{1+x_c^2}}\exp(-\sqrt{\tilde{\epsilon}}\sqrt{1+x_c^2}\omega_0|t-z/v|) \tag{3.9}$$

Correspondingly,

$$A_z = A_z^{(1)} + A_z^{(2)}, \quad \text{where}$$

$$A_z^{(1)} = \frac{2e\omega_0}{c}\Theta(t-z/v)\int_0^\infty \frac{dx}{(1+x^2)^{3/2}}J_0(\frac{\rho\omega_0 x}{\beta_c\gamma_c c})\sin[\frac{\omega_0(t-z/v)x}{\sqrt{1+x^2}}].$$

$$A_z^{(2)} = \frac{e\beta}{R}\exp(-\frac{\omega_0 R\gamma}{\beta_c\gamma_c c}), \quad R = [(z-vt)^2 + \rho^2/\gamma^2]^{1/2}.$$

We did not succeed in evaluating $A_z^{(1)}$ in a closed form. Instead, we consider particular cases when the condition $\tilde{\epsilon} >> 1$ can be realized.

Let β be finite and $\beta_c \to 0$. This corresponds to an optically dense medium. Then, $A_z^{(2)}$ is exponentially small whereas

$$A_z^{(1)} = \frac{2e\beta_c\gamma_c}{(\beta_c^2\gamma_c^2c^2(t-z/v)^2-\rho^2)^{1/2}}\Theta(t-z/v)\Theta[\beta_c\gamma_c c(t-z/v)-\rho]. \quad (3.10)$$

is confined to an infinitely narrow cone lying behind the moving charge. This equation is obtained by neglecting x_c^2 in the square roots in (3.9).
Let $\beta \to 1$, $\beta_c \to 1$ under the condition $\tilde{\epsilon} >> 1$ (that is, β is much closer to unity than β_c). This inequality is possible because of the γ factors in the definition of $\tilde{\epsilon}$. Then,

$$A_z^{(1)} = 2e\Theta(t-z/v)\frac{1-\cos[\omega_0(t-z/v)]}{ct-z}. \quad (3.11)$$

for small values of ρ. It is seen that VP exhibits oscillations in a half-space behind the moving charge.
More accurately, condition under which Eq. (3.11) is valid looks as $\rho\omega_0/\beta_c\gamma_c c << 1$. This means that for β_c fixed in the interval $0 < \beta_c < 1$, $A_z^{(1)}$ oscillates for $\rho << \beta_c\gamma_c c/\omega_0$.

8) Let $\tilde{\epsilon} << 1$. Then,

$$\omega_1^2 = \omega_0^2(1-\frac{\tilde{\epsilon}}{1+x^2}), \quad \omega_2^2 = \omega_0^2x^2(1+\frac{\tilde{\epsilon}}{1+x^2}), \quad x = \beta\gamma kc/\omega_0,$$

$$F_A^{(1)} = \frac{2\tilde{\epsilon}}{\omega_0}\frac{1}{(1+x^2)^2}\Theta(t-z/v)\sin[\omega_0(t-z/v)(1-\frac{\tilde{\epsilon}}{2}\frac{1}{1+x^2})],$$

$$F_A^{(2)} = \frac{1}{\omega_0 x}\exp(-\omega_0 x|t-z/v|).$$

$A_z^{(2)}$ coincides with the VP of a charge moving in vacuum:

$$A_z^{(2)} = \frac{ev}{[(z-vt)^2+\rho^2/\gamma^2]^{1/2}}.$$

As to $A_z^{(1)}$, it can be taken in an analytic form for $(t-z/v)\omega_0\tilde{\epsilon} << 1$:

$$A_z^{(1)} = \frac{e\rho\omega_0^2\beta\gamma}{c^2\beta_c^2\gamma_c^2}\Theta(t-z/v)\sin[\omega_0(t-z/v)]K_1(\rho\omega_0/\beta\gamma c). \quad (3.12)$$

The condition $\tilde{\epsilon} << 1$ can be realized in two ways. First, β_c can be finite but $\beta << 1$. In this case $A_z^{(1)}$ is confined to a narrow beam behind the moving charge:

$$A_z^{(1)} = e(\frac{\pi\rho\omega_0^3\beta^3\gamma^3}{2c^3})^{1/2}\frac{1}{\beta_c^2\gamma_c^2}\Theta(t-z/v)\sin[\omega_0(t-z/v)]\exp(-\frac{\rho\omega_0}{\beta\gamma c}). \quad (3.13)$$

On the other hand, the condition $\tilde{\epsilon} << 1$ can be fulfilled when β is close to 1, but β_c is much closer to it. Then,

$$A_z^{(1)} = \frac{e\omega_0\tilde{\epsilon}}{c}\Theta(t-z/v)\sin[\omega_0(t-z/v)]. \quad (3.14)$$

Thus, $A_z^{(1)}$ is small (due to the $\tilde{\epsilon}$ factor), but not exponentially small. This means that one should observe oscillations in the half-space behind the moving charge. Physically,

$\beta_c \approx 1$, $\beta \approx 1$, $\tilde{\epsilon} << 1$ corresponds to the motion in an optically rarefied medium (e.g., gas) with a charge velocity slightly smaller than the light velocity in medium.
We observe the a noticeable distinction between the cases $\beta \approx 1$, $\beta_c \approx 1$ corresponding to $\tilde{\epsilon} >> 1$ and $\tilde{\epsilon} << 1$. In both cases $A_z^{(1)}$ oscillates in the half-space behind the moving charge, but the amplitude of oscillations is considerably lesser for $\beta < \beta_c$ (due to the $\tilde{\epsilon}$ factor in (3.14)).
More precisely, conditions under which Eq. (3.14) is valid is $\rho\omega_0/\beta\gamma c << 1$. This means that for β fixed, VP oscillations should take place for small values of ρ.

9) Let the charge velocity exactly coincide with the light velocity in medium: $\beta = \beta_c$, $\tilde{\epsilon} = 1$. Then,

$$\frac{\omega_1^2}{\omega_0^2} = -\frac{x^2}{2} + x(1+x^2/4)^{1/2}, \quad \frac{\omega_2^2}{\omega_0^2} = \frac{x^2}{2} + x(1+x^2/4)^{1/2}.$$

Let $\beta = \beta_c \approx 1$. This corresponds to a fast charged particle moving in a rarefied medium. Then,

$$A_z^{(1)} = \frac{4}{3}\frac{e\omega_0}{c}\Theta(t-z/v)\sin[\omega_0(t-z/v)][K_0(\frac{\omega_0\rho}{\sqrt{2}\beta\gamma c}) - K_0(\sqrt{2}\frac{\omega_0\rho}{\beta\gamma c})],$$

$$A_z^{(2)} = \frac{e\beta}{R}\exp(-\omega_0 R/v), \quad R = [(z-vt)^2+\rho^2/\gamma^2]^{1/2}.$$

Thus, $A_z^{(2)}$ differs from zero in a neighbourhood of the current charge position, whereas $A_z^{(1)}$ describes the oscillations in the half-plane behind the moving charge. As γ is very large, $A_z^{(1)}$ as a function of ρ diminishes rather slowly: it decreases essentially when the radius $\rho \approx \sqrt{2}c\gamma/\omega_0$.

4 Numerical Results

In this section we, present the results of numerical calculations. We intend to consider the EMF distribution on the surface of a cylinder of the radius ρ (Fig. 4). This is a usual procedure in the consideration of VC effect (see, e.g., [7]).
For frequency-independent electric permittivity ($\epsilon = \epsilon_0$) there is no radiation for $\beta < \beta_c = \epsilon_0^{-1/2}$. For $\beta > \beta_c$, the energy flux is infinite on the surface of the Cherenkov-Mach cone. On the surface of C_ρ it equals zero for $z > -z_c$, $z_c = \sqrt{\frac{\beta^2}{\beta_c^2}-1}$ and acquires an infinite value in the place $z = -z_c$ where C_ρ is intersected by the above cone. Inside the Mach cone the electromagnetic strengths fall as r^{-2} at large distances and, therefore, do not contribute to the radial flux.

In what follows the results of numerical calculations will be presented in dimensionless variables. In particular, lengths will be expressed in units c/ω_0, time in units ω_0^{-1}, electromagnetic strengths in units $e\omega_0^2/c^2$, the Poynting vector $\vec{P} = (c/4\pi)(\vec{E}\times\vec{H}$ in units $e^2\omega_0^4/c^3$, etc. We evaluate the radial energy flux per unit length through the surface of a cylinder C_ρ (Fig. 4) coaxial with the z axis for the total time of motion. It is given by

$$W_\rho = 2\pi\rho\int_{-\infty}^{+\infty}S_\rho dt = \frac{2\pi\rho}{v}\int_{-\infty}^{+\infty}S_\rho dz, \quad S_\rho = \frac{c}{4\pi}(\vec{E}\times\vec{H})_\rho = -\frac{c}{4\pi}E_zH_\phi. \quad (4.1)$$

Substituting here E_z and H_ϕ we get for energy losses per unit length

$$W_\rho = \frac{e^2}{c^2} \int\limits_{\beta^2\epsilon>1} \omega d\omega (1 - \frac{1}{\epsilon\beta^2}). \tag{4.2}$$

Or, explicitly [17],

$$W_\rho = \frac{e^2}{c^2} \int\limits_{\omega_c}^{\omega_0} \omega d\omega (1 - \frac{1}{\epsilon\beta^2}) = -\frac{e^2\omega_0^2}{2c^2\beta_c^2\gamma_c^2}[1 + \frac{1}{\beta^2}\ln(1-\beta^2)] \tag{4.3}$$

for $\beta < \beta_c$ and

$$W_\rho = \frac{e^2}{c^2} \int\limits_{0}^{\omega_0} \omega d\omega (1 - \frac{1}{\epsilon\beta^2}) = \frac{e^2\omega_0^2}{2c^2}[-\frac{1}{\beta^2\gamma^2} + \frac{1}{\beta^2\beta_c^2\gamma_c^2}\ln(\gamma_c^2)] \tag{4.4}$$

for $\beta > \beta_c$. Similar expressions were obtained by E. Fermi [11]. he validity of Eq.(4.2) is also confirmed by the results obtained by Sternheimer [18] (whose equations pass into (4.2) in the limit $p \to 0$) and in the recent review by Ginzburg [19].
For $\beta \to 0$ the energy losses W_ρ tend to 0, while for $\beta \to 1$ (it is just this limit that was considered by Tamm and Frank [2]) they tend to the finite value $\frac{e^2\omega_0^2}{2c^2}\ln(1 + \frac{\omega_0^2}{\omega_L^2})$.
In Fig. 5, we present the dimensionless quantity $F = W_\rho/(e^2\omega_0^2/c^2)$ as a function of the particle velocity β. The numbers on curves mean β_c. Vertical lines with arrows divide a curve into two parts corresponding to the energy losses with velocities $\beta < \beta_c$ and $\beta > \beta_c$ and lying to the left and right of vertical lines, resp. We see that the charge uniformly moving in medium radiates at every velocity.
How is this flux distributed over the surface of C_ρ? For the definiteness we take $\beta_c = 0.75$ to which corresponds the refractive index $n = 1/\beta_c = 1.333$. This is close to the refractive index of the water ($n = 1.334$). The value of ρ is chosen to be $\rho = 10$ (in units of c/ω_0). In Fig. 6 it is shown how the quantity $\sigma_\rho = 2\pi S_\rho$ is distributed over the C_ρ surface for $\beta = 0.3$. It is seen that the radiation EMF (corresponding to the $A_z^{(1)}$ term in A_z) differs from zero only at large distances from the moving charge. The isolated oscillation in the neighbourhood of $z = 0$ corresponds to the EMF carried by the moving charge with itself. We refer to this part of EMF as to the non-radiation one. The integral of it taken over either z or t is equal to zero. Thus, it does not contribute to the total energy losses given by (4.1). Being originated from the $A_z^{(2)}$ term in A_z (see Eq.(2.7)), it approximately equals

$$\sigma_\rho^{(2)} = -\frac{c\beta e^2}{2\epsilon_0}(1 - \beta^2/\beta_c^2)^2 \frac{\rho(z - vt)}{[z^2 + \rho^2(1 - \beta^2/\beta_c^2)]^3} \tag{4.5}$$

As we have mentioned, this corresponds to the radial energy flux carried by the uniformly moving charge with the velocity $\beta < \beta_c$ in medium with a constant $\epsilon = \epsilon_0$. Due to its antisymmetry w.r.t. $z - vt$ the integral of it taken over either z or t is equal to zero.

If the distribution of the radiation flux on the surface of the sphere S (instead on the cylinder surface C_ρ, as we have done up to now) were considered, the radial radiation flux S_ρ would be confined to the narrow cone adjusted to the negative z semi-axis (Fig. 7). As it follows from Fig. 6, the solution angle θ_c of this cone equals approximately 3 degrees for $\beta_c = 0.75$ and $\beta = 0.3$.

When β grows, the relative contribution of the radiation term also increases. This is clearly demonstrated in Figs. 8 and 9 where the distributions of σ_ρ are presented for $\beta = 0.5$ and $\beta = 0.99$, resp. The energy flux distributions presented in Figs. 6, 8 and 9 consist in fact of many oscillations. This is shown in Fig. 10 where the magnified image of σ_ρ for $\beta = 0.99$ is presented. It turns out that the first maximum of the radiation intensity is located in the same place $z_c = -\rho\sqrt{\beta^2/\beta_c^2 - 1}$ where in the absence of dispersion the singular Cherenkov-Mach cone intersects the C_ρ surface.
In Fig. 11 there is shown the distribution of the energy flux on the C_ρ surface for $\beta_c = 0.1$ and $\beta = 0.99$. In this case $\tilde{c} >> 1$ and the radiation, similarly to the $\tilde{c} << 1$ case shown in Fig. 6, is confined to the narrow cone behind the moving charge but with a much larger amplitude. This completely agrees with a qualitative consideration of the previous section.

To detect the S_ρ component of $\vec{S}$, one should have a detector imbedded into a thin collimator and directed towards the charge motion axis. The collimator should be impenetrable for the γ quanta with directions different from the radial one. It follows from Figs. 6-11 that in a particular detector ($z = const$), rapid oscillations of the radiation intensity as a function of time should be observed (as all the physical quantities and, in particular, S_ρ depend on t and z via the combination $z - vt$). It should be asked why so far nobody observed these oscillations? From the $\beta = 0.99$, $\beta_c = 0.75$ case presented in Fig. 10 it follows that diffraction picture differs essentially from zero on the interval $-150 < z - vt < 0$, where z is expressed in units c/ω_0. The typical ω_0 value taken from the Frank book [7] is $\omega_0 \approx 6 \cdot 10^{15} sec^{-1}$. This gives $c/\omega_0 \approx 5 \cdot 10^{-6}$ cm. We see that the above interval is of the order $150 \cdot 5 \cdot 10^{-6} cm \approx 10^{-3} cm$. The rapidly moving charge ($v \approx c$) overcomes this distance for the time $10^{-3}c^{-1} \cdot sec \approx 3 \cdot 10^{-14} sec$. It follows from Fig. 10 that there are many oscillations in this time interval. Because of this, they hardly can be resolved experimentally.
Now we turn to experiments recently discussed in [12-14]. In them, for the electron moving in a gas with a fixed energy the radiation intensity was measured as a function of the gas pressure P. The latter is related to the gas density N_g by the well-known thermodynamic relation: $PV = kN_gT$, where V is the gas volume, T is its temperature and k is the Boltzmann constant. The quantities N_e, ω_L^2 and β_c used in section 2 are related to N_g as follows:

$$N_e = N_g \cdot Z, \quad \omega_L^2 = 4\pi N_e e^2/m \quad \beta_c^2 = \frac{\omega_0^2}{\omega_0^2 + \omega_L^2}.$$

Here Z is the atomic number of a gas. Let the gas pressure, at which $\beta_c = \beta$, be equal to p_c. In the experiments quoted above a sharp reduction of the radiation intensity was observed for the gas pressure $p \approx p_c/100$. To this gas pressure there corresponds $\tilde{c} << 1$ despite the fact that $\beta \approx \beta_c \approx 1$ (this is possible because of the γ factors in the definition of $\tilde{c}$).

To clarify the nature of this phenomenon we turn to Eqs. (4.3) and (4.4) which for a fixed β define energy losses per unit length as a function of β_c. The typical curves are shown in Fig. 12. The numbers on curves mean charge velocity. It follows that for $\beta = 0.99$ the radiation intensity diminishes approximately 60 times when β_c changes from 0.9 to 0.999. The corresponding distributions of the energy flux on the surface of C_ρ are shown in Figs. 13 and 14. It is seen that the intensity at maxima is almost 1000 times lesser for $\beta_c = 0.999$ than for $\beta_c = 0.9$. The intensity distribution is very sharp for

$\beta_c = 0.9$ and rather broad for $\beta_c = 0.999$. The physical reason for the sharp reduction of intensity is in the increase for $\beta_c > \beta$ of the region where the electromagnetic waves are damped. This agrees with qualitative estimates of section 3.

So far, we have evaluated S_ρ, the radial component of the Poynting vector. The integral $2\pi\rho\int S_\rho dz$ taken over C_ρ is the same for any ρ. It is equal to vW_ρ, where v is the charge velocity while the quantity W_ρ independent of ρ is defined by Eqs. (4.2)-(4.4). The Poynting vector $\vec{P}$ has another component, S_z. Both of them define the direction in which the radiatiion propagates. The distributions of $\sigma_z = 2\pi S_z$ on the surface of C_ρ are shown in Figs. 15-17 for the charge velocities $\beta = 0.3$, 0.5 and 0.99, resp. The isolated peak in the neighbourhood of $z = 0$ plane corresponds to EMF carried by the moving charge with itsef. Being originated from the second term in A_z it approximately equals to (for $\beta < \beta_c$)

$$\sigma_z^{(2)} \approx \frac{c\beta e^2\rho}{2\epsilon_0\gamma_n^4}\frac{1}{[(z-vt)^2+\rho^2/\gamma_n^2]^3},\ \gamma_n^2 = (1-\beta_n^2)^{-1},\ \beta_n = \beta/\beta_c.$$

It is seen that the qualitative behaviour of S_z is almost the same as S_ρ, however, the maxima of S_z are approximately twice of those of S_ρ. This means that the radiation is emitted more in the forward direction than in the transverse one. To observe S_z, one should orient the collimator with a detector inside it along the z axis. The collimator should be impenetrable for the γ quanta with directions non-parallel to the z axis. Again, the oscillations of intensity as a function of time should be detected during the charge motion.

To determine the major direction of the radiation, one should find surfaces on which the Poynting vector is maximal ([17]). Due to the axial symmetry these surfaces look like lines in ρ, z variables. We shall refer to these lines as to trajectories. The behaviour of these trajectories is quite different depending on whether $\beta > \beta_c$ or $\beta < \beta_c$. For $\beta > \beta_c$ the trajectories are not closed. When $z \to \infty$, ρ also tends to ∞. For $\beta < \beta_c$ the trajectories are closed. In the WKB approach, on a particular of the mentioned surfaces, the inclination of the Poynting vector towards the motion axis is given by ([17])

$$\cos\theta_P = \frac{S_z}{\sqrt{S_\rho^2+S_z^2}} = \frac{1}{\beta\sqrt{\epsilon(x)}}.$$

Here x is a parameter, $\epsilon(x) = 1+[\beta_c^2\gamma_c^2(1-x^2)]^{-1}$, $S_\rho = -cE_z H_\phi/4\pi$ and $S_z = cE_\rho H_\phi/4\pi$. For $\beta > \beta_c$, x changes from $x = 1$ for which ρ is zero, z is finite and $\theta_P = \pi/2$ up to $x = 0$ for which both ρ and z are infinite while $\cos\theta_P$ has the same value β_c/β as in the absence of dispersion.

For $\beta < \beta_c$ a particular trajectory intersects the motion axis two times: at $x = 1$ where z is finite and $\theta_P = \pi/2$ and at $x = \sqrt{1-\bar{\epsilon}}$ where z is finite and greater in absolute value than for $x = 1$, while $\theta_P = 0$ there. At the point of the trajectory where ρ is maximal the inclination of the Poynting vector towards the motion axis acquires the intermediate value

$$\cos\theta_P = \frac{1}{\beta}[1+\frac{1}{\beta_c^2\gamma_c^2(2-\sqrt{4-3\bar{\epsilon}})}]^{-1/2}$$

Consider now the energy flux per unit time through the entire $z = const$ plane. It is given

by

$$W_z = \int S_z\rho d\rho d\phi = \frac{c}{2}\int E_\rho H_\phi\rho d\rho$$

Substituting here E_ρ and H_ϕ from (2.7) and using the well-known orthogonality relation between the Bessel functions ($\int_0^\infty \rho d\rho J_m(k\rho)J_m(k'\rho) = \frac{1}{k}\delta(k-k')$),

one obtains

$$W_z = \frac{1}{2}e^2v^3\gamma^2\int k^2dkF_A(k,z-vt)\{\gamma^2F_\phi(k,z-vt) -$$

$$-\frac{2}{v^2}\frac{\omega_0}{\gamma_c^2\beta_c}\frac{1}{k^2+\omega_0^2/v^2\beta_c^2}\sin[\omega_0(t-z/v)/\beta_c]\},$$

where F_A and F_ϕ are given by Eqs. (2.8). It is not evident that W_z is positive definite. In Fig. 18 we present W_z as a function of z for $\beta_c = 0.75$ and $\beta = 0.99$. It is seen that W_z is almost constant in a very broad range of z except for the neighbourhood of the $z = const$ plane passing through the current charge position. The positivity of $W_z = 2\pi\int S_z\rho d\rho$ means that the energy flow of radiation follows for the moving charge and does not mean that S_z is also positive. This is illustrated in Fig. 19 where $\sigma_z = 2\pi S_z$ as a function of ρ is presented for a particular $z = const$ plane ($z = -800$). It is seen that S_z contains both positive and negative parts. This may be understood within the polarization formalism [17]. In it, the moving charge induces the time-dependent polarization of the medium. This in turn leads to the appearance of the radiation characterized by the Poynting vector $\vec{S} = \frac{c}{4\pi}(\vec{E}\times\vec{H})$. The positivity of S_z means that the part of the induced radiation flux follows for the moving charge. This fact has no relation to the well-known difficulty taking place for the radiation of the accelerated charge moving in vacuum where the solutions of the Maxwell equations corresponding to the energy flux directed inwards the moving charge are regarded as unphysical.

5 Conclusion

1. It is shown that a point charge moving uniformly in a dielectric medium with a standard choice (1.1) of electric permittivity should radiate at each velocity. The distributions of the radiated electromagnetic field differ drastically for a charge velocity v below and above some critical value v_c which depends on the medium properties and does not depend on the frequency (despite the fact that ω dispersion is taken into account). For $v < v_c$ the radiation flux is concentrated behind the moving charge on a sufficiently remote distance from the charge.

2. The electromagnetic field radiated by a charge uniformly moving in a dielectric medium with $\epsilon(\omega)$ given by (1.1) consists of many oscillations which should be observed experimentally.

3. The results of recent experiments [12-14] dealing with the Vavilov-Cherenkov radiation and indicating on the existence of the radiation below the Cherenkov threshold seems to be supported by the the present investigation.

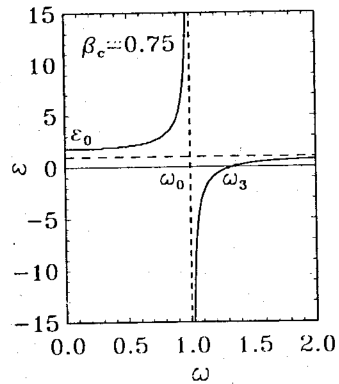

Fig. 1: For the free electromagnetic wave propagating in medium, the damping region where $\epsilon < 0$ corresponds to $\omega_0^2 < \omega^2 < \omega_3^2$.

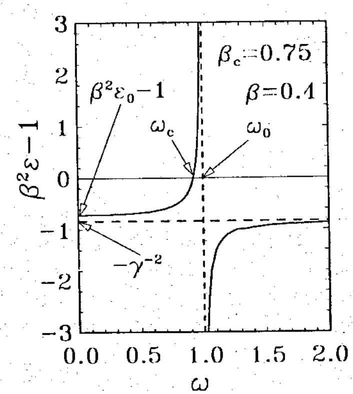

Fig. 2: For the electromagnetic field radiated by a charge uniformly moving in medium with velocity $v < v_c$, the damping region where $1 - \beta^2\epsilon > 0$ lies within the intervals $0 < \omega < \omega_c$ and $\omega_0 < \omega < \infty$.

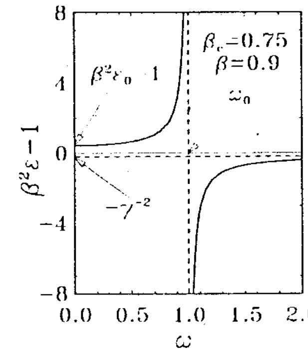

Fig. 3: For the electromagnetic field radiated by a charge uniformly moving in medium with velocity $v > v_c$, the damping region where $1-\beta^2\epsilon > 0$ extends from $\omega = \omega_0$ to $\omega = \infty$.

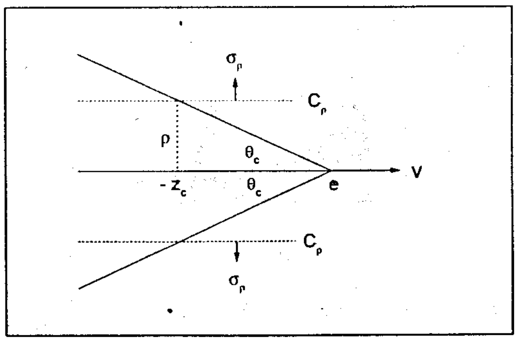

Fig. 4: Schematic presentation of the Cherenkov cone for a constant electric permittivity. The radiation field has a δ-type singularity on the surface of cone, the field inside the cone does not contribute to the radiation. On the surface of the cylinder C_ρ the electromagnetic field is zero for $z > -z_c$; σ_ρ is proportional to the radial energy flux through the cylinder surface ($\sigma_\rho = 2\pi S_\rho$).

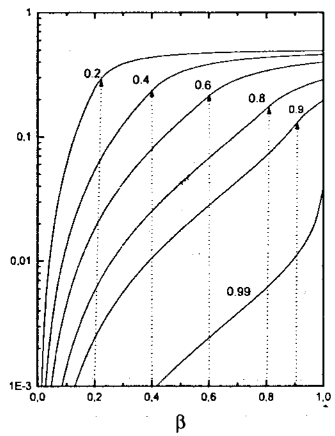

Fig. 5: The radial energy losses per unit length (in units $e^2\omega_0^2/c^2$) as a function of $\beta = v/c$. The number of a particular curve mean the critical velocity β_c.

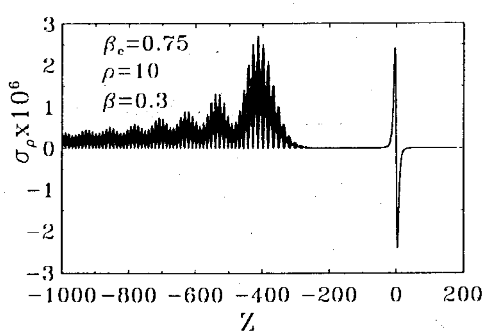

Fig. 6: The distribution of the radial energy flux on the surface of C_ρ for $\beta = 0.3$ and $\beta_c = 0.75$. The isolated oscillation in the neighbourhood $z = 0$ corresponds to the non-radiation field carried by a charge with itself. The radiation and non-radiation terms are of the same order.

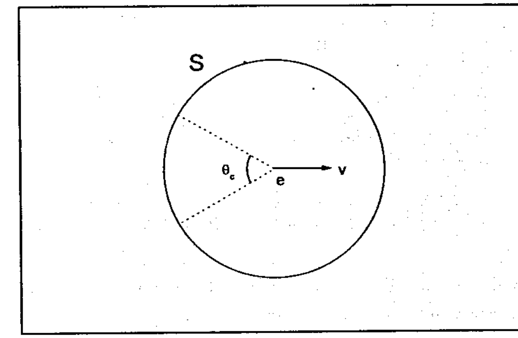

Fig. 7: The distribution of the radial energy flux on the surface of the sphere S_ρ for $\beta = 0.3$ and $\beta_c = 0.75$ is confined to the narrow cone with a the solution angle $\theta_c \approx 3^0$.

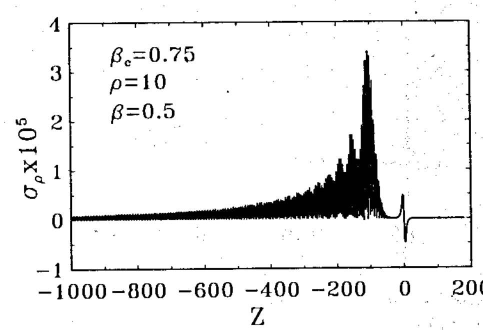

Fig. 8: The same as in Fig. 6, but for $\beta = 0.5$. The contribution of the non-radiation term relative to the radiation one is much smaller than for $\beta = 0.3$.

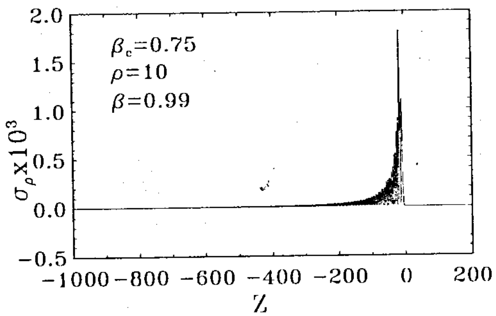

Fig. 9: The same as in Fig. 6, but for $\beta = 0.99$. The contribution of the non-radiation term relative to the radiation one is negligible.

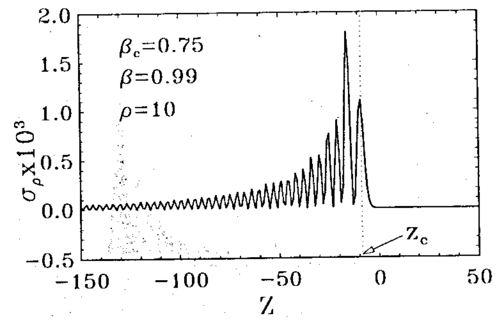

Fig. 10: Fine structure of $\beta = 0.99$ case. It seen that a seemingly continuous distribution of Fig. 9 consists of many peaks.

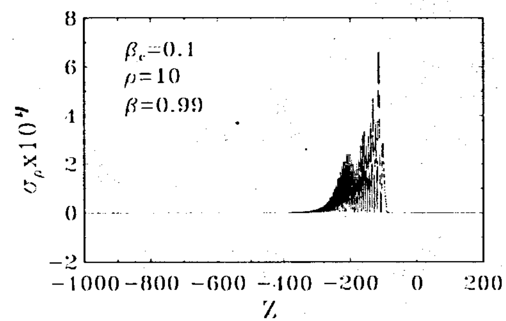

Fig. 11: The distribution of the radial flux on the surface of C_ρ for $\beta = 0.99$ and $\beta_c = 0.1$. As in Fig. 6, the radiation is confined to the narrow cone behind the moving charge, but with a much greater amplitude.

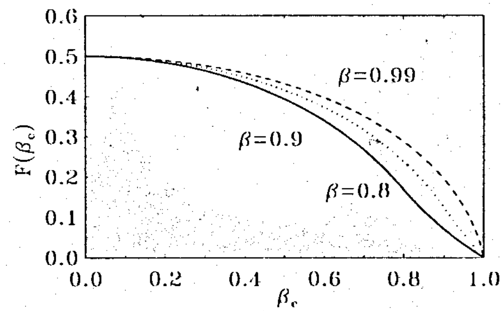

Fig. 12: The radial energy losses as a function of the critical velocity characterizing medium properties. Values of β_c close to 1 and 0 correspond to optically rarefied and dense media. Numbers of curves mean charge velocity β.

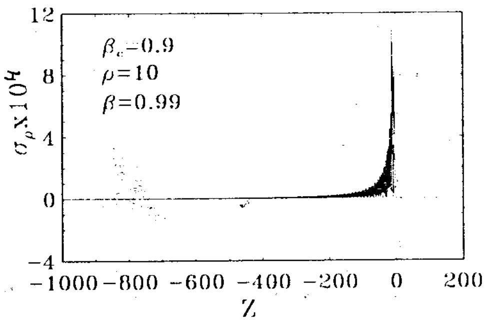

Fig. 13: The distribution of the radial energy flux on the surface of C_ρ, for the charge velocity ($\beta = 0.99$) slightly smaller than charge velocity in vacuum and for the critical velocity $\beta_c = 0.9$ slightly smaller than β. The intensity of radiation is concentrated near the $z = z_c$ plane.

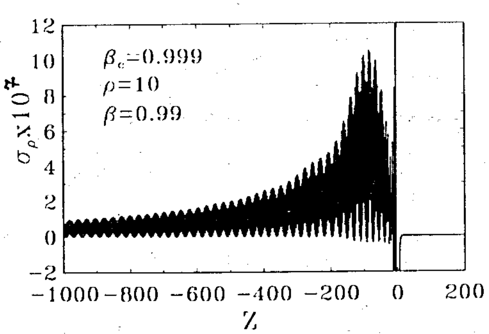

Fig. 14: The same as in Fig. 13, but for the critical velocity $\beta_c = 0.999$ only slightly greater than $\beta = 0.99$. The distribution of the radiation intensity is very broad and by three orders smaller than in Fig. 13.

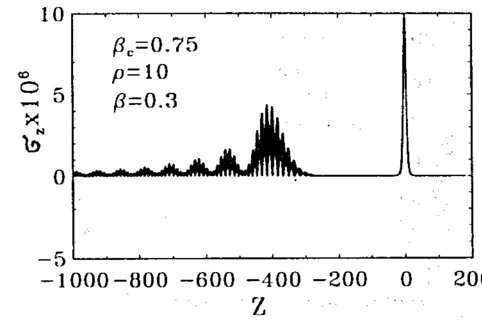

Fig. 15: The distribution of the energy flux σ_z along the motion axis on the surface of C_ρ for $\beta = 0.3$ and $\beta_c = 0.75$. The isolated peak in the neighbourhood of $z = 0$ corresponds to the non-radiation field carried by a charge with itself. The radiation and non-radiation terms are of the same order.

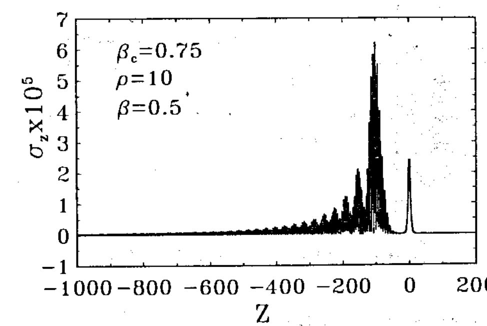

Fig. 16: The same as in Fig. 15, but for $\beta = 0.5$. The contribution of the non-radiation term relative to the radiation one is much smaller than for $\beta = 0.3$.

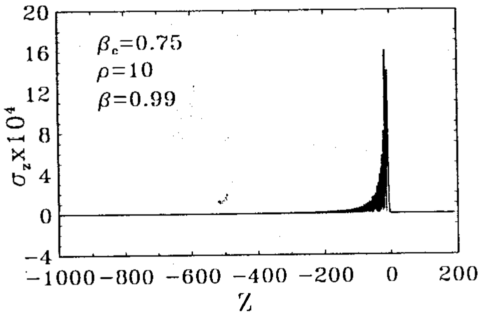

Fig. 17: The same as in Fig. 15, but for $\beta = 0.99$. The contribution of the non-radiation term relative to the radiation one is negligible.

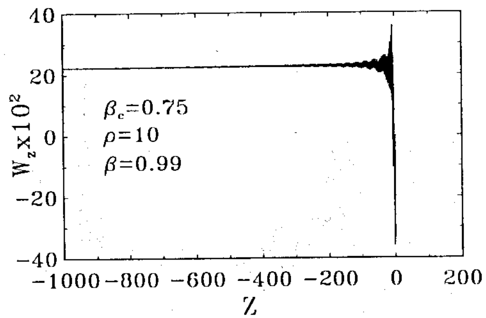

Fig. 18: The total integral energy flux W_z through the plane normal to the motion axis as a function of this plane position for $\beta = 0.99$ and $\beta_c = 0.75$.

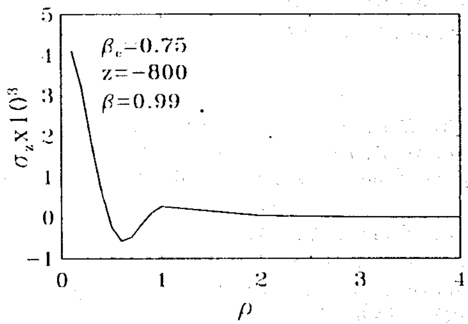

Fig. 19: The distribution of the energy flux in a particular ($z = -800$) plane normal to the motion axis as a function of the radial distance ρ. Positive and negative signs of σ_z correspond to the energy flow directed towards the moving charge and outwards it, resp.

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