СООБЩЕНИЯ
ОБЪЕДИНЕННОГО
ИНСТИТУТА
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

Дубна

98-125

E2-98-125

Kh.M.Beshtoev

OSCILLATIONS OF NEUTRAL *K* MESONS IN THE THEORY OF DYNAMICAL EXPANSION OF THE WEAK INTERACTION THEORY OR IN THE THEORY OF DYNAMICAL ANALOGY OF THE CABIBBO–KOBAYASHI–MASKAWA MATRICES

1998

1 INTRODUCTION

At the present time, the theory of electroweak interactions, has a status of theory which is confirmed with a high degree of precision. However, some experimental results (the existence of the quarks and the leptons families, etc.) did not get any explanation in the framework of the theory. One part of the electroweak theory is the existence of quark mixings introduced by the Cabibbo-Kobayashi-Maskawa matrices (i.e., these matrices are used for parametrization of the quark mixing).

In previous works [1] a dynamical mechanism of quark mixing by the use of four doublets of massive vector carriers of weak interaction $B^{\pm}, C^{\pm}, D^{\pm}, E^{\pm}$, i.e., expansion of the standard theory of weak interaction (the theory of dynamical analogy of the Cabibbo-Cobayashi-Maskawa matrices) working on the tree level, was proposed.

This work is devoted to the study of $K^0, \bar{K}^0$ oscillations in the theory of dynamical analogy of the Cabibbo-Cobayashi-Maskawa matrices.

At first, we will give the general elements of the theory of dynamical analogy of the Cabibbo-Cobayashi-Maskawa matrices, then the $K^0, \bar{K}^0$ oscillations will be considered in this theory.

2 The Theory of Dynamical Analogy of the Cabibbo-Kobayashi-Maskawa matrices

In the case of three families of quarks the current J^{μ} has the following form:

$$J^{\mu} = (\bar{u}\bar{c}\bar{t})_L \gamma^{\mu} V \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L \tag{1}$$

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix},$$

where V is Kobayashi-Maskawa matrix [2].

Объединенный институт ядерных исследований
БИБЛИОТЕКА

Mixings of the d, s, b quarks are not connected with the weak interaction (i.e., with $W^{\pm}, Z^o$ bosons exchanges). From equation (1) it is well seen that mixings of the d, s, b quarks and exchange of $W^{\pm}, Z^o$ bosons take place in an independent manner (i.e.. if matrix V were diagonal, mixings of the d, s, b quarks would not have taken place).

If the mechanism of this mixings is realized independently of the weak interaction ($W^{\pm}, Z^o$ boson exchange) with a probability determined by the mixing angles $\theta, \beta, \gamma, \delta$ (see below), then this violation could be found in the strong and electromagnetic interactions of the quarks as a clear violations of the isospin, strangeness and beauty. But, the available experimental results show that there is no clear violations of the number conservations in strong and electromagnetic interactions of the quarks. Then we must connect the non-conservation of the isospins, strangeness and beauty (or mixings of the d, s, b quarks) with some type of interaction mixings of the quarks. We can do it introducing (together with the $W^{\pm}, Z^o$ bosons) the heavier vector bosons $B^{\pm}, C^{\pm}, D^{\pm}, E^{\pm}$ which interact with the d, s, b quarks with violation of isospin, strangeness and beauty.

We shall choose parametrization of matrix V in the form offered by Maiani [3]

$$V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\gamma & s_\gamma \\ 0 & -s_\gamma & c_\gamma \end{pmatrix} \begin{pmatrix} c_\beta & 0 & s_\beta \exp(-i\delta) \\ 0 & 1 & 0 \\ -s_\beta \exp(i\delta) & 0 & c_\beta \end{pmatrix} \begin{pmatrix} c_\theta & s_\theta & 0 \\ -s_\theta & c_\theta & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$c_\theta = \cos\theta, s_\theta = \sin\theta, \exp(i\delta) = \cos\delta + i\sin\delta. \qquad (2)$$

To the nondiagonal terms in (2), which are responsible for mixing of the d, s, b- quarks and CP-violation in the three matrices, we shall make correspond four doublets of vector bosons $B^{\pm}, C^{\pm}, D^{\pm}, E^{\pm}$ whose contributions are parametrized by four angles $\theta, \beta, \gamma, \delta$. It is supposed that the real part of $Re(s_\beta \exp(i\delta)) = s_\beta \cos\delta$ corresponds to the vector boson $C^{\pm}$, and the imaginary part of $Im(s_\beta \exp(i\delta)) = s_\beta \sin\delta$ corresponds to the vector boson $E^{\pm}$ (the couple constant of E is an imaginary value !). Then, when $q^2 << m_W^2$, we get:

$$\tan\theta \cong \frac{m_W^2 g_B^2}{m_B^2 g_W^2},$$
$$\tan\beta \cong \frac{m_W^2 g_C^2}{m_C^2 g_W^2},$$
$$\tan\gamma \cong \frac{m_W^2 g_D^2}{m_D^2 g_W^2},$$
$$\tan\delta \cong \frac{m_W^2 g_E^2}{m_E^2 g_W^2}. \qquad (3)$$

If $g_{B^\pm} \cong g_{C^\pm} \cong g_{D^\pm} \cong g_{E^\pm} \cong g_{W^\pm}$, then

$$\tan\theta \cong \frac{m_W^2}{m_B^2},$$
$$\tan\beta \cong \frac{m_W^2}{m_C^2},$$
$$\tan\gamma \cong \frac{m_W^2}{m_D^2},$$
$$\tan\delta \cong \frac{m_W^2}{m_E^2}. \qquad (4)$$

Concerning the neutral vector bosons B^0, C^0, D^0, E^0, the neutral scalar bosons $B'^0, C'^0, D'^0, E'^0$and the GIM mechanism [4] we , can repeat the same arguments which were given in the previous work [1].

The proposed Lagrangian for expansion of the weak interaction theory (without CP-violation) has the following form:

$$L_{int} = i\sum_i g_i(J^{i,\alpha} A^i_\alpha + c.c.), \qquad (5)$$

where $J^{i,\alpha} = \bar\psi_{i,L}\gamma^\alpha T \varphi_{i,L}$,

$$T = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix},$$

$$i = 1 \quad i = 2 \quad i = 3$$

$$\psi_{i,L} = \begin{pmatrix} u \\ c \end{pmatrix}_L, \begin{pmatrix} u \\ t \end{pmatrix}_L, \begin{pmatrix} c \\ t \end{pmatrix}_L,$$

$$i=1 \quad i=2 \quad i=3$$

$$\varphi_{i,L} = \begin{pmatrix} d \\ s \end{pmatrix}_L, \begin{pmatrix} d \\ b \end{pmatrix}_L, \begin{pmatrix} s \\ b \end{pmatrix}_L.$$

The weak interaction carriers A^i_α, which are responsible for the weak transitions between different quark families are connected with the B, C, D bosons in the following manner:

$$A^1_\alpha \to B^\pm_\alpha, A^2_\alpha \to C^\pm_\alpha; A^3_\alpha \to D^\pm_\alpha. \tag{6}$$

Using the data from [5] and equation (4) we have obtained the following masses for $B^\pm, C^\pm, D^\pm, E^\pm$ bosons:

$$m_{B^\pm} \cong 169.5 \div 171.8 \text{ GeV},$$

$$m_{C^\pm} \cong 345.2 \div 448.4 \text{ GeV}, \tag{7}$$

$$m_{D^\pm} \cong 958.8 \div 1794 \text{ GeV},$$

$$m_{E^\pm} \cong 4170 \div 4230 \text{ GeV}.$$

3 Oscillations of Neutral $K^0, \bar{K}^o$ Mesons in the Theory of Dynamical Analogy of Kabibbo-Kobayashi-Maskawa Matrices

The oscillations of $K^0, \bar{K}^0$ mesons are characterized by two parametrs: angle mixing–θ and length– R or time oscillations–τ.

At first, we will consider mixings of K^0 mesons and then obtain the Lagrangian for mass differences of K^0_1, K^0_2 arising for existence of the weak interaction through B boson violating strangeness, and further mass differences of K^0_1, K^0_2 mesons is estimated. In the end, the general scheme of $K^0, \bar{K}^0$ meson oscillations is given.

In the further considerations, for transition from the standard model to our model, the following values are used for $sin\theta, cos\theta$ and G_F:

$$sin\theta \cong \frac{m^2_W g^2_B}{m^2_B g^2_W} \cong \frac{m^2_W}{m^2_B}.$$

$$cos^2\theta = 1 - sin^2\theta. \tag{8}$$

$$G^2_F = \frac{g^2_W}{32 m^2_W}.$$

A) Mixings of $K^0, \bar{K}^0$ mesons

Let us consider mixings of $K^0, \bar{K}^0$ mesons.

K^0 and $\bar{K}^0$ consist of $\bar{d}, s, d, \bar{s}$, quarks and have the same masses (it is a consequence of the CPT invariance) but their strangenesses are different $s_{K^0} = -1, s_{\bar{K}^0} = 1$. Since the weak interaction, through B boson exchanges, changes the strangeness, then the $K^0, \bar{K}^0$ are mixed.

The mixings of K^0 mesons can be considered using the following nondiagonal mass matrix of K^0 mesons:

$$\begin{pmatrix} m^2_{K^0} & m^2_{K^0\bar{K}^0} \\ m^2_{\bar{K}^0 K^0} & m^2_{\bar{K}^0} \end{pmatrix}. \tag{9}$$

Since $m^2_{K^0} = m^2_{\bar{K}^0}$, the angle θ' of $K^0, \bar{K}^0$ mixing, or the angle rotation for diagonalization of this matrix

$$\begin{pmatrix} m^2_1 & 0 \\ 0 & m^2_2 \end{pmatrix}.$$

given by the expression:

$$tg2\theta' = \frac{m^2_{K^0\bar{K}^0}}{m^2_{K^0} - m^2_{\bar{K}^0}}.$$

equal $(\theta' =)\frac{\pi}{4}$ and

$$m^2_{1,2} = \frac{1}{2}((m^2_{K^0} - m^2_{\bar{K}^0}) \pm \sqrt{(m^2_{K^0} - m^2_{\bar{K}^0})^2 + 4(m^2_{K^0\bar{K}^0})^2}). \tag{10}$$

$$m^2_1 - m^2_1 = m^2_{K^0\bar{K}^0}. \tag{11}$$

Then the following new states K_1^0, K_2^0 arise:

$$K_1^0 = \frac{K^0 + \bar{K}^0}{\sqrt{2}}, \quad K_2^0 = \frac{K^0 - \bar{K}^0}{\sqrt{2}} \tag{12}$$

Below the CP–invariance is supposed to be strong conservated (the consequences which arise of the CP–violation were discussed in work [6]), and then the following decays are possible: (CP parity of K_1^0 is $P(K_1^0) = +1$, and of $\bar{K}_2^0$ is $P(\bar{K}_2^0) = -1$)

$$K_1^0 \to 2\pi, \quad K_2^0 \to 3\pi. \tag{13}$$

B). A Lagrangian for Mass differences of K_1^0, K_2^0 Mesons and Estimation of the Value of this Difference

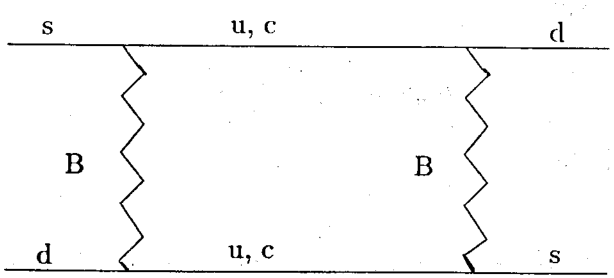
If the masses of K_1^0, K_2^0 mesons are the average value of Lagrangian L (the evident form of L will be obtainned below) then:

$$m_1^2 =< K_1^0 \mid L \mid K_1^0 >, \quad m_2^2 =< K_2^0 \mid L \mid K_2^0 >, \tag{14}$$

and, if to take into account equation (12) then:

$$m_1^2 - m_2^2 =< K^0 \mid L \mid \bar{K}^0 > + < \bar{K}^0 \mid L \mid K^0 > . \tag{15}$$

The calculation of the value $\Delta m = m_1 - m_2$ will be executed in the framework of the weak quark interactions using the following diagram:



where B is a boson changing the strangeness; u, c are quarks (for simplification the t quark is not taken into account).

We do not give here the details of the calculation on this diagram since they were widely discussed in literature [7,8]. Using the standard Feynman rules for the weak interaction (after integrating on the inside lines, twice using the Firtz rules and without the outside momenta), we obtain the following equation for the amplitude of $K^0 \to \bar{K}^0$ transition:

$$M(K^0 \to \bar{K}^0) = -\frac{G_F^2 m_c^2 sin^2\theta cos^2\theta}{8\pi^2} \bar{d}Q_\alpha s \bar{d}Q^\alpha s = \tag{16}$$

$$= G\bar{d}Q_\alpha s\bar{d}Q^\alpha s,$$

where $G = \frac{G_F^2 m_c^2 sin^2\theta cos^2\theta}{8\pi^2}$, $Q_\alpha = \gamma_\alpha(1-\gamma_5)$.

Then, the Lagrangian for this process with changing the strangeness on $\Delta s = -2$ is:

$$L_{\Delta s=-2} = -G\bar{d}Q_\alpha s\bar{d}Q^\alpha s, \tag{17}$$

and the Lagrangian with $\Delta s = +2$ is Lagrangian conjugated by Ermit to Lagrangian (17).

Using equations (14), (15) and quark Lagrangian (17), Δm is computed.

For this purpose the following phenomenological matrix element is used:

$$< 0 \mid \bar{d}Q_\alpha s \mid \bar{K}^0 >= \varphi_K f_K p_\alpha, \tag{18}$$

where p_α is four–momentum, φ_K is the wave function of K^0 meson and $f_K \cong 1.27 f_\pi$. If to take into account that the quarks have colour, then there appears the factor $(1 + \frac{1}{3})$.

So, the following equation for matrix element of L is obtained:

$$< K^0 \mid L \mid \bar{K}^0 > \cong \frac{8}{3} G m_K^2 f_K^2. \tag{19}$$

For the full matrix element of L, when also the inverse process $\bar{K}^0 \to K^0$ is taken into account , the following equation is obtained:

$$< K^0 \mid L \mid \bar{K}^0 > + < \bar{K}^0 \mid L \mid K^0 > \cong \frac{16}{3} G m_K^2 f_K^2,$$

and then for masse differences:

$$m_1^2 - m_2^2 = \frac{16}{3} G m_K^2 f_K^2,$$

or

$$\Delta m = (m_1 - m_2) \cong \frac{8}{3} G m_K f_K^2. \tag{20}$$

If to take into account equation (8), then the equation (20) for Δm is rewritten in the following form:

$$\Delta m = \frac{1}{96\pi^2} m_K f_K^2 m_c^2 (1 - \frac{m_W^4}{m_B^4})(\frac{g_B^2}{m_B^2})^2, g_W^2 \cong g_B^2, \tag{21}$$

i.e. Δm is inversely proportional to the mass of B boson, the carrier changing strangeness, in the fourth degree

$$\Delta m \sim \frac{1}{m_B^4}.$$

In the general case we obtain: the heavier mass of the carrier changing the strageness, the less is Δm :

$$\frac{1}{\Delta m} \sim m_B^4, \tag{22}$$

or the heavier mass of the carrier changing the strangeness, the more is the time (the length R) of oscillation:

$$\tau \sim m_B^4, R = \tau v, \tag{23}$$

where v is the velocity of the oscillating particle.

The $m^2_{K^0\bar{K}^0}$ in (9) and (11) or Δm in (20) characterize the time of $K^0 \leftrightarrows \bar{K}^0$ transitions or the mass value corresponding to this time. And at transitions from states $K^o, \bar{K}^o$ to states K_1^o, K_2^o the mass sum $m^2_{K^o} + m^2_{\bar{K}^o} = m^2_{K_1^o} + m^2_{K_2^o}$ is not changed since the weak interaction is γ_5 invariance interaction and it cannot generate masses [9].

C) The scheme of $K^0, \bar{K}^0$ oscillations

As an example of this oscillation we consider the oscillation of K^0 mesons created in the reaction $\pi^- + P \to K^0 + \Lambda$. At $t = 0$ we have the state $K^0(0)$, then in time $t \neq 0$ for $K^0(t)$, if to take into account equation (12), we get K^0:

$$\begin{aligned} K^0(t) &= \frac{1}{2}[(K^0 + \bar{K}^0) exp(-im_1 t - \frac{\Gamma_1 t}{2}) + (K^0 - \bar{K}^0) exp(-im_2 t - \frac{\Gamma_2 t}{2})] = \\ &= \frac{1}{2} K^0 exp(-im_2 t)[exp(-i\Delta m t - \frac{\Gamma_1}{2} t) + exp(-\frac{\Gamma_2}{2} t)] + \\ &+ \frac{1}{2} \bar{K}^0 exp(-im_1 t)[exp(i\Delta m t - \frac{\Gamma_1}{2} t) + exp(-\frac{\Gamma_2}{2} t)]. \end{aligned} \tag{24}$$

From (23) it is clear that on the background of K^0 meson decays, the oscillations of K^0 mesons take place [7,9].

4 Conclusion

The elements of the theory of dynamical expansion of the weak interaction theory working on the tree level, i.e., the theory of dynamical analogy of Cabibbo-Kobayashi-Maskawa matrices, were given.

The equation for mass difference of K_1^0, K_2^0 mesons or the length of $K^0, \bar{K}^0$ meson oscillations was calculated. In the framework of this theory the oscillations of $K^0, \bar{K}^0$ mesons which arise at violation of strangeness by B bosons were considered.

The general conclusion is: The length of $K^0, \bar{K}^0$ meson oscillations is proportional to the mass of B boson (which changes strangeness) in the fourth degree.

REFERENCES

1. Beshtoev Kh.M., JINR. E2-94-293. Dubna. 1994;
   Turkish Journ. of Physics 1996, 20, p.1245;
   JINR, E2-95-535, Dubna, 1995;
   JINR, P2-96-450, Dubna, 1996.
   JINR Commun. E2-97-210, Dubna. 1997.
2. Kobayashi M. and Maskawa K., Prog. Theor. Phys..1973, 49, p.652.
3. Maiani L., Proc.Int. Symp. on Lepton-Photon Inter..Hamburg, DESY, p.867.
4. Glashow S., Iliopoulos J. and Maiani L., Phys. Rev.. 1970, D2,p.1285.
5. Review of Particle Prop., Phys. Rev. 1992. D45. N 11.
6. Beshtoev Kh.M., JINR Commun. E2-93-167. Dubna, 1993. Chinese Journal of Phys. 1996, v.34, p.979.
7. Okun L.B., Leptons and Quarks, M., Nauka. 1990.
8. Buras A.J., Harlander M.K., MPI-PAE/TTh/92. TUM-T31-25/92.
9. Beshtoev Kh.M., Fiz. Elm. Chastitz At. Yadra, 1996, v.27, p.23.

Received by Publishing Department
on May 13, 1998.