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V.S.Barashenkov[1], M.Z.Yur'iev[2]

MULTITIME QUANTUM THEORY

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[1]E-mail: barashenkov@lcta30.jinr.dubna.su
[2]The company INTERPROM, Moscow, Russia

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1. Introduction

Six-dimensional theory with the equal number of space and time co-ordinates $\hat{\mathbf{x}} = x_1, x_2, x_3, t_1, t_2, t_3$ [1] becomes apparent as a development of the relativity taking into account a subsequent symmetrization of space and time and, independently, as an attempt to bystep the difficulties of the Lorentz transformations in the theories with faster-than-light velocities (see papers [1] - [7] where one can find a more detailed bibliography).

From a physical viewpoint such an approach is based on a hypothesis that our universe has been created possessing an accidental "time arrow" determined by the evolution of physical processes at the first moments of their existence. The following inflationary expansion destroyed spacetime correlations of remote regions of the world and every such a region has now its own time arrow $\hat{t}$ declined in a general case to the primary "relict" one. Since all processes and all bodies in such parts of our universe have the same, parallel one another, time trajectories, we do not observe two additional time co-ordinates and perceive the surrounding world as one-temporal. The energy conservation law and the time irreversibility prevent any change of body time trajectories. In each such a case a body (or bodies) with compensating energy components $E_i \leq 0$, i.e. moving backward in time have to be present (see, e. g. Fig. 1 where a decay of a body into two components with distinct time trajectories is shown). Such phenomena are forbidden and the multidimensionality of time remains hidden for us.

[1] In what follows the tree-dimensional vectors in x- and t-subspaces will be denoted, respectively, by bold symbols and by a hat, six-dimensional vectors will be marked by bold symbols with a hat. In manuscripts it is convenient to use the notations $\bar{x}$, $\hat{x}$ and $\hat{\bar{x}}$. All matrices will be denoted by capital letters. The "six-dimensional nabla" $\hat{\boldsymbol{\nabla}} = (\nabla, \hat{\nabla})$, where time operator $\hat{\nabla} = (-\partial/\partial t_1, -\partial/\partial t_2, -\partial/\partial t_3)$. We suppose also that co- and contravariant vectors are distinguished by the sign of their space components, e. g., $(\hat{\mathbf{x}})_\mu = (\mathbf{x}, -c\hat{t})^T_\mu$, $(\hat{\mathbf{x}})^\mu = (\mathbf{x}, c\hat{t})^{T\mu}$. So, a scalar product $\hat{\mathbf{a}}\hat{\mathbf{b}} = \mathbf{ab} - \hat{a}\hat{b}$. As a rule, we shall also suppose that the Latin and Greek indices take values $k = 1, ..., 3, \mu = 1, ..., 6$ and the constants $\hbar = c = 1$.

The performed investigations convince that such an approach is logically consistent and has no disagreement with the known experimental facts [8].

One can wait an appearance of bodies (or rays) with "different time" in regions with strong gravity where the concept of energy itself loses its sense and the energy conservation law becomes inexact [9] or in microscopic quantum phenomena where the energy conservation and even time irreversibility (at very small Δx and Δt) can be violated virtually.

The investigation of such phenomena demands a development of multitime quantum mechanics. The first steps on this way have been done in papers [10, 11]. The goal of our paper is to analyze the solutions of the Dirac equation in the general case of an arbitrary particle time trajectory and to develop a theory of the second quantization of spinor and electromagnetic fields.

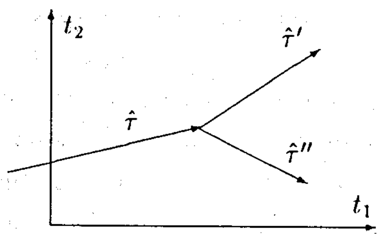

Fig. 1. The creation of a component with the energy $\hat E' = \hat\tau' E \geq 0$ is accompanied, without fail, by the creation of a compensating component with the energy $\hat E'' = \hat\tau'' E \leq 0$.

In the next section the solutions for the Dirac equation are considered. Sec. 3 is devoted to the deduction of multitime nonrelativistic Pauli and Schrödinger equations. Secs. 4 and 5 define commutation rules and discuss the problem of the positiveness of the field energy and of the wave function norm. The last section summarizes the main results.

2. Four spinor states

Following the paper [10], let us write the multitime eighty-component Dirac equation with electromagnetic field in the form

$$(i\hat\gamma\hat{\boldsymbol\nabla} + e\hat\gamma\hat{\mathbf A} - m)\Psi = 0. \tag{1}$$

Here

$$\gamma_i = \begin{pmatrix} O & \Sigma_i \\ \Sigma_i & O \end{pmatrix} + \delta_{4i}\begin{pmatrix} I_4 & -I_4 \\ I_4 & -I_4 \end{pmatrix}, \quad \Sigma_i = \begin{pmatrix} O & \sigma_i \\ \sigma_i & O \end{pmatrix}.$$

$$\Sigma_4 = \begin{pmatrix} I_2 & O \\ O & -I_2 \end{pmatrix}, \quad \Sigma_5 = \begin{pmatrix} -iI_2 & O \\ O & iI_2 \end{pmatrix}, \quad \Sigma_6 = \begin{pmatrix} O & I_2 \\ -I_2 & O \end{pmatrix}.$$

σ_i are the known Pauli matrices, I_n is $n \times n$ unit matrix.

Let us replace

$$\Psi(\hat{\mathbf x}) = \Phi(\hat{\mathbf x})e^{-im\hat\tau\hat t}, \tag{2}$$

where $\Phi = (\phi_1, ..., \phi_8)^T$ is the eighty-component Dirac spinor, $\hat\tau$ is a constant unit vector ($\hat\tau^2 = 1$). For a free particle this vector defines a constant direction of its time trajectory (in this case the function ϕ is time independent). When $\hat A \neq 0$ and the direction of the particle time trajectory is changed, $\hat\tau$ is a vector (the tangent) characterizing the t-trajectory direction at some arbitrary chosen moment t_o. So, eq. (1) can be rewritten as

$$\left[i\hat\gamma\hat{\boldsymbol\nabla} + e\hat\gamma\hat{\mathbf A} - m(1-\Theta)\right]\Phi = 0 \tag{3}$$

with the matrices

$$\Theta = \hat\gamma\hat\tau = \begin{pmatrix} \tau_1 I_2 & \Theta_{23} \\ -\Theta_{23} & -\tau_1 I_2 \end{pmatrix}, \quad \Theta_{23} = \hat\Sigma\hat\tau - \Sigma_4\tau_1 = \begin{pmatrix} -i\tau_2 I_2 & \tau_3 I_2 \\ -\tau_3 I_2 & i\tau_2 I_2 \end{pmatrix}.$$

Later on it will be convenient to split the Dirac wave function into two four-dimensional components: $\Phi = (\Phi', \Phi'')^T$. In this case eq. (3) is splitted also into two ones:

$$D\Phi' - [i\nabla_4 + eA_4 - m(1+\tau_1)]\,\Phi'' = 0, \tag{4}$$

$$D\Phi'' - [i\nabla_4 + eA_4 - m(11-\tau_1)]\,\Phi' = 0, \tag{5}$$

where

$$D = \hat{\Sigma}(i\hat{\nabla} + e\hat{\mathbf{A}}) - \Sigma_4(i\nabla_4 + eA_4) + m\Theta_{23}. \tag{6}$$

Now we suppose that the particle momentum is directed along z-axis, i.e. $\mathbf{p} = (0,0,1)$, and the field $\hat{\mathbf{A}} = 0$, then taking into account the relations

$$i\nabla\Phi' = p\Phi', \quad i\hat{\nabla}\Phi' = -\hat{\tau}E\Phi' \tag{7}$$

and the similar relations for Φ'' eqs. (4) and (5) can be split into eighty equations: four equations for the odd components ϕ_{2n+1} and four ones for the even ϕ_{2n}. Each group of the equations has two independent solutions. For example, the equations of the "odd group"

$$(E - m\tau_1 + p\tau_3)\phi_1 - ip\tau_2\phi_3 + im\tau_2\phi_5 - (m\tau_3 + p\tau_1)\phi_7 = 0, \tag{8}$$

$$ip\tau_2\phi_1 + (E - \tau_1 - p\tau_3)\phi_3 + (m\tau_3 - p\tau_1)\phi_5 - im\tau_2\phi_7 = 0 \tag{9}$$

have a solution

$$\phi_1 = 1, \quad \phi_3 = \phi_{2n} = 0, \quad \phi_5 = i\tau_2 gE, \quad \phi_7 = g(p + E\tau_3), \tag{10}$$

where $g = 1/(m + E\tau_1)$ and a solution obtained from (10) by the substitution

$$p \to -p, \quad \phi_1 \to \phi_3, \quad \phi_3 \to \phi_1, \quad \phi_5 \to -\phi_7, \quad \phi_7 \to -\phi_5\,.$$

The "even group" equations and their two solutions coincide with the odd ones by the transformation

$$p \to -p, \quad \phi_{2n} \to \phi_{2n+1}\,.$$

A complete set of the independent solutions Φ_s for a positive energy E is presented in the Table I. There is an analogous set for $E < 0$ [2].

In contrast to the one-time theory where the scalars

$$\bar{\Phi}_s\Phi_s = \Phi_s^{+}\gamma_4\Phi_s = (m/E)\Phi_s^{+}\Phi_s \neq 0$$

in six-dimensional spacetime the frame independent quantity $\bar{\Phi}_s\Phi_s = \Phi_s^{+}\Gamma\Phi_s$ with the matrix

$$\Gamma = i\gamma_4\gamma_5\gamma_6 = \begin{pmatrix} -\Sigma_0 & 0 \\ 0 & \Sigma_0 \end{pmatrix}, \quad \Sigma_0 = \begin{pmatrix} O & I_2 \\ I_2 & O \end{pmatrix}$$

is zero:

$$\bar{\Phi}_s\Phi_r = -N\delta_{r\ell}, \quad N = 2mg, \quad \ell = s + (-1)^{s+1}, \tag{11}$$

therefore it is more convenient to use the linear combinations

$$\Psi_{1,2} = (2N)^{-1/2}(\Phi_1 \pm \Phi_2), \quad \Psi_{3,4} = (2N)^{-1/2}(\Phi_3 \pm \Phi_4). \tag{12}$$

By means of the formulae (11), or using Table II where Ψ-functions are presented, one can prove that the relativistic-invariant scalar products of these functions

$$\bar{\Psi}_s\Psi_r = \eta_s\delta_{sr}. \tag{13}$$

$$\eta_s = \begin{cases} -1, & s = 1,4 \\ 1, & s = 2.3\,. \end{cases} \tag{14}$$

[2] The solutions found by Cole [11] correspond to the particular case when the particle moves along t_3-axis: $\hat{\tau} = (0,0,1)$. By this condition

$$\Phi_{1,2}^{Cole} = \lambda^{-1}(\Phi_1 \mp \Phi_2), \quad \Phi_{3,4}^{Cole} = \lambda^{-1}(\Phi_3 \mp \Phi_4),$$

where $\lambda = [(E+p)/m]^{-1/2}$ and the Φ-functions are presented in the Table I

Table I

Solutions Φ_s and $\bar{\Phi}_s$ for the multitime Dirac equation (1) with $\hat{\mathbf{A}} = 0$ and $E \geq 0$.

No	I	II	III	IV
ϕ_1	1	0	0	0
ϕ_2	0	0	1	0
ϕ_3	0	1	0	1
ϕ_4	0	0	0	0
ϕ_5	$igE\tau_2$	$g(p - E\tau_3)$	0	$-g(p + E\tau_3)$
ϕ_6	0	0	$ig\tau_2$	0
ϕ_7	$g(p + E\tau_3)$	$-ig\tau_2 E$	0	$-ig\tau_2 E$
ϕ_8	0	0	$g(-p + E\tau_3)$	
$\bar{\phi}_1$	0	-1	0	0
$\bar{\phi}_2$	0	0	0	-1
$\bar{\phi}_3$	-1	0	0	0
$\bar{\phi}_4$	0	0	-1	0
$\bar{\phi}_5$	$g(p + E\tau_3)$	$ig\tau_2 E$	0	0
$\bar{\phi}_6$	0	0	$g(-p + E\tau_3)$	$igE\tau_2$
$\bar{\phi}_7$	$-igE\tau_2$	$g(p - E\tau_3)$	0	0
$\bar{\phi}_8$	0	0	$-ig\tau_2 E$	$-g(p + E\tau_3)$

Table II

The functions $U_s = 2(mg)^{1/2}\Psi_s$ and $\bar{U}_s = 2(mg)^{1/2}\bar{\Psi}_s$ with the norm $\bar{U}_s U_r = 4mg\eta_s\delta_{sr}$ for the four spin states and a positive energy $E > 0$.

s:	I	II	III	IV
u_1	1	1	0	0
u_2	0	0	1	1
u_3	1	-1	0	0
u_4	0	0	1	-1
u_5	$g(p + E_{23}^-)$	$g(-p + E_{23}^+)$	0	0
u_6	0	0	$g(-p + E_{23}^-)$	$g(p + E_{23}^+)$
u_7	$g(p - E_{23}^-)$	$g(p + E_{23}^+)$	0	0
u_8	0	0	$g(-p - E_{23}^-)$	$g(-p + E_{23}^+)$
$\bar{u}_1$	-1	1	0	0
$\bar{u}_2$	0	0	-1	1
$\bar{u}_3$	-1	-1	0	0
$\bar{u}_4$	0	0	-1	-1
$\bar{u}_5$	$g(p + E_{23}^+)$	$g(p - E_{23}^-)$	0	0
$\bar{u}_6$	0	0	$g(-p + E_{23}^+)$	$g(-p - E_{23}^-)$
$\bar{u}_7$	$g(p - E_{23}^+)$	$g(-p - E_{23}^-)$	0	0
$\bar{u}_8$	0	0	$g(-p - E_{23}^+)$	$g(p - E_{23}^-)$
$\Psi\Psi$	-1	1	-1	1
$\mathcal{S}$	-1	1	1	-1
$\mathcal{T}$	1	-1	1	-1

Here $E_{23}^{\pm} = E(i\tau_2 \pm \tau_3)$.

Note should be taken that in the multitime theory not only particle energy E but also the probability $w \sim \bar{\Psi}\Psi$ can be negative, i.e. we deal with a Hilbert space with an indefinite metric.

Now we may define spin matrices [11]

$$\mathcal{S} = (i\gamma_2\gamma_3), i\gamma_3\gamma_1, i\gamma_1\gamma_2) = \sigma \cdot I_4 \qquad (15)$$

and

$$\hat{\mathcal{T}} = (-i\gamma_5\gamma_6, -i\gamma_6\gamma_4, -i\gamma_4\gamma_5) = -\Gamma\hat{\gamma} \qquad (16)$$

which satisfied the two symmetrical Pauli relations

$$\mathcal{S}_i\mathcal{S}_k = \varepsilon_{ikell}\mathcal{S}_\ell + \delta_{ik}I_8, \quad \mathcal{T}_i\mathcal{T}_k = \varepsilon_{ikell}\mathcal{T}_\ell + \delta_{ik}I_8.$$

By means of these matrices we calculate spatial and temporal particle helicities presented in the Table II:

$$S = \bar{\Psi}_s\mathcal{S}(\mathbf{p}/p)\Psi_s = \pm(p/m)\bar{\Psi}_s\Psi_s, \qquad (17)$$

where the signs plus and minus correspond respectively to the spin states $s = 1, 2$ and $s = 3, 4$,

$$T = \bar{\Psi}_s\hat{\mathcal{T}}\hat{\tau}\Psi_s = -(E/m)\bar{\Psi}_s\Psi_s. \qquad (18)$$

In these expressions we have taken into account that Dirac equation for a plane wave $\hat{\mathbf{p}}\hat{\gamma}\Psi = -m\Psi$, therefore

$$\bar{\Psi}\gamma_\mu\Psi = -m^{-1}\bar{\Psi}(\gamma_\nu\gamma_\mu + \gamma_\mu\gamma_\nu)p^\nu\Psi = -m^{-1}p^\mu\bar{\Psi}\Psi.$$

The distinction particles with different temporal helicities has to become apparent in the interactions which change the time trajectories. In other cases these particles are indistinguishable.

3. Nonrelativistic approximation

Let us return to eqs. (4) — (6). By analogy with the one-time theory we suppose that $\Phi'' \ll \Phi'$. Then we may disregard the term $(i\nabla_4 + eA_4)$ in the equation for Φ'' and the equation for Φ' can be represent in the form

$$[P_4 + m(1 - \tau_1)]\,\Phi + \frac{1}{m(1+\tau_1)}(\hat{\Sigma}\hat{\mathbf{P}} + \Sigma_4 P_4 + m\Theta_{23})^2\Phi = 0, \qquad (19)$$

where $\hat{\mathbf{P}} = i\hat{\nabla} + e\hat{\mathbf{A}}$ and we denoted $\Phi = \Phi'$.

Taking into account the properties of the matrices $\hat{\Sigma}$ one can prove the relations :

$$(\boldsymbol{\Sigma}(i\nabla + e\mathbf{A}))^2\,\Phi = (\left[i\nabla + e\mathbf{A})^2 - e\sigma\mathbf{H}\right]\Phi, \quad \mathbf{H} = \nabla \times \mathbf{A};$$

$$\tilde{\Sigma}(i\tilde{\nabla} + e\tilde{A} + m\tilde{\tau})^2\Phi = \left[-(i\tilde{\nabla} + e\tilde{A} + m\tilde{\tau})^2 - eT_1G_1\right]\Phi, \quad \hat{G} = -\hat{\nabla} \times \hat{A},$$

where here and in what follows $\tilde{X} = (X_5, X_6)$ is a two-dimensional time vector,

$$T_1 = i\Sigma_5\Sigma_6 = \begin{pmatrix} 0 & I_2 \\ I_2 & 0 \end{pmatrix}$$

is the matrix of the temporal spin (16);

$$\left[\boldsymbol{\Sigma}(i\nabla + e\mathbf{A})\tilde{\Sigma}(\tilde{A} - m\tilde{\tau}) + \tilde{\Sigma}(\tilde{A} - m\tilde{\tau})\boldsymbol{\Sigma}(i\nabla + e\mathbf{A})\right]\Phi =$$

$$\left[ie\boldsymbol{\Sigma}\tilde{\Sigma}(\nabla\tilde{A} - \tilde{\nabla}\mathbf{A})\right]\Phi = ie\boldsymbol{\Sigma}_i\tilde{\Sigma}_{3+k}\mathcal{E}_{ik}\Phi = e\sigma_i(\mathcal{E}_{i3}T_2 - \mathcal{E}_{i2}T_3)\Phi,$$

where $\hat{\mathcal{E}} = \mathbf{A}\hat{\nabla} - \nabla\hat{A}$, the subscript $k = 2, 3$ and

$$T_2 = \begin{pmatrix} 0 & iI_2 \\ -iI_2 & 0 \end{pmatrix}, \quad T_3 = \begin{pmatrix} I_2 & 0 \\ 0 & I_2 \end{pmatrix}$$

are components of the temporal spin (16).

Now eq. (19) can be rewritten in the form

$$[P_4 + m(1 - \tau_1)]\,\phi_1 + (i\nabla + e\mathbf{A})^2 - e\sigma\mathbf{H} -$$

$$- (i\tilde{\nabla} + e\tilde{A} + m\tilde{\tau})^2 - eT_1G_1 + \sigma_i(\mathcal{E}_{i3}T_2 - \mathcal{E}_{i2}T_3)\Phi. \qquad (20)$$

As it follows from Tables I and II, the condition $\Psi'' \ll \Psi'$ demands a smallness of τ_2 and τ_3, i.e. $1 + \tau_1 \simeq 2$, $1 - \tau_1 = \tilde{\tau}^2/(1 + \tau_1) \sim 0$. We also suppose a smallness of $\hat{A}^2$ and the time derivatives $\hat{\nabla}^2\psi$, $\hat{\nabla}\hat{A}$. Hence, we have the four-component wave equation:

$$i\frac{d\Psi}{dt} = \left[(1/2m)(i\nabla + e\mathbf{A})^2 - (e/2m)\boldsymbol{\sigma}\mathbf{H} + eA_4 + \right.$$

$$(ie/2m)\sigma_k(\mathcal{E}_{i3}T_2 - \mathcal{E}_{i2}T_3)]\,\Psi. \qquad (21)$$

That is the multitime generalization of Pauli equation.

The time derivative in the left side can be presented in a more symmetrical form:

$$\frac{d\Psi}{dt} = \frac{d\Psi}{dt_k}\frac{dt_k}{dt}\tau_k = \hat{\tau}\hat{\nabla}\Psi = \frac{d\Psi}{d\hat{\tau}}.$$

One should note that eq (21) does not contain **explicitly** the vector component τ_2 and τ_3. Another peculiarity of this equation consists in a non-Hermitity of the Hamiltonian due to the non-Hermitity of the field matrix $\mathcal{E}_{ik}$: in general case $\mathcal{E}_{ik} \neq \mathcal{E}_{ki}$ [3]. From the physical viewpoint it means that the additional (temporal) field components change the direction of the particle time trajectory $\hat{\tau}$ and, respectively, the energy vector $\hat{E} = E\hat{\tau}$. The time dependent energy is described by a non-Hermitian operator.

4. Quantization of the spinor field

Let us consider all quantities in the reference frame with the t_1-axis which is parallel to the time trajectory of a group (an ensemble) of free, non-interacting spinor particles when $\hat{\tau} = (1,0,0)^T$. In this case the spinor $\Psi(\hat{\mathbf{x}})$ can be developed in a four-dimensional Fourier integral

$$\Psi(\hat{\mathbf{x}}) = (2\pi){-}3/2\sum_{s=1}^{4}\int d^4p\,\delta(\hat{\mathbf{p}}^2+m^2)A_s(\hat{\mathbf{p}})U_s(\hat{\mathbf{p}})e^{i\hat{\mathbf{p}}\hat{\mathbf{x}}}, \qquad (22)$$

where U_s are the plane wave solutions of Dirasc equation represented in Table II. A transition to other co-ordinate systems can be done by means of the extended Lorentz transformations [12, 13].

Taking into account the properties of δ-function we get

$$\Psi(\hat{\mathbf{x}}) = (2\pi)^{-3/2}\sum_{s=1}^{4}\int\frac{{}^3p}{2E_p}\times$$

$$\left(A_s(\mathbf{p})U_s(\mathbf{p},E_p)e^{i\mathbf{px}-iE_pt} + A_s'(\mathbf{p},)V_s(\mathbf{p},E_p)e^{i\mathbf{px}+iE_pt}\right), \qquad (23)$$

[3] That can be illustrated by simple examples in six-dimensional mechanics [7].

with $E_p = (\mathbf{p}^2+m^2)^{1/2}$. The spinor V_s is the solution of Dirac equation for $E_p < 0$.

After the change **the sign of** $\mathbf{p}$ in the second term of the right part of eq. (23)

$$\Psi(\hat{\mathbf{x}}) = (2\pi)^{-3/2}\sum_{s=1}^{4}\int\frac{{}^3p}{2E_p}\times$$

$$\left(A_s(\mathbf{p})U_s(\mathbf{p})e^{i\hat{\mathbf{p}}\hat{\mathbf{x}}} + B_s^+(\mathbf{p})V_s(-\mathbf{p})e^{-i\hat{\mathbf{p}}\hat{\mathbf{x}}}\right). \qquad (24)$$

where $\hat{\mathbf{p}}\hat{\mathbf{x}} = \mathbf{px} - E_pt$ and we denote $B_s^+(\mathbf{p}) = A_s(-\mathbf{p})$.

Analogously

$$\bar{\Psi}(\hat{\mathbf{x}}) = (2\pi)^{-3/2}\sum_{s=1}^{4}\int\frac{{}^3p}{2E_p}\times$$

$$\left(A_s^+(\mathbf{p})\bar{U}_s(\hat{\mathbf{p}})e^{-i\hat{\mathbf{p}}\hat{\mathbf{x}}} + B_s(\hat{\mathbf{p}})\bar{V}_s(-\mathbf{p})e^{i\hat{\mathbf{p}}\hat{\mathbf{x}}}\right). \qquad (25)$$

As it is noted by Boyling and Cole [11], in contrast to the customary one-time theory in the multitime case the mass hyperboloid $bhP = m^2$ is connected, i.e. here there is no a gap between positive and negative energies. The situation is analogous to the continuous transition in each other of positive and negative momentum components p_i (Fig. 2). Due to this peculiarity in general case one cannot distinguish uniquely the positive- and negative-frequency field states. However, if we take into account the condition **of time** irreversibility $\hat{\tau} \simeq 0$, then in this part of the time sub-space all energies $E_i = E\tau_i \simeq 0$ and the region with $+E$ and $-E$ are separated uniquely.

The six-dimensional momentum-energy vector and electric charge

$$\hat{\mathbf{P}} = -\frac{i}{2}\int d^4x\,T_{\mu k}\tau^k = -\frac{i}{2}\int d^4x\,T_{\mu 1} =$$

$$\frac{1}{2}\int d^4x\left(\bar{\Psi}\gamma_1\hat{\boldsymbol{\nabla}}\Psi - \hat{\boldsymbol{\nabla}}\bar{\Psi}\gamma_1\Psi\right) =$$

$$\sum_{s=1}^{4}\eta_s\int d^3p\,N_p\hat{\mathbf{p}}\left[A_s^+(\mathbf{p})A_s(\mathbf{p}) - B_s(\mathbf{p})B_s^+\mathbf{p})\right]. \qquad (26)$$

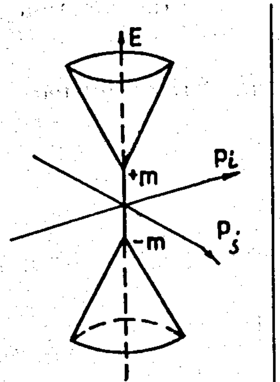

Fig. 2. In the one-time world positive and negative energies are separated by the gap $\Delta E = 2m$. There are no gaps in the momentum plains (p_i, p_j).

$$Q = q\int d^3x\, \bar\Psi\gamma_4\Psi =$$

$$\sum_{s=1}^{4}\eta_s\int d^3p\, N_p\left[A_s^+(\mathbf{p})A_s(\mathbf{p}) + B_s(\mathbf{p})B_s^+\mathbf{p})\right], \tag{27}$$

with the uniquely separated amplitudes A and B. Here $N_p = 1/E_p(E_p + m)$ and the relations

$$\bar U_s(\mathbf{p})\gamma_4 U_r(\mathbf{p}) = (E_p/m)\bar U_s(\mathbf{p})U_r(\mathbf{p}) = 4E_p(E_P + m)^{-1}\eta_s\delta_{sr}, \tag{28}$$

$$\bar V_s(\mathbf{p})\gamma_4 V_r(\mathbf{p}) = (E_p/m)\bar V_s(\mathbf{p})V_r(\mathbf{p}) = N_p\eta_s\delta(sr), \tag{29}$$

$$U_s(\mathbf{p})\gamma_4 V_r(\mathbf{p}) = \bar U_s(\mathbf{p})\gamma_4 V_r(\mathbf{p}) = \bar U_s(\mathbf{p})\gamma_4\bar V_r(\mathbf{p}) = 0 \tag{30}$$

have been used (see eq. (18)).

If we introduce the normalized quantities $a_s = A_s N_p^{1/2}$, $b_s = b_s N_p^{1/2}$ and define the commutation rules

$$[a_s^+(\mathbf{p}), a_r(\mathbf{p}')]_+ = [b_s^+(\mathbf{p}), b_r(\mathbf{p}')]_+ = \eta_s\delta_{rs}\delta(\mathbf{p} - \mathbf{p}') \tag{31}$$

$$[a_s(\mathbf{p}'), a_r(\mathbf{p}')]_+ = [b_s(\mathbf{p}), b_r(\mathbf{p}')]_+ = [a_s(\mathbf{p}), b_r(\mathbf{p}')]_+ = [a_s^+(\mathbf{p}), b_r(\mathbf{p}')]_+ = 0 \tag{32}$$

then

$$\hat{\mathbf{P}} = \sum_{s=1}^{4}\int d^3p\,\hat{\mathbf{p}}\left[n^+(\mathbf{p}) + n^-(\mathbf{p})\right] + \hat{\mathbf{P}}_o \tag{33}$$

$$Q = q\sum_{s=1}^{4}\int d^3p\left[n^+(\mathbf{p}) - n^-(\mathbf{p})\right] + Q_o \tag{34}$$

what allows one to conclude that, as in the customary one-time theory, $n^+ = a_s^+(\mathbf{p})a_r(\mathbf{p}')$ and $n^- = b_s^+(\mathbf{p})b_r(\mathbf{p}')$ are the numbers of particles and antiparticles. The infinite quantities $\hat{\mathbf{P}}_o$ and Q_o are vacuum momentum-energy and vacuum charge.

We see that independently of the sign of the norm $\bar\Psi\Psi$ the particle energy $(\hat P - \hat P_o)\hat\tau$ is of positive quantity. This result is independent on the choice of the co-ordinate frame.

5. Quantum relations for the electromagnetic field

Similar to the above considered case of spinor field the potential of a non-interacting electromagnetic field can be also developed in a four-dimensional Fourier integral

$$\hat{\mathbf{A}} = (2\pi)^{-3/2}\int d^4p\,\delta(\hat{\mathbf{p}}^2)\hat{\mathbf{A}}_{\hat{\mathbf{p}}}e^{i\hat{\mathbf{p}}\hat{\mathbf{x}}} =$$

$$(2\pi)^{-2/3}\int\frac{d^3p}{2p}(\hat{\mathbf{A}}(\mathbf{p})e^{i\hat{\mathbf{p}}\hat{\mathbf{x}}} + \hat{\mathbf{A}}^*(-\mathbf{p})e^{-i\hat{\mathbf{p}}\hat{\mathbf{x}}}) \tag{35}$$

with $p = |\mathbf{p}|$, $\hat{\mathbf{A}}^*(\mathbf{p}) = \hat{\mathbf{A}}(-\mathbf{p})$. As a consequence of the extended Lorentz condition [14, 15] scalar product $\hat{\mathbf{p}}\hat{\mathbf{A}}(\mathbf{p}) = 0$.

Let us start from the expression for the momentum-energy tensor

$$T_{\mu\nu} = \frac{1}{4\pi}\left(\frac{\partial A_\alpha}{\partial x_\nu}\frac{\partial A_\alpha}{\partial x_\mu} - \frac{1}{2}\delta_{\mu\nu}\frac{\partial A_\alpha}{\partial x_\beta}\frac{\partial A_\alpha}{\partial x_\beta}\right). \tag{36}$$

The corresponding six-dimensional momentum-energy vector

$$P_\mu = -i\int d^3x\, T_{\mu,3+k}\tau^k =$$

$$\frac{1}{4\pi}\int\frac{d^3p\,d^3p'}{pp'}\delta(\mathbf{p} - \mathbf{p}')\left(p'_\mu\hat p\hat\tau - (1/2)\hat{\mathbf{p}}\hat{\mathbf{p}}'\delta_{\mu,3+k}\tau^k\right)\times$$

$$\left[\hat{\mathbf{A}}(\mathbf{p})\hat{\mathbf{A}}^+(\mathbf{p}') + \hat{\mathbf{A}}^+(\mathbf{p})\hat{\mathbf{A}}(\mathbf{p}')\right] =$$

$$\frac{1}{4\pi}\int d^3p(p_\mu/p)\left[\hat{\mathbf{A}}(\mathbf{p})\hat{\mathbf{A}}^+(\mathbf{p})+\hat{\mathbf{A}}^+(\mathbf{p})\hat{\mathbf{A}}(\mathbf{p})\right], \quad (37)$$

where we took into account the relation

$$p'_\mu\hat{p}\hat{\tau}-(1/2)\hat{\mathbf{p}}\hat{\mathbf{p}}'\delta_{\mu 3+k}\tau^k=p_\mu\hat{p}\hat{\tau}=p_\mu p.$$

In classical (non-quantum) electrodynamics the field energy $E=\hat{\mathbf{P}}\hat{\tau}$ has negative terms connected with temporal field components (for $\nu>3$, see also [14, 15]). One should note that in the ordinary one-time theory we encounter also a similar negative term corresponding to a scalar component $A_4(\mathbf{p})$. In this case, due to Lorentz condition, the negative energy is compensated by the contribution of the longitudinal component $A_3(\mathbf{p})$. In a multidimensional variant the similar compensation must also take place, but the components A_5 and A_6 are left uncompensated and give a negative contribution to the field energy. In this case we must demand the additional condition

$$\hat{\mathbf{A}}(\mathbf{p})\hat{\mathbf{A}}^+(\mathbf{p})+\hat{\mathbf{A}}^+(\mathbf{p})\hat{\mathbf{A}}(\mathbf{p})=|\hat{\mathbf{A}}(\mathbf{p})|^2\geq 0, \quad (38)$$

i.e. the vector $\mathbf{A}$ must be a quantity of the space-like type. Only by this condition the classical field energy is a positive quantity.

If we introduce now normalized six-dimensional amplitudes $\hat{\mathbf{a}}(\mathbf{p})=(4\pi p)^{1/2}\hat{\mathbf{A}}(\mathbf{p})$ obeying the commutation in relations

$$[a^+_\mu(\mathbf{p}),a_\nu(\mathbf{p}')]=g_{\mu\nu}\delta(\mathbf{p}-\mathbf{p}') \quad (39)$$

$$[a_\mu(\mathbf{p}),a_\nu(\mathbf{p}')]=[a^+_\mu(\mathbf{p}),a^+_\nu(\mathbf{p}')]=0 \quad (40)$$

with the metric tensor $g_{\mu\nu}$ [4] than the momentum-energy vector

$$\hat{\mathbf{P}}=\int d^3p\,\hat{\mathbf{p}}\left[\mathbf{a}^+(\mathbf{p})\mathbf{a}(\mathbf{p}-\hat{a}^+(\mathbf{p})\hat{a}(\mathbf{p}\right]=$$

$$\sum_{k=1}^{3}\int d^3p\,\hat{\mathbf{p}}\,[n_k(\mathbf{p})+n_{3+k}(\mathbf{p})]+\hat{\mathbf{P}}_{o\mu}, \quad (41)$$

[4] Remember that we use the metric $g_{\mu\nu}=\delta_{\mu\nu}$ for $\mu,\nu\leq 3$ and $g_{\mu\nu}=-\delta_{\mu\nu}$ for $\mu,\nu>3$. As in the customary one-time theory, such a method of the quantization assumes that the undefinite metric in Hilbert space is used.

where the positive quantities n_ν are the numbers of photons with the polarization of the type ν.

6. Conclusion

We see that like the known one-time theory, the field theory with three-dimensional time vector can be formulated in Hamiltonian form and allows a quatization in the Hilbert space with an indefinite metric after which the theory describes a system of particles with positive energies. The non-Hermitity of the Hamiltonian reflects an alteration of the directions of particle time trajectories $\hat{t}_i$ and the energy vectors $\hat{E}_i$.

One should stress that the considered quantization procedure in the particular co-ordinate system with $\hat{\tau}=(1,0,0)^T$ is applicable only to free fields. By taking into account an interaction when particles time trajectories are changed we must use an arbitrary chosen co-ordinate frame.

The next goal is to consider this difficult problem and to develop S-matrix theory of interacting fields. That is necessary for investigation of processes in small spacetime intervals where the time irreversibility and the classical momentum-energy conservation laws are violated and a creation of virtual particles with various time trajectories becomes possible.

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