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# ASYMPTOTICS OF PION ELECTROMAGNETIC FORM FACTOR IN SCALE INVARIANT QUARK MODEL

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# ASYMPTOTICS OF PION ELECTROMAGNETIC FORM FACTOR IN SCALE INVARIANT QUARK MODEL

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The behaviour of hadron electromagnetic form factors at large momentum transfer is attracting now a considerable attention which is also stimulated by the fact that the behaviour of proton and pion form factors is well described by the "quark counting" rule [1]

$$F_a(t) \sim t^{1-n_a}, \tag{1}$$

where $n_a$ is the number of quarks in the hadron considered, $t$ - the momentum transfer squared. In the present paper the asymptotic behaviour of the electromagnetic form factor of pion, treated as a bound state of two quarks, is considered on the basis of known methods of finding the asymptotic behaviour of Feynman diagrams. The quarks are supposed to interact through intermediate vector or zero spin gluon $\varphi_A$ :

$$\mathcal{L}_{int} = 4\pi g \sum_a :\bar{\psi}^a(x)\Gamma^A\psi^a(x)\varphi_A(x): \qquad \Gamma^A = 1, i\gamma_5, \gamma^\mu, \tag{2}$$

where $a$ are the indices distinguishing the quarks. It is also assumed that the interaction (2) is scale invariant at small distances, i.e., that the case of finite renormalization of coupling constant $g$ is realized [2,3].

## 1. Form factors of composite particles and Green functions

Let us consider, following the Mandelstam's paper [4] the 5-point truncated Green function

$$(2\pi)^4\delta^4(p_1+p_2+p_1'+p_2'+Q)\, R^\mu(p_1,p_2,p_1',p_2',Q) =$$
$$\langle 0|\, T\{\eta^a(p_1')\,\bar{\eta}^b(p_2')\,J^\mu(Q)\,\bar{\eta}^a(p_1)\,\eta^b(p_2)\}|0\rangle. \tag{1.1}$$

Here $J^{\mu}$, $\eta$ and $\bar{\eta}$ are the electromagnetic and quark currents, respectively, all taken in the momentum representation. The T-product of operators in momentum representation should be understood as a Fourier transform to the corresponding expression in a coordinate space. 4-point Green's function $g(p_1, p_2, p_1', p_2')$ defined by the equality $(2\pi)^4 \delta^4(p_1+p_2+p_1'+p_2') g(p_1, p_2, p_1', p_2') =$

$$= \langle 0 | T\{ \eta^a(p_1') \bar{\eta}^b(p_2') \bar{\eta}^a(p_1) \eta^b(p_2)\} | 0 \rangle \tag{1.2}$$

should have a pole at $s \equiv (p_1+p_2)^2 = M^2$ if there exists a bound state having the mass $M$, composed by the particles with momenta $p_1, p_2$. Near this pole $g(\{p_i\})$ can be represented in the form /4,5/

$$g(p_1, p_2, p_1', p_2') = i \frac{\chi_P\left(\frac{p_1'-p_2'}{2}\right) \bar{\chi}_P\left(\frac{p_1-p_2}{2}\right)}{s - M^2}, \tag{1.3}$$

where $\chi_P\left(\frac{p_1'-p_2'}{2}\right)$ is the truncated Bethe-Salpeter wave function

$$\theta(P_0)(2\pi)^4 \delta^4(p_1'+p_2'+P) \chi_P\left(\frac{p_1'-p_2'}{2}\right) = \langle 0 | T\{\eta^a(p_1') \bar{\eta}^b(p_2')\} | P \rangle. \tag{1.4}$$

In the following we use the notations $p_1-p_2 = 2r$; $p_1+p_2 = P$; $p_1'+p_2' = -P'$. The function $R^{\mu}$ has a double-pole singularity: at $s_1 \equiv P^2 = M^2$ and at $s_2 \equiv P'^2 = M^2$. Near these poles the representation



$$(2\pi)^4 \delta^4(P-P'+Q) R^{\mu}(P, P', r, r') =$$

$$= i^2 \frac{\chi_{P'}(r') \bar{\chi}_P(r)}{(s_1-M^2)(s_2-M^2)} \langle P' | J^{\mu}(Q) | P \rangle \tag{1.5}$$

is valid. The matrix element $\langle P', \sigma' | J^{\mu}(Q) | P, \sigma \rangle$ can be expanded into independent structures $A_i^{\mu}(P, P', \sigma, \sigma')$, the number of which depends on the spin of a composite particle.

$$\langle P', \sigma' | J^{\mu}(Q) | P, \sigma \rangle = e (2\pi)^4 \delta^4(P-P'+Q) \sum_i T_i(Q^2) A_i^{\mu}(P, P', \sigma, \sigma'). \tag{1.6}$$

The coefficients $T_i(Q^2)$ are defined to be the form factors of the particle in question. Pion, as a zero spin particle has only one form factor:

$$\langle P' | J^{\mu}(Q) | P \rangle = e (2\pi)^4 \delta^4(P-P'+Q)(P^{\mu}+P'^{\mu}) F_{\pi}(Q^2). \tag{1.7}$$

We define $F_{(\pi)}(Q^2)$ by the formula

$$(2\pi)^4 \delta^4(P-P'+Q) \delta_{\sigma\sigma'} F_{(\pi)}(Q^2) = \frac{1}{e(P+P')^2} \langle P', \sigma' | (PJ) + (P'J) | P, \sigma \rangle \tag{1.8}$$

The asymptotic form of the function $g(p_i)$ understood as a sum of asymptotic forms of all relevant Feynman diagrams in the region $t \equiv |(rr')| \gg P^2 \sim m_q^2$ has a Regge-like behaviour

$$g(t, s) \simeq g_0^2 K(s) \Gamma(-\alpha(s)) t^{\alpha(s)} \tag{1.9}$$

that is, $g(t, s)$ has a poles at $\alpha(s) =$ 0,1,2... and in the neighbourhood of such a pole the following representation

$$g(t,s)\Big|_{s\simeq M^2} = \frac{g_c^2 K(s)\, t^{\alpha(s)}}{\Gamma(1+\alpha(s))\,\alpha'(s)(s-M^2)} \tag{1.10}$$

takes place. Consequently, when $t \gg s \sim m_q^2$

$$i\,\chi_p(r')\bar{\chi}_p(r) = \frac{g_c^2 K(s)\, t^{\alpha(s)}}{\alpha'(s)\,\Gamma(1+\alpha(s))} = \lambda(s) t^{\alpha(s)}. \tag{1.11}$$

For the investigation of the $F_{(\pi)}(Q^2)$ behaviour for $|Q^2| \gg s, m_q^2$ we shall consider the behaviour of $R^\mu$ in the region

$$|rr'| \gg |Pr'| \sim |P'r| \gg |Q^2| \gg s, m_q^2$$

$$\frac{Q^2 (rr')}{(Pr')(P'r)} \simeq 1 . \tag{1.12}$$

In this region the asymptotic forms of the functions $\chi_p(r')\bar{\chi}_p(r)$ and $\chi_{p'}(r')\bar{\chi}_p(r)$ coincide, because from $P'^2 = P^2 = M^2$ it follows that

$$-\lambda^2 |rr'|^{2\alpha(s)} \simeq \{\chi_{p'}(r')\chi_p(r)\}^2 = \{\chi_{p'}(r')\bar{\chi}_p(r)\}\{\chi_p(r')\bar{\chi}_{p'}(r)\} \tag{1.13}$$

and in view of symmetry between $P$ and $P'$

$$i\,\chi_{p'}(r')\bar{\chi}_p(r) \simeq |rr'|^{\alpha(s)} \frac{g_0^2 K(s)}{\alpha'(s)\,\Gamma(1+\alpha(s))} . \tag{1.14}$$

Thus, from (1.5),(1.8),(1.14) it follows that

$$R^\mu(p_1, p_2, p_1', p_2', Q) =$$
$$= ie\, \frac{g_c^2 K(s)\, |rr'|^{\alpha(s)}}{\alpha'(s)\,\Gamma(1+\alpha(s))} (P^\mu + P'^\mu) \frac{F_{(\pi)}(Q^2)}{(S_1 - M^2)(S_2 - M^2)} + \dots \tag{1.15}$$

The conclusion is that for finding the asymptotic behaviour of



the form factor one has to consider the asymptotic behaviour of $R^\mu$ in the region (1.12) and to single our the Regge factor

$$\frac{g_0^2 K(s)\, |rr'|^{\alpha(s)}}{\alpha'(s)\,\Gamma(1+\alpha(s))\,(S_1 - M^2)(S_2 - M^2)} .$$

## 2. The asymptotic methods in the theory of Feynman diagrams

We shall use the $\alpha$- representation of a graph ( see, e.g. /2,6/ ):

$$A(p_1, \dots p_n) = g^N \int_0^{i\infty} \frac{\prod_\sigma d\alpha_\sigma}{\mathcal{D}^2(\alpha)} G(\alpha, p) \exp\left\{ \frac{Q(\alpha,p)}{\mathcal{D}(\alpha)} - \sum_\sigma \alpha_\sigma (m_\sigma^2 - i\varepsilon) \right\}, \tag{2.1}$$

where $\mathcal{D}(\alpha)$ is the sum of all trees of the graph ( its "determinant") $Q(\alpha, p)$ is determined by the expression

$$Q(\alpha, p) = \sum_{k+\ell=n} B(i_1, \dots i_k | j_1, \dots j_\ell)(p_{i_1} + \dots p_{i_k})^2, \tag{2.2}$$

where $B(i_1, \dots i_k | j_1, \dots j_\ell)$ is the sum of all the 2-trees of a graph, the vertices $i_1, \dots i_k$ of which belong to one component, $j_1, \dots j_\ell$ - to the other, the nonenumerated vertices may belong to any of the components. The summation in (2.2) turns over all the possible separations of the vertices, in which the external momenta $p_1, \dots p_n$ enter, into two groups. In a more compact form

$$Q(\alpha, p) = \sum_r S_r\, A_{S_r}(\alpha), \tag{2.3}$$

where $S_r = (p_{i_1} + \dots p_{i_k})^2$ . The $A_{S_r}(\alpha)$ construction can be illustrated by the ladder graph. Let $S_1 = (p_1 + Q)^2$ ( fig.1). One has to cut the graph into two connected parts

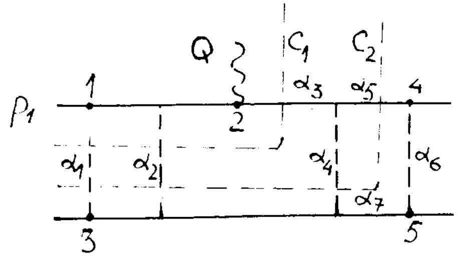


Fig.1

(cutting the lines only) to separate the vertices 1 and 2 from the rest of "external" vertices 3,4,5. Each of these separations corresponds to the product of $\alpha_\sigma$ parameters of cut lines, multiplied by determinants of forementioned parts ( e.g. $\alpha_1\alpha_2\alpha_3\mathcal{D}_1$ for the cut $C_1$ and $\alpha_1\alpha_2\alpha_4\alpha_5$ for $C_2$, $\mathcal{D}_1=\alpha_4+\alpha_5+\alpha_6+\alpha_7$ ). The summation over all such possible separations forms the coefficient A.

The five-leg diagram ( fig.1) depends on 10 independent invariants. Four invariants $t_1=-2(P'P_1)$; $t_2=-2(PP_1')$; $t_3=2(P_1P_1')$; $t_4=-Q^2$ are large in comparison with the remaining ones: $S_1$, $S_2$, $P_i^2$ . This gives

$$Q(\alpha,p)=\sum_{k=1}^{4} t_k\tilde{A}_k(\alpha)+I(\alpha,S_1,S_2,P_i^2). \qquad (2.4)$$

Functions $\tilde{A}_k(\alpha)$ may change their signs.

The preexponential factor $G(\alpha,p)$ is due to the numerators of spinor propagators.[7] To every spinor line there corresponds the quantity

$$\hat{f}_\sigma=m_\sigma+\frac{1}{\alpha_\sigma\mathcal{D}(\alpha)}\sum_{i=1}^{n}B(b_\sigma,i|e_\sigma)\hat{p}_i\,, \qquad (2.5)$$



where "$b_\sigma$" and "$e_\sigma$" are initial and end points of the spinor line $\sigma$ . $G(\alpha,p)$ is given by the expression

$$G(\alpha,p)=\prod_{\sigma<\tau}(1+R_{\sigma\tau})Y\hat{f}_{\sigma_1}\ldots\hat{f}_{\sigma_K} \qquad (2.6)$$

in which the operator $Y$ arranges in a proper order the quantities $\hat{f}_\sigma$ and the vertex matrices $\gamma_\mu$, $\gamma_5$ ; and the pairing operator $R_{\sigma\tau}$ replaces $\ldots\hat{f}_\sigma\ldots\hat{f}_\tau\ldots$ by $\ldots\gamma_\mu\ldots\gamma^\mu\ldots$ ( with the summation over $\mu$ ) and adds the factor $r_{\sigma\tau}$

$$r_{\sigma\tau}=\frac{B(b_\sigma,b_\tau|e_\sigma,e_\tau)-B(b_\sigma,e_\tau|b_\tau,e_\sigma)}{2\alpha_\sigma\alpha_\tau\mathcal{D}(\alpha)}. \qquad (2.7)$$

The 2- trees entering the equations (2.5),(2.7) are also constructed by the diagram separation.

Every diagram can be represented as a sequence of two-particle irreducible subgraphs-kernels ( fig.2). Projecting the contribution of

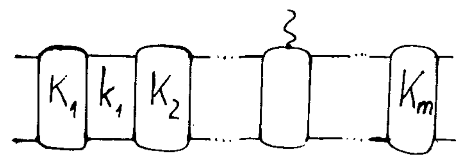


Fig.2

every kernel and interkernel lines on the complete set of projection operators $R_S$, $R_V$, $R_T$, $R_A$ and $R_P$ /2/ one can write

$$G^{\ell n}(\alpha, t_i, s_k) = f(\alpha) \sum_{\ell_i} K_1^{\ell \ell_1} k_1^{\ell_1 \ell_1'} K_2^{\ell_1' \ell_2} \dots k_{m-1}^{\ell_{m-1} \ell_{m-1}'} K_m^{\ell_{m-1}' n}, \tag{2.8}$$

where $\ell_i = S, V, T, A, P$ and $K^{\ell m} = (\Gamma^{\ell})^{\alpha}_{\beta} K^{\beta\delta}_{\alpha\gamma} (\Gamma^{n})^{\gamma}_{\delta}$

The pion, being "unnatural" particle $(\sigma P = -1)$ may give a projection only to $P$- and $A$- states ( T-state projection leads to the pion "conspirator", i.e. $0^+$ particle, which is not observed). One can consider either P or A projections of $G(\alpha, p)$. We choose the P-projection. Notice, however, that the $K^{PA}$ - transition is nonzero. The asymptotic behaviour $F_\pi \sim 1/Q^2$ as it will be shown, is just due to this circumstance.

Hence, for the $G_{PP}$-projection one can obtain

$$G_{PP}(\alpha, p) = \sum_{k_1, \dots k_4} G^{(k_1 \dots k_4)}(\alpha, s_1, s_2, p_i^2) \prod_{i=1}^{4} t_i^{k_i}. \tag{2.9}$$

To investigate the asymptotic behaviour of amplitudes $R(t_i, s_k)$ in the region (1.12) it is suitable to introduce the Mellin transform $\Phi(j_i, s_k)$ with respect to each of the large variables $t_i$

$$R(t_1 \dots t_a; s_1 \dots s_6) = \frac{1}{(2\pi i)^a} \left\{ \prod_{i=1}^{a} \int_{\delta_i - i\infty}^{\delta_i + i\infty} dj_i \, t_i^{j_i} \right\} \Phi(j_1, \dots j_a; s_1 \dots s_6). \tag{2.10}$$

Using the expressions (2.1), (2.4) and (2.9) it is easy to obtain the representation for

$$\Phi(j_1, \dots j_4; s_1, s_2, p_i^2) = g^N \sum_{k_i} \prod_{i=1}^{4} \Gamma(k_i - j_i) \int_0^{i\infty} \frac{\prod_\sigma d\alpha_\sigma}{D^2(\alpha)} G^{(k_1 \dots k_4)}(\alpha, s, p_i^2) \cdot \tag{2.11}$$

$$\cdot \left( \frac{\tilde{A}_i(\alpha)}{D(\alpha)} \right)^{j_i - k_i} \varepsilon(A_i) \exp\left\{ \frac{I(\alpha, s, p_i^2)}{D(\alpha)} - \sum_\sigma \alpha_\sigma (m_\sigma^2 - i\varepsilon) \right\},$$



where $\varepsilon(\tilde{A}_i) = [\theta(\tilde{A}_i) \pm \theta(-\tilde{A}_i)] \dfrac{e^{-i\pi j_i} \pm 1}{2}$

is the signature factor appeared because of the sign indefiniteness of $\tilde{A}_1$, $\tilde{A}_2$, $\tilde{A}_3$.

Besides the poles of functions $\Gamma(k_i - j_i)$ lying to the right of the integration contour, the function $\Phi(j_i, s_k)$ may possess left-lying poles resulting from the integration over $\alpha$ in the region, where $\tilde{A}(\alpha) \approx 0$. The position of these poles does determine the asymptotic behaviour of $R(t_i, s_k)$. $\tilde{A}(\alpha)$ can approach zero either on the edge of the integration region, when $\alpha_\sigma \to 0$ for the lines $\sigma$, forming such a connected subgraph $V$, the contraction of which into the point "kills" the dependence of the diagram on $t_i$ ( endpoint singularity) or as a result of cancellation of opposite sign terms entering $\tilde{A}_i(\alpha)$ at nonzero $\alpha$ ( pinch singularity). The latter contributes only to the negative signature amplitudes and is of no interest for us.

The simultaneous vanishing of the $\alpha$-parameters is convenient to describe with the help of the scaling [2]:

$$\alpha_\sigma = \lambda_V \beta_\sigma \quad \prod_\sigma d\alpha_\sigma = \lambda_V^{\ell-1} \prod_\sigma d\beta_\sigma \, d\lambda_V \, \delta(1 - \sum_\sigma \beta_\sigma). \tag{2.12}$$

If the contraction of $V$ into the point kills the dependence on variables $t_{i_1} \dots t_{i_c}$ ( i.e. $V$ is $t_{i_1} \dots t_{i_c}$ - subgraph) this should result in the appearance of the factor $\lambda_V^{j_{i_1} + \dots j_{i_c} - j_c - 1}$ and the integration over $\lambda_V \sim 0$ ( the "contraction" or asymptotic regime of $V$ ) gives a pole

$( j_{i_1} + \dots j_{i_c} - j_c )^{-1}$. As a consequence $\Phi(j, S)$ may be represented in the form

$$\Phi(j_i, S_k) = \frac{C_V(j_i, S_k)}{j_{i_1} + \dots j_{i_c} - j_0} + R_V(j_i, S_K). \tag{2.13}$$

The first term corresponds to the asymptotic regime of the subgraph, $R_V$ is a contribution of noncontracted $V$. The functions C and R may also possess the leading singularities, i.e., the poles at $j_{i_1} + \dots + j_{i_c} = j_c$ due to the contraction of other subgraphs.

The asymptotic contribution of any diagram can be obtained by combination of all the possibilities of asymptotic and non-asymptotic regimes of subgraphs. Due to $\delta(1 - \Sigma \beta_\sigma)$ only those subgraphs are allowed to be simultaneously in the asymptotic regime, which either have no common lines or are wholly one inside another.

Divergent parts of the diagram do increase the order of the pole $( j_{i_1} + \dots + j_{i_c} - j_0 )^{-n}$, but only when they are inside the contracted $t_{i_1} \dots t_{i_c}$ subgraph, i.e. when all distances inside the divergent part are small. To sum these additional poles one may, consequently, apply the renormalization group methods. The assumption of finite charge renormalization results in the shift of the pole position to the left at the distance, equal to the half sum of anomalous dimensions of the external lines

$$j_0 = \frac{1}{2}\left[4 - M(1+\varepsilon_\varphi) - B(1+\varepsilon_q)\right] - d, \tag{2.14}$$



where $\varepsilon_q$, B and $\varepsilon_\varphi$, $M$ are respectively the anomalous dimensions and the number of quark and gluon external lines entering $V$. The meaning of the number $d$ will be clarified later.

From (2.14) it is evident that for the leading singularities the t-subgraphs with minimal number of external lines are responsible.

Besides, the renormalized charge $g$ inside the contracted subgraph $V$ must be replaced by the "bare" coupling constant $g_0$. The experimental situation indicates to the smallness of $g_c^2$ and anomalous dimensions. Therefore for the investigation of contracted subgraphs the weak coupling constant approximation is justified.

## 3. The structure of pion trajectory

The methods scatched in § 2 have been applied to the investigation of the 4-point function $g(t,s)$ asymptotic behaviour /2/:

$$g_{k\ell}(t,s) = \frac{1}{2\pi i} \int_{\delta - i\infty}^{\delta + i\infty} dj \; t^j \, \Gamma(-j) \, h_{k\ell}(j, s, m^2). \tag{3.1}$$

The result for $h_{pp}(j, s, m^2)$ has the form

$$h_{pp}(j, s, m^2) = j \, C_{p\tau}(s) \left[U(j) - B(s)\right]^{-1}_{\tau\tau} \tilde{C}_{\tau p}(s), \tag{3.2}$$

where $C$, $\tilde{C}$ are the contributions of the most left and the most right nonasymptotic objects ( fig.3)

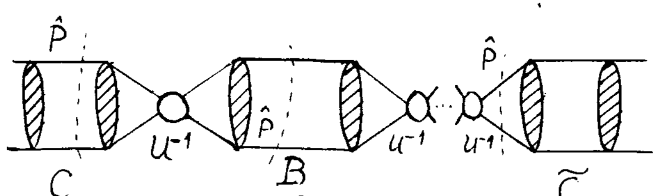

Fig3

The factor j is due to the fact that the preexponent for the senior asymptotic contributions for $G_{PP}$ is of the form

$$G_{PP} = \sum_{n=1} t^n \; G^{(n)}(\alpha, s) \; . \tag{3.3}$$

This results in factors $\frac{\Gamma(n-j)}{\Gamma(-j)}$ ( see (2.11)) "killing" the poles at $j = 0, 1 \ldots \; n-1$.

Nonasymptotic contributions $C, \tilde{C}$ are projected on the T-states just with the aim to get the senior asymptotic behaviour. To obtain the Regge trajectory possessing the zero spin state one should consider only the contributions with the preexponents without factors t, pick out of them the contributions with the leading asymptotic behaviour and sum up them. Each of these contributions has a pole at $-j = 1 + 2\varepsilon_q$ , i.e. at a point by unity further to the left of the value, dictated by the dimensions of external lines. The quantity $d$ in (2.14) reflects just this circumstance.

The summation, analogous to one used in /2/ gives

$$h_{PP}(j, s, m^2) = C_{PN}(s)\left[\tilde{u}(j+1) - \breve{B}(s)\right]^{-1}_{NN'} \tilde{C}_{N'P}(s) , \tag{3.4}$$

where $N, N'$ means P- or A-states. Nonasymptotic objects $\tilde{B}_{AP}(s), \tilde{C}_{AP}(s), B_{PA}(s), C_{PA}(s)$ have the following structure ( fig.3): there is an odd number of $\hat{P}$



values in each object, and the cuts, giving the factor corresponding to any $\hat{P}$ value ( see (2.5) ) do not touch the lines outside the object in question.

## 4. The structure of contributions and the factorization problem

The large variables in the region (1.12) are $t_1, t_2, t_3, t_4$ which are related by $\frac{t_1 t_2}{t_3 t_4} \simeq 1$. It is easy to see that for asymptotic form in this region the most essential are subgraphs $V_M$ ( fig.4), the contraction of which kills the dependence on all the large variables and therefore generates the pole $(j_1 + j_2 + j_3 + j_4 + 1 + 2\varepsilon_q)^{-1} \equiv (J + 1 + 2\varepsilon_q)^{-1}$ and subgraphs $V_L, V_R$, generating the poles $(j_1 + j_3 + 1 + 2\varepsilon_q)^{-1} \equiv (j_\lambda + 1 + 2\varepsilon_q)^{-1}$ and $(j_2 + j_3 + 1 + 2\varepsilon_q)^{-1} \equiv (j_\rho + 1 + 2\varepsilon_q)^{-1}$, respectively. The assumption $g_c^2 \ll 1$ permits us to use the ladder approximation inside the contracted subgraphs . The dependence on $t_4 = -Q^2$ being killed only by $V_M$ contraction, in all essential contributions the photon is entering the contracted subgraph. Consequently only diagrams of fig.4 type are essential.

The 2-trees of some subgraph $V$ of a diagram will be denoted as $B(i_1 \ldots i_k | j_1 \ldots j_\ell ; V)$ . 2-trees of the type fig.4 have the form

$$B(N_L + N_R) = B'(N_L)\, B''(N_R) , \tag{4.1}$$

where $B'(N_L), B''(N_R)$ are the 2-trees of the left and the right parts of the diagram respectively. The following notations are suitable also

$$B(\lambda_1,\mu|\lambda_2,\lambda_4;\Lambda_L)=L_s \quad B(\lambda_1,\lambda_4|\lambda_2;\mu;\Lambda_L)=L_u$$
$$B(\mu|\lambda_1,\lambda_2,\lambda_4;\Lambda_L)=L \quad B(\lambda_4|\lambda_1,\lambda_2,\lambda_3;\Lambda_L)=L' \tag{4.2}$$
$$L_s-L_u=L_- \quad L_s+L_u=L_+$$

with the change $\lambda\to\rho$ , $L\to R$ for the left part. In these notations one can rewrite the exponent $Q(\alpha,p)$ in the form:

$$Q(\alpha,p)=t_4(L+L_s)(R+R_s)+t_1L_-(R+R_s)+$$
$$+t_2(L+L_s)R_-+t_3R_-L_-+I(\alpha,s_1,s_2,p_i^2). \tag{4.3}$$

According to (1.8) all the contributions are divided into two groups depending on the momentum $\hat P$ or $\hat P'$ in the photon vertex. Let $V_M^{(P')}$ be the maximal contracted subgraph of the p'-type. $V_M$ is the central part of the diagram and the kernels outside the $V_M$ make up the left and right "Regge" parts. Simple but cumbersome considerations give the following result for the preexponent structure in the approximation $g_0^2\ll 1$ (however, $g_0^2\ln(-G^2)\lesssim 1$ ): the $\mu\rho_3$ line contributes $\hat P(L+L_s-\frac{L_-}{2})$ to the preexponent, the remaining spinor lines of the central part contribute to the pairings $\tau_{\sigma\sigma'}$ as it is shown on fig.4.

The central part must be projected on the AA-component, i.e. the left Regge-part gives an extra $\hat P$ , the right one gives an extra $\hat P'$ . The pole corresponding to $V_M$-contraction is, as a result, at the point dictated by the line dimensions, that is $\alpha$ =0 for $V_M$ , $\alpha$ =1 for $V_L, V_R$ .



Finally, the preexponent of the leading contribution has the form

$$G(\alpha,p)=2(PP')\,G_L(\alpha,s_1,p_i^2)\frac{(L+L_s)\,\mathcal{D}(N_R)}{\mathcal{D}(\alpha)}\cdot$$
$$\cdot\prod_i\frac{D_L^i(\alpha)D_R^i(\alpha)}{-2\,\mathcal{D}(\alpha)}\;G_R(\alpha,s_2,p_i'^2) \tag{4.5}$$

where $\mathcal{D}(\alpha)$ is the determinant of the whole diagram, $\mathcal{D}(N_R)$ is that for the right part of the graph, $D_L^i$ , $D_R^i$ are the determinants of left and right, in respect of i-th pairing, components of the graph, and $(G_L)_{PA}$, $(G_R)_{AP}$ are the preexponents of left and right 4-leg diagrams. Using (2.11), (4.4), (4.5) one can rewrite the Mellin transform $\Phi(j_i,s_k)$ as

$$\Phi(j_i,s_k)=\sum_{V_M}g_0^{N(V_M)}\prod_{i=1}^{3}\Gamma(-j_i)\,\Gamma(1-j_4)\int\frac{\prod_\sigma d\alpha_\sigma}{\mathcal{D}^2(\alpha)}\cdot$$
$$\cdot\,\tilde G(\alpha,s_k,p_i^2)\left(\frac{L+L_s}{D}\right)^{J-j_\lambda}\left(\frac{L_-}{D}\right)^{j_\lambda}\left(\frac{R+R_s}{D}\right)^{J-j_s-1}\left(\frac{R_-}{D}\right)^{j_\rho}, \tag{4.6}$$

where

$$\tilde G=G_L\,G_R\,D(N_R)\prod_i\frac{D_L^iD_R^i}{-2D}\quad;$$

For subsequent considerations one has to know the factorization properties /2,3/ of functions $A$, $A_s$, $A_-$ ( $A=R,L$ ) Using the expression (4.3) one can prove that

$$A_-(V+\bar V)=A_-(V)\,A_-(\bar V)$$
$$A(V+\bar V)=A(V)\mathcal{L}(\bar V)+A_s(V)A(\bar V)+A_u(V)A'(\bar V) \tag{4.7}$$
$$A_s(V+\bar V)=A_s(V)\,A_s(\bar V)+A_u(V)\,A_u(\bar V)$$

( $V$ is chosen to be on the left from $\bar{V}$ for $R$ and on the right for $L$ ). The subgraph $V$ being contracted, can be considered in the ladder approximation ( $g_c^2 \ll 1$). In this case

$$A(V+\bar{V}) + A_S(V+\bar{V}) =$$
$$[A(V) + a(V)]\mathcal{D}(\bar{V}) - a(V)[\mathcal{D}(\bar{V}) - A_S(\bar{V}) - A(V)]. \tag{4.8}$$

For the planar graphs $A_S = A_u \equiv a$ . However, the subgraph $\bar{V}$ itself contains the "Regge-tail", and, consequently, should have at least one contracted subgraph $V_A$ ( $V_R$ on fig.5).

When $\lambda_{V_R} \to 0$

$$\frac{A_S(V_R)}{\mathcal{D}(\bar{V})} \sim \frac{A(V_R)}{\mathcal{D}(\bar{V})} \sim \frac{A_u(V_R)}{\mathcal{D}(\bar{V})} \sim \lambda_{V_R}\,;\quad \frac{\mathcal{D}(V_R)}{\mathcal{D}(\bar{V})} \approx \frac{1}{\mathcal{D}(V_1)} \tag{4.9}$$

and it is easy to get from (4.7)

$$\frac{1}{\mathcal{D}(\bar{V})}[\mathcal{D}(\bar{V}) - A(\bar{V}) - A_S(\bar{V})] \to \frac{\mathcal{D}(V_1) - A(V_1)}{\mathcal{D}(V_1)} \tag{4.10}$$

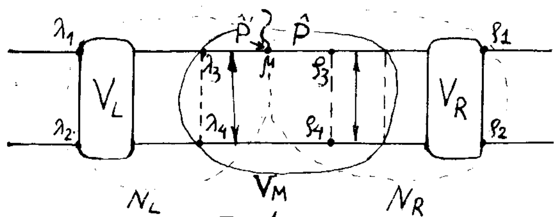


Fig.4

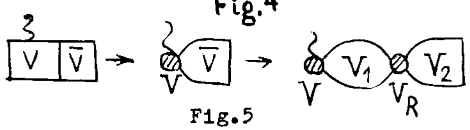


Fig.5

For the subgraph $V_1$ , both the ends of which are contracted $\mathcal{D}(V_1) - A(V_1) = A'(V_1)$ . For the combination $\mathcal{A}(\ell, j) = \left(\frac{A+A_S}{\mathcal{D}}\right)^{\ell+1}\left(\frac{A_-}{\mathcal{D}}\right)^j$ in (4.6) one can obtain, using (4.7) the following rule for separating the contributions from



$V$ and $\bar{V}$ ( $j+\ell \equiv \mathcal{J}$):

$$\mathcal{A}(V+\bar{V}; \ell, j) = \sum_{n=0}^{\infty} \frac{\Gamma(n-\ell+1)}{n!\,\Gamma(1-\ell)}\, \mathcal{A}(V; \ell-n, j+n)\, \frac{A_-^j(\bar{V})[\mathcal{D}(\bar{V}) - A(\bar{V}) - A_S(\bar{V})]^n}{\mathcal{D}^{\,j+n}} \tag{4.11}$$

where the formula

$$(\alpha-\beta)^\ell = \sum_{n=0}^{\infty} \frac{\Gamma(n-\ell)}{\Gamma(-\ell)\,n!}\, \alpha^{\ell-n}\beta^n \tag{4.12}$$

was used which is valid for the case of our interest $\mathrm{Re}\,\ell < 0$ when $|\alpha| > |\beta|$ . One can easily verify that this requirement is fullfilled. If $V+\bar{V}$ belongs to the central part, then $\mathcal{D}(\bar{V}) - A_S(\bar{V}) - A(\bar{V}) = A'(\bar{V})$ ( remind that the $\bar{V}$'s end adjoint to $V$ is contracted) . Formula (4.11) can be rewritten in the form

$$\mathcal{A}(V+\bar{V}) \underset{V}{=} \mathcal{A}(V) \otimes A'(\bar{V}) \tag{4.13}$$

( The sign $\underset{V}{=}$ shows that this equality is valid when $\lambda_V \sim 0$). When $V_2$ is contracted, it follows from (4.9) that

$$A'(V_1+V_2) \underset{V_2}{=} A'(V_1)\,E(V_2) \qquad E(V; j) \equiv \left\{\frac{A_-(V)}{\mathcal{D}(V)}\right\}^j \tag{4.14}$$

Thus, using formulae (4.13), (4.14) one can express the contribution of an object with m+1 kernels in terms of the contribution of an object with $m$ kernels. The summation over the number of kernels and over all possible sorts of them gives the following result ( see fig.6)

$$\Phi(j_i, S_k) = H^{(L)}_{PN}(j_\lambda, S_1, \rho^2_{1,2})\, \bar{W}_{NN'}(j_i, S_1, S_2)\, H^{(R)}_{N'P}(j_\rho, S_2, \rho^2_{1,2}) \tag{4.15}$$

$$W_{NN'}(j_i,S_1,S_2)=\beta^{(L)}_{NM}(S_1)\otimes M_{MM'}(j_i)\otimes\beta^{(R)}_{M'N'}(S_2)+ \tag{4.16}$$
$$+\gamma^{(L)}_{NM}(S_1)K_{MM'}(j_i)\otimes\beta^{(R)}_{M'N'}(S_2)+\beta^{(L)}_{NM}(S_1)\otimes K_{MM'}(j_i)\gamma^{(R)}_{M'N'}$$

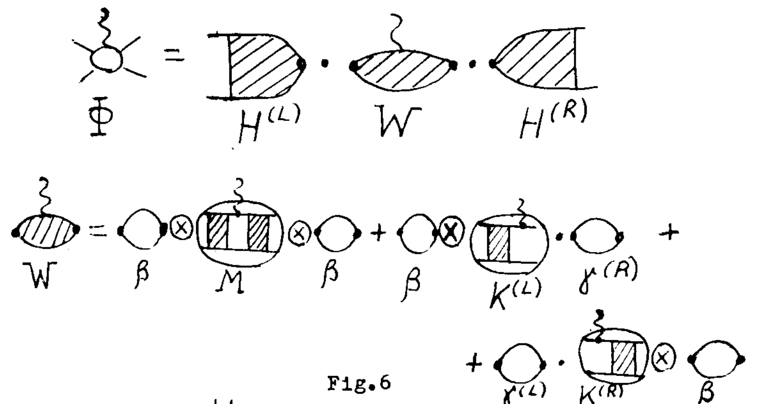


Fig.6

The structure of $H$ coincides with that of the 4-leg diagram:

$$H^{(L)}=C(S_1)\left[\tilde{u}(j_\lambda+1)-\tilde{B}(S_1)\right]^{-1}$$
$$H^{(R)}=\left[\tilde{u}(j_\rho+1)-\tilde{B}(S_2)\right]^{-1}\tilde{C}(S_2) \tag{4.17}$$

Using new integration variables $j_\lambda, j_\rho, J, j_3$ one can rewrite (2.10)

$$R(t_1,t_2,t_3,t_4)=\frac{C(S_1)}{(2\pi i)^4}\int\frac{dj_\lambda\, dj_\rho\left(\frac{t_1}{t_4}\right)^{j_\lambda}\left(\frac{t_2}{t_4}\right)^{j_\rho}\tilde{C}(S_2)}{\left[\tilde{u}(j_\lambda+1)-\tilde{B}(S_1)\right]\left[\tilde{u}(j_\rho+1)-\tilde{B}(S_2)\right]} \tag{4.18}$$
$$\int dJ\, dj_3\,\Gamma(j_3-j_\lambda)\Gamma(j_3-j_\rho)\Gamma(-j_3)\Gamma(1+j_\lambda+j_\rho-j_3-J)W(j_\lambda,j_\rho,J)\,t_4^J\left(\frac{t_3t_4}{t_1t_2}\right)^{j_3}$$



The $j_3$- integration contour encircles the points $j_3=0,1\ldots\max\{\mathrm{Re}\,j_\lambda,\mathrm{Re}\,j_\rho\}$. The poles $[\tilde{u}(j+1)-\tilde{B}(s)]^{-1}$ are on the left from the $j_\lambda$-, $j_\rho$- integration contours. The residues at these poles give the asymptotic behaviour of $R(t_i)$ with respect to the variables $(t_1/t_4)$, $(t_2/t_4)$:

$$R(t_i)\simeq\left(\frac{t_1}{t_4}\right)^{\alpha(S_1)}\left(\frac{t_2}{t_4}\right)^{\alpha(S_2)}\frac{C(S_1)}{\tilde{u}'[\alpha(S_1)]}\sum_{n=0}^{\max\{\alpha(S_1),\alpha(S_2)\}}\Gamma(n-\alpha(S_1))\frac{\Gamma(n-\alpha(S_2))}{n!}\cdot \tag{4.19}$$
$$\cdot\left(\frac{t_3t_4}{t_1t_2}\right)^n\int dJ\,t_4^J\,\Gamma(\alpha(S_1)+\alpha(S_2)-n-J+1)W(\alpha(S_1),\alpha(S_2),J)\cdot$$
$$\cdot\frac{\tilde{C}(S_2)}{\tilde{u}'[\alpha(S_2)]}$$

The amplitude $R$ has a double-pole singularity at $S_1=S_2=M^2$

$$R(t_i)\simeq\frac{1}{[\alpha'(M^2)]^2}\frac{1}{(S_1-M^2)(S_2-M^2)}\left(\frac{t_1t_2}{t_4^2}\right)^{\alpha(M^2)}\cdot\sum_n\ldots\int dJ\ldots \tag{4.20}$$

The summation over n does not affect the asymptotic behaviour because of $t_3t_4/t_1t_2\simeq 1$. $W(\alpha,\alpha,J)$ has singularities near the point $J=-1-2\varepsilon_q$,

$$\int dJ\,t_4^J\,W(\alpha,\alpha,J)\simeq(-Q^2)^{-1-2\varepsilon_q}f(\alpha,Q^2). \tag{4.21}$$

As to the function $f(Q^2)$, we can only say now, that it is slow varying function, depending on $g_0^2$ and $\ln(-Q^2)$. It is shown in the Appendix that $f(Q^2)=\varphi^2(Q^2)$ and the Mellin transform of the function $\varphi(Q^2)$ possesses the poles at $j = -\frac{Z g_c^2}{n(n+1)}$, accumulating at zero ( $Z$ =1 for the spin 0 gluons and $Z = -2$ for vector gluons). But the relative weight of these poles depends not only on small distance physics, but also on the large distance one, this fact being reflected by parameters $\beta$ in (A.21). That is why it seems impossible to sum these poles, but one can consider expressions (A.19)-(A.22) as a starting point for further approximations. Using (1.15) and (4.18) one obtains

$$F_{(\pi)}(Q^2, \alpha(M^2)) \simeq \frac{k(\alpha)\, f(Q^2)}{-Q^2\ \alpha'(M^2)}\left(-\frac{m_q^2}{Q^2}\right)^{\alpha(M^2)+2\varepsilon_q} \tag{4.22}$$

This formula corresponds to the quark counting rule /1/ and gives also the natural correction $(-Q^2)^{-\alpha(M^2)}$ for the spin $\alpha(M^2)$ of a composite particle, just as in the nonrelativistic case.

For the pion ($\alpha$=0) in the parton region ( $g_c^2 \ln(-Q^2) \ll 1$ ) according to (A.23)

$$F_\pi(Q^2) = \frac{C}{-Q^2\ \alpha'(m_\pi^2)} \tag{4.23}$$

where C=O(1). Thus, in this case, the dimensional parameter, compensating the $Q^2$ to make $F_\pi(Q^2)$ dimensionless, is the inclination of pion trajectory $\alpha' \equiv \frac{d\alpha}{ds}$ .

5. Concluding remarks

The method of Feynman diagrams turned out to be a rather effective tool for consistent relativistic consideration of form factor asymptotic behaviour of the simplest composite system.



The important role in the consideration plays the assumption about the weakness of the effective quark interaction at small distances. In particular, just this property is responsible for the existence of the parton region, $g_c^2 \ln(-Q^2) \ll 1$, where the "quark counting" rules are valid, $F_\pi(Q^2) \sim (Q^2)^{-1}$. It predicts also the deviation from these rules at larger $Q^2$ .

The breaking of the Bjorken scaling in deep-inelastic $\mu p$- scattering /8/ , discovered at FNAL allows one to expect these deviations in the region $|Q^2| \sim$ 50-100(GeV/c)$^2$ .

The consistent consideration of more complicated system seems to be difficult for the present from the technical point of view . But there is no doubt, however, that for these systems also the asymptotic behaviour in the parton region $g_c^2 \ln(-Q^2) \ll 1$ is determined by the scale dimension of the corresponding t-subgraph, i.e., is given by the quark counting rules.

The analysis carried out shows, that although the scale degree of form factor asymptotic behaviour is governed by the small distance dynamics, the function $f(Q^2)$ . due to the absence of simple factorization depends on the wave function properties at large distances.

For the deep inelastic scattering it is possible to get simple factorization when some ( the most natural, indeed) variables are chosen. It is just the reason for the success of application of the renormalization group (RG) and Wilson expansions methods for the investigation of this process. It is interesting, in our view, to obtain the results of the present paper by the renormalization group.

In conclusion we want to thank A.T.Filippov and I.F.Ginzburg for useful discussions and valuable remarks.

## APPENDIX

### The investigation of the central part contribution

The formula (4.16) can be rewritten in more detail

$$W(j_\lambda, j_\rho, \mathcal{J}) = \sum_{k,n=0}^{\infty} \left\{ \frac{\widetilde{M}(j_\rho+n, j_\lambda+k, \mathcal{J})}{\Gamma(1-\ell_\rho)\,\Gamma(-\ell_\lambda)} + \frac{\widetilde{M}(j_\lambda+k, j_\rho+n, \mathcal{J})}{\Gamma(1-\ell_\lambda)\,\Gamma(-\ell_\rho)} \right\} \frac{\beta(n,S_2)\,\beta(k,S_1)}{n!\,k!} + \sum_{n=0}^{\infty} \left\{ \frac{\widetilde{K}(j_\rho+n)}{\Gamma(1-\ell_\rho)} \gamma(\ell_\lambda, S_1) \frac{\beta(n,S_2)}{n!} + \frac{K(j_\lambda+n)}{\Gamma(1-\ell_\lambda)} \gamma(\ell_\rho, S_2) \frac{\beta(n,S_1)}{n!} \right\}, \tag{A.1}$$

where

$$\widetilde{M}(j_\rho, j_\lambda, \mathcal{J}) = \Gamma(1-\ell_\rho)\,\Gamma(-\ell_\lambda)\,M(j_\rho, j_\lambda, \mathcal{J})$$
$$\widetilde{K}(j) = \Gamma(1-\ell)\,K(j). \tag{A.2}$$

The contraction of the $M$- or $K$- type subgraph gives a pole $C(j_\lambda, j_\rho, \mathcal{J})\,(\mathcal{J}+1+2\varepsilon_q)^{-1}$ . $M$ in (A.1) is supposed to correspond to the case when in the photon vertex there is $\hat{p}$ . The coefficient $C_M$ for a $M$- contribution is determined by the sum of the following contributions:

1. The subgraph $M_1 \in M$ is contracted, the contribution from the left kernels of a complementary subgraph $M_2 = M \setminus M_1$ being $b(j_\lambda, k)$ from the right ones- $b(j_\rho, n)$ . One must also take into account that there can be no left or right kernels:

$$C_M^{(1)} = M_1 \otimes b + b \otimes M_1 + b \otimes M_1 \otimes b. \tag{A.3}$$

2. Inside $M$ the right subgraph $K$ is contracted. The contribution from the left part of the graph ( which cannot be asymptotical due to the superfluous factor $L + L_S$ in (4.6) ) is $C(j_\lambda)$ . Hence

$$C_M^{(2)} = c\,(K \otimes b + K). \tag{A.4}$$

Taking into account that $b = O(g_0^2)$ and leaving the senior power in $g_0^2$ , we get

$$C_M = M_1 \otimes b + b \otimes M_1 + cK. \tag{A.5}$$

In more detail

$$\tau\,\widetilde{M}(j_\lambda, j_\rho, \mathcal{J}) = \sum_{n=0}^{\infty} \frac{\widetilde{M}(j_\lambda, j_\rho+n, \mathcal{J})}{n!}\, b(j_\rho, n) + \sum_{k=0}^{\infty} \frac{\widetilde{M}(j_\lambda+k, j_\rho, \mathcal{J})}{k!}\, b(j_\lambda, k) + \Gamma(-\ell_\lambda)\, c(j_\lambda)\, \widetilde{K}(j_\rho, \mathcal{J}), \tag{A.6}$$

where the notation $\tau = \mathcal{J} + 1 + 2\varepsilon_q$ is introduced.

For investigation of (A.6) one requires to know the function $K(j, \mathcal{J})$. The coefficient $C_K$ is formed by the contribution from a subgroup $K_1 \in K$ and by the contribution from $K \setminus K_1$, which equals $b(j_\rho, n)$. Consequently

$$\tau\,\widetilde{K}(j, \mathcal{J}) = \Gamma(1-\ell)\,R(j, \mathcal{J}) + \sum_{n=0}^{\infty} \frac{\widetilde{K}(j+n, \mathcal{J})}{n!}\, b(j, n) \tag{A.7}$$

The term $R(j, \mathcal{J})$ is due to the minimal $t_1 t_2 t_3 t_4$ - subgraph. The straightforward calculations give in the lowest order in $g_0^2$ the values for $b$, $\varepsilon_q$, $R(j, \mathcal{J})$; $c$:

$$b(j,n) = -Z g_0^2 \frac{\Gamma(j+1)}{\Gamma(j+n+3)} \qquad \varepsilon_q = \frac{|Z| g_0^2}{2} \tag{A.8}$$

$$R(j,\mathcal{J}) = \frac{Z g_0^2}{j+1} \qquad c(j,\mathcal{J})\Big|_{\mathcal{J}=-1} = -\frac{Z g_0^2}{j+1}, \tag{A.9}$$

where $Z$ =1 for zero spin gluons and $Z$ =-2 for vector gluons. From equation (A.7) and equalities (A.8),(A.9) there follows the equation:

$$\tau(j+1)(j+2)F(j,\mathcal{J}) = \frac{\Gamma(1-\ell)}{\Gamma(j+2)} - Z g_0^2 \sum_{n=0}^{\infty} F(j+n,\mathcal{J}), \tag{A.10}$$

where $F(j,\mathcal{J}) = \dfrac{\tilde{K}(j,\mathcal{J})}{Z g_0^2 \Gamma(j+3)}$.

Since $\Gamma(1-\ell)/\Gamma(j+2) = 1 + O(g_0^2)$ in the neighbourhood of the point $\tau$ =0, equation (A.10) can be simplified:

$$(j+1)(j+2)F(j,\mathcal{J}) = \frac{1}{\tau} + \nu(\nu+1)\sum_{n=0}^{\infty} F(j+n,\mathcal{J}), \tag{A.11}$$

where $\nu(\nu+1) = -\dfrac{Z g_0^2}{\tau} = \varkappa$ . From equation (A.11) there follows the recurrent relation

$$(j+3)(j+2)F(j+1,\mathcal{J}) = [(j+2)(j+1) - \nu(\nu+1)]\, F(j,\mathcal{J}) \tag{A.12}$$

from which one can obtain the solution for F:

$$F(j,\mathcal{J}) = \frac{\Gamma(j-\nu+1)\,\Gamma(j+\nu+2)}{\Gamma(j+2)\,\Gamma(j+3)} f(j,\mathcal{J}) \equiv$$

$$\equiv \mathcal{P}(j,\varkappa)\, f(j,\mathcal{J}). \tag{A.13}$$



Moreover $f(j,\mathcal{J}) = f(j+1,\mathcal{J})$ . From (A.11) and the Dugoll formula ( /9/ , § 14) it follows that for integer $j$ ( which we are interested in) $f(j,\mathcal{J}) = \frac{1}{\tau}$. The function $\mathcal{P}(j,\varkappa)$ for integer $j$ satisfies the equation

$$\frac{\varkappa}{(j+1)(j+2)} \sum_{n=0}^{\infty} \mathcal{P}(j+n,\varkappa) = \mathcal{P}(j,\varkappa) - \frac{1}{(j+1)(j+2)} \tag{A.14}$$

and is a moremorphic function of a parameter $\varkappa$ , having the poles at $\varkappa = (j+n+1)(j+n+2)$, n=0,1,2 ... . Equation (A.6) with account of (A.8), (A.10), (A.13) and the fact that $\ell \simeq -1-j$ can be rewritten in the form

$$\Phi(j_\lambda, j_\rho, \tau) - \frac{\varkappa}{(j_\rho+1)(j_\rho+2)} \sum_{n=0}^{\infty} \Phi(j_\lambda, j_\rho+n, \tau) - \frac{\varkappa}{(j_\lambda+1)(j_\lambda+2)} \sum_{k=0}^{\infty} \Phi(j_\lambda+k, j_\rho, \tau) = -\frac{\varkappa^2}{(j_\lambda+1)^2(j_\lambda+2)}, \tag{A.15}$$

where

$$\Phi(j_\lambda, j_\rho, \tau) = \Gamma(j_\lambda+3)\,\Gamma(j_\rho+3)\,\tilde{M}(j_\lambda, j_\rho, \mathcal{J}).$$

The solution of (A.15) is searched to be the sum of two terms:

$$\Phi(j_\lambda, j_\rho, \tau) = \varkappa \frac{\mathcal{P}(j_\rho,\varkappa)}{(j_\lambda+1)(j_\lambda+2)} + \Phi_1(j_\lambda, j_\rho, \varkappa) \tag{A.16}$$

$\Phi_1(j_\lambda, j_\rho, \varkappa)$ satisfies the equation which is symmetric with respect to the change $j_\lambda \leftrightarrow j_\rho$

$$\Phi_1(j_\lambda, j_\rho, \varkappa) - \frac{\varkappa}{(j_\rho+1)(j_\rho+2)} \sum_{n=0}^{\infty} \Phi_1(j_\lambda, j_\rho+n, \varkappa) - \tag{A.17}$$

$$- \frac{\varkappa}{(j_\lambda+1)(j_\lambda+2)} \sum_{k=0}^{\infty} \Phi_1(j_\lambda+k, j_\rho, \varkappa) = - \frac{\varkappa}{(j_\lambda+1)(j_\lambda+2)(j_\rho+1)(j_\rho+2)} .$$

The solution of (A.17) can be reprsented as a double integration in the space of complex variables $\varkappa_1, \varkappa_2$ over the hypersurface $\Sigma$, the poles of $\mathcal{P}(j, \varkappa_1)$, $\mathcal{P}(j, \varkappa_2)$ being inside it, the points $\varkappa_1 = 0$, $\varkappa_2 = 0$ and the ones satisfying the relation $\frac{1}{\varkappa} = \frac{1}{\varkappa_2} + \frac{1}{\varkappa_1}$ outside it:

$$\Phi_1(j_\lambda, j_\rho, \tau) =$$

$$= \frac{1}{(2\pi i)^2} \iint_\Sigma d\varkappa_1 d\varkappa_2 \, \frac{\mathcal{P}(j_\lambda, \varkappa_1)\, \mathcal{P}(j_\rho, \varkappa_2)}{\frac{1}{\varkappa_1} + \frac{1}{\varkappa_2} - \frac{1}{\varkappa}} \left(\frac{1}{\varkappa} - \frac{1}{\varkappa_1}\right)\left(\frac{1}{\varkappa} - \frac{1}{\varkappa_2}\right). \tag{A.18}$$

One can easily verify the formula (A.18), substituting it into the equation (A.17) and taking into aocount that

$$\mathcal{P}(j, 0) = [(j+1)(j+2)]^{-1} \quad \text{and} \quad \int_\Gamma \mathcal{P}(j, \varkappa_2)\, d\varkappa_2 = 0 ,$$

where $\Gamma$ is the integration contour remaining after the integration over $\varkappa_1$

Collecting together all the terms entering the (A.1) one can obtain the expression for the asymptotic behaviour of form factor:

$$F_{(\pi)}(Q^2) = \left(-\frac{Q^2}{m_q^2}\right)^{-\alpha(M^2) - 2\varepsilon_q} \frac{\varphi^2(\alpha, Q^2)\,\alpha!}{-Q^2 \alpha'(M^2)(\alpha+1)\{g_0^2 \tilde{u}'[\alpha(M^2)]\}} .$$

$$\cdot \sum_{n=0}^{\alpha} \frac{(2\alpha - n + 1)!}{n!\, [(\alpha-n)!]^2} , \tag{A.19}$$



where

$$\varphi(\alpha, Q^2) = \frac{1}{2\pi i} \int (-Q^2)^\tau \frac{d\tau}{\tau}\, \xi(\alpha, \tau) \tag{A.20}$$

and

$$\xi(\alpha, \tau) = \sum_{n=0}^{\infty} \frac{\Gamma(\alpha+n+3)}{n!} \beta(n, \alpha, M^2)\, \mathcal{P}\left(\alpha+n, -\frac{Z g_0^2}{\tau}\right). \tag{A.21}$$

To derive (A.19) the equality

$$\sum_{k=0}^{\infty} \frac{\Gamma(\alpha+k+1)}{\Gamma(\alpha+1)\, k!} \beta(k, \alpha, M^2) = \gamma(\alpha, M^2) \tag{A.22}$$

is to be used. (A.22) follows from $j + \ell = -1$ and from the fact that

$$\gamma(j, s) = \int \prod_\sigma d\alpha_\sigma \left(\frac{R+R_s}{D}\right)^{-j-1} \zeta_j(\alpha_\sigma, s)\ ; \beta(k, j, s) = \int \prod_\sigma d\alpha_\sigma \left(1 - \frac{R+R_s}{D}\right)^k \zeta_j(\alpha_\sigma, s),$$

where $\zeta_j(\alpha_\sigma, s)$, some known function. The function $\mathcal{P}(\alpha, -\frac{Z g_0^2}{\tau})$ has the poles, aocumulating at $\tau = 0$. The same one can say about $\xi(\alpha, \tau)$. In the parton region $g_0^2 \ln(-Q^2) \ll 1$ we can restrict ourselves to consider the lowest approximation $\mathcal{P}(j, \varkappa) \approx [(j+1)(j+2)]^{-1}$. Then $\varphi(\alpha, Q^2) = \varphi(\alpha)$ and from (A.19) it follows that

$$F_{(\pi)}(Q^2, \alpha(M^2)) = \frac{k(\alpha)}{-Q^2 \alpha'(M^2)} \left(-\frac{m_q^2}{Q^2}\right)^{\alpha(M^2)}, \tag{A.23}$$

where $k(\alpha) = O(1)$.

References:

1. V.A.Matveev, R.M.Muradyan, A.N.Tavkhelidze. Lett. Nuovo Cim., 7, 719 (1973).

2. A.V.Efremov, I.F.Ginzburg. Fortschr.d.Physik, 22, 575 (1974).

3. A.V.Efremov, Yad.Fiz. 19, 196 (1974); JINR, E2-7864, Dubna (1974).

4. S.Mandelstam. Proc.Roy.Soc., A233, 248 (1955).

5. R.N.Faustov et al. JINR, E2-8126, Dubna (1974).

6. N.N.Bogolubov, D.V.Shirkov, Introduction to the Theory of Quantized Fields, Nauka, Moscow, 1973.

7. I.F.Ginzburg, A.V.Efremov, V.G.Serbo. Yad.Fiz., 9, 451 (1969).

8. C.Chang et al. Preprint MSU/CSC-23 (1975).

9. H.Bateman, A.Erdelyi, Higher transcendental Functions. McGraw-Hill, New York, 1953.

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