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THE BALMER-LIKE FORMULA FOR MASS DISTRIBUTION OF ELEMENTARY PARTICLE RESONANCES

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1 Introduction

The merits of contemporary quark models (the Standard Model) in describing properties of elementary particles are impressive. However, it is well known that [1] the only uncertain aspect of the Standard Model is the mechanism that gives elementary particles their masses. In the simplest version of the model, these masses depend on constants that specify the strength of the interaction of various elementary particles with a new kind of field but these constants are just free parameters of the theory. It seems that the mass distribution of elementary particle resonances is a fundamental problem of modern theory until we have a final theory of forces and matter, particles and fields. Therefore, a systematic analysis of existing experimental data is desirable to establish simple phenomenological rules for the mass distribution of resonances.

2 General results

One of the most remarkable corner-stones for foundation of the quantum theory was the Balmer formula for the spectrum of a hydrogen atom. It is our purpose here to demonstrate that the Balmer-like formula for the mass distribution of elementary particle resonances can be obtained from a systematic analysis of all available experimental data.

Some resonances have a dominant decay channel, and we suggest that the momentum of this channel should manifest itself in properties of decays through other channels. For example, it is known that the pion $\pi^{\pm}$ decays through the muon and neutrino with a probability about unity and asymptotic momentum $P_1 = 29.7918 MeV/c$. We are unable to explain this property of the pion; we only suggest that it is fundamental for comprehension of the structure of some resonances. To establish some common properties of the mass distribution of elementary particle resonances, we check the usefulness of a commensurable principle [2] of decay asymptotic momenta. In other words, we check the following ratio:

$$P_n = nP_1, \quad n = 1, 2, 3, ... \tag{1}$$

where $P_1 = 29.7918$ Mev/c.

The masses of resonances are calculated by the formula

$$M_{th} = \sqrt{m_a^2 + P_n^2} + \sqrt{m_b^2 + P_n^2} = \sqrt{m_a^2 + n^2 P_1^2} + \sqrt{m_b^2 + n^2 P_1^2}, \tag{2}$$

where m_a and m_b are masses of decay products of the resonance to be considered. The results and corresponding experimental data [3] are given in Table 1. A more complete analysis has been made in [4] and contains a few hundred of resonances.

We can see from the table that such a simple phenomenological method describing the experimental data within the accuracy of measurements is unusual and unexpected for this branch of physics. The results of the χ^2 calculations are given in Fig. 1 as a function of decay momentum P including about 4 hundred experimental data. Well pronounced deep minimum χ^2 is found at $P_1 = 29.79$ MeV/c which corresponds to the above-mentioned decay momentum. If we change $P_1 = 29.79$ MeV/c to $P_1 = 29.79 \pm 0.3$ MeV/c, the magnitude of χ^2 will increase three times. Therefore the mass distribution of

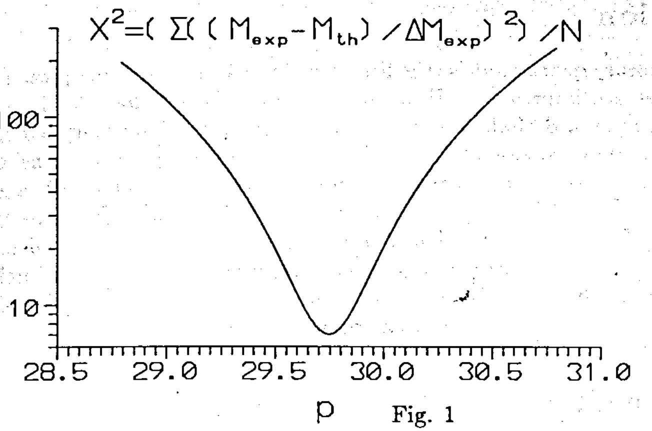

Fig. 1

resonances is a sensitive function of the basic decay momentum and the good description of the experimental data to be observed is not accidental. It means that this observation manifests the simple physics of resonances remaining unnoticed.

By the way, the phenomenological analysis of elementary particle and resonance mass distribution was popular in early days. For example, authors of [5] come to the conclusion that the appearance of quanta of energy (mass) is possible only in the case when the action achieves the magnitude multiple of the Planck constant $\hbar$.

Of course, there arises a question: what is the role of resonance decay channels with a very small probability? We know from nuclear physics (see, for example, [6]) that basic vectors with small (large) weights in the ground states of nuclei become the dominant (small) ones in highly excited states. The same phenomenon is expected in the physics of resonances. Let us consider as a basic channel the following decay channel $\pi^0 \to \mu^\pm e^\mp$ with P_1=26.1299 MeV/c and fraction $\approx 10^{-8}$%. The results of calculations are presented in Table 2.

Tables 1, 2 contain rich information, and it is possible to make many fundamental conclusions based on them. We think that the results contained in the tables convincingly demonstrate the empirical fact of commensurability of resonance decay product momenta within the accuracy of existing experimental data. In other words, resonance decay product momenta are quantized. It is clear from the tables that commensurability of momenta does not depend on the type of interaction between resonance decay products, quantum numbers of resonances and type of particles. Moreover, commensurability of momenta is justified for all considered resonances. It seems to be a universal property of resonances, and many periodic structures are just governed by this property. A main question arises here: is this commensurability precise or approximate? We do not know the answer but think that it is as precise as the one in solids and crystals. If so, the common structure of elementary particles must be analogous to that of solids and crystals. An excellent possibility for prediction of new resonances and verification of masses of the existing ones arises in any case.

Table 1. Invariant masses of resonances decaying through binary channels with momenta $P_n = n * 29.7918$ MeV/c, $\Delta M = \mid M_{exp} - M_{th} \mid$.

resonances	decay channels	P_{exp}	n	P_{exp}/n	M_{exp}	M_{th}	ΔM
$\pi^\pm$	$\mu^\pm\nu_\mu$	29.79	1	29.79	139.56995	139.56995	—
$\rho(770)$	$\pi^\pm\pi^\mp$	358	12	29.83	768.5 ± 0.6	767.56	0.94
$\rho(1450)$	$\pi^\pm\pi^\mp$	719	24	29.96	1465 ± 25	1456.99	8.01
$\rho(2150)$	$\pi^\pm\pi^\mp$	984.15	33	29.82	~ 1988	1985.97	2.03
$f_1(1285)$	$a_0(980)\pi^0$	235.01	8	29.38	1282.2 ± 0.7	1285.87	3.67
$f_1(1420)$	$a_0(980)\pi^0$	356.10	12	29.68	1426.8 ± 2.3	1428.59	1.79
$f_1(1510)$	$a_0(980)\pi^0$	not seen	14	29.79	1512 ± 4	1506.67	5.33
$f_2(1270)$	$\pi^\pm\pi^\mp$	622.03	21	29.62	1275 ± 5	1282.01	7.01
$f_2(1430)$	$\pi^\pm\pi^\mp$	701.26	24	29.22	1436^{+26}_{-16}	1456.99	20.99
$f_2(1565)$	$\pi^\pm\pi^\mp$	769.95	26	29.61	1565 ± 20	1574.12	9.12
$f_2(1640)$	$\pi^\pm\pi^\mp$	807.02	27	29.89	1638 ± 6	1632.79	5.21
$f_2(1810)$	$\pi^\pm\pi^\mp$	896.70	30	29.89	1815 ± 12	1809.17	5.83
$f_2(1950)$	$\pi^\pm\pi^\mp$	988.19	33	29.95	~ 1996	1985.97	10.03
$f_2(2150)$	$\pi^\pm\pi^\mp$	1015.45	37	29.87	~ 2226	2222.19	3.81
$\eta(1295)$	$a_0(980)\pi^0$	246.54	8	30.82	1295 ± 4	1285.87	9.13
$\eta(1440)$	$a_0(980)\pi^0$	346.80	12	28.9	1415 ± 10	1428.59	13.59
$\eta(1760)$	$a_0(980)\pi^0$	not seen	20	29.79	1760 ± 11	1760.84	0.84
$\eta(2225)$	$a_0(980)\pi^0$	not seen	30	29.79	~ 2221	2232.82	11.82
$a_0(980)$	$K^\pm\overline{K^\pm}$	89.11	3	29.70	1003.3 ± 7	1003.40	0.10
$a_0(980)$	$K^\pm\overline{K^\pm}$	119.80	4	29.95	1016 ± 10	1015.71	0.29
$a_2(1320)$	$K^0K^\pm$	443.32	15	29.55	1330 ± 11	1334.76	4.76
$a_4(2040)$	$K^0_SK^-$	812.19	27	30.08	1903 ± 10	1889.68	13.32
$a_4(2040)$	$K^0_SK^-$	891.46	30	29.72	2040 ± 30	2044.01	4.01
B(1876)	pp	29.94	1	29.94	1877.5 ± 0.5	1877.49	0.01
B(1880)	pp	—	2	—	not seen	1880.32	—
B(1885)	pp	94.31	3	31.44	1886 ± 1	1885.04	0.96
B(1890)	pp	120.67	4	30.17	1892	1891.62	0.38
B(1900)	pp	142.29	5	28.49	1898 ± 1	1900.05	2.05
X(1900)	$p\overline{p}$	138.91	5	27.78	1897 ± 1	1900.05	3.05
B(1910)	pp	180.62	6	30.10	1911	1910.30	0.70
B(1920)	pp	207.76	7	29.68	1922	1922.34	0.34
X(1920)	$p\overline{p}$	203.09	7	29.01	~ 1920	1922.34	2.34
B(1936)	pp	240.08	8	30.01	1936 ± 0.3	1936.14	0.14
X(1936)	$p\overline{p}$	240.08	8	30.01	$1937.3^{+1.3}_{-0.7}$	1936.14	1.16
B(1950)	pp	274.14	9	30.46	1951 ± 0.3	1951.66	0.66
B(1970)	pp	298.14	10	29.81	1969 ± 2	1968.87	0.13
X(1970)	$p\overline{p}$	296.48	10	29.65	1968	1968.87	0.87
B(1990)	pp	328.15	11	29.83	1989 ± 1	1987.71	1.29
B(2008)	pp	357.30	12	29.76	2008 ± 3	2008.15	0.15
X(2008)	$p\overline{p}$	361.49	12	30.12	2011 ± 7	2008.15	2.75
X(2030)	$p\overline{p}$	381.86	13	29.37	2026 ± 5	2030.12	4.12
B(2050)	pp	415.12	14	29.65	2052	2053.60	1.60

Continuation of Table 1

resonances	decay channels	P_{exp}	n	P_{exp}/n	M_{exp}	M_{th}	ΔM
B(2080)	pp	447.44	15	29.83	2079 ± 4	2078.51	0.49
X(2080)	$p\overline{p}$	448.60	15	29.91	2080 ± 10	2078.51	1.49
B(2105)	pp	477.97	16	29.87	2106 ± 4	2104.82	1.18
B(2130)	pp	502.80	17	29.58	2129 ± 5	2132.47	3.47
B(2160)	pp	—	18	—	not seen	2161.41	—
B(2190)	pp	566.45	19	29.81	2192 ± 3	2191.58	0.42

Table 2. Invariant masses of higher excited resonances decaying through binary channels with momenta $P_n = n * 26.1299$ MeV/c, $\Delta M = \mid M_{exp} - M_{th} \mid$.

resonances	decay channels	P_{exp}	n	P_{exp}/n	M_{exp}	M_{th}	ΔM
π^0	$\mu^\pm e^\mp$	26.1299	1	26.1299	134.9764	134.9764	—
$D_2^*(2460)^0$	$\mu^\pm e^\mp$	1227.18	47	26.11	2458.9 ± 2.0	2460.75	1.85
$B_J^*(5732)$	$\mu^\pm e^\mp$	2848.02	109	26.13	5698 ± 12	5698.28	0.28
$B_{sJ}^*(5850)$	$\mu^\pm e^\mp$	2925.55	112	26.12	5853 ± 13	5855.00	2.00
$\Upsilon(1S)$	$\mu^\pm e^\mp$	4729.59	181	26.13	9460.37 ± 0.21	9460.20	0.17
$\omega(782)$	e^-e^+	390.97	15	26.06	781.94 ± 0.12	783.90	1.96
$X(1097)$	e^-e^+	548.5	21	26.12	1097^{+16}_{-19}	1097.46	0.46
$K^*(1410)$	$e^\pm \nu_e$	706.00	27	26.15	1412 ± 12	1411.02	0.98
$\rho(1450)$	e^-e^+	732.5	28	26.16	1465 ± 25	1463.28	1.72
$f_2(1565)$	e^-e^+	782.5	30	26.08	1565 ± 20	1567.79	2.80
τ	$e^\pm\gamma$	888.5	34	26.13	$1777^{+0.30}_{-0.27}$	1776.83	0.17
$X(1830)$	e^-e^+	915.00	35	26.14	~ 1830	1829.09	0.91
$\pi_2(2090)$	e^-e^+	1045.00	40	26.13	2090 ± 29	2090.39	0.39
$f_0(2200)$	e^-e^+	1098.50	42	26.15	2197 ± 17	2194.91	2.09
$K_2(2250)$	$e^\pm \nu_e$	1123.50	43	26.13	2247 ± 17	2247.17	0.17
$D_2^*(2460)^0$	e^-e^+	1229.45	47	26.16	2458.9 ± 2	2456.21	2.69
$D_2^*(2460)^\pm$	$e^\pm \nu_e$	1229.50	47	26.16	2459 ± 4	2456.21	2.79
$f_2(2510)$	e^-e^+	1255	48	26.15	2510 ± 30	2508.47	1.53
$\eta_c(1S)$	e^-e^+	1489.90	57	26.14	2979.8 ± 2.1	2978.81	0.99
B^0	e^-e^+	2639.6	101	26.13	5279.2 ± 1.8	5278.24	0.96
$B^\pm$	$e^\pm \nu_e$	2639.45	101	26.13	5278.9 ± 1.8	5278.24	0.64
$B_J^*(5732)$	e^-e^+	2849.00	109	26.14	5698 ± 12	5696.32	1.68
$B_J^*(5732)$	$e^\pm \nu_e$	2849.00	109	26.14	5698 ± 12	5696.32	1.68
$B_{sJ}^*(5850)$	e^-e^+	2926.5	112	26.13	5853 ± 15	5853.10	0.10
$B_{sJ}^*(5850)$	$e^\pm \nu_e$	2926.5	112	26.13	5853 ± 13	5853.10	0.10
$\Upsilon(10860)$	e^-e^+	5432.5	208	26.12	10865 ± 8	10870.05	5.04
$f_1(1285)$	$\phi\gamma$	236	9	26.22	1282.2±0.7	1281.36	0.84
$f_1(1285)$	$a_0(980)\pi^0$	234	9	26	1282.2±0.7	1282.38	0.18
$a_2(1320)$	$\eta'(958)\pi^0$	287	11	26.09	1318.1±0.7	1317.51	0.59
K_S^0	$\pi^0\pi^0$	209	8	26.13	497.672±0.031	497.66	0.01
$K_2^*(1430)^\pm$	$K^\pm\gamma$	627	24	26.13	1425.4±1.3	1425.24	0.16
$D^\pm$	$\overline{K}^0\pi^\pm$	862	33	26.12	1869.3±0.5	1869.11	0.19
D^0	$\overline{K}^0 f_0(980)$	549	21	26.14	1864.5±0.5	1863.97	0.53

Continuation of Table 2.

resonances	decay channels	P_{exp}	n	P_{exp}/n	M_{exp}	M_{th}	ΔM
D^0	$\phi\rho^0$	260	10	26	1864.5±0.5	1864.08	0.42
$D_s^\pm$	$f_0(980)\pi^\pm$	732	28	26.14	1968.5±0.6	1967.82	0.68
$D_s^\pm$	$\eta'(958)\rho^\pm$	470	18	26.11	1968.5±0.6	1968.03	0.47
$D_{s1}(2536)^\pm$	$D^0K^\pm$	392	15	26.13	2535.35±0.34	2535.6	0.25
$B^\pm$	$e^\pm\gamma$	2639	101	26.13	5278.9±1.8	5278.24	0.66
B^0	$e^\pm\mu^\mp$	2639	101	26.13	5279.2±1.8	5280.35	1.15
$\eta_c(1S)$	$K^*(892)^\pm\overline{K}^\mp$	1307	50	26.14	2979.8±2.1	2978.38	0.42
$\psi(2S)$	$\gamma\chi_{c0}(1P)$	261	10	26.1	3686.00±0.09	3686.38	0.38
$\chi_{b0}(1P)$	$\gamma\Upsilon(1S)$	391	15	26.07	9859.8±1.3	9860.44	0.64
$\Upsilon(2S)$	$\gamma\chi_{b1}(1P)$	131	5	26.2	10023.3±0.31	10023.41	0.11
$\Sigma(1385)^0$	$\Lambda\pi^0$	208	8	26	1383.7±1.0	1383.93	0.23
Λ_c^+	$\Xi(1530)^0K^+$	471	18	26.17	2284.9±0.6	2284.25	0.65
$\Lambda_c(2593)^+$	$\Lambda_c^+\pi^0$	261	10	26.1	2593.6±1.0	2593.9	0.30
Ξ_c^+	$\Sigma^+\overline{K}^*(892)^0$	653	25	26.12	2465.6±1.4	2465.89	0.29
Ξ_c^0	Ω^-K^+	522	20	26.1	2470.3±1.8	2471.11	0.89
$\Xi_c(2645)$	$\Xi_c^+\pi^-$	107	4	26.75	2643.8±1.8	2642.18	1.62

Table 3. The masses of resonances grouped near the masses of $\Omega^-(1672.45 \pm 0.29)$ and $\tau(1777^{+0.30}_{-0.27})$

baryons	M_{exp}^{baryon}	mesons	M_{exp}^{meson}
Ω^-	1672.45 ± 0.29	τ	$1777^{+0.30}_{-0.27}$
$\omega(1600)$	1663 ± 12	X(1775)	1776 ± 13
$\omega_3(1670)$	1673 ± 12	$f_J(1710)$	1768 ± 14
$\pi_2(1670)$	1676 ± 6	$\pi(1800)$	$1775 \pm 7 \pm 10$
$\phi(1680)$	1677 ± 12	$\rho(1700)$	1780
$\rho_3(1690)$	1679 ± 11	$K_2(1700)$	~ 1780
$\rho_3(1690)$	1673 ± 9	$K_3^*(1780)$	1779 ± 11
$\rho(1700)$	1659 ± 25	$\Delta(1750)P_{31}$	1778.4 ± 9
$K^*(1680)$	$1677 \pm 10 \pm 32$	$\Delta(1620)S_{31}$	1786.7 ± 2
$N(1650)S_{11}$	1672	$\Delta(1900)S_{31}$	1780
$N(1675)D_{15}$	1673	$\Delta(1905)F_{35}$	$1787^{+6.0}_{-5.7}$
$N(1680)F_{15}$	1674 ± 12	$\Delta(1910)P_{31}$	1790
$N(1700)D_{13}$	1670 ± 10	$\Lambda(1800)S_{01}$	1767
$N(1710)P_{11}$	1670	$\Lambda(1810)P_{01}$	1780 ± 20
$N(1720)P_{13}$	1675	$\Sigma(1750)S_{11}$	1785 ± 12
$\Delta(1600)P_{33}$	1672 ± 15	$\Sigma(1750)P_{11}$	1770 ± 20
$\Delta(1620)S_{31}$	1672 ± 5	$\Sigma(1775)D_{11}$	1777 ± 5
$\Delta(1700)D_{33}$	1672	$\Xi(1820)D_{13}$	1770
$\Lambda(1670)S_{01}$	1670.8 ± 1.7	—	
$\Sigma(1670)P_{11}$	1671 ± 2	—	
$\Sigma(1670)D_{11}$	1671 ± 3	—	
$\Sigma(1670)Bumps$	1670 ± 4	—	

We are able to interpret some of the resonances as radial excitations. For example, $\rho(770)$, $\rho(1450)$ and $\rho(2150)$-mesons decay through two pions with momenta 358, 719 and 1065 MeV/c, i.e. their momenta are commensurable.

Obviously, the accuracy of resonance masses to be predicted depends on the accuracy of decay product masses used in calculations. Therefore, our predictions of resonance masses are to be considered as preliminary ones. We ask readers to send information about more accurate contemporary experimental resonance mass data. We will be indebted for this.

We have established that experimental values of masses of PP and $\overline{P}P$ resonances [7] coincide within the accuracy of experimental data. This remarkable observation was highlighted in our earlier works [8, 9]. But the systematic analysis of experimental data brings us to a more general conclusion to be switched on below after discussing some examples.

In Table 3 we have collected the masses of resonances grouped near the masses of τ-lepton and Ω-hyperon. The masses of resonances coincide within the accuracy of experimental data. This observation is valid for many resonances independently of their type, quantum numbers, interactions, and so on. If so, good perspectives for a combined systematic analysis of this type of resonances are opened.

3 Nucleon structure

The excited states of nucleons have been investigated in a large number of experiments. The main information of the masses, widths and elasticities for N and Δ resonances comes from the partial-wave analysis of $N\pi$, $N\eta$, ΛK and ΣK data sets (for details see [3]). We would like to propose that the properties of the ground state of nucleon can be studied from the experiments performed for extraction of information about the excited states of nucleons.

The proton is practically stable (mean lifetime $\tau > 1.6 * 10^{25}$ years — independent of the decay mode [3]). According to the minimal $SU(5)$ Grand Unified Theory (see minireview [10], page 1673) it can decay via different channels. For example, $p \rightarrow e^+\pi^0$, $p \rightarrow \mu^+\gamma$, $p \rightarrow \nu K^*(892)^+, \ldots$. But the proton does not decay despite the possibility from energy-momentum conservation law.

Let us consider the problem of particle's stability at least. According to our hypothesis [7], a stable particle (proton, neutron) represents a complex ideal wave resonator. Each individual resonator has the frequency ω_i and uncertainty in frequency $\Delta\omega_i$. A wave function of eigen oscillations of a combination of coupled resonators can be represented in the form:

$$\Psi = \sum_i a_i(\omega_i\vec{r})\sqrt{\frac{1}{2\pi}\frac{\Delta\omega_i}{(\omega-\omega_i)^2+\Delta\omega_i^2/4}}, \tag{3}$$

where $a_i(\omega_i\vec{r})$ is the radial part of the partial wave function.

The channel energy distribution of a resonator can be calculated by the formula

$$\int \Psi^2 d\vec{r} = \int \left(\sum_i a_i(\omega_i\vec{r})\sqrt{\frac{1}{2\pi}\frac{\Delta\omega_i}{(\omega-\omega_i)^2+\Delta\omega_i^2/4}}\right)^2 d\vec{r}. \tag{4}$$

So, two conclusions follow from (4):

1) If all $\omega_i = \omega_0$ and $\Delta\omega_i = \Delta\omega_0 \rightarrow 0$ (or $\tau \rightarrow \infty$ —resonator with ideally reflecting walls),

$$\int \Psi^2 d\vec{r} = \int \left(\frac{1}{2\pi}\frac{\Delta\omega_0}{(\omega-\omega_0)^2+\Delta\omega_0^2/4}\right)\left(\sum_i a_i(\omega_0\vec{r})\right)^2 d\vec{r} =$$
$$\frac{1}{2\pi}\frac{\Delta\omega_0}{(\omega-\omega_0)^2+\Delta\omega_0^2/4} \rightarrow \delta(\omega-\omega_0). \tag{5}$$

It seems that just this case of a system of resonators corresponds to stable systems (like proton and neutron), when all the frequencies of a system of resonators are ideally coordinated and equal to each other. In other words, all channel motions in the stable systems are exactly synchronous. Amplitudes are coherently summarized. Apparently, the root of the riddle about stability of some particles (like proton, neutron, electron) lies in it.

2) If $\omega_i = \omega_0 n_1^i/n_2^i$ (where n_1^i and n_2^i are integer numbers, there can arise beating phenomena well known in the wave theory. This case corresponds to hadron resonances such as $f_0(400-1200)$ and ρ-mesons, Δ-isobar,... with a large width.

These resonances are combinations of resonances with a small width. An experiment gives us information about the envelope of intensities of resonance excitation; fine structures will be found when the accuracy increases. The situation is in full analogy with giant resonances in atomic nuclei [9], and there are many examples in atomic spectroscopy. The beating phenomenon is known in microcosm. Now it is a generally accepted view point that appearance of magic numbers (a structure possessing a large binding energy) in atomic nuclei, atoms, and metallic clusters is conditioned by beating [11].

4 The conservation laws and the Regge-like trajectories

The well-known results in high-energy physics indicate that there is a profound connection between spins and masses of strongly interacting elementary particles, hadrons. The spin J of some baryons and mesons appears to be nearly proportional to the square of their mass M: $M^2 \propto J$. The correlation between spin and mass of experimentally known low mass hadrons is represented by a straightline Regge trajectory [12] in a Chew-Frautschi [13] plot. These trajectories are remarkably linear and approximately parallel (such trajectories of mesons going up $J = 6$ and of baryons up to $J = 15/2$); i.e., the angular momentum is a linear function of the square of the particle mass [3].

The general formula which connects the maximal spin J and mass M of heavy hadrons was obtained in [14] by using simple arguments of dimensional analysis and similarity principle

$$J = \hbar(\frac{M}{m_p})^{1+1/\zeta}, \tag{6}$$

where m_p is the proton mass, the number ζ takes values ζ=1, 2, 3 and characterizes the spatial dimensionality of hadrons.

The case $\zeta = 1$ describes one-dimensional string-like hadrons and corresponds to the well-known straight-line trajectory for ordinary hadrons and hadronic resonances [13]

$$J = \hbar(\frac{M}{m_p})^2. \tag{7}$$

The case ζ=2 corresponds to two-dimensional disk-like hadrons

$$J = \hbar(\frac{M}{m_p})^{3/2}. \tag{8}$$

Finally, ζ=3 corresponds to the case of three-dimensional or spherical hadrons

$$J = \hbar(\frac{M}{m_p})^{4/3}. \tag{9}$$

An analysis [14] of the data on the rotation of cosmic objects to be observed shows that all of them can be classified into two groups, in which spin-mass relations are given by formulae (8) and (9), respectively.

The first one includes clusters of galaxies, single galaxies, globular, and open star clusters and perhaps stellar associations and super-associations. The angular momentum-mass distributions for these objects are described by equation (8).

The second group of objects described by equation (9) includes single stars, planets, and asteroids.

The plot [14] $\lg J$ versus $\lg M$ shows a remarkable regularity, and the theoretical lines describe not only the shape, but also absolute values in tremendous mass and spin intervals (the mass interval is about 34 orders of magnitude, the corresponding interval for angular momenta covers about 60 orders of magnitude) without invoking arbitrary parameters. Therefore, Muradian's approach incorporates in a natural way fundamental quantum mechanical constants $\hbar$ and m_p.

It is worthwhile to note following the conclusion of R.M. Muradian [14] that equations (7-9) can be obtained from (2) by using the Bohr-Sommerfeld quantization conditions

$$Pr = n\hbar \equiv J\hbar, \tag{10}$$

and the assumption that the masses of an n-dimensional rotational object can be written in the form

$$m = \rho r^{\zeta}, \tag{11}$$

where ρ is the constant density of the object (note that R.M. Muradian used $Pr = J\hbar/2$). Therefore equation (2) acquires the form

$$M = \sqrt{m^2 + \frac{P^2}{c^2}} = \sqrt{(\rho r^{\zeta})^2 + \frac{J^2\hbar^2}{c^2 r^2}}. \tag{12}$$

The minimization of this expression over r provides the Regge-like trajectories (7-9) with the accuracy of a constant factor. This result is remarkable because the observed and well-established Regge-like trajectories in micro- and macrosystems were obtained from the first principles.

Let us discuss an example of a hydrogen atom. The electrostatic force between the proton and electron leads to the formation of bound states of the hydrogen atom. Following Bohr, we require the equality of the Coulomb and centrifugal forces

$$\frac{e^2}{r^2} = \frac{mv^2}{r}, \tag{13}$$

and using Bohr's quantization condition (10) we obtain the admissible values of r, or the Bohr radii:

$$r = \frac{n^2\hbar^2}{me^2} = n^2 a_1; \quad a_1 = \frac{\hbar^2}{mc^2}, \tag{14}$$

where a_1 is the radius of the first Bohr orbit. The momentum of an electron on an n-th Bohr orbit equals

$$P = \frac{me^2}{nh} = \frac{P_1}{n}, \quad P_1 = \frac{me^2}{h}, \tag{15}$$

where P_1 is the momentum of the electron on the first Bohr orbit.

Therefore, we can conclude from equations (14) and (15) that the electron momenta and orbits in a hydrogen atom are quantized. Note that the minimal value of the orbit radius and the maximal value of the momentum depend on the electron reduced mass m. If we should consider other systems, for a example $e^{\pm}e^{\mp}$, $\mu^{\pm}e^{\mp}$, then the corresponding minimal orbit radius and maximal momentum are scaled according to the considered reduced masses. The same conclusions can be obtained from quantum theory. We have chosen the simplest way. One can see that equation (15) for the electron momentum quantization on an n-th Bohr orbit is the same (inverted) in the analytic form as the momentum quantization for decay products of elementary particle resonances (1).

It is easy to find the energy of a hydrogen atom, which is equal to the kinetic and potential energies (in the nonrelativistic limit):

$$E = \sqrt{m^2c^4 + P^2c^2} - mc^2 - \frac{e^2}{r} = -\frac{e^2}{2r} = -\frac{e^2}{2n^2 a_1}. \tag{16}$$

We note the well-known fact that Bohr solved the problem of quantization of a hydrogen atom in 1913 [15], long before the creation of quantum theory.

Sommerfeld [16] generalized Bohr results to the relativistic case

$$E = mc^2 \left(1 + \frac{\alpha^2}{\left(n_r + \sqrt{n_\phi^2 - \alpha^2}\right)^2}\right)^{-1/2} - mc^2, \tag{17}$$

where n_r and n_ϕ are radial and azimuthal quantum numbers, $\alpha = e^2/\hbar c$ is the fine-structure constant. It is worthwhile to note that the Sommerfeld results coincide exactly with the Dirac results obtained later if one puts $n_\phi = j + 1/2$ where j is the orbital angular momentum plus spin of an electron.

To conclude a short excursion of the history of physics we can say that the exactness of the Bohr and Sommerfeld results for a hydrogen atom in nonrelativistic and relativistic cases is surprising because these results have been obtained within classical equations of motion and Ehrenfest adiabatic invariants. This coincidence cannot be accidental.

In further discussion, we will employ on the hypothesis that conservation laws of energy-momentum and Ehrenfest's adiabatic invariant and also the resonator principle for standing waves of any physical nature are common for all hierarchic systems.

The analogy between the relativistic form of a free particle Hamilton function and dispersion relations for standing waves in a hollow metallic resonator (waveguide) has been discussed long ago (for a recent review see, for example, paper [17]). It seems that the underlined analogy has a deep physical reason.

Let us carry out a visual comparative analysis of quantization of a classical field for the string displacement $q(t,x)$ and scalar field described by the same equations [18]. The classical field of displacement is described by the equation

$$\frac{\partial^2 q(t,x)}{\partial t^2} - \frac{\partial^2 q(t,x)}{\partial x^2} = 0, \quad (18)$$

where the linear density of the string and the velocity of spreading of oscillations are equal to unity. The following relations are present when both ends of the string are fixed: $q(t,0) = q(t,a) = 0$. Then, the classical eigenstates of equation (18) are

$$q(t,x) \sim exp[\pm i\omega_n t] sin(k_n x), \quad k_n a = n\pi. \quad (19)$$

The Klein-Gordon equation is a relativistic analog of (18)

$$\frac{\partial^2 \phi(t,\vec{r})}{\partial t^2} - \Delta\phi(t,\vec{r}) + [m^2 + V(\vec{r})]\phi(t,\vec{r}) = 0. \quad (20)$$

It describes a neutral scalar field where m is the particle mass and $V(\vec{r})$ is the potential of an external field interacting with the field ϕ. When $V = 0$ and the field ϕ is defined in the interval $[0,a]$ with zero boundary conditions $\phi(t,0) = \phi(t,a) = 0$, the eigenfunctions and eigenfrequencies in the one-dimensional case acquire the form

$$\phi_n(t,x) \sim exp[\pm i\omega_n t] sin(k_n x), \quad k_n a = n\pi, \quad \omega_n = \sqrt{m^2 + k_n^2} = \sqrt{m^2 + \frac{n^2\pi^2}{a^2}}. \quad (21)$$

So, the quantized fields of displacements and eigenfrequencies of a classical string and eigenfunctions and eigenfrequencies of a relativistic scalar field have the similar analytical form. They coincide when $m = 0$. This is a consequence of the identity of appropriate equations and boundary conditions. Moreover, the formulas for eigenfrequencies of a classical string and a quantum relativistic scalar field coincide with those for eigenfrequencies of classical resonators. For example, the eigenfrequencies of the cavity-resonators having a cylindrical form with the radius R and with the length d are equal to

$$\omega_n = \frac{\pi}{\mu\varepsilon}\sqrt{(\frac{x_{\nu j}}{\pi R})^2 + \frac{n^2}{d^2}}, \quad (22)$$

where $x_{\nu j}$ are solutions of the equations

$$J_\nu(x_{\nu j}) = 0 \ or \ J'_\nu(x_{\nu j}) = 0, \quad (23)$$

while $\nu \geq 0$ and j=1, 2, 3,...

The mass formula (2) for resonances can be rewritten in the following form

$$M_{th} = \sqrt{m_a^2 + P_n^2} + \sqrt{m_b^2 + P_n^2} = \sqrt{\frac{1}{\Lambda_C^2(a)} + \frac{n^2}{\Lambda_D^2}} + \sqrt{\frac{1}{\Lambda_C^2(b)} + \frac{n^2}{\Lambda_D^2}}, \quad (2a)$$

where Λ_C and $\Lambda_D = 1/P_1$ are the Compton and de Broglie wave lengths, respectively.

The similitude of analytical forms for eigenfrequencies of cavity-resonators for micro- and macrosystems, invariant masses of elementary particle resonances and eigenvalues for hydrogen atoms is not accidental but represents the general law of the resonator principle. The Regge-like trajectories being fundamental for elementary particle physics provide a powerful instrument for understanding the mass and spin distributions of astrophysical objects [14], the velocity and orbit distributions of planets and their satellites in the Solar system [8, 19] without any free parameters independent of the type of interactions, and so on. The Regge-like trajectories, as it has been shown above, have been obtained by using the two invariants: the energy-momentum and the Ehrenfest adiabatic invariant.

Furthermore, the ratio of the average kinetic energy of a system to its frequency $\overline{E}_{kin}/\nu$, according to the Boltzmann theorem [20], is the Ehrenfest adiabatic invariant, and this affirmation does not depend on the type of interaction of constituents belonging to the whole system. Actually, lord Rayleigh pointed out the fact in 1902 that in some sine-like oscillating systems (standing waves in an adiabatically decreasing cavity, a transversely oscillating string inside a narrow shrinking ring) adiabatic changes occur so that the correlation between energy and frequency remains fixed [21].

Our discussions may seem old-fashioned. But the coincidence of predictions following from the Ehrenfest adiabatic invariant quantization condition with true results of quantum theory obtained from analytic calculations is extraordinary. When some parameters of a system change adiabatically, the Ehrenfest adiabatic invariant is a constant of motion in the classical and quantum mechanics. Therefore, quantization of this invariant leads to the known results of quantum theory. Apparently, the Ehrenfest adiabatic invariant is a universal invariant for periodic motions including resonances of elementary particles. We have checked this inference using a systematic analysis of experimental data for asymptotic momenta P of decay products of elementary particle resonances. We have used the Bohr-Sommerfeld quantization rule (as a special case of the Ehrenfest adiabatic invariant quantization rule)

$$P_n = n\hbar/r_n = nP_0 \ or \ P_n = P_0/n. \quad (1a)$$

5 The second Kepler law and the Planck constant $\hbar$

The history of adiabatic invariants has approximately two stages. The first stage corresponded to the time when the main question of theory was: what type of quantities are the adiabatic invariants ? The clarification of this question was very important for solving the problem of quantization in the old quantum theory. The main postulate of old quantum theory formulated by Ehrenfest (for details see [20]) stated that only the adiabatic invariants should be quantized. The second stage has been started recently and the main question is: how exact are adiabatic invariants? Answers can be found in the monograph [22]. The adiabatic invariants seem to be reduced to the exact one under definite conditions (for details see [22].

An important question of physics is to establish the conservation laws of motion. If the number of those laws is large enough, then they can describe the motion of a system in an adequate way. As an interesting example, we consider the ground state of a hydrogen atom in a classical way

$$H = -\frac{e^2}{r} + \frac{p^2}{2m}. \tag{24}$$

If we introduce the quantity $f = rv$, the invariant of motion according to the second Kepler law, then

$$p = mv = \frac{mvr}{r} = \frac{mf}{r}, \tag{25}$$

and equation (24) is rewritten in the following form

$$H = -\frac{e^2}{r} + \frac{mf^2}{2r^2}. \tag{26}$$

The minimum of (26) will be achieved at

$$r_0 = \frac{mf^2}{e^2}. \tag{27}$$

From (26) and (27), we obtain

$$H_{min} \equiv E_{min} = -\frac{e^4}{2mf^2}. \tag{28}$$

The value of the invariant of motion f can be calculated from (28), if we use the experimental value [3] for the ground state of a hydrogen atom. The result is equal to

$$f = 12.8808885 * 10^{-22} c^2\ s^{-1}. \tag{29}$$

Let us calculate the quantity

$$mf = mvr = 6.5821220 * 10^{-22} MeV * s, \tag{30}$$

which is exactly equal to the Planck constant $\hbar$.

This result is amazing: we have used the classical Hamiltonian for a hydrogen atom using the second Kepler law, then we have found the minimum of this Hamiltonian. Equating this minimal value of the Hamiltonian to the ground state energy of a hydrogen atom, we have calculated the electron sectorial velocity f. As a final result, we obtain that the action is equal to the Planck constant

$$mf = mvr = \hbar, \tag{31}$$

and we come to the Bohr quantization condition for the ground state of a hydrogen atom. It means that the Planck constant $\hbar$ is the Ehrenfest adiabatic invariant for the ground state of a hydrogen atom.

Therefore the Planck constant $\hbar$ and the Bohr quantization condition for the ground state of a hydrogen atom have been deduced from classical mechanics by using only two invariants: the conservation of energy and the second Kepler law. It seems that this result is exact and fundamental. But the question arises: is it possible to reproduce the quantum mechanical results from classical mechanics for excited states of a hydrogen atom ?

6 The predictions of resonance masses

The above-presented method is able to describe the existing experimental data. Even a short comparison our calculations with experimental data of masses of the elementary particle resonances suggests simple ideas for experimental searches of nonobserved ones. This is obvious. We would like in this section to discuss another idea for this purpose. Let us consider a few dominant decay channels of the resonances:

$$\Xi^- \rightarrow \Lambda\pi^-$$

with fraction 99.887 ±0.035%,

$$\Xi(1530)^0 \rightarrow \Xi^-\pi^+$$

with fraction 100%,

$$\Sigma^0 \rightarrow \Lambda\gamma$$

with fraction 100%,

$$\Sigma^- \rightarrow n\pi^-$$

with fraction 99.848 ± 0.005%. The masses of heavier resonances were calculated by the formula:

$$M_{th} = \sqrt{m_1^2 + P_n^2} + \sqrt{m_2^2 + P_n^2} = \sqrt{m_1^2 + n^2P_1^2} + \sqrt{m_2^2 + n^2P_1^2}. \tag{32}$$

where m_1, m_2 and P_1 are the masses and momenta of decay products from one of the dominant channels cited above. The results of our calculations and the corresponding experimental data [3] are illustrated in Figures 2-9. The X-axis characterizes the families of resonances (baryonic or mesonic) and Y-axis represents their masses (in MeV). The figures show that momenta P_1 to be proposed generate the families of resonances with different quantum numbers. We think that the results given in the figures convincingly demonstrate the empirical fact that resonance decay product momenta and their masses are quantized. It is clear that commensurability of momenta does not depend on the type of interaction between resonance decay products, quantum numbers of resonances, and the type of particles. An excellent possibility for the prediction of new resonances and verification of masses of existing ones arises in any case.

At the beginning of this paper we have mentioned about hypothetical decay channels of a proton. We decided to investigate some of these channels, for example, $p \rightarrow \nu K^*(892)^+$. The masses of p, ν, $K^*(892)^+$ are known. So we are able calculate the decay momentum P_1 and then to evaluate the masses of excited states of a proton using the formula analogous to (32) where m_1, m_2 are the masses of hypothetical decay

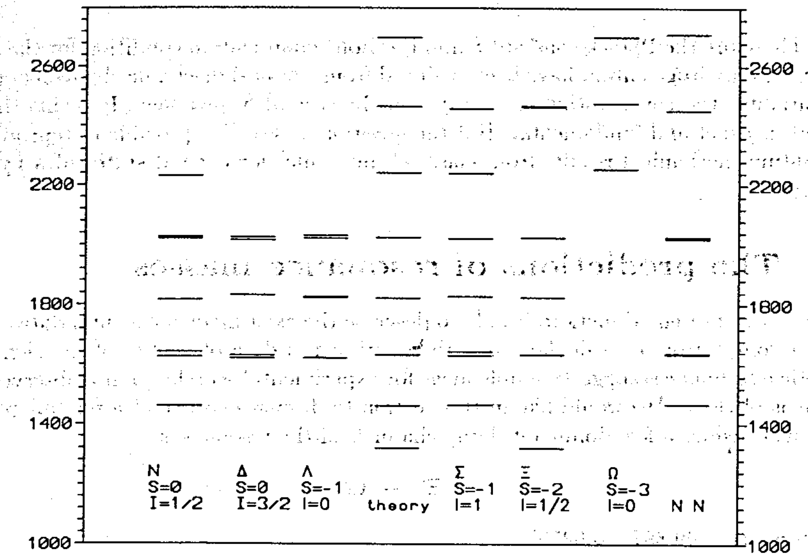

Figure 2. The mass distribution of baryonic resonances with momenta multiples of 139.57 MeV/c. The basic momentum is taken from the channel $\Xi^- \to \Lambda\pi^-$ with fraction 99.887 ±0.035%.

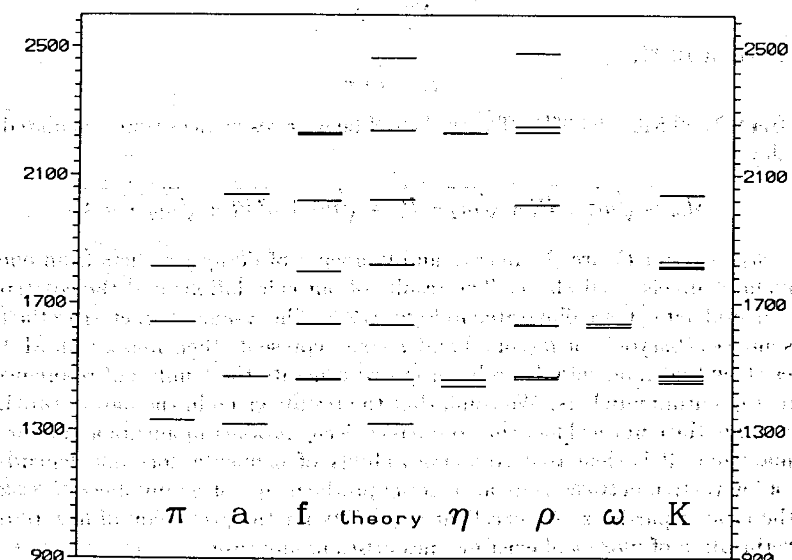

Figure 3. The mass distribution of mesonic resonances with momenta multiples of 139.57 MeV/c. The basic momentum is taken from the channel $\Xi^- \to \Lambda\pi^-$ with fraction 99.887 ±0.035%.

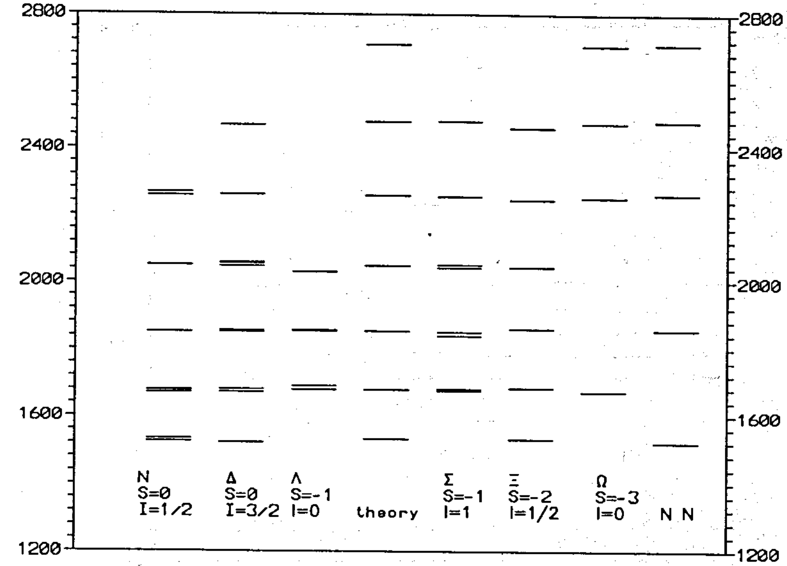

Figure 4. The mass distribution of baryonic resonances with momenta multiples of 146.55 MeV/c. The basic momentum is taken from the channel $\Xi(1530)^0 \to \Xi^-\pi^+$ with fraction 100%.

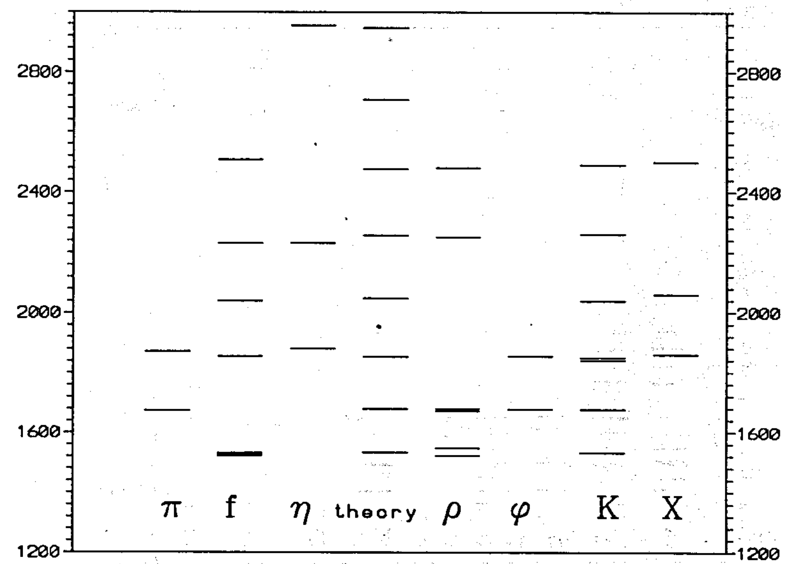

Figure 5. The mass distribution of mesonic resonances with momenta multiples of 146.55 MeV/c. The basic momentum is taken from the channel $\Xi(1530)^0 \to \Xi^-\pi^+$ with fraction 100%.

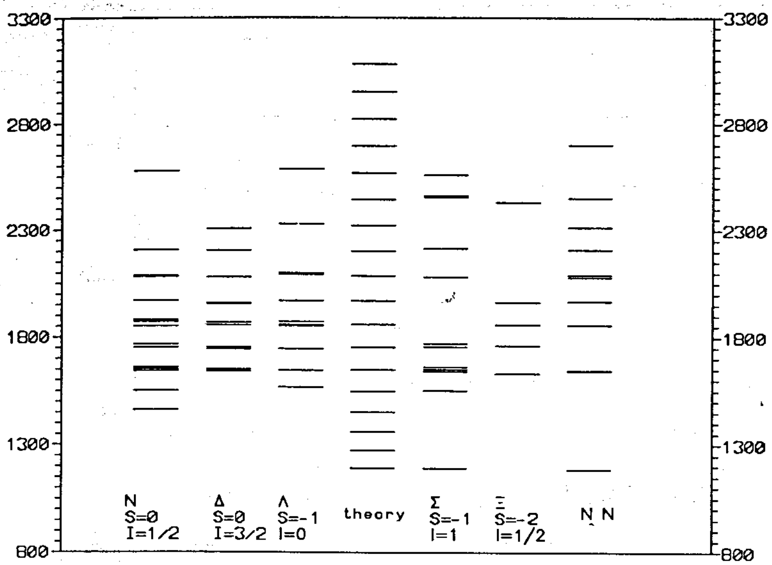

Figure 6. The mass distribution of baryonic resonances with momenta multiples of 74.39 MeV/c. The basic momentum is taken from the channel $\Sigma^0 \to \Lambda\gamma$ with fraction 100%.

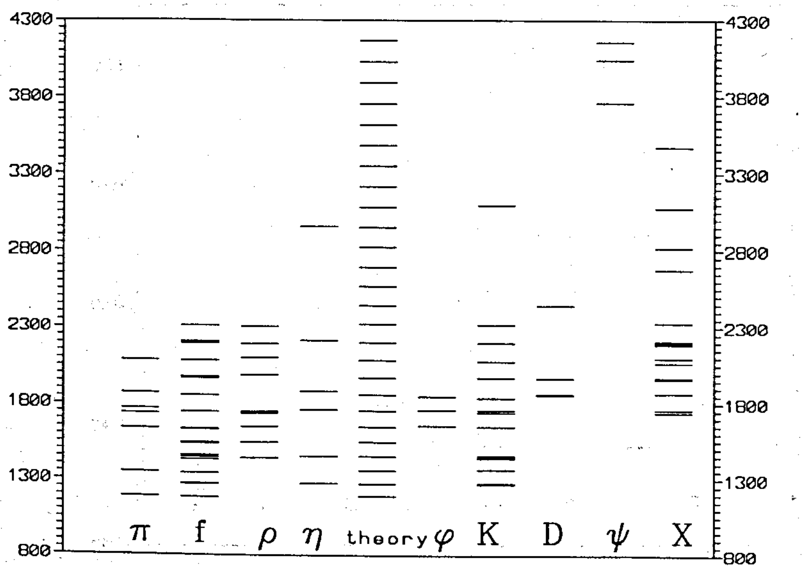

Figure 7. The mass distribution of mesonic resonances with momenta multiples of 74.39 MeV/c. The basic momentum is taken from the channel $\Sigma^0 \to \Lambda\gamma$ with fraction 100%.

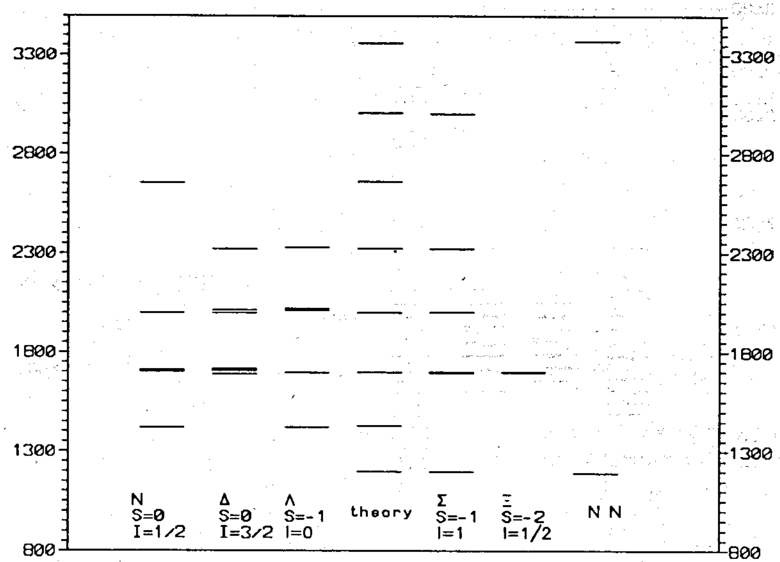

Figure 8. The mass distribution of baryonic resonances with momenta multiples of 193.07 MeV/c. The basic momentum is taken from the channel $\Sigma^- \to n\pi^-$ with fraction $99.848 \pm 0.005\%$.

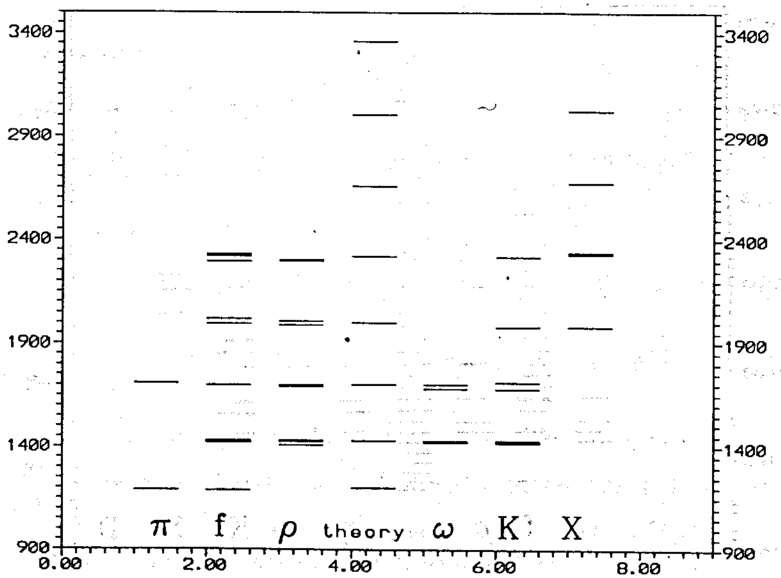

Figure 9. The mass distribution of mesonic resonances with momenta multiples of 193.07 MeV/c. The basic momentum is taken from the channel $\Sigma^- \to n\pi^-$ with fraction $99.848 \pm 0.005\%$.

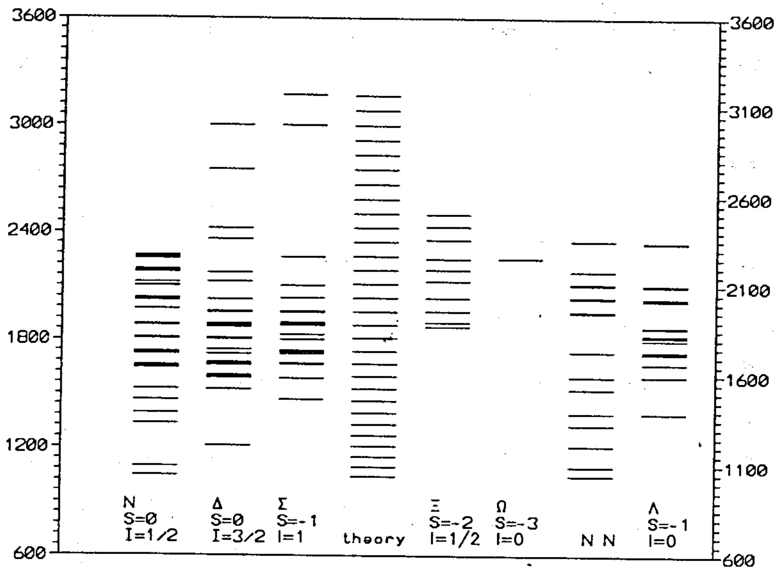

Figure 10. The mass distribution of baryonic resonances with momenta multiples of 45.52 MeV/c. The basic momentum is taken from the hypothetical channel $p \to \nu K^{*}(892)^{+}$.

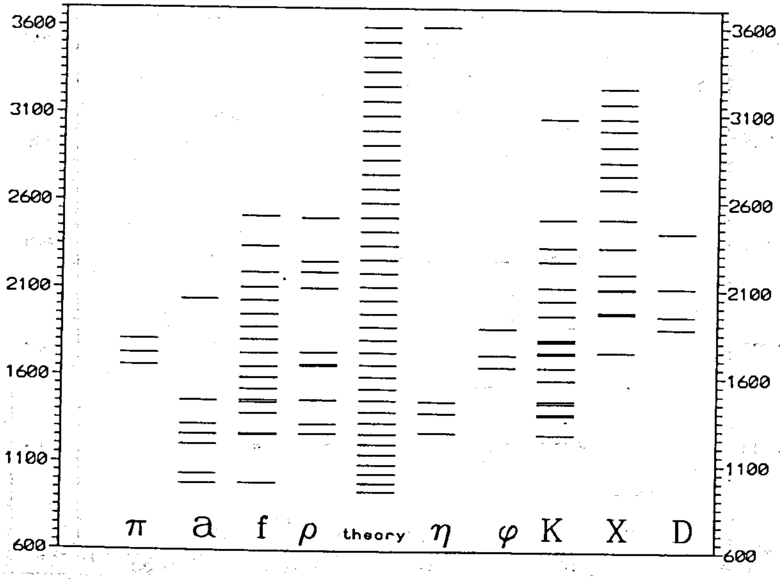

Figure 11. The mass distribution of mesonic resonances with momenta multiples of 45.52 MeV/c. The basic momentum is taken from the hypothetical channel $p \to \nu K^{*}(892)^{+}$.

particles (ν and $K^{*}(892)^{+}$) and P_1 is the momentum of their relative motion. The results of calculations and the corresponding experimental data are illustrated in Figure 10, 11. There is a good correlation between the experimental data and theoretical calculations. Moreover, there are many new predicted resonances.

This last example have demonstrated clearly the possibility to extract some information about the inner structure of nucleon using experimental data for excited nucleon states. This possibility will be discussed in future publications.

7 Conclusion

In conclusion we are able to say that we have established the Balmer-like parameter-free formula for masses of elementary particle resonances in accordance with the systematic analysis of experimental data.

The use of formula (2) is so simple that one can check all our results. The interest of our results is not only in their closeness to the experimental data, but also in the derivation of formula (2) from the two invariants: the conservation law of energy-momentum and the Ehrenfest adiabatic invariant.

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