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PROPER TIME DYNAMICS
IN GENERAL RELATIVITY
AND CONFORMAL UNIFIED THEORY

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1 Introduction

The notion of "time", in general relativity, is many-sided [1, 2, 3].

General relativity is invariant with respect to general coordinate transformations including the reparametrizations of the "initial time-coordinate" $t \mapsto t' = t'(t)$.

The Einstein observer, in GR, measures the proper time as the invariant geometrical interval.

The Hamiltonian reduction [1] of cosmological models inspired by GR [1, 2, 3] reveals the internal dynamical "parameter of evolution" of the Dirac invariant sector of physical variables [4, 5, 6, 7, 8]. In cosmological models this "evolution parameter" is the cosmic scale variable, and the relation between an invariant geometrical interval and dynamical "evolution parameter" (the "proper time" dynamics) describes data of the observational cosmology (the red shift and Hubble law).

In this paper we would like to generalize the Hamiltonian reduction with internal evolution parameter to the case of field theories of gravity.

For researching the problem of "time" in a theory with the general coordinate transformations [1], one conventionally uses [9, 10] the Dirac-ADM parametrization of the metric [11] and the Lichnerowicz conformal invariant variables [12] constructed with the help of the scale factor (i.e. the determinant of the space metric).

The Dirac-ADM parametrization is the invariant under the group of kinematic transformations. The latter contains the global subgroup of the reparameterization of time $t \mapsto t' = t'(t)$. The Hamiltonian reduction of such the time-reparameterization invariant mechanical systems is accompanied by the conversion of one of the initial dynamical variables into parameter of evolution of the corresponding reduced systems. York and Kuchar [9, 10] pointed out that such variable in GR (which is converted in the evolution parameter) can be proportional to the trace of the second form.

In the contrast with [9, 10], we suppose that the second form can be decomposed on both global excitation and local one.

The ADM-metric and the Lichnerowicz conformal invariant variables allow us [13, 14] to extract this evolution parameter of the reduced system, in GR, as the global component of the scale factor.

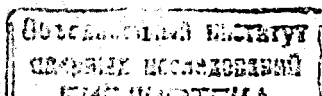
The main difficulty of the Hamiltonian reduction in GR is the necessity of separation of parameters of general coordinate transformations from invariant physical variables and from quantities including the parameter of evolution and proper time.

Recently, this separation was fulfilled in the cosmological Friedmann models [7, 8] with the use of the Levi-Civita canonical transformation [15, 16, 17], which allows one to establish direct relations between the Dirac observables of the generalized Hamiltonian approach and the Friedmann ones in the observational cosmology (the red shift and the Hubble law) expressed in terms of the proper time.

It has been shown that in this way one can construct the normalizable wave function of the Universe so that the variation of this function under the proper time leads to the "red shift" measured in observational cosmology [8].

We show that the Hamiltonian reduction of GR distinguishes the conformal time as more preferable than the proper time from the point of view of the correspondence principle and causality [18]. The usage of the conformal time (instead of the proper one) as a measurable interval can be argued in the conformal unified theory (CUT) [19, 20] based on the standard model of fundamental interactions where the Higgs potential is changed by the Penrose-Chernikov-Tagirov Lagrangian for a scalar field [21].

The content of the paper is the following. In Section 2, we use a model of classical mechanics with the time reparametrization invariance to introduce definitions of all times



used in the extended and reduced Hamiltonian systems. Section 3 is devoted to special relativity to emphasize the main features of relativistic systems with the frame of reference of an observer. In Section 4, we consider the Friedmann cosmological models of expanding Universe to find the relation between the evolution parameter in the reduced Hamiltonian system and the proper time of the Einstein-Friedmann observer. In Section 5, a dynamical parameter of evolution is introduced in GR as the global component of the space metric, and an equation for the proper time in terms of this dynamical parameter is derived. Section 6 is devoted to the construction of conformal invariant theory of fundamental interactions to analyze similar dynamics of the proper time in this theory.

2 Classical mechanics

We consider a reparametrization invariant form of classical mechanics system

$$W^E [p_i, q_i; p_0, q_0 | t, N] = \int_{t_1}^{t_2} dt \left(-p_0 \dot{q}_0 + \sum_i p_i \dot{q}_i - N H_E(q_0, p_0, q_i, p_i) \right), \quad (1)$$

where

$$H_E(q_0, p_0, q_i, p_i) = [-p_0 + H(p_i, q_i)] \quad (2)$$

is the extended Hamiltonian.

The action (1) was constructed from

$$W^R [p_i, q_i | q_0] = \int_{q_0(1)}^{q_0(2)} dq_0 \left[\sum_i p_i \frac{dq_i}{dq_0} - H(p_i, q_i) \right] \quad (3)$$

by the introduction of a "superfluous" pair of canonical variables (p_0, q_0) and the Lagrange factor (N) .

The reduction of the extended system (1) to (3) means the explicit solution of the equations for "superfluous" canonical variables and the Lagrange factor

$$\frac{\delta W}{\delta N} = 0 \Rightarrow -p_0 + H(p_i, q_i) = 0, \quad (4)$$

$$\frac{\delta W}{\delta q_0} = 0 \Rightarrow \dot{p}_0 = 0, \quad (5)$$

$$\frac{\delta W}{\delta p_0} = 0 \Rightarrow dq_0 = N dt \equiv dT. \quad (6)$$

Equation (4) is a constraint; eq. (5), is the conservation law; and eq. (6) establishes the relation between the evolution parameter of the reduced system (3) and the "Lagrange time", which can be defined for any time reparametrization invariant theory with the use of the Lagrange factor

$$dT = N dt. \quad (7)$$

The "Lagrange time" is invariant $(T(t') = T(t))$.

In the considered case, these two times, q_0 and T , are equal to each other due to the equation for "superfluous" momenta. However, in the following, we shall mainly consider opposite cases.

Here, we would like to emphasize that any time reparametrization invariant theory contains three times:

M) the "mathematical time" (t) (with a zero conjugate Hamiltonian (4) as a constraint), this time is not observable,
 L) the "Lagrange time" T (7) constructed with the help of the Lagrange factor,
 D) the dynamical "parameter of evolution" of the corresponding reduced system (3), which coincides in this case with the "superfluous" variable (q_0) .

The last two times are connected by the equation of motion for the "superfluous" momentum.

3 Relativistic mechanics

Let us consider the relativistic mechanics with the extended action

$$W^E [p_i, q_i; p_0, q_0 | t, N] = \int_{t_1}^{t_2} dt \left(-p_0 \dot{q}_0 + \sum_i p_i \dot{q}_i - \frac{N}{2m} [-p_0^2 + p_i^2 + m^2] \right). \quad (8)$$

In this theory, one usually solves the constraint $-p_0^2 + p_i^2 + m^2 = 0$ with respect to the momentum with negative sign in the extended Hamiltonian. As a result we get

$$\frac{\delta W}{\delta N} = 0 \Rightarrow (p_0)_\pm = \pm \sqrt{p_i^2 + m^2}, \quad (9)$$

so that the conjugate (superfluous) variable converts into the evolution parameter of the corresponded reduced systems described by the actions:

$$W^R_{(\pm)} [p_i, q_i | q_0] = \int_{q_0(1)}^{q_0(2)} dq_0 \left[\sum_i p_i \dot{q}_i \mp \sqrt{p_i^2 + m^2} \right]. \quad (10)$$

The latter correspond to two solutions of the constraint.

The variation of action (8) with respect to the "superfluous" momentum p_0 gives

$$\frac{\delta W}{\delta p_0} = -\frac{dq_0}{dt} + N \frac{p_0}{m} = 0, \Rightarrow T_{(q_0)\pm} = \pm \int_0^{q_0} dq_0 \frac{m}{\sqrt{p_i^2 + m^2}}. \quad (11)$$

On the solutions of the equations of motion (11) represents Lorentz transformation of the proper time q_0 of a particle into the proper time T of an observer: $T = q_0 \sqrt{1 - v^2}$.

In this theory we have again three times:

M) the "mathematical time" (t) (with a zero conjugate Hamiltonian as a constraint), this time is not observable,

L) the "Lagrange time" T constructed with the help of the Lagrange factor and given by (11); this time coincides with the proper time of an observer,

D) the dynamical "parameter of evolution" of the corresponding reduced system (10), which coincides with the proper time of a particle.

In contrast with the mechanical system considered above, the evolution parameter (D) differs from the "Lagrange time" (L) which coincides with proper time of the Einstein-Poincaré observer. The latter is defined as the measurable time interval in SR.

4 Classical and quantum cosmological models

We consider the cosmological model inspired by the Einstein-Hilbert action with an electromagnetic field [2, 3, 5, 6, 7, 8]

$$W = \int d^4x \sqrt{-g} \left[-\frac{{}^{(4)}R(g)}{16\pi} M_{Pl}^2 - \frac{1}{4} F_{\mu\nu}(A) F^{\mu\nu}(A) \right]. \quad (12)$$

If we substitute the Friedmann-Robertson-Walker (FRW) metric with an interval

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = a_0^2(t) [N_c^2 dt^2 - \gamma_{ij} dx^i dx^j]; \quad {}^{(3)}R(\gamma^c) = \frac{6k}{r_0^2} \quad (13)$$

into the action, this system reduces to the set of oscillators. It is described by the action in the Hamiltonian form [6, 8]

$$W^E [p_f, q_f; p_0, a_0 | t, N_c] = \int_{t_1}^{t_2} dt \left(-p_0 \dot{a}_0 + \frac{1}{2} \frac{d}{dt} (p_0 a_0) + \sum_f p_f \dot{q}_f - N_c \left[-\frac{p_0^2}{4} + \mathbf{h}^2(a_0) \right] \right), \quad (14)$$

where

$$\mathbf{h}^2(a_0) = -\frac{k a_0^2}{r_0^2} + H_M(p_f, f), \quad (15)$$

the variable a_0 is the scale factor of metric (13), $k = +1, 0, -1$ stands for the closed, flat and open space with the three-dimensional curvature ($6kr_0^{-2}$). We kept also the time-surface term which follows from the initial Hilbert action [6].

The equation of motion for the matter "field" corresponds to the conservation law

$$\frac{d}{dt} H_M(p_f, f) = 0. \quad (16)$$

Let us consider the status of different times (M, L, D) in the theory.

M) The main peculiarity of the considered system (12) is the invariance with respect to reparametrizations of the initial time

$$t \mapsto t' = t'(t). \quad (17)$$

This invariance leads to the energy constraint and points out that the initial time t is not observable.

L) The "Lagrange time" t_F of the extended system (14) coincides with conformal time η [8] of the Einstein-Friedmann observer who moves together with the Universe and measures the "proper time" interval $ds|_{dx=0}$

$$dt_F = a_0 N_c dt = a_0 d\eta. \quad (18)$$

D) The reduction of the extended system (14) by resolving the constraint $\frac{\delta W}{\delta N_c} = 0$ with respect to the momentum with negative sign in the extended Hamiltonian distinguishes the scale factor as the dynamical parameter of evolution of the reduced system [5, 2, 3, 6].

The constraint

$$-\frac{p_0^2}{4} + \mathbf{h}^2 = 0 \quad (19)$$

has two solutions

$$(p_0)_\pm = \pm 2\mathbf{h} \quad (20)$$

that correspond to two actions of the reduced system (like in relativistic mechanics considered in Section 3). The substitution of (20) into eq. (14) leads to the action

$$W_\pm^R [p_f, f | a_0] = \int_{a_0(1)}^{a_0(2)} da_0 \left[\sum_f p_f \frac{df}{da_0} \mp 2\mathbf{h} \pm \frac{d}{da_0} (a_0 \mathbf{h}) \right] \quad (21)$$

with the evolution parameter a_0 .

We can see the equation of motion for "superfluous" momentum p_0 of the extended system (14)

$$\frac{\delta W}{\delta p_0} = 0 \Rightarrow p_0 = 2 \frac{da_0}{N dt} = 2 \frac{da_0}{d\eta} = 2a' \quad (22)$$

(together with constraint (20)) establishes the relation between the conformal and proper times (18) of the observer and the evolution parameter a_0 (similar to (6) and (11))

$$\eta_\pm = \pm \int_0^{a_0} da \mathbf{h}^{-1}; \quad dt_F = a_0(\eta) d\eta. \quad (23)$$

Those times can be calculated for concrete values of the integral of motion

$$H_M = E_c. \quad (24)$$

Equation (23) presents the Friedmann law [22] of the evolution of "proper time" with respect to the "parameter of evolution" a_0 .

The extended system describes the dynamics of the "proper time" of an observer with respect to the evolution parameter.

This proper time dynamics of an observer of the Universe was used by Friedmann [22] to describe expansion of Universe. This expansion is connected with the Hubble law

$$Z = \frac{a_0(t_F - D)}{a_0(t_F)} - 1 \simeq D H_{Hub}(t_F) + \dots \quad (25)$$

where $H_{Hub}(t_F)$ is the Hubble parameter and D is the distance between Earth and the cosmic object radiating photons.

To reproduce this proper time dynamics the variation principle applied to the reduced system (21) should be added by the convention about measurable time of an observer (18).

In particular, to get direct relation to the observational cosmology (25) of the Wheeler-DeWitt [23] wave function based on the quantum constraint

$$\left[-\frac{p_0^2}{4} + \mathbf{h}^2 \right] \Psi_{WDW}(a_0 | f) = 0; \quad \left(\dot{p}_0 = \frac{d}{ida_0} \right) \quad (26)$$

equation (26) should be added by the convention of an observer about the measurable time interval (18). In this context, it has been shown [8] that there are the Levi-Civita type canonical transformations [15] of "superfluous" variables

$$(p_0, a_0) \rightarrow (H, \eta)$$

for which the constraint (19) becomes linear

$$-H + H_M = 0. \quad (27)$$

The conformal time of the observer coincides with the evolution parameter, and the new reduced action completely coincides with the conventional quantum field theory action of matter fields in the flat space

$$W_{\pm}^R[p_f, f|\eta] = \int_{\eta(1)}^{\eta(2)} d\eta \left[\sum_f p_f \frac{df}{d\eta} \mp H_M(p_f, f) \right]. \quad (28)$$

In this case, the WDW equation (26) of the new extended system coincides with the Schrödinger equation of the reduced system (refft)

$$\pm \frac{d}{id\eta} \Psi_{\pm}(\eta|f) = H_M \Psi_{\pm}(\eta|f). \quad (29)$$

We can get the spectral decomposition of the wave function of Universe and anti-Universe over "in" and "out" solutions and eigenfunctions of the operator H_M with the quantum eigenvalues E ($H_M < E|f \rangle = E < E|f \rangle$)

$$\Psi_+(\eta_+|f) = \sum_E \left[e^{i\bar{W}_E^{(+)}(\eta_+)} \langle E|f \rangle \theta(\eta_+) \alpha_{(in)}^{(+)} + e^{-i\bar{W}_E^{(+)}(\eta_+)} \langle E|f \rangle^* \theta(-\eta_+) \alpha_{(out)}^{(-)} \right] \quad (30)$$

$$\Psi_-(\eta_-|f) = \sum_E \left[e^{i\bar{W}_E^{(-)}(\eta_-)} \langle E|f \rangle \theta(\eta_-) \beta_{(out)}^{(-)} + e^{-i\bar{W}_E^{(-)}(\eta_-)} \langle E|f \rangle^* \theta(-\eta_-) \beta_{(in)}^{(+)} \right], \quad (31)$$

where $\bar{W}_E^{(\pm)}(\eta)$ is the energy part of the reduced actions (21) [6, 8]

$$W_E^{(\pm)}(\eta_{\pm}) = \mp \int_{a_0(1)}^{a_0(2)} da_0 \left[2h - \frac{d}{da_0}(a_0 h) \right] \equiv E\eta_{\pm}, \quad (32)$$

$\alpha_{(in)}^{(+)}, \alpha_{(out)}^{(-)}$ are operators of creation and annihilation of the Universe (Ψ_+) with the conformal time $\eta_{(+)}$ and $\beta_{(in)}^{(+)}, \beta_{(out)}^{(-)}$ are the ones for the anti-Universe (Ψ_-) with the conformal time $\eta_{(-)}$ (23).

If we recall the convention (18) of an observer and variate the wave function (30) with respect to the proper time t_F , we get the red shift energy E/a_0 forming the Hubble law. This wave function has simple interpretation, the same time of evolution as in the classical theory, and bears direct relation to the observable red shift.

We have got the renormalizable function of the Universe, as we excluded the superfluous variables from the set of variables of the reduced system.

To obtain this clear quantum theory, we should use the Einstein-Hilbert action (12), conformal invariant observables, and the Levi-Civita prescription for the Hamiltonian reduction, which leads to the conventional matter field theory in the flat space with the conformal time of an observer.

One can say that the Hamiltonian reduction reveals the preference of the conformal time from the point of view of the principle of correspondence with quantum field theory in the flat space (28) [8].

5 General relativity

5.1 Variables

The purpose of the present paper is to analyze the problem of "proper time" dynamics in the exact Einstein-Hilbert-Maxwell theory

$$W^E(g, A) = \int d^4x \sqrt{-g} \left[-\frac{\mu^2}{6} {}^{(4)}R - \frac{1}{4} F_{\mu\nu}(A) F^{\mu\nu}(A) \right]; \quad \left(\mu = M_{Pl} \sqrt{\frac{3}{8\pi}} \right), \quad (33)$$

where M_{Pl} is the Planck mass.

The initial points of our analysis are the (3+1) foliation of the four-dimensional manifold [11]

$$(ds)^2 = g_{\mu\nu} dx^\mu dx^\nu = N^2 dt^2 - g_{ij}^{(3)} \check{d}x^i \check{d}x^j; \quad (\check{d}x^i = dx^i + N^i dt) \quad (34)$$

and the Lichnerowicz conformal invariant variables [12]

$$N_c = \|g^{(3)}\|^{-1/6} N; \quad g_{ij}^c = \|g^{(3)}\|^{-1/3} g_{ij}^{(3)}; \quad (\|g^c\| = 1); \quad \bar{a} = \mu \|g^{(3)}\|^{1/6} \quad (35)$$

which are convenient for studying the problem of initial data [1, 9, 10] and the Hamiltonian dynamics.

With this notation the action (33) reads

$$W_{[\bar{a}, g^c, A]}^E = \int d^4x \left[-N_c \frac{\bar{a}^2}{6} R^{(4)}(g^c) + \bar{a} \partial_\mu (N_c \partial^\mu \bar{a}) - \frac{1}{4} F_{\mu\nu}(A) F^{\mu\nu}(A) \right]. \quad (36)$$

In the first order formalism, the action (33) in terms of the variables (34), (35) has the form

$$W^E = [P_A, A; P_g, g^c, \bar{P}_a, \bar{a}|t] = \int_{t_1}^{t_2} dt \int d^3x \left[\sum_{f=g,A} P_f D_0 f - \bar{P}_a D_0 \bar{a} - N_c \mathcal{H} + S \right], \quad (37)$$

where

$$\mathcal{H} = -\frac{\bar{P}_a^2}{4} + 6 \frac{P_g^2}{\bar{a}^2} + \frac{\bar{a}^2}{6} \bar{R} + \mathcal{H}_A; \quad \left(\mathcal{H}_A = \frac{1}{2} P_A^2 + \frac{1}{4} F_{ij} F^{ij} \right)$$

is the Hamiltonian density, \bar{R} is a three-dimensional curvature

$$\bar{R} = R^{(3)}(g_{ij}^c) + 8\bar{a}^{-1/2} \Delta \bar{a}^{1/2}; \quad \Delta \bar{a} = \partial_i (g_c^{ij} \partial_j \bar{a}), \quad (38)$$

S is the surface terms of the Hilbert action (33), P_A, P_g, \bar{P}_a are the canonical momenta, and

$$D_0 \bar{a} = \partial_0 \bar{a} - \partial_k (N^k \bar{a}) + \frac{2}{3} \bar{a} \partial_k N^k, \quad D_0 g_{ij}^c = \partial_0 g_{ij}^c - \nabla_i N_j - \nabla_j N_i + \frac{2}{3} \partial_k g_{ij}^c N^k \quad (39)$$

$$D_0 A_i = \partial_0 A_i - \partial_i A_0 + F_{ij} N^j \quad (40)$$

are the quantities invariant (together with the factor dt) under the kinematic transformations [13]

$$t \rightarrow t' = t'(t); \quad x^k \rightarrow x'^k = x'^k(t, x^1, x^2, x^3), \quad N_i \rightarrow N'_i \dots \quad (41)$$

In this theory we also have three "times":

M) The invariance of the theory (37) under transformations (41) (in accordance with our analysis of the problem in the previous Sections) means that the "mathematical time" t is not observable.

L) The invariant ‘‘Lagrange time’’ defined by the Lagrange factor N_c

$$dT_c(x, t) = N_c(x, t) dt \quad (42)$$

coincides with the measurable proper time in ADM parametrization (34) within the factor \bar{a}/μ :

$$dT(x, t) = ds|_{dx=0} = \frac{\bar{a}(x, t) dT_c(x, t)}{\mu} \quad (43)$$

D) The dynamical parameter of evolution of the reduced physical sector as ‘‘superfluous’’ variable of the extended system (37) - a generalization of scale factor a_0 in cosmology.

For the choice of the ‘‘superfluous’’ variable in GR we use the results of papers [13] where it has been shown that the space scale $\bar{a}(\bar{x}, t)$ contains the global factor ($a_0(t)$)

$$\bar{a}(x, t) = a_0(t) \lambda(x, t) \quad (44)$$

which depends only on time and it does not convert into a constant with any choice of the reference frame in the class of kinematic transformations, where we impose the constraint

$$\int d^3x \lambda(x, t) \frac{D_0 \lambda(x, t)}{N_c} = 0 \quad (45)$$

which diagonalizes the kinetic term of the action (37).

The new variables (44) require the corresponding momenta P_0 and P_λ . We define decomposition of \bar{P}_a over the new momenta P_0 and P_λ

$$\bar{P}_a = \frac{P_\lambda}{a_0} + P_0 \frac{\lambda}{N_c \int d^3x \frac{\lambda^2}{N_c}}; \quad \left(\int d^3x \lambda(x, t) P_\lambda \equiv 0 \right), \quad (46)$$

so that to get the conventional canonical structure for the new variables:

$$\int d^3x (\bar{P}_a D_0 \bar{a}) = \dot{a}_0 \int d^3x \bar{P}_a \lambda + a_0 \int d^3x \bar{P}_a D_0 \lambda \stackrel{\text{def}}{=} \dot{a}_0 P_0 + \int d^3x P_\lambda D_0 \lambda. \quad (47)$$

The substitution of (46) into the Hamiltonian part of the action (37) extracts the ‘‘superfluous’’ momentum term

$$\int d^3x N_c \bar{P}_a^2 = P_0^2 \left[\int d^3x \frac{\lambda^2}{N_c} \right]^{-1} + \frac{1}{a_0^2} \int d^3x N_c P_\lambda^2. \quad (48)$$

Finally, the extended action (37) acquires the structure of the extended cosmological model (14)

$$W^E[P_f, f; P_0, a_0|t] = \int_{t_1}^{t_2} dt \left(\left[\int d^3x \sum_{f=g,c,\lambda} P_f D_0 f \right] - \dot{a}_0 P_0 + \frac{P_0^2}{4} \left[\int d^3x \frac{\lambda^2}{N_c} \right]^{-1} - \int d^3x N_c \mathcal{H}_F \right), \quad (49)$$

where \mathcal{H}_F is the Hamiltonian \mathcal{H} without the ‘‘superfluous’’ momentum part:

$$\mathcal{H}_F = \frac{1}{a_0^2} \left[-\frac{P_\lambda^2}{4} + 6 \frac{P_g^2}{\lambda^2} \right] + a_0^2 \frac{\lambda^2}{6} \bar{R} + \mathcal{H}_A. \quad (50)$$

For simplicity we neglect the space-surface term.

5.2 Reduction

Now we can eliminate the ‘‘superfluous’’ variables a_0, P_0 resolving the constraint

$$\int d^3x N_c \frac{\delta W}{\delta N_c} = 0 \Rightarrow \frac{P_0^2}{4} = \left(\int d^3x N_c \mathcal{H}_F \right) \left(\int d^3x \frac{\lambda^2}{N_c} \right) \quad (51)$$

with respect to the momentum P_0 . This equation has two solutions that correspond to two reduced systems with the actions

$$W_\pm^R(P_f, f|a_0) = \int_{a_0(1)}^{a_0(2)} da_0 \left[\sum_{f=\lambda, g, c, A} P_f D_a f \mp \left(\int d^3x N_c \mathcal{H}_F \right)^{1/2} \left(\int d^3x \frac{\lambda^2}{N_c} \right)^{1/2} \right] \quad (52)$$

with the parameter of evolution a_0 , where

$$D_a f = \frac{D_0 f}{\dot{a}_0} \quad (53)$$

is the covariant derivative with the new shift vector N^k and vector field A_μ , which differ from the old ones, in (39), by the factor $(\dot{a}_0)^{-1}$.

The local equations of motion of systems (52) reproduce the invariant sector of the initial extended system and determine the evolution of all variables (P_f, f) with respect to the parameter a_0

$$(P_f(x, t), f(x, t), \dots) \rightarrow (P_f(x, a_0), f(x, a_0), \dots). \quad (54)$$

The actions (52) are invariant under the transformations $N_c(x, t) \rightarrow N'_c = f(t) N_c$. In other words, the lapse function $N_c(x, t)$ can be determined up to the global factor depending on time:

$$N_c(x, t) = N_0(t) \mathcal{N}(x, t). \quad (55)$$

This means that the reduced system loses the global part of the lapse function which forms the global time of an observer

$$N_0 dt = d\eta; \quad (\eta(t') = \eta(t)) \quad (56)$$

like the reduced action of the cosmological model lost the lapse function which forms the conformal time of the Friedmann observer of the evolution of the Universe considered in the previous Section).

We call quantity (56) the global conformal time. We can define the global lapse function $N_0(t)$ using the second integral in eq. (51)

$$\int d^3x \frac{\lambda^2}{N_c} \stackrel{\text{def}}{=} \frac{l_0}{N_0(t)}, \quad (57)$$

where l_0 is the constant which can be chosen so that $\mathcal{N}(x, t)$ and $\lambda(x, t)$ in the Newton approximation have the form

$$\mathcal{N}(x, t) = 1 + \delta_N(x) + \dots; \quad \lambda(x, t) = \mu(1 + \delta_\lambda(x) + \dots) \quad (58)$$

where $\delta_N, \delta_\lambda(x)$ are the potentials of the Newton gravity.

5.3 The proper time dynamics

To research the evolution of the system with respect to the global conformal time of an observer (56), we shall use the short notation

$$\int d^3x N_c \mathcal{H}_F = l_0 N_0 h^2(a_0) \equiv l_0 N_0 \left[\frac{k_A^2}{a_0^2} + h_R^2 + a_0^2 \Gamma^{-2} \right], \quad (59)$$

where k_A^2 and Γ^{-2} correspond to the kinetic and potential parts of the graviton Hamiltonian in eq. (50), h_R^2 is the electromagnetic Hamiltonian.

The equations for superfluous variables P_0 , a_0 and global lapse function (which are omitted by the reduced action (52)) have the form

$$N_0 \frac{\delta W^E}{\delta N_c} = 0 \Rightarrow (P_0)_\pm = \pm 2l_0 h(a_0), \quad (60)$$

$$\frac{\delta W^E}{\delta a_0} = 0 \Rightarrow P'_0 = l_0 \frac{d}{da_0} h^2(a_0); \quad (f' = \frac{d}{d\eta} f), \quad (61)$$

$$\frac{\delta W^E}{\delta P_0} = 0 \Rightarrow a'_0 = \frac{P_0}{2l_0}. \quad (62)$$

These equations lead to the conservation law

$$\frac{(k_A^2)'}{a_0^2} + (h_R^2)' + a_0^2 (\Gamma^{-2})' = 0 \quad (63)$$

and to the Friedmann-like evolution of global conformal time of an observer (56)

$$\eta_{(\pm)}(a_0) = \pm \int_0^{a_0} da h^{-1}(a). \quad (64)$$

The integral (64) can be computed, if we know a solution of the reduced system of equations (54) as functions of the parameter of evolution a_0 . To get this equations, we should change, in eq. (52), N_c by $\mathcal{N}(x, t)$ (as we discussed above).

The conservation law (63) allows us to verify that the red shift and the Hubble law for our observer

$$Z(D) = \frac{a(t_F)}{a(t_F - D)} - 1 = D \cdot H_0 + \dots; \quad \left(t_F(\eta) = \int_0^\eta d\eta' a_0(\eta') \right) \quad (65)$$

reproduce the evolution of the Universe in the standard cosmological models (with the FRW metrics); if we suppose the dominance of the kinetic part of the Hamiltonian or the potential one, in accordance with the a_0 -dependence of this Hamiltonian.

In the first case ($k_A^2 \neq 0$, $h_R = \Gamma^{-1} = 0$), we get the Misner anisotropic model [5]; in the second case, the Universe filled with radiation ($k_A^2 = 0$; $h_R \neq 0$; $\Gamma^{-1} \neq 0$). In both the cases, the quantities (k_A , h_R , Γ^{-1}) play the role of conserved integrals of motion which are constants on solutions of the local equations.

The "Lagrange time" differential (42) is

$$dT_c(x, t) = \mathcal{N}(x, \eta) d\eta. \quad (66)$$

In the quantum theory, the integrals of motion become conserved quantum numbers (in accordance with the correspondence principle). Each term of the spectral decomposition

of the wave function over quantum numbers can be expressed in terms of the proper time of an observer to distinguish "in" and "out" states of the Universe and anti-Universe with the corresponding Hubble laws (87).

Attempts [8] to include an observer into the reduced scheme (by the Levi-Civita canonical transformation [15, 16, 17] of the extended system variables to the new ones for which the new "superfluous" variable coincides with the proper time) show that the conformal time and space observables are more preferable than proper time and space. The conformal time leads, in the flat space limit, to the quantum field theory action [6] and does not violate causality [18] (in contrast with the proper one). The conformal space interval does not contain singularity at the beginning of time [6, 7, 8]. In the next Section we try to remove these defects changing only the convention of measurable intervals and keeping the physics of the reduced system unviolated.

6 Conformal Unified Theory (CUT)

6.1 The formulation of the theory

Our observer in his (3+1) parametrization of metric can see that the Einstein-Hilbert theory, in terms of the Lichnerowicz conformal invariant variables (35), completely coincides with the conformal invariant theory of the Penrose-Chernikov-Tagirov (PCT) scalar field with the action (except the sign)

$$W^{PCT}[\Phi, g] = \int d^4x \left[-\sqrt{-g} \frac{\Phi^2}{6} R^{(4)}(g) + \Phi \partial_\mu (\sqrt{-g} \partial^\mu \Phi) \right] \quad (67)$$

if we express this action also in terms of the Lichnerowicz conformal invariant variables:

$$\varphi_c = \|\|g^{(3)}\|\|^{1/6} \Phi; \quad g_{\mu\nu}^c = \|\|g^{(3)}\|\|^{-1/3} g_{\mu\nu}; \quad \sqrt{-g^c} = N_c. \quad (68)$$

From (67) we get the action

$$W^{PCT}[\varphi_c, g_c] = \int d^4x \left[-N_c \frac{\varphi_c^2}{6} R^{(4)}(g^c) + \varphi_c \partial_\mu (N_c \partial^\mu \varphi_c) \right], \quad (69)$$

which coincides with the Einstein action (36) if we replace \bar{a} with φ_c . However, in contrast with the Einstein theory, the observables in PCT theory are conformal-invariant quantities, in particular, an observer measures the conformal invariant interval

$$(ds)_c^2 = g_{\mu\nu}^c dx^\mu dx^\nu = N_c^2 dt^2 - g_{ij}^{(3)c} \check{d}x^i \check{d}x^j \quad (70)$$

with the conformal time η and the conserved volume of the conformal invariant space (as $\|\|g_{ij}^{(3)c}\|\| = 1$).

Following refs. [24, 19, 20, 25], we can identify the PCT-scalar field with the modulus of the Higgs dublet and add the matter fields as the conformal invariant part of the standard model (SM) for strong and electroweak interactions with the action

$$W^{SM}[\phi_{Hc}, n, V, \psi, g_c] = \int d^4x \left(\mathcal{L}_0^{SM} + N_c [-\varphi_{Hc} F + \varphi_{Hc}^2 B - \lambda \varphi_{Hc}^4] \right), \quad (71)$$

where \mathcal{L}_0^{SM} is the scalar field free part of SM expressed in terms of the conformal invariant variables of the type of (68) [20], B and F are the mass terms of the boson and fermion fields respectively:

$$B = Dn(Dn)^*; \quad F = (\bar{\psi}_L n) \psi_R + h.c. \quad (72)$$

They can be expressed in terms of the physical fields (V_j^p, ψ_β^p), in the unitary gauge,

$$B = V_j^p \hat{Y}_{ij} V_j^p; F = \bar{\psi}_\alpha^p \hat{X}_{\alpha\beta} \psi_\beta^p \quad (73)$$

which absorb the angular components (n) of scalar fields (here $\hat{Y}_{ij}, \hat{X}_{\alpha\beta}$ are the matrices of coupling constants).

We have introduced the rescaled scalar field φ_{Hc}

$$\varphi_{Hc} = \chi \varphi_c \quad (74)$$

in order to ensure a correspondence with ordinary SM notation. The rescaling factor χ must be regarded as a new coupling constant which coordinates weak and gravitational scales [19]. (The value of χ is very small number of order of $\frac{m_w}{M_{Pl}}$ where m_w is the mass of weak boson W .)

The conformally invariant unified theory (CUT) of all fundamental interactions

$$W^{CUT}[\phi_c, V^p, \psi^p, g_c] = W^{PCT}[\phi_c, g_c] + W^{SM}[\phi_c, V^p, \psi^p, g_c] \quad (75)$$

does not contain, in the Lagrangian, any dimensional parameters.

6.2 Reduction

We can apply, to CUT, the analysis of the notions of "times" in the previous Sections.

The scalar field in CUT acquires the feature of the scale factor component of metric with the negative kinetic energy and the evolution parameter a_0 can be extracted from the scalar field. It is convenient to use for global component the denotations

$$\varphi_c(x, t) = \varphi_0(t) a(x, t); \quad N = N_0(t) \mathcal{N}(x, t) \quad (76)$$

so that the expression for the extended action has the form

$$W^{CUT}(P_f, f; P_0, \varphi_0|t) = \int_{t_1}^{t_2} \left(\int d^3x \sum_{f=a, g_c, F_{SM}} P_f D_0 f - P_0 \dot{\varphi}_0 - N_0 \left[-\frac{P_0^2}{4V_0} + H_f[\varphi_0] \right] \right) dt, \quad (77)$$

where F_{SM} is the set of the SM fields,

$$H_f[\varphi_0] = \int d^3x \mathcal{N} \mathcal{H}(p_f, f, \varphi_0) = h_{CUT}^2(\varphi_0) V_0, \quad V_0 = \int d^3x \frac{a^2}{\mathcal{N}} \quad (78)$$

is the Hamiltonian of the local degrees of freedom, the Newton perturbation theory for a, \mathcal{N} begins from unit ($a = 1 + \dots, \mathcal{N} = 1 + \dots$), (the time-surface term is omitted).

The reduction means that we consider the extended action (77) onto the constraint

$$\frac{\delta W^E}{\delta N_0} = 0 \Rightarrow (P_0)_\pm = \pm 2\sqrt{V_0 H_f}. \quad (79)$$

The reduced action

$$W_\pm^R(P_f, f|\varphi_0) = \int_{\varphi_1=\varphi_0(t_1)}^{\varphi_2=\varphi_0(t_2)} d\varphi_0 \left\{ \left(\int d^3x \sum_f P_f D_\varphi f \right) \mp 2\sqrt{V_0 H_f} \right\} \quad (80)$$

is completed by the proper time dynamics.

6.3 The proper time dynamics

The equations of global dynamics (which are omitted by the reduced action (80)) have the form

$$\frac{\delta W^E}{\delta N_0} = 0 \Rightarrow (P_0)_\pm = \pm 2V_0 h_{CUT}(\varphi_0), \quad (81)$$

$$\frac{\delta W^E}{\delta \varphi_0} = 0 \Rightarrow P'_0 = V_0 \frac{d}{d\varphi_0} h_{CUT}^2(\varphi_0); \quad (f' = \frac{d}{d\eta} f), \quad (82)$$

$$\frac{\delta W^E}{\delta P_0} = 0 \Rightarrow \left(\frac{d\varphi_0}{d\eta} \right)_\pm = \frac{(P_0)_\pm}{2V_0} = \pm h_{CUT}(\varphi_0), \quad (83)$$

where the effective Hamiltonian density functional has the form

$$h_{CUT}^2 = \frac{k_A^2}{\varphi_0^2} + h_R^2 + \mu_F^2 \varphi_0 + \Gamma_B^{-2} \varphi_0^2 + \Lambda \varphi_0^4, \quad (84)$$

in correspondence with the new terms in the CUT action.

These equations lead to the Friedmann-like evolution of global conformal time of an observer

$$\eta(\varphi_0) = \int_0^{\varphi_0} d\varphi h_{CUT}^{-1}(\varphi), \quad (85)$$

and to the conservation law

$$\frac{(k_A^2)'}{\varphi_0^2} + (h_R^2)' + (\mu_F^2)' \varphi_0 + (\Gamma_B^{-2})' \varphi_0^2 + (\Lambda)' \varphi_0^4 = 0. \quad (86)$$

The red shift and the Hubble law in the conformal time version

$$z(D_c) = \frac{\varphi_0(\eta_0)}{\varphi_0(\eta_0 - D_c)} - 1 \simeq D_c H_{Hub}; \quad H_{Hub} = \frac{1}{\varphi_0(\eta)} \frac{d}{d\eta} \varphi_0(\eta) \quad (87)$$

reflects the alteration of size of atoms in the process of evolution of masses [26, 8].

In the dependence on the value of φ_0 , there is dominance of the kinetic or the potential part of the Hamiltonian (84), (86) and different stages of evolution of the Universe (85) can appear: anisotropic ($k_A^2 \neq 0$) and radiation ($h_R^2 \neq 0$) (at the beginning of the Universe), dust ($\mu_F^2 \neq 0$; Γ_B^{-2}) and De-Sitter $\Lambda \neq 0$ (at the present time).

In perturbation theory, the factor $a(x, t) = (1 + \delta_a)$ represents the potential of the Newton gravity (δ_a). Therefore, the Higgs-PCT field, in this model, has no particle-like excitations (as it was predicted in paper [19]). Let us show that the scalar field as a function of the proper time (η) (85) gives the Newton coupling constant at present time η_0 .

6.4 Cosmic Higgs vacuum

Let us show that value of the scalar field in CUT is determined by the present state of the Universe with observational density of matter ρ_{Un} and the Hubble parameter H_{Hub} .

For an observer, who is living in the Universe, a state of "vacuum" is the state of the Universe at present time: $|Universe\rangle = |Lab.vacuum\rangle$, as his unified theory pretends to describe both observational cosmology and any laboratory experiments.

In correspondence with this definition, the Hamiltonian (78) can be split into the large (cosmological - global) and small (laboratory - local) parts

$$H_f[\varphi_0] = \rho_{Un} V_0 + (H_f - \rho_{Un} V_0) = \rho_{Un}(\varphi_0) V_0 + H_L, \quad (88)$$

where the global part of the Hamiltonian $\rho_{U_n}(\varphi_0)V_0$ can be defined as the "Universe" averaging

$$\langle Universe | H_f | Universe \rangle = \rho_{U_n} V_0, \quad (89)$$

so that the "Universe" averaging of the local part of Hamiltonian (88) is equal to zero

$$\langle Universe | H_L | Universe \rangle = 0. \quad (90)$$

Let us suppose that the local dynamics (H_L) can be neglected if we consider the cosmological sector of the proper time dynamics (60), (61), (62)

$$\frac{\delta W^E}{\delta N_0} = 0. \Rightarrow p_0 = 2V_0 \sqrt{\rho_{U_n} + \frac{H_L}{V_0}} = 2V_0 \sqrt{\rho_{U_n}} + \frac{H_L}{\sqrt{\rho_{U_n}}} + o\left(\frac{1}{V_0}\right) \quad (91)$$

$$\frac{\delta W^E}{\delta P_0} = 0. \Rightarrow \left(\frac{d\varphi_0}{d\eta}\right)_{\pm} = \pm \sqrt{\frac{H_f}{V_0}} = \sqrt{\rho_{U_n} + \frac{H_L}{V_0}} = \sqrt{\rho_{U_n}} + o\left(\frac{1}{V_0}\right). \quad (92)$$

The evolution of the proper time of an observer with respect to the evolution parameter φ_0 determines the Hubble "constant"

$$H_{H_{ub}} = \frac{1}{\varphi_0(\eta_0)} \frac{d\varphi(\eta_0)}{d\eta_0} = \frac{\sqrt{\rho_{U_n}(\varphi_0)}}{\varphi_0(\eta_0)}. \quad (93)$$

The last equality follows from eq.(92) and gives the relation between the present-day value of scalar field and the cosmological observations:

$$\varphi(\eta = \eta_0) = \frac{\sqrt{\rho_{U_n}(\eta_0)}}{H_{H_{ub}}(\eta_0)}. \quad (94)$$

If $\rho_{U_n} = \rho_{cr}$, where

$$\rho_{cr} = \frac{3H_{H_{ub}}^2 M_{Pl}^2}{8\pi}, \quad (95)$$

as it is expected in the observational cosmology. The substitution of (95) into (94) leads to the value of scalar field

$$\varphi(\eta = \eta_0) = M_{Pl} \sqrt{\frac{3}{8\pi}}, \quad (96)$$

what corresponds to the Newton law of gravity.

6.5 The dust Universe

The present-day Universe is filled in by matter with the equation of state of the dust at rest. This means the "vacuum" averaging of the mass term in the SM Hamiltonian is equal to the mass of the Universe M_D , while other terms can be neglected:

$$\rho_{U_n} V_0 = \varphi_0(\eta) \langle Univ. | \int d^3x N \alpha \bar{\psi}_\alpha X_{\alpha\beta} \psi_\beta | Univ. \rangle \stackrel{\text{def}}{=} M_D = \varphi_0(\eta) \langle n_b \rangle V_0, \quad (97)$$

where $\langle n_b \rangle$ is the conserved integral of motion. In this case, the proper time dynamics is described by eq. (92) with the density

$$\rho_{U_n}(\varphi_0) = \varphi_0 \langle n_b \rangle; \quad \frac{d\varphi_0}{d\eta} = \sqrt{\varphi_0 \langle n_b \rangle}. \quad (98)$$

We get the evolution law for a scalar field

$$\varphi_0(\eta) = \frac{\eta^2}{4} \langle n_b \rangle \quad (99)$$

and the Hubble parameter $H_{H_{ub}}(\eta)$

$$H_{H_{ub}} = \frac{1}{\varphi_0} \frac{d\varphi_0}{d\eta} = \frac{2}{\eta}. \quad (100)$$

The barion density

$$\rho_b = \Omega_0 \rho_{U_n}; \quad (\rho_{U_n} = \frac{3H_{H_{ub}}^2 M_{Pl}^2}{8\pi}) \quad (101)$$

is estimated from experimental data on luminous matter ($\Omega_0 = 0.01$), the flat rotation curves of spiral galaxies ($\Omega_0 = 0.1$) and others data [27] ($0.1 < \Omega_0 < 2$).

We should also take into account that these observations reflect the density at the time of radiation of a light from cosmic objects $\Omega(\eta_0 - \text{distance}/c)$ which was less than at the present-day density $\Omega(\eta_0) = \Omega_0$ due to increasing mass of matter. This effect of retardation can be roughly estimated by the averaging of $\Omega(\eta_0 - \text{distance}/c)$ over distances (or proper time)

$$\gamma = \frac{\eta_0 \Omega_0}{\int_0^{\eta_0} d\eta \Omega(\eta)}. \quad (102)$$

For the dust stage the coefficient of the increase is $\gamma = 3$. Finally, we get the relation of the cosmic value of the Planck "constant" and the GR one

$$\frac{\varphi(\eta = \eta_0)}{(M_{Pl})} \sqrt{\frac{8\pi}{3}} = \sqrt{\gamma \Omega_0(\text{exp})} / h = \omega_0, \quad (103)$$

where $h = 0.4 \div 1$ is observational bounds for the Hubble parameter.

From data on Ω_0 we can estimate ω_0 : $\omega_0 = 0.04$ (luminous matter), $\omega_0 = 0.4$ (flat rotation curves of spiral galaxies), and $0.4 < \Omega_0 < 9$ (others data [27]) for lower values of h ($h = 0.4$).

6.6 The local field theory

As we have seen in cosmological models there is a Levi-Civita canonical transformation to new variables for which the "Lagrange time" coincides with the evolution parameter and the extended system converts into a conventional field theory. In general case it is difficult to find the exact form of this LC transformation. However we can proof the equivalence of our reduced system with conventional field theory with measurable conformal time in next order of the expansion in V_0^{-1} - the inverse volume of the system

The next order of the decomposition of (80) over V_0^{-1} defines the action for local excitations

$$p_0 d\varphi_0 = 2V_0 \sqrt{\rho_{U_n}(\varphi_0)} d\varphi_0 + H_L(\varphi_0) \frac{d\varphi_0}{\sqrt{\rho_{U_n}(\varphi_0)}} + o\left(\frac{1}{V_0}\right), \quad (104)$$

where $\rho_{U_n}(\varphi_0)$ is determined by the global equation (93), and

$$\frac{d\varphi_0}{\sqrt{\rho_{U_n}(\varphi_0)}} = d\eta_0 \quad (105)$$

in accordance with eq.(92). The reduced action (80) in the lowest order in V_0^{-1} (91) has the form of conventional field theory without the global time-reparametrization group symmetry

$$W_{(+)}^R(p_f, f|\bar{\varphi}_0) = W_{(+)}^G(\bar{\varphi}_0) + W_{(+)}^L(p_f, f|\bar{\varphi}_0) \quad (106)$$

$$W_{(+)}^L(p_f, f|\bar{\varphi}_0) = \int_{\eta_1}^{\eta_2} d\eta \left(\int d^3x \sum_f p_f D_\eta f - H_L(p_f, f|\bar{\varphi}(\eta)) \right). \quad (107)$$

Really, an observer is using the action for description of laboratory experiments in a very small interval of time in the comparison with the lifetime of the Universe η_0

$$\eta_1 = \eta_0 - \xi; \eta_2 = \eta_0 + \xi; \xi \ll \eta_0, \quad (108)$$

and induring this time-interval $\varphi_0(\eta)$ can be considered as the constant

$$\varphi_0(\eta_0 + \xi) \approx \varphi_0(\eta_0) = M_{Pl} \sqrt{\frac{3}{8\pi}}. \quad (109)$$

In this case we got the σ -model version of the standard model [19].

7 Conclusion

In the paper we discussed the status of measurable interval of time — “proper time” in the scheme of the Hamiltonian reduction of GR and conformal unified theory (CUT) invariant with respect to general coordinate transformations.

This invariance means that GR and CUT represent an extended systems (ES) with constraints and “superfluous” variables. To separate the physical sector of invariant variables and observables from parameters of general coordinate transformations, one needs the procedure of the Hamiltonian reduction which leads to an equivalent unconstraint system where one of “superfluous” variables becomes the dynamical parameter of evolution.

We have pointed out this “superfluous” variable for considered theories (which converts into the evolution parameter of the reduced system) using the experience of cosmological models and the Lichnerowich conformal invariant variables.

The dynamics of proper time of an observer with respect to the evolution parameter of the reduced system is described by the equation of ES for the “superfluous” canonical momentum.

Just this “superfluous” equation of ES determines the “red shift” and Hubble law in cosmological models, GR, and CUT. To reproduce the Hubble law in quantum theory, the reduced scheme of quantization of GR and cosmology should be added by the convention of an observer about measurable time interval. Normalizability of a wave function is achieved by removing the “superfluous” variable from the set of variables of the reduced system.

From the point of view of the principles of causality and correspondence with the field theory in the flat space the considered Hamiltonian reduction of GR prefers to treat the conformal time as measurable.

We formulated the conformal invariant theory of fundamental interactions where an observer measures the conformal time and space intervals. This theory unifies gravitation with the standard model for strong and electroweak interactions and has no any dimensional parameters in the Lagrangian. In fact, in practice, only the ratios of dimensional quantities are the subject of experimental tests. Roughly speaking Planck mass is nothing but a multiplicity of the proton mass.

We described the mechanism of appearance of mass scale using as the example the dust stage of the evolution of the Universe and have shown that the value of scalar field at present time can be determined by the cosmological data: density of matter and the Hubble constant expressed in terms of the Planck mass.

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