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Resently some models have been proposed in which the forces between quarks are increasing with the relative distance that results in the confinement of quarks inside a particle and their unobservability in the free state.

We shall consider the problem of quark confinement in the framework of the quasipotential approach[1], namely by using the Kadyshevsky equation[2]. In quasipotential equations, in contrast to the Bethe-Salpeter equation, the momenta of all the particles belong to the mass shell. Therefore it is convenient to pass here to the relativistic configurational representation (RCR), introduced earlier[3] in the framework of the Kadyshevsky approach. The difference of the RCR from the nonrelativistic coordinate representation consists in application here of the Shapiro transformation[4] instead of the conventional Fourier transformation. The Shapiro transformation has the meaning of the expansion over the principal series (PS) of the unitary irreducible representations of the Lorntz group SO(3.1) - the group of motions of the mass shell hyperboloid $p_0^2 - \vec{p}^2 = M^2$.

With notations[3] this expansion for the wave function of relative motion reads

$$\Psi(\vec{p}) = \int \xi(\vec{p},\vec{r})\,\Psi(\vec{r})\,d\vec{r} \quad ; \quad \xi(\vec{p},\vec{r}) = \left(\frac{p_0 - \vec{p}\vec{n}}{M}\right)^{-1-irM} \qquad (1)$$
$$\vec{r} = r\vec{n} \quad ; \quad \vec{n}^2 = 1 .$$

Here $\vec{p}$ is the quark momentum in the c.m.s. $(\vec{p}_1 = -\vec{p}_2 = \vec{p})$. The parameter r defines eigenvalues χ^2 of the Casimir operator of the SO(3.1). $\hat{C} = \frac{1}{4} M_{\mu\nu} M^{\mu\nu}$ ($M_{\mu\nu}$ - are the generators of the SO(3.1))

$$\hat{C}\,\xi(p,r) = \chi^2\,\xi(p,r)\;;\quad \chi^{2} = \frac{1}{M^2} + r^2 \quad (0 < r \le \infty) \tag{2}$$

and, as was shown in[3], it has the meaning of a relativistic generalization at the relative coordinate. In the quasipotential equation written in the RCR the transforms of the Feynman propagators in the new r -space play the role of potentials. Thus, to the propagator $\frac{1}{(p-k)^2}$, describing the massless gluon exchange there corresponds the attractive relativistic Coulomb potential[3]

$$V(r) = -\frac{1}{4\pi r}\,\mathrm{cth}\,\pi r M \tag{3}$$

Due to the proven in[5] equality $\langle r_c^2\rangle = 6\frac{\partial F(t)}{\partial t}\Big|_{t=0} = \{\hat{C}F(t)\}\Big|_{t=0}$ the invariant mean square radius of a particle has the meaning of the average of the eigenvalue of the SO(3.1) Casimir operator $\hat{C} = \chi^2$ over the transforms F(2) of the form factor $F(t)$ in the RCR. In the case when these distributions $F(r)$ in the new r -space are the functions of constant sign, the relativistic coordinate r describes the distances larger than the Compton wave length.

The transition to the distances, smaller than the Compton wave length, may be achieved, following[5], by including into the wave function expansion the supplementary series (SS), characterized by the subsequent values of the Casimir operator $\hat{C} \to \chi^2 = \frac{1}{M^2} - \rho^2$, where $0 \le \rho \le \frac{1}{M^2}$. The coordinate ρ is reckoned beginning from the boundary of the sphere to its center, and the value $\rho = \frac{1}{M}$ corresponds to the origin $\chi^2 = 0$.

For the SS the analogs of the plane waves of PS $\xi(\vec{p},\vec{r})$ are the functions $\zeta(\vec{p},\vec{\rho}) = \left(\frac{p_0 - \vec{p}\vec{n}}{M}\right)^{-1-\rho M}$ $\left(0 < \rho \le \frac{1}{M}\right)$ which formally can be found from $\xi(\vec{p},\vec{r})$ by the change $r \to i\rho$. The expansion of $\Psi(\vec{p})$ with account of SS for the states with $l=0$ has the form:

$$\Psi_{l=0}(p) = 4\pi\int_0^\infty \frac{\sin rM\chi}{rM\,\mathrm{sh}\chi}\,\Psi(r)\,r^2dr + 4\pi\int_0^{1/M}\frac{\mathrm{sh}\,\rho M\chi}{\rho M\,\mathrm{sh}\chi}\,\Psi(\rho)\,\rho^2 d\rho \tag{4}$$

Consider now the analog of the relativistic Coulomb potential for distances smaller than $1/M$. Passing in (3) to the SS through the change $r \to i\rho$ we arrive at the potential (see Fig.1)

$$V(\rho) = \frac{1}{4\pi\rho}\,\mathrm{ctg}\,\pi\rho M\;; \quad 0 < \rho \le \frac{1}{M} , \tag{5}$$

confining quarks inside the sphere with $R^2 = \chi^{-2} = \frac{1}{M^2}$. The operator of the free Hamiltonian $\hat{H}_0$ for the plane waves of the SS $\hat{H}_0\,\zeta(\vec{p},\vec{\rho}) = 2E_p\,\zeta(\vec{p},\vec{\rho})$; $E_p = M\,\mathrm{ch}\chi = \sqrt{M^2+\vec{p}^{\,2}}$

$$\hat{H}_0 = 2M\,\mathrm{ch}\frac{1}{M}\frac{\partial}{\partial\rho} + \frac{2}{\rho}\,\mathrm{sh}\frac{1}{M}\frac{\partial}{\partial\rho} - \frac{\Delta_{\theta,\varphi}}{\rho^2}\,e^{\frac{1}{M}\frac{\partial}{\partial\rho}} \tag{6}$$

as in the case of [3] is the finite-difference operator. The solution of the quasipotential equation with the potential (5)

$$\left(\hat{H}_0 + V(\rho)\right)\Psi_\rho(\vec{p}) = 2E_\rho\,\Psi_\rho(\vec{p}) \tag{7}$$

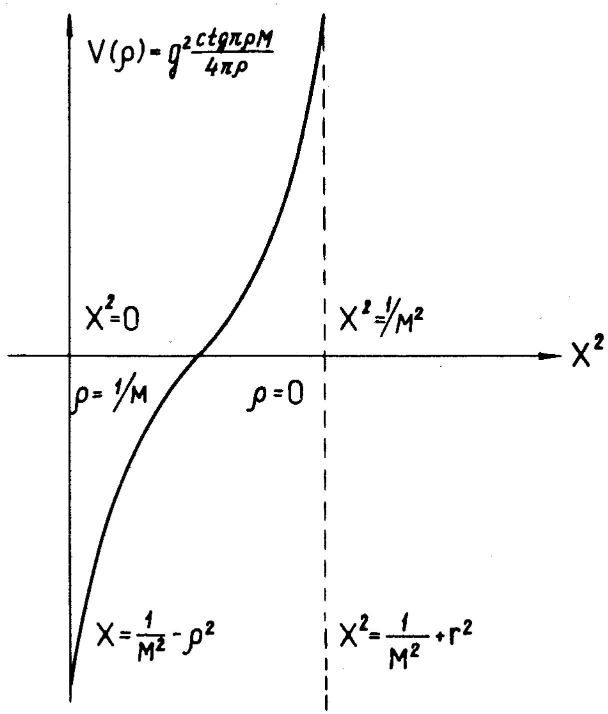

Fig.1

in the domain $0 \le X^2 < \frac{1}{(2M)^2}$, where $ctg\,\pi\rho M < 0$ and the $M_{bound} \equiv 2E_q = 2M\cos x$, for the states with $\ell=0$ has the form

$$\Psi_{q,\ell=0}(\rho) = (e^{-ix}\sin x)\cdot e^{-ix\rho}\rho\cdot \exp\left[x\cdot\frac{ctg\,\pi\rho M}{2\sin x}\right]\cdot$$

$$\cdot F\left(1+\rho M,\ 1+i\,\frac{ctg\,\pi\rho M}{2\sin x};2;2ie^{-ix}\sin x\right). \tag{8}$$

The function $ctg\,\pi\rho M$ in (5), constant with respect to the operation of the finite-difference differentiation (cf./3/), plays a role of the effective interaction constant in equation (6). The requirement of the regularity of the solution at $X^2=0\ (\rho=1/M)$ leads to the condition $\sin 2x = x$, which determines two energy levels. One with $M_{bound} \equiv 2E_q = 1.38M$, another with $M_{bound} \equiv 2E_q = 2M$. In the region $\frac{1}{(2M)^2} \le X^2 < \frac{1}{M^2}$, where $ctg\,\pi\rho M > 0$ and $2E_q = 2M\,ch\chi \geq 2M$, the wave function can be obtained from (8) by the change $x \to -i\chi$. The requirement of the regularity at $X^2=\frac{1}{M^2}\ (\rho=0)$ leads to another condition $2sh\chi\, e^{-\chi}=\chi$, that determines the third level with $M_{bound} \equiv 2E_q = 2.93M$. Therefore in the quark-antiquark system, moving in the field of potential (5) in the state with $\ell=0$ there are possible three energy levels, or three excited states of one particle (for example ρ,ρ' and ρ'').

The functions of SS $\zeta(\vec{p},\vec{\rho})$ do not belong to the class of square- integrable functions/7/. This leads to necessity to include into the definition of the scalar product of the wave functions (8) in the momentum space the regularing kernel $k[(p-k)^2]$, i.e.,

$$(\Psi_1, \Psi_2) = \int \Psi_1 (\vec{p})\, K\left[(p-k)^2\right] \Psi_2 (\vec{k}) \frac{d^3\vec{p}}{p_0} \frac{d^3\vec{k}}{k_0} .$$

The questions of the normalization of the wave functions (8) and the description of the meson spectrum and Ψ -particles in our model with the quark confining potential (5) will be the subject of the next publications.

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