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SCALAR MESONS IN THE NAMBU—JONA-LASINIO MODEL WITH THE 't HOOFT INTERACTION

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1. Introduction

During the last years noticeable progress has been achieved in both experimental and theoretical investigations of scalar mesons. The low-mass sigma meson appeared in the last Review of Particle Properties (1996) [1]. A new theoretical analysis of experimental data on the low-mass sigma meson has been completed in the papers [2, 3]. Scalar mesons in the NJL model with the 't Hooft interaction were investigated in [4, 5]. In the papers [5] the coupling of the $\bar{q}q$ states to the two-pion continuum and a model of confinement were also included into consideration.

Here we continue these investigations in the framework of the standard NJL model with the 't Hooft interaction, following the papers [4]–[6].

Our work is organized as follows. In Sec. 2, we describe the characteristics of our NJL model with the 't Hooft interaction. We define the main parameters of this model - the cut-off parameter Λ and the constituent u-quark mass m_u using the experimental values of the pion decay coupling constant F_π=93 MeV, the ρ-meson coupling constant $g_\rho = 6.14$ ($\frac{g_\rho^2}{4\pi} \approx 3$), describing the decay $\rho \to 2\pi$, and the relation $g_\rho = \sqrt{6} g_\sigma$. In Sec. 3, we describe the masses of the isovector and strange mesons. In Sec. 4, we calculate the quark-loop contributions to the masses of the η, η', σ and f_0 mesons. In Sec. 5, the numerical estimations of the quark-loop contributions to the meson masses are carried out. In Sec. 6 the strong decay widths of the scalar mesons are estimated. Sec. 7 contains discussion and the conclusion.

2. The NJL model with the 't Hooft interaction

The U(3)×U(3) version of the NJL model supplemented by the 't Hooft interaction takes the form

$$\begin{aligned} L &= \bar{q}(i\hat{\partial} - m^0)q + \frac{G}{2}\sum_{i=0}^{8}[(\bar{q}\lambda_i q)^2 + (\bar{q}i\gamma_5\lambda_i q)^2] - \\ &\quad -K\left\{\det[\bar{q}(1+\gamma_5)q] + \det[\bar{q}(1-\gamma_5)q]\right\} \end{aligned} \tag{1}$$

where λ_i (i=1,...,8) are the Gell-Mann matrices and $\lambda^0 = \sqrt{\frac{2}{3}}\mathbf{1}$, with $\mathbf{1}$ being the unit matrix; m^0 is a current quark mass matrix with diagonal elements m_u^0, m_d^0, m_s^0 ($m_u^0 \approx m_d^0$). The Lagrangian (1) can be rewritten in the form (see [6])

$$\begin{aligned} L = \bar{q}(i\hat{\partial} - m^0)q + \frac{1}{2}\sum_{i=1}^{9}[G_i^{(-)}(\bar{q}\tau_i q)^2 + G_i^{(+)}(\bar{q}i\gamma_5\tau_i q)^2] + \\ +G_{us}^{(-)}(\bar{q}\lambda_u q)(\bar{q}\lambda_s q) + G_{us}^{(+)}(\bar{q}i\gamma_5\lambda_u q)(\bar{q}i\gamma_5\lambda_s q) \end{aligned} \tag{2}$$

where

$$\tau_i = \lambda_i \quad (i = 1, ..., 7), \quad \tau_8 = \lambda_u = (\sqrt{2}\lambda_0 + \lambda_8)/\sqrt{3},$$

$$\tau_9 = \lambda_s = (-\lambda_0 + \sqrt{2}\lambda_8)/\sqrt{3},$$
$$G_1^{(\pm)} = G_2^{(\pm)} = G_3^{(\pm)} = G \pm 4Km_sI_1(m_s),$$
$$G_4^{(\pm)} = G_5^{(\pm)} = G_6^{(\pm)} = G_7^{(\pm)} = G \pm 4Km_uI_1(m_u),$$
$$G_u^{(\pm)} = G \mp 4Km_sI_1(m_s), \quad G_s^{(\pm)} = G, \quad G_{us}^{(\pm)} = \pm 4\sqrt{2}Km_uI_1(m_u). \quad (3)$$

Here m_u and m_s are the constituent quark masses and

$$I_n^\Lambda(m_i) = \frac{N_c}{(2\pi)^4}\int d_e^4k \frac{\theta(\Lambda^2 - k^2)}{(k^2 + m_i^2)^n}. \quad (4)$$

Here we have used the Euclidean space and the cut-off parameter Λ.

Let us define the parameters m_u and Λ, following the papers [7, 8, 9]. We shall use the four equations

1) the Goldberger-Treiman relation

$$g_{\pi\bar{q}q} = \frac{m_u}{F_\pi}, \quad (5)$$

2) the relation between g_ρ and $g_{\sigma\bar{q}q}$ [7]-[10]

$$g_\rho = \sqrt{6}g_{\sigma\bar{q}q}, \quad (6)$$

3) the relation between $g_{\sigma\bar{q}q}$ and $g_{\pi\bar{q}q}$, which was obtained by taking into account the $\pi - a_1$ transitions (see [8, 9])

$$g_{\pi\bar{q}q} = Z^{-1/2}g_{\sigma\bar{q}q}, \quad Z = \left(1 - \frac{6m_u^2}{M_{a_1}^2}\right)^{-1}, \quad (7)$$

where M_{a_1} is the a_1-meson mass;

4) the expression for $g_{\sigma\bar{q}q}$ through a logarithmically divergent integral [7, 8]

$$g_{\sigma\bar{q}q}^2 = [4I_2^\Lambda(m_u)]^{-1}. \quad (8)$$

From equations (5)-(7) it is possible to express the constituent u-quark mass through the observable values $F_\pi = 93$ MeV, $g_\rho \approx 6.14$ and $M_{a_1} = 1230$ MeV

$$m_u^2 = \frac{M_{a_1}^2}{12}\left[1 - \sqrt{1 - \left(\frac{2g_\rho F_\pi}{M_{a_1}}\right)^2}\right], \quad m_u = 280 \text{ MeV}. \quad (9)$$

Then, from (6) and (8) we define

$$\Lambda = 1250 \text{ MeV}. \quad (10)$$

3. The masses of the isovector and strange mesons

Now let us first consider bosonization of the diagonal parts of the Lagrangian (2) including the isovector and strange mesons. In Sec. 4, we complete the bosonization of the last part of the Lagrangian (2) containing the nondiagonal terms.

After renormalization of the meson fields we obtain [7, 8]

$$L(\pi, K, a_0, K_0^*) = -\frac{g_\pi^2}{2G_\pi}\vec{\pi}^2 - \frac{g_K^2}{G_K}K^2 - \frac{g_{a_0}^2}{2G_{a_0}}\vec{a}_0^2 - \frac{g_{K_0^*}^2}{G_{K_0^*}}K_0^{*2} -$$
$$-i\mathrm{Tr}\ \ln\left\{1 - \frac{1}{i\hat{\partial} - M}\left[\sum_{i=1}^{7}(g_{\phi_i}i\gamma_5\lambda_i\phi_i + g_{\sigma_i}\lambda_i\sigma_i)\right]\right\}, \quad (11)$$

where ϕ_i and σ_i are the pseudoscalar and scalar fields, respectively, $\vec{\pi}^2 = \pi^{0^2} + 2\pi^+\pi^-$, $K^2 = K^0\bar{K}^0 + K^+K^-$, $\vec{a}_0^2 = a_0^{0^2} + 2a_0^+a_0^-$, $K_0^{*2} = \bar{K}_0^*K_0^* + K_0^{*+}K_0^{*-}$

$$G_\pi = G + 4Km_sI_1(m_s),$$
$$G_K = G_\pi - 4K(m_sI_1(m_s) - m_uI_1(m_u)),$$
$$G_{a_0} = G_\pi - 8Km_sI_1(m_s), \quad (12)$$
$$G_{K_0^*} = G_\pi - 4K(m_sI_1(m_s) + m_uI_1(m_u)),$$

$$g_{a_0}^2 = [4I_2(m_u)]^{-1}, \quad g_{K_0^*}^2 = [4I_2(m_u, m_s)]^{-1},$$
$$I_2(m_u, m_s) = \frac{N_c}{(2\pi)^4}\int d_e^4k \frac{\theta(\Lambda^2 - k^2)}{(k^2 + m_u^2)(k^2 + m_s^2)} =$$
$$= \frac{3}{(4\pi)^2(m_s^2 - m_u^2)}\left[m_s^2 \ln\left(\frac{\Lambda^2}{m_s^2} + 1\right) - m_u^2 \ln\left(\frac{\Lambda^2}{m_u^2} + 1\right)\right], \quad (13)$$
$$g_\pi = Z_\pi^{1/2}g_{a_0}, \quad g_K = Z_K^{1/2}g_{K_0^*}, \quad Z_\pi \approx Z_K \approx 1.44 .$$

Then, in the one-loop approximation, the following expressions for the meson masses are obtained [8]

$$M_\pi^2 = g_\pi^2\left[\frac{1}{G_\pi} - 8I_1(m_u)\right],$$
$$M_K^2 = g_K^2\left[\frac{1}{G_K} - 4[I_1(m_u) + I_1(m_s)]\right] + Z(m_s - m_u)^2,$$
$$M_{a_0}^2 = g_{a_0}^2\left[\frac{1}{G_{a_0}} - 8I_1(m_u)\right] + 4m_u^2, \quad (14)$$
$$M_{K_0^*}^2 = g_{K_0^*}^2\left[\frac{1}{G_{K_0^*}} - 4[I_1(m_u) + I_1(m_s)]\right] + (m_u + m_s)^2.$$

The first relation is used to define the parameter G_π. For the experimental values $M_{\pi^0} = 135$ MeV we have

$$G_\pi = 4.92 \text{ GeV}^{-2}. \quad (15)$$

4. The masses of the η, η', σ and f_0 mesons

The nondiagonal part of the Lagrangian (2) has the form

$$\begin{aligned}\Delta L &= \frac{1}{2}\left\{G_u^{(+)}(\bar{q}i\gamma_5\lambda_u q)^2 + 2G_{us}^{(+)}(\bar{q}i\gamma_5\lambda_u q)(\bar{q}i\gamma_5\lambda_s q) + G_s^{(+)}(\bar{q}i\gamma_5\lambda_s q)^2 + \right.\\ &\left. + G_u^{(-)}(\bar{q}\lambda_u q)^2 + 2G_{us}^{(-)}(\bar{q}\lambda_u q)(\bar{q}\lambda_s q) + G_s^{(-)}(\bar{q}\lambda_s q)^2\right\} = \\ &= (\bar{q}i\gamma_5\lambda_\alpha q)T^P_{\alpha\beta}(\bar{q}i\gamma_5\lambda_\beta q) + (\bar{q}\lambda_\alpha q)T^S_{\alpha\beta}(\bar{q}\lambda_\beta q), \quad (\alpha = u,s) \quad (\beta = u,s)\end{aligned} \tag{16}$$

where

$$\begin{aligned} G_u^{(+)} &= G_\pi - 8Km_sI_1(m_s), \quad G_u^{(-)} = G_\pi, \\ G_s^{(\pm)} &= G_\pi - 4Km_sI_1(m_s), \quad G_{us}^{(\pm)} = \pm 4\sqrt{2}Km_uI_1(m_u), \end{aligned} \tag{17}$$

$$T^{P(S)} = \frac{1}{2}\begin{pmatrix} G_u^{(\pm)} & G_{us}^{(\pm)} \\ G_{us}^{(\pm)} & G_s^{(\pm)} \end{pmatrix} \tag{18}$$

After bosonization we obtain

$$\begin{aligned}\Delta\bar{L} &= -\frac{g_{\eta_\alpha}g_{\eta_\beta}}{4}\eta_\alpha(T^P)^{-1}_{\alpha\beta}\eta_\beta - \frac{g_{\sigma_\alpha}g_{\sigma_\beta}}{4}\sigma_\alpha(T^S)^{-1}_{\alpha\beta}\sigma_\beta - \\ &-i\,\mathrm{Tr}\ln\left\{1 + \frac{1}{i\hat{\partial} - M}[g_{\eta_\alpha}i\gamma_5\lambda_\alpha\eta_\alpha + g_{\sigma_\alpha}\lambda_\alpha\sigma_\alpha]\right\}\end{aligned} \tag{19}$$

where

$$(T^{P(S)})^{-1} = \frac{2}{G_u^{(\pm)}G_s^{(\pm)} - (G_{us}^{(\pm)})^2}\begin{pmatrix} G_s^{(\pm)} & -G_{us}^{(\pm)} \\ -G_{us}^{(\pm)} & G_u^{(\pm)} \end{pmatrix}, \tag{20}$$

$$g_{\sigma_u} = g_{\sigma\bar{u}u}, \quad g_{\sigma_s} = [4I_2^\Lambda(m_s)]^{-1/2}, \quad g_{\eta_u} = g_{\pi\bar{u}u}, \quad g_{\eta_s} = Z^{1/2}g_{\sigma_s}. \tag{21}$$

From the Lagrangian (19), in the one-loop approximation, the following expressions for the mass terms are obtained

$$\begin{aligned}\Delta\bar{L}^{(2)} &= -\frac{g_{\eta_\alpha}g_{\eta_\beta}}{4}\eta_\alpha(T^P)^{-1}_{\alpha\beta}\eta_\beta - \frac{g_{\sigma_\alpha}g_{\sigma_\beta}}{4}\sigma_\alpha(T^S)^{-1}_{\alpha\beta}\sigma_\beta + \\ &+4I_1(m_u)(g^2_{\eta_u}\eta_u^2 + g^2_{\sigma_u}\sigma_u^2) + 4I_1(m_s)(g^2_{\eta_s}\eta_s^2 + g^2_{\sigma_s}\sigma_s^2) - \\ &-2(m_u^2\sigma_u^2 + m_s^2\sigma_s^2) = -\frac{1}{2}\left\{\eta_\alpha M^P_{\alpha\beta}\eta_\beta + \sigma_\alpha M^S_{\alpha\beta}\sigma_\beta\right\}\end{aligned} \tag{22}$$

where

$$\begin{aligned} M^P_{uu} &= g^2_{\eta_u}\left(\frac{1}{2}(T^P)^{-1}_{uu} - 8I_1(m_u)\right), \\ M^P_{ss} &= g^2_{\eta_s}\left(\frac{1}{2}(T^P)^{-1}_{ss} - 8I_1(m_s)\right), \\ M^P_{us} &= \frac{1}{2}g_{\eta_u}g_{\eta_s}(T^P)^{-1}_{us}, \end{aligned} \tag{23}$$

$$\begin{aligned} M^S_{uu} &= g^2_{\sigma_u}\left(\frac{1}{2}(T^S)^{-1}_{uu} - 8I_1(m_u)\right) + 4m_u^2, \\ M^S_{ss} &= g^2_{\sigma_s}\left(\frac{1}{2}(T^S)^{-1}_{ss} - 8I_1(m_s)\right) + 4m_s^2, \\ M^S_{us} &= \frac{1}{2}g_{\sigma_u}g_{\sigma_s}(T^S)^{-1}_{us}. \end{aligned} \tag{24}$$

After diagonalization of the Lagrangian (22) we find masses of the pseudoscalar and scalar mesons η, η', σ and f_0

$$M^2_{(\eta,\eta')} = \frac{1}{2}\left[M^P_{ss} + M^P_{uu} \mp \sqrt{(M^P_{ss} - M^P_{uu})^2 + 4(M^P_{us})^2}\right], \tag{25}$$

$$M^2_{(\sigma,f_0)} = \frac{1}{2}\left[M^S_{ss} + M^S_{uu} \mp \sqrt{(M^S_{ss} - M^S_{uu})^2 + 4(M^S_{us})^2}\right]. \tag{26}$$

Let us define the mixing angle for the pseudoscalar mesons

$$\begin{aligned}\eta_s &= \eta\cos\bar{\theta} + \eta'\sin\bar{\theta}, \\ \eta_u &= -\eta\sin\bar{\theta} + \eta'\cos\bar{\theta}, \quad \bar{\theta} = \theta - \theta_0\end{aligned} \tag{27}$$

where $\theta_0 \approx 35.3°$ is the ideal mixing angle (ctg $\theta_0 = \sqrt{2}$) and θ is the singlet-octet mixing angle

$$\mathrm{tg}\ 2\bar{\theta} = \frac{2M^P_{us}}{-M^P_{ss} + M^P_{uu}}. \tag{28}$$

For the scalar mesons we use the relations

$$\begin{aligned}\sigma_u &= \sigma\cos\bar{\phi} + f_0\sin\bar{\phi}, \\ \sigma_s &= -\sigma\sin\bar{\phi} + f_0\cos\bar{\phi}, \quad \bar{\phi} = \theta_0 - \phi\end{aligned} \tag{29}$$

where ϕ is the singlet-octet mixing angle and

$$\mathrm{tg}\ 2\bar{\phi} = \frac{2M^S_{us}}{M^S_{ss} - M^S_{uu}}. \tag{30}$$

5. Numerical estimations of quark-loop contributions to meson masses

Using for the parameters m_s and K the values

$$m_s = 425\ \mathrm{MeV}, \quad K = 13.3\ \mathrm{GeV}^{-5} \tag{31}$$

we obtain the following estimations for the masses of the pseudoscalar and scalar mesons

$$\begin{aligned} M_\pi &= 135\ \mathrm{MeV}, \quad M_K = 495\ \mathrm{MeV}, \\ M_\eta &= 520\ \mathrm{MeV}, \quad M_{\eta'} = 1000\ \mathrm{MeV}. \end{aligned} \tag{32}$$

$$M_\sigma = 550 \text{ MeV}, \quad M_{f_0} = 1130 \text{ MeV},$$
$$M_{a_0} = 810 \text{ MeV}, \quad M_{K_0^*} = 960 \text{ MeV}, \tag{33}$$
$$\theta = -19^\circ, \quad \phi = 24^\circ.$$

Note that the pion and kaon masses are the input parameters in our model. The experimental data are [1]

$$M_{\pi^0} = 134.9764 \pm 0.0006 \text{ MeV}, \quad M_{\pi^\pm} = 139.6 \text{ MeV},$$
$$M_{K^+} = 493.677 \pm 0.016 \text{ MeV}, \quad M_{K^0} = 497.672 \pm 0.031 \text{ MeV},$$
$$M_\eta = 547.45 \pm 0.19 \text{ MeV}, \quad M_{\eta'} = 957.77 \pm 0.14 \text{ MeV},$$
$$\theta \approx -20^\circ \quad [11]. \tag{34}$$

$$M_{\sigma_0(400-1200)} = 400 - 1200 \text{ MeV}, \quad M_{f_0(980)} = 980 \pm 10 \text{ MeV},$$
$$M_{a_0} = 983.5 \pm 0.9 \text{ MeV}, \quad M_{K_0^*} = 1429 \pm 6 \text{ MeV}. \tag{35}$$

Comparing the theoretical results with the experimental data we can see that we have obtained satisfactory results for the pseudoscalar mesons and for the octet-singlet mixing angle of the $(\eta\eta')$ mesons. However, for the scalar mesons we have got the masses smaller than the experimental data (except for the f_0 meson).

6. Strong decays of the scalar mesons.

Now, let us show what the information concerning the strong decays of the scalar and pseudoscalar mesons we can obtain from the Lagrangian (11) and (19). These Lagrangians allow us to get the following expressions for the scalar-pseudoscalar meson vertices, describing the corresponding strong decays of the scalar mesons

$$\begin{aligned}
g_{\sigma\pi\pi} &= \frac{2m_u^2 Z^{1/2}}{F_\pi} \cos\bar{\phi} \\
g_{f_0\pi\pi} &= \frac{2m_u^2 Z^{1/2}}{F_\pi} \sin\bar{\phi} \\
g_{a_0\eta\pi} &= \frac{2m_u^2 Z^{1/2}}{F_\pi} \sin\bar{\theta} \\
g_{K_0^{*+}K^-\pi^0} &= \frac{2m_u^2 Z^{1/2}}{F_\pi} \\
g_{K_0^{*+}K^0\pi^-} &= \frac{2\sqrt{2}m_u m_s Z^{1/2}}{F_\pi}
\end{aligned} \tag{36}$$

With these vertices one obtains the following decay widths

$$\begin{aligned}
\Gamma_{\sigma\to\pi\pi} &\approx 700 \text{ MeV}, & \Gamma_{f_0\to\pi\pi} &\approx 20 \text{ MeV}, \\
\Gamma_{a_0\to\eta\pi} &\approx 130 \text{ MeV}, & \Gamma_{K_0^*\to K\pi} &\approx 330 \text{ MeV}.
\end{aligned} \tag{37}$$

These values are in qualitative agreement with the experimental data

$$\begin{aligned}
\Gamma^{exp}_{\sigma\to\pi\pi} &\sim (600 - 1000) \text{ MeV}, & \Gamma^{exp}_{f_0\to\pi\pi} &\sim (30 - 78) \text{ MeV}, \\
\Gamma^{exp}_{a_0\to\eta\pi} &\sim (50 - 100) \text{ MeV}, & \Gamma^{exp}_{K_0^*\to K\pi} &\approx (263 \pm 26 \pm 21) \text{ MeV}.
\end{aligned} \tag{38}$$

7. Conclusion

Our calculations have shown that the 't Hooft interaction allows us to describe the masses of pseudoscalar meson's masses and their singlet-octet mixing angle in satisfactory agreement with the experiment[2]. For the scalar meson masses we have also obtained results more close to experimental data than in the NJL model without the 't Hooft interaction. However, the masses of the a_0 and especially the K_0^* mesons are noticeably less then the experimental ones.

In order to get satisfactory result for the f_0 meson it is necessary to take into account the mixing of the f_0 and σ mesons with the glueball state (see [12]). The problem with masses of a_0 and K_0^* mesons could be solved in the framework of the four-quark $(\bar{q}^2q^2)$ MIT-bag model [13] or the $\bar{K}K$ molecule model [14]. However, it is interesting to note that in spite of a very rough description of the scalar meson's masses our model allows us to obtain a qualitatively true picture of the strong decay widths of the scalar mesons.

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