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POSSIBLE MANIFESTATION OF LONG RANGE FORCES IN HIGH-ENERGY HADRON COLLISIONS

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1 Introduction

It is well known since long time that for the description of the high energy peripheral scattering [1] [1] it is necessary to use the impact representation according to which the scattering amplitude may be expressed in the form

$$\begin{aligned} A(s,t) &= i\, s\, F^{AA'BB'}(q), \qquad s=(p_A+p_B)^2, \\ t &= q^2=(p_A-p'_A)^2 \simeq -\boldsymbol{q}^2, \quad \boldsymbol{q}^2 \ll s. \end{aligned} \tag{1}$$

The quantity $F(q)$ may be expressed in terms of a 2-dimensional integral on the transverse (with respect to the initial particles momenta in the c. of m. frame $\boldsymbol{p}_A = -\boldsymbol{p}_B$) components of the virtual photons (gluons) exchanged by particles A and B of the product of *impact factors* Φ^{AB} assigned to each of the incoming particles.

$$F^{AA'BB'}(\boldsymbol{q}) = \frac{1}{(2\pi)^2}\int \frac{d^2k}{\boldsymbol{k}^2(\boldsymbol{q}-\boldsymbol{k})^2}\Phi^{AA'}(\boldsymbol{k},\boldsymbol{q})\Phi^{BB'}(\boldsymbol{k},\boldsymbol{q}), \tag{2}$$

where $\Phi^{(a,b)}(\boldsymbol{k},\boldsymbol{q})$ are the impact factor of the colliding hadrons.

The universal quantities represented by the impact factors Φ_i with $i=A,B$ describe, in a quantum mechanical sense, the probability of finding any possible state of one hadron i surrounded by a cloud of quark pairs and gluons. These states, due to a Lorentz boost, live long and the virtual components of each hadron become real on-shell particles due to the interaction of the hadrons and manifest themselves as hadron jets moving along the directions close to the initial particles. Such quantities may be computed by means of perturbation theory.

The form of the scattering amplitude in eq.(2) corresponds to the minimal number of exchanged vector mesons (two) in the scattering channel. By taking into account the exchange of particles like fermions, scalars would lead to contributions to the scattering amplitude which, at large s, would behave as s^α with $\alpha < 1$ and therefore to less dominant ones. The requirement of gauge invariance on the two Compton scattering blobs involving the two exchanged photons (gluons) for each particle A and B impose [1] the property for the impact factors

$$\Phi(\boldsymbol{k},\boldsymbol{q})|_{\boldsymbol{k}=0} = \Phi(\boldsymbol{k},\boldsymbol{q})|_{\boldsymbol{k}=\boldsymbol{q}} = 0. \tag{3}$$

This property has the important effect of providing the infrared convergence of the expression for $F(q)$ and, in particular, the finiteness of $F(0)$. The quantity $F(0)$ can be related to the cross-section for two jet production in high-energy hadron collisions when one vector particle, photon or gluon, is exchanged (see. Fig.1) (optical theorem).

[1] Differently from hard and diffractive.

$$\text{Im } A^{AA,BB}(s,0) = s\sigma(s)_{tot}^{AB\to jet\ jet}. \quad (4)$$

In the lowest order of perturbation theory the total cross-section $\sigma(s)_{tot}^{AB\to jet\ jet}$ does not depend [2] on s. By taking into account the first-order of perturbation theory radiative corrections leads [22] to a contribution to σ_{tot} proportional to $\frac{\alpha_s}{\pi}\ln\frac{s}{m^2}$ and, as was shown in the papers [20, 21] with higher orders of p.t. taken into account will produce unitarity-violation results for the cross section i.e. $\sigma(s) \simeq \sigma_0(\frac{s}{m^2})^{\omega_0}$ with $\omega_0 = \frac{4N}{\pi}\alpha_s \ln 2$. Let us recall here that the natural hypothesis of short range forces between hadrons leads to a so-called Froissart restriction on the growth of the total cross-section of high energy hadron-hadron collisions $\sigma(s) < m^{-2}(\ln\frac{s}{m^2})^2$. We will not discuss here the problem of restoring the unitarity (see for example [24]).

Our note concerns the form of the q^2 corrections to the scattering amplitude. One may see that the property of the impact factors will not provide the infrared convergence of the terms of order $(q^2)^n$, $n = 1, 2, 3, \ldots$ in the series expansion of $F(q)$ at $q^2 = 0$.

$$F(q) \simeq F(0) + \frac{q^2}{m^2}(F_1 + F_2 \ln\frac{-q^2}{m^2}) + \cdots. \quad (5)$$

The aim of this paper is to attract the attention to the logarithmic $\ln -q^2/m^2$ term appearing in eq.(5) in the low q^2 expansion of F. m represents the charachteristic hadron mass or the constituent quark mass. Strictly speaking we see that the Taylor expansion, due to an logarithmic singularity for small q^2 is not possible. The consequence of the presence of this singularity is the appearance of the long range forces (LRF) between hadrons namely by considering the Fourier transform of the formula.

As well as at peripheral high-energy kinematics of scattering the transverse momentum effectively 2-dimensional $q^2 = -\boldsymbol{q}^2 < 0$ we may formulate the consequences of the presence of terms of order $\boldsymbol{q}^2 \ln\frac{\boldsymbol{q}^2}{m^2}$ in the scattering amplitude in terms of impact parameter of the scattering (distance ρ between hadrons in the scattering point). The Fourier transform of (5) gives [25]:

$$\int d^2\rho e^{i\boldsymbol{q}\cdot\boldsymbol{\rho}} \simeq 2\pi\delta^2(\boldsymbol{\rho}), \quad (6)$$
$$\lim_{\epsilon\to 0}\int d^2q e^{i\boldsymbol{q}\cdot\boldsymbol{\rho} - |q|\epsilon}|q|^{2\nu} = (\frac{4}{\rho^2})^{1+\nu}\Gamma^2(1+\nu)\sin\pi\nu.$$

(here and further we denote by q the module of two-dimension vector $\boldsymbol{q}$) one may conclude the presence of a potential between the hadrons acting in the transverse plan of the form:

[2]The total cross section remains finite when energy goes to infinity and includes the production of two small transverse momentum jets.

$$V(\rho) \simeq \int d^2q q^2 e^{i\boldsymbol{q}\cdot\boldsymbol{\rho}} \ln\frac{q^2}{m^2} = \frac{16\pi}{\rho^4}. \quad (7)$$

This potential may produce long range forces between the hadrons in a similar way as they arise in the interactions of neutral objects [7] (for review see [8]).

The similar nature leads to singular potential for interactions of neutral atoms. We may consider this term $\frac{16\pi}{|\rho|^4}$ as the potential of the interaction in the transverse plane.

Below we compute the quantities $F(0)$ and F_2 for different processes. We discuss here two additional topics. The first one concerns the influence of the confinement phenomenon. At small q^2 the coupling constant becomes large [9] and all the considerations based on a perturbative approach are not valid any more. Nevertheless the functional form of the amplitude Eq.(5) will remain the same and the value of the coupling constant will not influence the analytical properties of the scattering amplitude in the q^2 plane. The fact of the presence of the $q^2\ln\frac{-q^2}{m^2}$ term may be shown in any number of loop-correction to Compton scattering of gluon on hadron A, B. The essential point here is the colorless exchange in the scattering channel provided by the two gluon state. Supposing the colorless initial and final particles A, A' the only possible combination of color state parameters a, b of the gluon fields A^a_μ and A^b_ν describing the Compton scattering amplitude $Ag^{(a)} \to A'g^{(b)}$ is of the form $trt^at^b = \frac{1}{2}\delta^{ab}$ which corresponds to colorless state of the two gluon scattering channel state This fact allows us to consider gluons as massless particles, which is a main feature of the QCD theory. Let us notice that, by assuming the non zero gluon mass, as is often assumed in phenomenological applications [10], would lead to the disappearance of the long range force effect. Really, by replacing the denominators in eq.(2) by the substitution $\boldsymbol{k}^2 \to \boldsymbol{k}^2 + \mu^2; (\boldsymbol{q}-\boldsymbol{k})^2 \to (\boldsymbol{q}-\boldsymbol{k})^2 + \mu^2$ would result in an absence of contributions proportional to $\ln\frac{\boldsymbol{q}^2}{m^2}$. A colorless state of any number of gluons $n \geq 2$ may be realized and manifest itself in the scattering of colorless states $AA'BB'$.It is necessary to note that QCD cannot be build in terms of massive gluons.

The second point we want to mention is the relation with the phenomenon of the logarithmic branch-point in the q^2 plane with the low-energy theorem [11]. According to the theorem the Compton scattering amplitude as a function of the photon energy ω for small values of ω may be expanded in a Laurent-type series and the first terms of this expansion may be expressed in terms of charge, electrical dipole moments and of the value of the anomalous magnetic moment of the target on which the photon is scattered. Radiative corrections to the Compton scattering amplitude [12] contain the terms of the form $\omega^2\ln\frac{\omega}{m}$ the presence of which points out the logarithmic branch point at $\omega = 0$ and breaks the possibility to expand the Compton scattering amplitude around the point $\omega = 0$. It is interesting to note that a contribution of this form will result in a non-trivial time-dependence of the dynamics of the Compton scattering process:

$$\int d\omega e^{i\omega t}\omega^2 \ln\frac{\omega}{m} \simeq t^{-3}.$$

Corrections to the Compton scattering amplitude due to the emission of massless virtual photons give small contributions of the order $\frac{\alpha}{\pi}$ to the scattering amplitude. The main contributions of this type for the Compton scattering on the proton arise from the emission of virtual massive pions and vector mesons and will not contain such contributions which diverge logarithmically at small photon energies. This fact provides the validity of the Low theorem and the possibility of testing it in experiments.

Here we will not touch the discussion of manifestation of LRF in deviations from Newton gravity, charmonium physics, and other topics, refering to the series of papers [15].

We discuss in this paper a possibilities provided by high energy hadron collisions experiments. Really, compared with the nonrelativistic case the contributions to the hadron-hadron scattering amplitude from exchange by vector mesons,pions,when the reggeization effect is taken into account becomes negligible compared with the ones provided by Pomeron exchange. The Pomeron can not be associated with any observed hadron.More of that,as was shown in the paper [4] that it have rather square root singularity in the angular momentum plane, associated with the massless colorless state of gluons.

The existence of the LRF are thus is the a consequence of the massless nature of the Pomeron (soft version of Pomeron is relevant here). Will be good to verify this possibility on experiment.

The paper is organized as follows: In the second part we describe polarized photon–photon scattering, and the consequent attraction forces due vacuum polarization fluctation. The explicit dependence on polarization is obtained. In section 3 we will consider pion-pion scattering through their dissociation into quarks and the proton-proton scattering in the phenomenological impact factor representation. In section 4 we will discuss the generalization of our resuts to higher orders QCD perturabation theory (PT).

The possibility of experimental observation of long-range forces effects,the region of applicability of LRF phenomenon, the problem of existing the massless colorless gluon states are discussed in the concluding section.In appendices we give some details of calculations,and put some numerical estimates.

2 Photon-photon scattering

The explicit expression of the impact factor of the photon was obtained in QED in [1]. Using it and applying the obvious changes for the case of gluon exchange we obtain:

$$\Phi^{\gamma(e_A)\gamma(e_{A'})}(k,q) = -2\alpha_s^2\int_0^1 dx\int_0^1 dy \times\left\{\frac{(8XYe_A\cdot q e_{A'}\cdot q - q^2 e_A\cdot e_A(1-2Y(1-4X)))y^2}{m^2+Xy^2q^2} - \frac{8XYe_A\cdot Q e_{A'}\cdot Q - Q^2 e_A\cdot e'_A(1-2Y(1-4X))}{m^2+XQ^2}\right\} \quad (8)$$

where $X=x(1-x)$, $Y=y(1-y)$ and $Q=k-qy$. e_A, $e_{A'}$ are the polarization vectors of the initial and final photons. One may see explicitely that (8) regards the properties (3). In the case of QCD (the exchange of two gluons is to be considered) an addictional factor appears for the color group SU(N): $\Phi^{\gamma\gamma}_{QCD}=\Phi^{\gamma\gamma}_{QED}\cdot Tr(t^a t^b) = 1/2\delta^{ab}\Phi^{\gamma\gamma}_{QED}$; $a,b=1\cdots N^2-1$.

The straightforward but tedious calculations using the expansion:

$$\frac{\Phi_1(k,q)\Phi_2(k,q)}{k^2(k-q)^2} = \frac{1}{(k^2)^2}(\Phi_1^{(0)}(k)+q\cdot\Phi_1^{(1)}(k))(\Phi_2^{(0)}(k) \quad (9)$$

$$+q\cdot\Phi_2^{(1)}(k))(1+\frac{2q\cdot k}{k^2}-\frac{q^2}{k^2}+\frac{4(q\cdot k)^2}{(k^2)^2}) = \frac{\Phi_1^{(0)}(k)\Phi_2^{(0)}(k)}{(k^2)^2}$$

$$+\frac{1}{(k^2)^2}\Big[q\cdot\Phi_1^{(1)}(k)q\cdot\Phi_2^{(1)}(k)+(\Phi_1^{(0)}(k)q\cdot\Phi_2^{(1)}(k)$$

$$+(\Phi_2^{(0)}(k)q\cdot\Phi_1^{(1)}(k))\frac{2q\cdot k}{k^2}+\Phi_1^{(0)}(k)\Phi_2^{(0)}(k)(\frac{4(q\cdot k)^2}{(k^2)^2}-\frac{q^2}{k^2})\Big] \quad (10)$$

and the subsequent averaging by azimuthal angle lead to the result for the scattering amplitude:

$$A^{\gamma(e_A)\gamma(e_B)\to\gamma(e_{A'})\gamma(e_{B'})} = \frac{isC^2}{2\pi m_1 m_2}\{I_1(\lambda)\alpha+I_2(\lambda)\beta$$

$$+\frac{q^2}{m_1 m_2}\ln\frac{q^2}{m_1 m_2}[C_1\alpha+C_2\beta+C_3\gamma+C_4\delta]\} \quad (11)$$

where $C=2\alpha^2$, (QED); $C=2\alpha\alpha_s(\frac{N^2-1}{4})$, (QCD); $\lambda=\frac{m_1}{m_2}$, $C_1=-\frac{290}{1944}$, $C_2=-\frac{5}{1944}$, $C_3=-\frac{4}{1944}$ and $C_4=\frac{2}{1944}$.

The quantities α, β, γ, δ represents the combinations of polarization vectors:

$$\begin{aligned}\alpha &= e_A\cdot e_{A'}e_B\cdot e_{B'} \quad (12)\\ \beta &= e_A\cdot e_{B'}e_B\cdot e_{A'}+e_{A'}\cdot e_B e_{B'}\cdot e_A\\ \gamma &= e_A\cdot e_{A'}n\cdot e_B n\cdot e_{B'}+e_B\cdot e_{B'}n\cdot e_A n\cdot e_{A'}\\ \delta &= e_A\cdot e_B n\cdot e_{A'}n\cdot e_{B'}+e_{A'}\cdot e_{B'}n\cdot e_A n\cdot e_B\\ &+ e_A\cdot e_{B'}n\cdot e_{A'}n\cdot e_B+e_{A'}\cdot e_B n\cdot e_A n\cdot e_{B'}\end{aligned}$$

where $\boldsymbol{n} = \frac{\boldsymbol{q}}{q}$ and the functions $\phi_{1,2}$ of the ratio of masses of the quark (lepton) pairs $\lambda = \frac{m_1}{m_2}$ are:

$$I_{1,2}(\lambda) = \frac{1}{\lambda}\int_0^\infty \frac{dx}{x^3}\Phi_{1,2}(x,\tilde{x})\;;\tilde{x} = x\lambda, \tag{13}$$

where

$$\Phi_1 = \frac{2x^2\tilde{x}^2}{9}[2(h_1+h_2)(\tilde{h}_1+\tilde{h}_2) - h_2\tilde{h}_2], \tag{14}$$
$$h_{1,2} = h_{1,2}(x), \qquad \tilde{h}_{1,2} = h_{1,2}(\tilde{x}),$$

$$\Phi_2 = \frac{2}{9}x^2\tilde{x}^2 h_2\tilde{h}_2, \tag{15}$$

and

$$h_1(x) = \frac{2}{x\sqrt{4+x^2}}L, \quad h_2(x) = \frac{1}{x^2} - \frac{2}{x^3\sqrt{4+x^2}}L, \tag{16}$$
$$L = \ln\frac{\sqrt{4+x^2}+x}{\sqrt{4+x^2}-x}.$$

Functions $I_{1,2}(\lambda)$ are tabulated in the Table.

The first term in parentheses (11) may be computed at $\lambda = 1$ and $e'_A = e^*_A$, $e'_B = e^*_B$ with the known result of the calculation of the cross section of creation of two lepton pairs from two polarized photons [13]:

$$\sigma^{\gamma\gamma\to 2e2\bar{e}} = i\frac{\alpha^4}{\pi m^2}\{\frac{175}{36}\xi_3 - \frac{19}{18} + [(e_A\cdot e_B)^2 - \frac{1}{2}][\frac{7}{18}\xi_3 - \frac{1}{3}]\}. \tag{17}$$

One may see from table that (11) and (17) are on numerical accordance at $\lambda = 1$.

Note that the right hand part of equation (11) for the case $\lambda \neq 1$ is to be multiplied by a factor 2 which takes into account the possibility to replace pairs of quarks (or leptons) ($m_1 \leftrightarrow m_2$).

3 Pseudoscalar mesons small angle scattering. Phenomenological approach

Let un now consider the pseudoscalar mesons high energy scattering when they belong to the nonet of pseudoscalar. Each one of the mesons dissociates into $q\bar{q}$ pairs (see Fig.2,b). The coupling constants wich describe such a dissociation,g_π,may be found using the Goldberger-Treiman relation: $g_\pi = \frac{m_u}{F_\pi} = 3$.For kaon desintegration $g_K = \frac{m_u+m_s}{2F_K} = 3.3$. The amplitude will have the form (1,2):

$$A(s,q) = \frac{is}{4\pi^2}\int\frac{d^2\boldsymbol{k}}{\boldsymbol{k}^2(\boldsymbol{q}-\boldsymbol{k})^2}\Phi_1(\boldsymbol{q},\boldsymbol{k})\Phi_2(\boldsymbol{q},\boldsymbol{k}) \tag{18}$$

The one loop Feynman diagrams describing the Compton scattering of virtual gluons on quarks are drawn in the figure 1.Pion desintegration impact factor have a form(see Appendix A):

$$\Phi_{1,2}(\boldsymbol{k},\boldsymbol{q}) = -c_\pi f_\pi, \quad c_{\pi_{1,2}} = \frac{\alpha_s g^2_{\pi_{1,2}}}{\pi}\sqrt{\frac{N^2-1}{4N^2}}, \tag{19}$$
$$f_\pi = \int_0^1\int_0^1 dxdy\frac{C(x,y)(\boldsymbol{k}^2 - \boldsymbol{k}\cdot\boldsymbol{q})}{(1-X)[(1-X)m^2_{1,2} + Y\boldsymbol{k}^2]},$$

where $X = x(1-x)$, $Y = y(1-y)$ and $C(x,y) = 1/2(1-X) + XY$; $N = 3$ is the number of colors. Factor $\frac{N^2-1}{4N^2}$ results from traces on color matices of group SU^c_3, $m_i, i = 1,2$ are the constituent masses of the quarks.

Substituting $\Phi_{1,2}$ into the expression for the scattering amplitude given above 2 and expandinbg in the small q limit we obtain

$$A(s,q) = \alpha_s^2 g^2_{\pi_1} g^2_{\pi_2}\frac{is}{72\pi^3 m_1 m_2}(I(\lambda) + \frac{C_1^2\boldsymbol{q}^2}{2m_1m_2}\ln(\frac{\boldsymbol{q}^2}{m_1m_2})), \tag{20}$$

where $C_1 = \frac{1}{9} + \frac{8\pi}{27\sqrt{3}}$ and the function $I(\lambda)$ (tabulated in Table) is presented in Appendix A.

Again the first term in the parentheses (20) describes the four jet production cross section in $\pi\pi$ scattering and the second term is responsible for LRF.

Consideration of the proton-proton and of the pion proton scattering cannot be treated analytically since the impact factor for the proton is unknown. Not even in the quark model. In this case we may use a phenomenological approach [14]. Let us define

$$\Phi(\boldsymbol{k},\boldsymbol{q}) = F(\frac{\boldsymbol{q}^2}{4}) - F((\boldsymbol{k} - \frac{\boldsymbol{q}}{2})^2), \tag{21}$$

with the evident fullfillment of the property $\Phi(0,\boldsymbol{q}) = \Phi(\boldsymbol{q},\boldsymbol{q}) = 0$.

A realistic choice is $F_i(k^2) = \frac{1}{1+c_ik^2}$ [14], where the quantities c_i may be expressed in terms of the charge radius of the hadron ρ: $c = 1/6 < \rho^2 >$.

A straightforward calculation gives:

$$\frac{1}{\pi}\int\frac{d^2\boldsymbol{k}\Phi_1\Phi_2}{\boldsymbol{k}^2(\boldsymbol{q}-\boldsymbol{k})^2} = c_1c_2(\frac{\ln(c_1/c_2)}{c_1-c_2} + \frac{1}{2}\boldsymbol{q}^2\ln(\frac{\boldsymbol{q}^2}{m_1m_2})). \tag{22}$$

Comparision of (22) and (20) for the case of pion-pion scattering with the value of charge radius $\rho = 0.7fm$ give $\alpha_s = 0.2$. This quantity do not contradict to the recent calculations [9], where was obtained $\alpha_s(0) = 1.4$ taking into account the rapid decrease of $\alpha_s(k^2)$ and the fact that essential values of momenta are $\sqrt{<k^2>} = 300MeV$.

4 Higher Orders of Perturbation Theory

A natural questions is how these results can be modified by higher order QCD corrections. In the paper of Balitski and Lipatov [2] it was obtained in first order QCD corrections of scattering photon-photon amlitude. Their resusult may be expressed in terms of impact factor in rather cumbersome form, wich nevertheless was shown to be consinstent with general BFKL approach [3, 4].

For the case of $\gamma\gamma$ scattering in the paper by [2] was calculated the first order of PT QCD correction.

$$A^{(1)}(s,q) = is\frac{3\alpha_s}{2}\ln\frac{s}{m_1m_2}I^{(1)AA'BB'}(q), \tag{23}$$

where

$$I^{(1)AA'BB'}(q) = \int\frac{d^2k}{\pi}\int\frac{d^2k_1}{\pi}\frac{\Phi^{AA'}(k,q)}{k^2(k-q)^2}\Big[-q^2\frac{\Phi^{BB'}(k_1,q)}{k_1^2(k_1-q)^2}$$
$$+\frac{\Phi^{BB'}(k_1,q)}{k_1^2(k_1-q)^2}\frac{k^2(k_1-q)^2+k_1^2(k-q)^2}{(k_1-k)^2}$$
$$-\frac{\Phi^{BB'}(k_1,q)}{(k-k_1)^2}\Big[\frac{k^2}{k_1^2+(k_1-k)^2}+\frac{(k-q)^2}{(k_1-q)^2+(k_1-k)^2}\Big]\Big]. \tag{24}$$

Using the property of the photons impact factor $\Phi^{AA'}(k,q) = \Phi^{AA'}(q-k,q)$ and making the substitution $k' \to k-k'$ one may see that the sum of the second and third term in the curly brackets contains the impact factors in the combination $\Phi^{BB'}(k-k',q)+\Phi^{BB'}(k',q)-\Phi^{BB'}(k,q)$. From this one may see that this term does not contain the contribution of the form $q^2\ln^2\frac{q^2}{m^2}$. Such a type of contributions arise from the first term in curly brackets which have a factorized form. Using for $\Phi^{AA'}(k,q)$ the approximate expression by neglecting the q^2 terms

$$\begin{aligned}\Phi^{AA'}(k,q) &= \frac{c_1}{3m_1^2}\Big[-4e_A\cdot e_{A'}(k^2-k\cdot q)+e_A\cdot ke_{A'}\cdot k \\ &- \frac{1}{2}(e_A\cdot ke_{A'}\cdot q+e_{A'}\cdot ke_A\cdot q)\Big],\end{aligned} \tag{25}$$

where $c_1 = \frac{4}{3}\alpha\alpha_s\sqrt{\frac{N^2-1}{4}}$, we obtain that the integral of $\Phi^{CC'}$ is

$$\int\frac{d^2k}{\pi}\frac{\Phi^{AA'}(k,q)}{k^2(q-k)^2} = \frac{7}{6}c_1(e_A\cdot e_{A'})\ln\frac{q^2}{m_1^2}, \tag{26}$$

and the relevant contribution to $I^{(1)AA'BB'}(q)$ have the form

$$-\frac{q^2c_1^2}{m_1^2m_2^2}\ln^2\frac{q^2}{m_1m_2}\frac{49}{36}. \tag{27}$$

For the finite contributions in the limit $q^2 \to 0$ we obtain (see for details Appendix B)

$$\begin{aligned}2\int\frac{d^2k}{\pi k^2}\frac{d^2k'}{\pi(k-k')^2}\Phi^{AA'}(k,0)\Big[\frac{\Phi^{BB'}(k',0)}{k'^2}-\frac{\Phi^{BB'}(k,0)}{k'^2+(k'-k)^2}\Big] &= \\ = \frac{c_1^2}{m_1m_2}[f_1(\lambda)\alpha+f_2(\lambda)\beta].\end{aligned} \tag{28}$$

Functions $f_{1,2}$ are tabulated for different values of λ (see Table).

For the first order QCD correction to the $\gamma\gamma \to \gamma\gamma$ anmplitude we obtain

$$\begin{aligned}A^{(1)}_{\gamma\gamma\to\gamma\gamma}(s,q) = \frac{is}{m_1m_2}\frac{3\alpha_sc_1^2}{2}\ln\frac{s}{m_1m_2}[f_1(\lambda)\alpha+f_2(\lambda)\beta \\ -\frac{q^2}{m_1m_2}\frac{49}{36}\ln^2\frac{m_1m_2}{q^2}\beta]\end{aligned} \tag{29}$$

The similar calculation for the amplitude of the proton-proton scattering in the phenomenological ansatz (21) gives

$$A^{(1)}_{pp\to pp}(s,q) = is\frac{3\alpha_s}{2}\sqrt{c_1c_2}\ln\frac{s}{m_1m_2}(2H(\lambda)-q^2\sqrt{c_1c_2}\,\ln^2\frac{q^2}{m_1m_2}) \tag{30}$$

where $H(\lambda)$ is given in Appendix B. Deriving the last expression we use the same form for first order QCD corrections as for the $\gamma\gamma \to \gamma\gamma$ scattering.[3]

5 Conclusion

A lot of attention was paid to discussions of van-der Waals-like forces between hadrons [15], where mostly the nonrelativistic interaction of hadrons was considered. Accepting the existence of them as a consequence of nonanalyticity of the scattering amplitude at $t = 0$ and the possibility to create zero mass state in the scattering channel.Some rigorous restrictions on the strength of it was obtained considering deviation from Newtonian gravity and the Coulomb low of electromagnetism.

There is the general belief (which was not proved rigorously) that the colorless state of two (or more) gluons cannot be massless. Phenomenological applications to radiative J/Ψ decay for example will be successful considering gluons similarly to quarks as a constituent objects with mass $m_g = 800MeV$. We agree that point of view for the cases when the invariant mass square of gluons is positive. For the case of its negative values we do not see the grounds for considering gluon as a massive object. This fact may be recognized as the fact of absence of the vacuum polarization amplitudes in any order of PT. As a consequence one cannot obtain

[3] L. Lipatov, private communication.

the Breit-Wigner type formula for the Green function,describing the colorless state of gluons with the negative invariant mass square.

Contrary, the Pomeron concept, which provides the non-vanishing cross sections of the high-energy hadron collisions is more consistent with the interpretation of it as a result of interaction of a massless fields similar to Coulomb field [5]. In this way we may consider LRF as a halo of a Pomeron.

As we already mentioned above the presence of a branch point in the q^2 plane of the scattering amplitude at $q^2 = 0$ may be proved in any order of PT.

Furthermore from the analysis of the radiative corrections of lowest order we may expect a result of the form:

$$A(s,q^2) = \frac{is}{m_1m_2}[F_1(\frac{s}{m^2}) + \frac{q^2}{m_1m_2}\ln\frac{q^2}{m_1m_2} \times F_2(\frac{\alpha_s}{\pi}\ln\frac{s}{m^2}\ln\frac{q^2}{m^2})] = A + \Delta A. \quad (31)$$

The simplest choice $F_2(z) = R\exp(Cz)$ give:

$$\Delta A = iR\frac{s}{m_1m_2}\cdot\frac{q^2}{m_1m_2}\ln\frac{q^2}{m_1m_2}\cdot(\frac{q^2}{m_1m_2})^{C\frac{\alpha_s}{\pi}\ln\frac{s}{m^2}} \quad (32)$$

for the case $-C\frac{\alpha_s}{\pi}\ln\frac{s}{m^2} \simeq 1$ their contribution to the scattering amplitude at $q^2 \to 0$ will be essential and may be the reason for the structure in $d\sigma_{el}/dt$ at small $-t \simeq 10^{-3}Gev^2$ [16] [4].

The coefficient before $q^2\ln(q^2/m^2)$ therms may be (at least in a phenomenological approach) expressed in therms of charge radii of hadrons or their polarizabilities (similarly in the Casimir–Polder effect [7]) (see discussion after (22)).

In the case of gluon exchange one may speak about the color polarizabilities of hadrons which differs from the corresponding quantities in QED by the replacement $\alpha \to \alpha_s$ and some color factors.

In a recent review of Lipatov [17] was obtained general solution of the BFKL equations, in coordinate space using the conformal properties of it. Both our statements about ρ^{-4} character of additional terms in the scattering amplitude and the role of radiative corrections where confirmed.

As was pointed out in paper [14] the presence of $q^2\ln(q^2/m^2)$ therms may be interpreted as the existence of an additional slope of the differential cross section at $q^2 = 0$:

$$\frac{d\sigma^{\pi\pi}}{dq^2}/\frac{d\sigma^{\pi\pi}}{dq^2}(q^2=0) \simeq G_\pi^4(q^2)e^{-B_{eff}^{(0)}\cdot q^2}, \quad (33)$$
$$B_{eff}^{(0)}(q^2) = \frac{C}{2m^2}\ln\frac{m^2}{q^2}|_{q^2\to 0}$$

[4] V.A.Nikitin informed us that their analysis with the resolution in the transverse momentum square of order $10^{-4}GeV^2$ any structure is absent.

where $m \simeq \sqrt{6/<r_\pi^2>} \simeq 400Mev$ and $G_\pi(q^2)$ is the pion form factor. Factor $exp(-B_{eff}^{(0)}\cdot q^2)$ describes the dependence of quark-quark amplitude on the momentum transfer ($\frac{d\sigma^{qq}}{dq^2} \simeq e^{-Bq^2}$). following authors [14] we underline that it would be very interesting to measure experimentally the variation of $B_{eff}(q^2)$ over a wide range of $q^2 \simeq 0.4Gev^2$ ($B_{eff} \simeq 1Gev^{-1}$) to $q^2 \simeq 0.04Gev^2$ ($B_{eff} \simeq 4Gev^{-2}$) and to smaller $q^2 \leq m_\pi^2$.

One may expect the distances $\rho > 0.5fm$ may be considered as the minimal ones to search LRF. At distances of order 10 fermi the LRF, presumably will be modified due to complicate structure of vacuum.

Clearly,it is impossible to measure the effects of LRF (4) at distances of order of some fermi in hadrons collisions due to much bigger sizes of bunches.Nevertheless the heavy ultra-relativistic ions collisions may be more convenient for this aim. Really,the sizes of nucleus of order some fermi and the tail of one–pion exchange (considered as a main mechanism of nucleons interaction in the nucleus) between nucleons in different nucleus (see the paper R. Willey in [15]) will be suppressed due to peripherical kinematics by factor $\frac{m_\pi^2}{s}$.

We also note that the effect of infrared singularity of gluon propagators which produce $q^2\ln\frac{q^2}{m^2}$ terms may be also investigated in the process of diffraction dissociation $pp \to Xp$, $\pi p \to Xp$ and in the processes of inelastic photon-photon scattering of the kind $\gamma^*\gamma^* \to VV'$ [18].

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Appendix A

We give here some details of calculation of the pion impact factor. The pion-pion scattering amplitude is described by Feynman diagrams drawn in Fig.2. Using the Sudakov parametrization technique we decompose the loop momenta as

$$k = \alpha\tilde{p_2} + \beta\tilde{p_1} + k\ ,\ k_1 = \alpha_1\tilde{p_2} + \beta_1\tilde{p_1} + k_1\ ;\tilde{p}_2 = p_1 - p_2\frac{m_1^2}{s};$$
$$p_i^2 = 0; s = 2p_1p_2 \gg p_i^2 = m_i^2, d^4k = \frac{s}{2}d\alpha d\beta d^2k, \quad (34)$$

and substitute $g^{\mu\nu} = g_{\perp}^{\mu\nu} + \frac{2}{s}(p_1^\mu p_2^\nu + p_1^\nu p_2^\mu)$ in the scattering amplitude

$$M = i\int \frac{d^4k}{(2\pi)^4}\frac{1}{2}\frac{g^{\mu\nu}g^{\mu_1\nu_1}}{k^2(q-k)^2}T^{(1)ab}_{\mu\mu_1}T^{(2)ab}_{\nu\nu_1}. \tag{35}$$

Using the specific of the high energy forward scattering kinematics

$$\begin{aligned} &s = (p_1+p_2)^2 \gg -(p_1-p_1')^2 \simeq \boldsymbol{q}^2\ ;\ \beta_1 \simeq \mathcal{O}(1)\\ &k^2 \simeq -\boldsymbol{k}^2\ ;\ (k-q)^2 \simeq -(\boldsymbol{k}-\boldsymbol{q})^2,\\ &g^{\mu\nu} \simeq \frac{2}{s}p_2^\mu p_1^\nu. \end{aligned} \tag{36}$$

we obtain

$$\begin{aligned} M &= is\frac{1}{4\pi}\int \frac{d^2k}{\pi \boldsymbol{k}^2(\boldsymbol{q}-\boldsymbol{k})^2}\Phi_1(\boldsymbol{k},\boldsymbol{q})\Phi_2(\boldsymbol{k},\boldsymbol{q}) \\ \sqrt{4N^2(N^2-1)}\Phi_1(\boldsymbol{k},\boldsymbol{q}) &= \frac{1}{s^2}\int d(s\alpha)T^{(1)aa}_{\mu\mu_1}\ ;\ p_2^\mu p_2^{\mu_1}\\ \sqrt{4N^2(N^2-1)}\Phi_2(\boldsymbol{k},\boldsymbol{q}) &= \frac{1}{s^2}\int d(s\beta)T^{(2)}_{\nu\nu_1}p_1^\nu p_1^{\nu_1}. \end{aligned} \tag{37}$$

For definiteness we show the method of calculation of the contribution to Φ_1 from the diagrams in Fig.2(a,b). It may be written in the form:

$$\begin{aligned} \Delta\Phi_{1\,a,b} &\sim \int\frac{ds\alpha}{2\pi i}\int\frac{ds\alpha_1}{2\pi i}\int d\beta_1\int d^2k_1\frac{1}{s^2d_1d_2d_3}\\ &\times\left[\frac{1}{-s\alpha(1-\beta_1)+i0}+\frac{1}{s\alpha(1-\beta_1)+i0}\right]\\ &\times \mathrm{Tr}\,\gamma_5(p_1'-k_1+m)p_2(p_1-k_1-k+m)p_2(p_1-k_1+m)\\ &\times\gamma_5(-k_1+m), \end{aligned} \tag{38}$$

where the denominators of the internal quark propagators are

$$\begin{aligned} d_1 &= k_1^2-m^2+i0 = s\alpha_1\beta_1-\boldsymbol{k}_1^2-m^2+i0,\\ d_2 &= (p_1-k_1)^2-m^2+i0 = -s\alpha_1(1-\beta_1)-\boldsymbol{k}_1^2-m^2\beta_1^2+i0\ ;\\ d_3 &= (p_1'-k_1)^2-m^2+i0 = -s\alpha_1(1-\beta_1)-\boldsymbol{k}_1^2\\ &\quad -2\boldsymbol{q}\boldsymbol{k}_1-\beta_1(m^2+\boldsymbol{q}^2)+i0\,. \end{aligned} \tag{39}$$

The analysis of location of the poles in the α_1 plane shows that the non-zero contribution arises from the $0<\beta_1<1$ region of the β integration. Closing the α_1 complex contour of integration around the pole of the d_1 denominator we obtain:

$$\begin{aligned} &d_2 = -\frac{1}{\beta_1}[\boldsymbol{k}_1^2+m^{*2}],\quad d_3 = \frac{1}{\beta_1}[(\boldsymbol{k}_1+\boldsymbol{q}\beta_1)^2+m^{*2}],\\ &m^{*2} = m^2(1-\beta_1(1-\beta_1)). \end{aligned} \tag{40}$$

After the integration over α and α_1 one obtains

$$\Delta\Phi_{1\,a,b} = -c_\pi\int_0^1 d\beta_1\int\frac{d^2k}{\pi}\frac{m^2+k_1(k_1+q\beta_1)}{[k_1^2+m^{*2}][(k_1+q\beta_1)^2+m^{*2}]}. \tag{41}$$

A similar procedure for the contribution of the Feynman diagrams Fig.1(ef) leads to the expression

$$\Delta\Phi_{1\,c} = c_\pi\int_0^1 d\beta_1\int\frac{d^2k}{\pi}\frac{m^2+k_1(k_1+k-q\beta_1)}{[k_1^2+m^{*2}][(k_1+k-q\beta_1)^2+m^{*2}]}. \tag{42}$$

Performing the k_1 integration (the contributions of eqs.(41),(42) is separately ultraviolet divergent but their sum is finite) we obtain for the sum

$$\begin{aligned} \Phi_1 &= 2c_\pi\int_0^1 d\beta_1\int_0^1 dy\Big[\frac{m^2-Y(k-q\beta_1)^2}{m^{*2}+Y(k-q\beta_1)^2}\\ &- \frac{m^2-Y\beta_1^2q^2}{m^{*2}+Yq^2\beta_1^2}+\ln\frac{m^{*2}+Yq^2\beta_1^2}{m^{*2}+Y(k-q\beta_1)^2}\Big], \end{aligned} \tag{43}$$

with $Y = y(1-y)$.

Performing the y integration by parts for the logarithmic term and renaming $\beta_1\to x$ we obtain:

$$\begin{aligned} \Phi_1(\boldsymbol{k},\boldsymbol{q}) &= -c_\pi\int_0^1 dx\int_0^1 dy\frac{m^2C(X,Y)}{[m^2(1-X)+Y(\boldsymbol{k}-\boldsymbol{q}x)^2]}\\ &\times\frac{(\boldsymbol{k}^2-2\boldsymbol{k}\boldsymbol{q}x)}{[m^2(1-X)+Y\boldsymbol{q}^2x^2]},\quad C(X,Y)=\frac{1}{2}-\frac{1}{2}X+XY. \end{aligned} \tag{44}$$

One may see from this expression that $\Phi(0,q) = \Phi(q,q) = 0$.

The value of $\Phi_1(\boldsymbol{k},0)$ may be expressed in a 1-fold integral, convenient for numerical integration:

$$\Phi_1(\boldsymbol{k},0) = \Phi(\rho) = -c_\pi f(\rho), \tag{45}$$

$$f(\rho) = \int_0^1\frac{dt}{3+t^2}\{1-t^2+\frac{L(\rho,t)[(3+t^2)\rho^2-(1-t^2)(3+t^2+\rho^2)]}{2\rho\sqrt{3+t^2+\rho^2}}\}, \tag{46}$$

$$L(\rho,t) = \ln\frac{\sqrt{3+t^2+\rho^2}+\rho}{\sqrt{3+t^2+\rho^2}-\rho},\quad \rho = \sqrt{\frac{k^2}{m_1^2}}. \tag{47}$$

In therms of functions $f(s)$ the contribution to the scattering amplitude at $q^2=0$ may be written in the form:

$$A(s,0)\simeq\frac{is}{m_1m_2}\frac{g_{\pi_1}^2g_{\pi_2}^2}{18\pi}\left(\frac{\alpha_s}{2\pi}\right)^2\cdot I(\lambda)\ ;\ \lambda=\frac{m_1}{m_2}\ ; \tag{48}$$

$$I(\lambda) = I(1/\lambda) = \frac{2}{\lambda}\int_0^\infty\frac{dx}{x^3}f(x)f(\lambda x). \tag{49}$$

The results of the numerical integration for $I(\lambda)$ are presented in Table.

Appendix B

Details of the calculation of the finite part of the QCD correction to $A_1^{\gamma\gamma\to\gamma\gamma}$

We may first perform the angular averaging of the second term in eq.(28). We obtain by using the tabulated result:

$$\int_0^{2\pi} \frac{d\phi}{2\pi(a+b\cos\phi)} = (a^2-b^2)^{-\frac{1}{2}}\ ; a^2 > b^2. \tag{50}$$

that

$$\int_0^{2\pi} \frac{d\phi}{2\pi(k-k')^2[k'^2+(k-k')^2]} = \frac{1}{k'^2}\left[\frac{1}{|k'^2-k^2|} - \frac{1}{\sqrt{4(k'^2)^2+(k^2)^2}}\right].$$

The singularity in the k'^2 integration at the point $k'^2 = k^2$ is compensated in the sum of the contributions of the first and of the second term in eq.(28). So we may introduce the auxiliary parameters ϵ and $\sigma \ll k^2$ and we obtain for the second term:

$$\int \frac{d^2k'}{\pi} \frac{1}{(k-k')^2[k'^2+(k-k')^2]} = \int_\epsilon^{k^2-\sigma} \frac{dk'^2}{k'^2(k^2-k'^2)} \tag{51}$$
$$+\int_{k^2+\sigma}^{\infty} \frac{dk'^2}{k'^2(k'^2-k^2)} - \int_\epsilon^\infty \frac{dk'^2}{k'^2\sqrt{4(k'^2)^2+(k^2)^2}} = \frac{2}{k^2}\ln\frac{k^2}{\sigma}.$$

The average over the k azimuthal angle can be performed independently. The second term then gives (the subsequent integration over x and y is implied)

$$\mathcal{I}_{II} = -2\int_0^\infty \frac{dk^2 \ln\frac{k^2}{\sigma}}{(1+X\frac{k^2}{m_1^2})(1+Y\frac{k^2}{m_2^2})} \tag{52}$$
$$\cdot[(1+X+Y+\frac{1}{2}XY)\alpha + \frac{1}{2}XY\beta]\ ;$$

where α and β are defined in(12)
when averaging on the angle ϕ' we use the expression

$$\int_0^{2\pi} \frac{d\phi_n}{2\pi}(ne_A)(ne_{A'})(ne_B)(ne_{B'}) = \frac{1}{8}(\alpha+\beta), \tag{53}$$

$$\int_0^{2\pi} \frac{d\phi_n}{2\pi}(ne_A)(ne_{A'})|e_B||e_{B'}|\sin(ne_B)\sin(ne_{B'}) = \tag{54}$$
$$= \frac{1}{8}(3\alpha-\beta).$$

For the first term we obtain:

$$\mathcal{I}_I = \int \frac{dk^2}{1+X\frac{k^2}{m_1^2}} \int \frac{dk'^2}{1+Y\frac{k'^2}{m_2^2}} \int_0^{2\pi} \frac{d\phi'}{2\pi}\left\{\frac{1}{|k^2-k'^2|}\right. \tag{55}$$
$$\times\left[-e_Ae_{A'}(1+2X) + \frac{(k^2+k'^2)2X}{k^2+k'^2+|k^2-k'^2|}(e_An')(e_{A'}n')\right]$$
$$\left.+\frac{2X|e_A||e_{A'}|\sin(e_An')\sin(e_{A'}n')}{k^2+k'^2+|k^2-k'^2|}\right\}[-e_Be_{B'}(1+2Y)$$
$$+2Y(n'e_B)(n'e_{B'})],\quad n' = \frac{k'}{|k'|}.$$

After the angular integration and the integration over k'^2 with the restriction $|k^2-k'^2| > \sigma$ we obtain:

$$\begin{aligned}\mathcal{I}_I &= -\mathcal{I}_{II} + \int dk^2 \frac{(1+X+Y)\beta + \frac{1}{2}XY(\alpha+\beta)}{(1+X\frac{k^2}{m_1^2})(1+Y\frac{k^2}{m_2^2})}\ln\frac{(1+X\frac{k^2}{m_1^2})^2}{X\frac{k^2}{m_1^2}} \\ &+ \frac{XY}{2}\frac{\alpha-\beta}{(1+Y\frac{k^2}{m_2^2})}[(\frac{m_1^2}{Xk^2}+1)\ln(1+X\frac{k^2}{m_1^2}) - \ln(\frac{Xk^2}{m_1^2})].\end{aligned} \tag{56}$$

Coefficients in front of β and α in the final expression eq.(29) are

$$f_1(\lambda) = \int_0^1\int_0^1 \frac{dxdy}{\sqrt{XY}}(2(1+X)(1+Y)H(\eta) - XY(H(\eta)-f_3(\eta))),$$
$$f_2(\lambda) = \int_0^1\int_0^1 dxdy\sqrt{XY}(H(\eta)-f_3(\eta)),\quad \eta = \lambda^2\frac{Y}{X} \tag{57}$$

and

$$H(z) = H(\frac{1}{z}) = \frac{\sqrt{z}}{z-1}(Li_2(1-\frac{1}{z}) - Li_2(1-z)); \tag{58}$$
$$f_3(z) = f_3(\frac{1}{z}) = \sqrt{z}(\frac{1}{z}(Li_2(1)-Li_2(1-z))$$
$$+Li_2(\frac{1}{z} - \ln z\ln(1-\frac{1}{z})).$$

These functions are given in the Table.

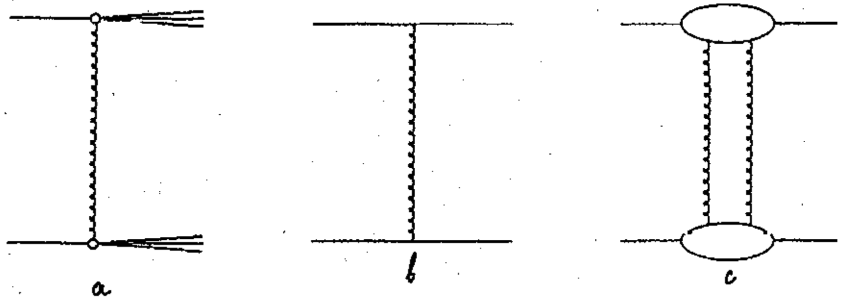

Fig. 1. Feynman graphs describing the nonvanishing in the high energy limit contributions to the total cross section; a) typical two-jet production process caused by spin unity state in the scattering channel; b) Pomeron exchange; c) two gluons exchange.

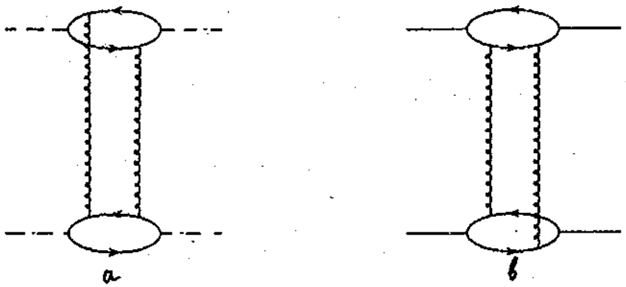

Fig. 2. Typical Feynman diagrams relevant in impact factor calculation: a) photon-photon scattering; b) pion-pion scattering.

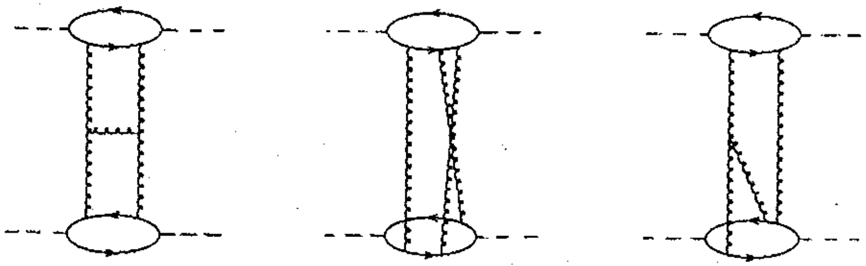

Fig. 3. The types of Feynman diagrams,relevant in the calculation of the lowest order correction to photon impact factor.

Table. Numerical values of integrals for different mass ratios.

λ	1	2	10	100	200	1000
$f_1(\lambda)$	46.4	45.1	33.7	13.2	90.7	34.4
$f_2(\lambda)$	−.194	−.179	−.0869	−.0165	−.00938	−.00241
$H(\lambda)$	2.	1.99	1.85	1.39	1.23	.859
$I(\lambda)$	1.42	1.34	.798	.215	.135	.0420
$I_1(\lambda)$	.329	.353	.254	.0779	.0499	.0162
$I_2(\lambda)$	.0168	.0156	.00807	.00164	.00095	.00025

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