ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

Дубна

E2-97-87

V.S.Barashenkov[1], A.B.Pestov[2], M.Z.Yur'iev[3]

TIME MULTIDIMENSION IN GRAVITY

Submitted to «General Relativity and Gravity»

[1]E-mail: barashenkov@lcta30.jinr.dubna.su
[2]E-mail: pestov@theor.jinr.dubna.su
[3]Firm YURIDOR, Moscow, Russia

1997

1 Introduction

The hypothesis of time multidimension is a direct generalization of Einsteinian special relativity assuming more symmetry of the time and space coordinates when time is considered as a three-dimensional subspace and every event is characterized by a six-dimensional vector [1]

$$\mathbf{x} = (x_1, \quad x_2, \quad x_3, \quad ct_1, \quad ct_2, \quad , t_3) \tag{1}$$

Dirac, Fock and Podolsky introduced a proper time for each material body [1], Tomonaga was the first who introduced that time for every spatial point $\mathbf{x}$ [2]. Though these generalizations themselves did not discover any new physical effects, they improved the theory and allowed one to formulate the condition of compatibility for equations of motion excluding superluminal velocities and to develop a consistent renormalization procedure. It is interesting to further follow the way of the space-time symmetrization and to consider time as a three-dimensional vector [3, 4, 5, 6][2].

The analysis has shown that this approach is at variance with all the presently known experimental facts [6]. In particular, the difficulties with negative energies mentioned by Dorking and Demers [8, 9] can be avoided by means of the Principle of time irreversibility [10, 11]. Nevertheless, we don't observe macroscopic bodies moving along time trajectories distinguishing from our one because the energy necessary for creation of such objects is huge [3]. Bodies with diverse t-trajectories would appear in cosmic cataclysms where enormous amounts of energy are produced. And which is more, in very strong gravitational fields the concept of energy itself loses its sense and the energy conservation law becomes inexact. All that must influence the properties of emitted gravitational waves which

[1] In what follows the Latin and Greek indices take the values $k = 1, ..., 3$, $\mu = 1, ..., 6$). All matrices will be denoted by capital letters.

[2] Several authors used multitime hypothesis in connection with attempts to bypass the difficulties in nonlocal theories [7]

acquire the "time component" [3].

How that affects the behavior of gravitation detectors which are now under the construction in many countries? The discussion of this question is the main goal of the present consideration. It should be noted that both aspects of the problem, the discovery of the time multi-dimension or, on the contrary, the proof of inconsistency that hypothesis, are very important.

Section 2 is devoted to calculations of the metric tensor determining a plane gravitational wave in six-dimensional space-time. We show that in contrast to the customary one-time gravity the transversal tensor components are mutually independent. In sect. 3 where tidal forces are considered it is shown that due to this fact the "height" and "width" of a gravitation detector have to oscillate also independently. The observation of such a detector behavior may be considered as an indication of the time multidimension. An interesting analogy with electrodynamics is drawn and a possibility of longitudinal components of gravitational waves is considered. In the last sect. we discuss results of our consideration.

2 Plane gravitational waves

According to the standard procedure (see, e. g., [14]) we represent the metric tensor as a sum

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \tag{2}$$

where

$$\eta = \begin{pmatrix} -\mathbf{I} & \mathbf{O} \\ \mathbf{O} & \mathbf{I} \end{pmatrix}, \tag{3}$$

$\mathbf{I}$ is the three-dimensional unit matrix and $h_{\mu\nu}$ is a small addition.

[3]One can expect the appearance of objects with a "turned time" on the level of microscopic space-time intervals where energy necessary for their creation is about their rest-mass. This aspect demands special investigation, however, one can predict that if the deviation of time trajectories is significant, then the time of an interaction of such particles with their surrounding is very short [12, 13]

The Ricci tensor

$$R_{\mu\nu} = \frac{1}{2}\Box_6 h_{\mu\nu} + \frac{1}{2}\left(\frac{\partial^2}{\partial x^\mu \partial x^\sigma}F^\sigma_\nu + \frac{\partial^2}{\partial x^\nu \partial x^\sigma}F^\sigma_\mu\right), \tag{4}$$

where $h = \eta^{\mu\nu}h_{\mu\nu}$, the operator $\Box_6 \equiv \partial^2/\partial x^\mu \partial x_\mu$ and

$$F^\sigma_\mu = h^\sigma_\mu - \frac{1}{2}h\delta^\sigma_\mu \tag{5}$$

The expression in the brackets can be turned into zero if we impose the condition

$$\partial F^\sigma_\nu / \partial x_\sigma = 0 \tag{6}$$

by means of the gauge transformation

$$h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu \tag{7}$$

when

$$F_{\mu\nu} \to F_{\mu\nu} - \partial_\mu \xi_\nu - \partial_\nu \xi_\mu + \eta_{\mu\nu}\partial_\sigma \xi^\sigma. \tag{8}$$

To prove that $F_{\mu\nu}$, besides eq. (6), is constrained so by the conditions

$$F \equiv \eta^{\mu\nu}F_{\mu\nu} = 0, \quad F_{\mu\nu}U^\nu = 0 \tag{9}$$

where U_μ are components of a constant unit vector ($U^\mu U_\mu = 1$) defining completely our gauge, we pass to the momentum space:

$$F_{\mu\nu} = \int e^{ik_\sigma y^\sigma} F_{\mu\nu}(k) d^6 k. \tag{10}$$

Then relations (6), (9) and wave equation

$$\Box_6 h_{\mu\nu} = \Box_6 F_{\mu\nu} = 0. \tag{11}$$

are reduced to the system of algebraic equations

$$\eta^{\mu\nu}F_{\mu\nu}(k) = 0, \tag{12}$$

$$k^\sigma F_{\sigma\nu}(k) = 0, \tag{13}$$

$$U^\sigma F_{\sigma\nu}(k) = 0, \tag{14}$$

$$(k_\sigma k^\sigma)F_{\mu\nu}(k) = 0. \tag{15}$$

Turning the axes one can reduce the momentum vector to the form $\mathbf{k} = (1,0,0,1,0,0)$. Hence eqs. (12) — (15) can be rewritten as follows

$$F_{\mu 1} = F_{\mu 4}, \tag{16}$$

$$F_{41} = F_{42} = F_{43} = F_{45} = F_{46} = 0, \tag{17}$$

$$F_{11} + F_{22} + F_{33} - F_{44} - F_{55} - F_{66} = 0. \tag{18}$$

Vector U^μ is taken here in the form $U^\mu = (0,0,0,1,0,0)$.

We satisfy these equations with the help of the gauge transformation (8) which has now the form

$$F_{\mu\nu}(k) \to F_{\mu\nu}(k) - ik_\mu\xi_\nu(k) - ik_\nu\xi_\mu(k) + i\eta_{\mu\nu}k_\sigma\xi^\sigma(k) \tag{19}$$

or for the chosen above special form of the components k^μ

$$F_{42} \to F_{42} - i\xi_2 \quad , \quad F_{43} \to F_{43} - i\xi_3, \tag{20}$$

$$\to F_{45} = F_{45} - i\xi_5 \quad , \quad F_{46} \to F_{46} - i\xi_6. \tag{21}$$

Being given by $\xi_2, \xi_3, \xi_5, \xi_6$ one can turn $F_{42}, F_{43}, F_{45}, F_{46}$ into zero. Now we must satisfy only the equations $F_{41} = 0$ and (18).

Using again the transformation (19) we get

$$F/4 \to F/4 + i\xi_1 - i\xi_4 \quad , \quad F_{41} \to F_{41} - i\xi_1 - i\xi_4. \tag{22}$$

If we assume $F = F_{41} = 0$, then we obtain a consistent system of equations for the calculations of ξ_1 and ξ_4.

So, we convinced ourselves that the tensor

$$\mathbf{h} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & h_{22} & h_{23} & 0 & h_{25} & h_{26} \\ 0 & h_{23} & h_{33} & 0 & h_{35} & h_{36} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & h_{25} & h_{35} & 0 & h_{55} & h_{56} \\ 0 & h_{26} & h_{36} & 0 & h_{56} & h_{66} \end{pmatrix}, \tag{23}$$

where $h_{22} + h_{33} - h_{55} - h_{66} = 0$, satisfies all necessary conditions.

In comparison with the customary one-time theory where every plane gravitational wave in the linear approximation is completely defined only be two quantities h_{23} and $h_{22} = -h_{33}$ the multitime wave depends on nine independent quantities.

3 Tidal forces

Such forces acting on particles of a body moving along a geodesic curve, i. e. "free-falling" together with an observer in the considered gravitational field, are described by the so-called deviation equation [15]:

$$d^2L^\mu/d\tau^2 + R^\mu{}_{4\sigma 4}L^\sigma = 0 \tag{24}$$

Here $\mathbf{L}$ is the vector linking two points with the same proper time on closely related geodesic curves [4], $\mathbf{R}$ is the curvature tensor.

The displacement vector can be represent as a sum of two terms

$$L^\mu = \ell^\mu + \Delta\ell^\mu \tag{25}$$

where ℓ^μ is a constant component, $\Delta\ell^\mu$ is a small addition changing during the motion. In this case eq. (24) assumes the form

$$d^2\Delta\ell^\mu/d\tau^2 + R^\mu{}_{4\sigma 4}\ell^\sigma = 0. \tag{26}$$

(We omitted the term $R^\mu{}_{4\sigma 4}\Delta\ell^\sigma$ since we confine ourselves only to the first approximation).

Using the above considered (transverse, traceless) gauge one can write

$$R^\mu{}_{4\sigma 4} = \eta^{\mu\mu}R_{\mu 4\sigma 4} = -(1/2)\eta^{\mu\mu}\partial^2 h_{\mu\sigma}/\partial\tau^2. \tag{27}$$

(the summation over μ is absent here). So,

$$d^2\Delta\ell^\mu/d\tau^2 = (\ell_k/2)\eta^{\mu\mu}\partial^2 h_{\mu\sigma}/\partial\tau^2\ell^\sigma. \tag{28}$$

[4]The proper (scalar) time $\tau = (t_1^2 + t_2^2 + t_3^2)^{-1/2}$ is counted along the body time trajectory [6].

Let us suppose that the considered body is a parallepiped with a cross section $\ell_2 \times \ell_3$. Inserting $\ell^\mu = \delta_{\mu k}\ell_k$, $\Delta\ell^\mu = \Delta\ell_k\delta_{\mu k}$ and $k = 2, 3$, into eq. (27) we get the equation determining the change of the body "width" and "height":

$$d^2\Delta\ell_k/d\tau^2 = -(\ell_k/2)d^2h_{kk}/d\tau^2. \tag{29}$$

Hence

$$\Delta l_k/l_k = -(1/2)h_{kk}, \quad k = 2, 3 \tag{30}$$

We see that there is the quantitative difference of one- and multi-time cases: in the latter there is no correlation of the detector "width" and "height" oscillations in a plane perpendicular to the direction of gravitational wave propagation . In the customary one-time theory the amplitudes of these oscillations are equal. That can be used for the experimental check of possible hidden multi-dimensionality of time in our world.

A few words about longitudinal waves. In the multi-time world plane electromagnetic waves possess longitudinal components [10]. Gravitational waves have also such components. Indeed, in the linear approximation (2) there is a remarkable analogy of the electromagnetic tensor $F_{\mu\nu}$ and the curvature tensor $R_{\alpha\beta\mu\nu}$. The latter can be splitted into three groups of components

$$\mathcal{E}_{ikj} = R_{i,j+3,k,j+3}, \quad \mathcal{H}_{ij} = (1/4)\varepsilon_{ikl}\,\varepsilon_{jmn}\,R^{klm,n+3} \tag{31}$$

$$\mathcal{G}_{ij} = (1/4)\varepsilon_{ikl}\,\varepsilon_{jmn}\,R^{k+3,l+3,m+3,n+3} \tag{32}$$

which are similar to the electric, magnetic, and "time-magnetic" fields

$$E_{ik} = F_{i,k+3}, \quad H_i = (1/2)\varepsilon_{ijk}\,F^{jk}, \tag{33}$$

$$G_i = -(1/2)\varepsilon_{ijk}\,F^{j+3,k+3} \tag{34}$$

(see [10, 16]). The fields $\mathcal{E}$, $\mathcal{H}$, $\mathcal{G}$ satisfy relations analogous to the generalized multi-time Maxwell equations [5] and, therefore, as in the case of electromagnetic field, the gravitational field should have the longitudinal components.

4 Conclusion

The multi-time gravitational waves differ in many aspects from one-time ones. In particular, interesting effects are stipulated by the longitudinal wave component. In fact, most of the differences cannot be observed at the level of recent experimental accuracy. For the time being the question is an unpretentious registration of gravitational pulses from any enormous cosmic events, therefore, one may hope to fix only the possible difference of the detector amplitude oscillations in perpendicular directions. However, the observation of only this effect would be an intrigue indication of the existence of hidden time properties and would be a powerful booster of further experimental investigations. The absence of the effect is also an important result making more precise our ideas about the most puzzle property of our world, the time.

We wish to thank B.F.Kostenko for the usefull discussions.

References

[1] P. A. M. Dirac, V. A. Fock,B. Podolsky. *Zs Sowjetunion* **2**(1932) 168.

[2] S. Tomonaga. *Prog. Theor. Phys.* **1** (1946) 27.

[3] E. A. B. Cole. *J. Phys. A: Math. Gen.* **13** (1980) 109.

[4] E. A. B. Cole, S. A. Buchanan. *J. Phys. A: Math. Gen.* 15(1982)L255.

[5]In particular, the identities

$$F_{\mu\nu} = -F_{\nu\mu}, \quad \partial_\mu F_{\nu\sigma} + \partial_\nu F_{\sigma\mu} + \partial_\sigma F_{\mu\nu} = 0$$

have the gravitational similarity

$$R_{\mu\nu\sigma\lambda} = -R_{\nu\mu\sigma\lambda} = -R_{\mu\nu\lambda\sigma} = R_{\sigma\lambda\mu\nu}, \quad R_{\mu\nu\sigma\lambda} + R_{\nu\sigma\mu\lambda} + R_{\sigma\mu\nu\lambda} = 0,$$

$$\partial_\tau R_{\mu\nu\sigma\lambda} + \partial_\mu R_{\nu\tau\sigma\lambda} + \partial_\nu R_{\tau\mu\sigma\lambda} = 0$$

[5] E. A. B. Cole.Proc. of the Sir Arthur Eddington Centenary Sump. *World Sc. Press* **2** (1985) 178.

[6] V. S. Barashenkov, M. Z. Yur'iev. Is the Hypothesis of Time Multi-Dimensionality at Variance with the Facts? *JINR E2-96-246*, Dubna, 1996; *Found. Phys.* (submitted).

[7] E. Recami. *Riv. Nuovo Cim.* **9** (1986) 1.

[8] J. Dorlind. *Amer. J. Phys.* **38** (1970) 539.

[9] P. Demers. Canad. J. Phys. **53** (1975) 687.

[10] V. S. Barashenkov, M. Z. Yur'iev.*Nuovo Cim. B*, in print.

[11] V. S. Barashenkov. Electrodynamics in Space with Multi-Dimensional Time. *JINR E2-96-10*, Dubna, 1996.

[12] E. A. B. Cole,I. M. Starr. *Let. Nuovo Cim.* **43**(1985)38.

[13] V. S. Barashenkov. M. Z. Yur'iev. Detection of ray's in multi-time world. *JINR E2-96-109*,Dubna, 1996.

[14] L. D. Landau, E. M. Lifshitz. The Classical Theory of Fields. *Addison-Wesley Reading, Massachusetts and Pergamon*, London, 1971.

[15] J. Weber.General Relativity and Gravitational Waves. New York, 1961.

[16] E. A. B. Cole.*Nuovo Cim. A* **60** (1980) 1.

Received by Publishing Department
on March 17, 1997.