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# THE ρ-MESON AND RELATED MESON WAVE FUNCTIONS IN QCD SUM RULES WITH NONLOCAL CONDENSATES

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# 1 Introduction

An important problem in the theory of strong interactions is to calculate, from the first principles of QCD, hadronic wave functions $\varphi_\pi(x)$, $\varphi_\rho(x)$, ..., $\varphi_N(x_1, x_2, x_3)$, *etc.* These phenomenological distributions of partons on the fraction $xP$ of a hadron momentum $P$ naturally appear as a result of applying "factorization theorems" to hard exclusive processes [1, 2, 3, 4]. They accumulate all the necessary information about non-perturbative long-distance dynamics of partons in hadrons.

The standard QCD sum rule (SR) calculation of light meson wave functions (WF's), firstly introduced by Chernyak and Zhitnitsky (C&Z) [5] and recently re-estimated by Ball and Braun (B&B) for $\rho$-meson [6], implicitly assumes that the correlation length $\Lambda$ of vacuum fluctuations is large compared to a typical hadronic scale $\sim 1/m_\rho$. Thus, one can replace the original nonlocal objects like $M(z^2) = \langle \bar{q}(0)E(0,z)q(z)\rangle$ [1] by the constant $\langle \bar{q}(0)q(0)\rangle$-type values. Based on this hypothesis, the well-known QCD SR approach [7] has been applied in [5] to calculate the first two moments $\langle \xi^N \rangle \equiv \int_0^1 \varphi(x)(2x-1)^N dx$ with $N=2$ and $N=4$ for WF's of light mesons. And just from these moments the whole WF's have been reconstructed, which are now referred to as C&Z WF's.

But now it is known that hadronic WF's are rather sensitive to the width of the function $M(z^2)$ [8, 9, 10] and the crucial parameter $\Lambda \cdot m_\rho \sim 1$. Therefore, one should use nonlocal condensates (NLC's) like $M(z^2)$ whose forms reflect the complicated structure of the QCD vacuum. Certainly, these objects can be subsequently expanded over the local condensates $\langle \bar{q}(0)q(0)\rangle$, $\langle \bar{q}(0)\nabla^2 q(0)\rangle$, *etc.* and one can return again to the standard SR by truncating this series (here $\nabla_\mu$ is the covariant derivative). Our strategy is to avoid such an expansion because we lose in this way an important physical property of non-perturbative vacuum – the possibility of vacuum quarks and gluons to flow through vacuum **with non-zero virtuality** $\langle k^2 \rangle \neq 0$. Indeed, the average virtuality of vacuum quarks $\lambda_q^2$ is not small and can be extracted from QCD SR analysis [8]

$$\lambda_q^2 \equiv \frac{\langle \bar{q}\nabla^2 q\rangle}{\langle \bar{q}q\rangle} = \frac{\langle \bar{q}\left(\sigma_{\mu\nu}G^{\mu\nu}\right)q\rangle}{2\langle \bar{q}q\rangle} = 0.4 \pm 0.1\,\mathrm{GeV}^2, \tag{1}$$

that is $\lambda_q^2$ is of an order of the typical hadronic scale [11] $m_\rho^2 \approx 0.6$ GeV$^2$. An estimate for $\lambda_q^2$ in the framework of the instanton liquid model [12] yield similar number.

Careful inspection of consequences of such an approach to QCD SR for pion WF [8, 9, 13] has revealed that the introduction of the correlation length $\Lambda \sim 1/\lambda_q$ into condensate distributions produces much smaller values for the first moments of pion WF than C&Z values. This leads to the shape of the pion WF strongly different from the C&Z shape and close to the asymptotic WF $\varphi^{as}(x) \equiv 6x(1-x)$ [4], *i.e.*, $\varphi_\pi(x) \approx \varphi^{as}(x)$. Later, this WF was confirmed by independent consideration of the QCD SR directly for $\varphi_\pi(x)$ based on the non-diagonal correlator [14] and on the advanced smooth distribution function for the quark nonlocal condensate. The final effect obtained by both the ways was due to the main physical reason – vacuum correlation length $\Lambda \sim 1/\lambda_q$ is of an order of $1/m_\rho$.

Our goal here is to show that in the case of QCD SR for the $\rho$-meson channel the situation is similar: all the predictions of the standard QCD SR (C&Z ones for the longitudinal case [5] and B&B ones for the transverse case [6] – we call them Local QCD SR) for moments $\langle \xi^{2N}\rangle_\rho$ with $N > 2$ could not be approved. We apply the NLC's formalism to calculate the diagonal correlators for $\rho$-meson currents, introduced in [5], and construct generalized SR including $O(\alpha_s)$-radiative corrections to obtain WF's of twist 2. The first ten moments of longitudinal WF's of $\rho$- and $\rho'$-mesons and of transverse WF's of $\rho$- and $b_1$-mesons are estimated (Table 2) and the models for them (see Figs.2–5) are suggested. As a byproduct we predict the lepton decay constant $f_{\rho'}$ and estimate the mass of $\rho'$-meson. The calculation technique is the same as in

[1] Here $E(0,z) = P\exp(i\int_0^z dt_\mu A_\mu^a(t)\tau_a)$ is the Schwinger phase factor required for gauge invariance.

Refs. [8, 9, 15]; therefore, the corresponding details are missed below, but we shall keep the connection with the pion case through the text.

## 2 Generalized sum rules for the $\rho$-meson channel vs the standard version

For the helicity-zero meson $M_{|\lambda|=0} = M_L$, $M_L = \rho,\ \rho' \ldots$, the leading-twist WF is defined as

$$\langle 0 \mid \bar{d}(z)\gamma_\mu u(0) \mid M_L(p)\rangle\Big|_{z^2=0} = f^L_{M_L} p_\mu \int_0^1 dx e^{ix(zp)}\, \varphi^L_{M_L}(x) \quad (2)$$

($\varepsilon^{\lambda=0}_\mu(p) \simeq p_\mu/m_{M_L}$ at $p_z \to \infty$, where $\varepsilon_\mu(p)$ is the polarization vector). Information about $\varphi^L(x)$ can be obtained from the correlator $I_{(n0)}(q^2)$ of vector currents $V_{(n)}(y)$, see *e.g.* [5]:

$$i\int dy e^{iqy}\langle 0|T\left\{V^{+}_{(0)}(y)V_{(n)}(0)\right\}|0\rangle = (zq)^{n+2}\, I_{(n0)},\ V_{(n)}(y) \equiv \bar{d}(y)\hat{z}\,(z\nabla)^n\, u(y), \quad (3)$$

where $z^2 = 0$. The corresponding formula for the unit helicity state, $M_\perp = M_{|\lambda|=1}$, $M_\perp = \rho,\ b_1 \ldots$, has the form:

$$\langle 0 \mid \bar{d}(z)\sigma_{\mu\nu}u(0) \mid M_\perp(p)\rangle\Big|_{z^2=0} = i f^T_{M_\perp}\left(\varepsilon^\perp_\mu(p)p_\nu - \varepsilon^\perp_\nu(p)p_\mu\right)\int_0^1 dx e^{ix(zp)}\, \varphi^T_{M_\perp}(x). \quad (4)$$

Moments of these functions $\varphi^T(x)$ are extracted from the correlator $J_{(n0)}(q^2)$ of tensor currents $T^\mu_{(n)}(y)$ [5, 6]:

$$i\int dy e^{iqy}\langle 0|T\left\{T^{\mu+}_{(0)}(y)T^\mu_{(n)}(0)\right\}|0\rangle = -2\,(zq)^{n+2}\, J_{(n0)},\ T^\mu_{(n)}(y) \equiv \bar{d}(y)\sigma^{\mu\alpha}z_\alpha\,(z\nabla)^n\, u(y). \quad (5)$$

This correlator $J_{(n0)}(q^2)$ contains the contribution from states with different parity (see analysis in [6]). Therefore, the presence of a contamination from $b_1$-meson $\left(J^{PC} = 1^{+-}\right)$ in the phenomenological part of SR is mandatory.

The theoretical, "condensate" parts for both the correlators — $I_{(n0)}$ in (3) and $J_{(n0)}$ in (5) — contain the same 6 universal elements as for the pion case. Their relative (to pion case) contributions $\Delta\Phi_\Gamma(x; M^2)$ to SR for WF's $\varphi_M(x)$ are collected in Table 1. The diagram origins of these elements $\Delta\Phi_\Gamma(x; M^2)$ are described in detail in [8, 9]. The direct SR formulation for WF allows one to construct immediately a "daughter SR" for any functional of $\varphi_M(x)$ (not only for moments $\langle\xi^N\rangle$). Let us write down the final SR's including WF's of $\rho$-meson and next resonances $\rho'$ and $b_1$ in phenomenological parts:

$$\left(f^L_\rho\right)^2 \varphi^L_\rho(x)e^{-m^2_\rho/M^2} + \left(f^L_{\rho'}\right)^2 \varphi^L_{\rho'}(x)e^{-m^2_{\rho'}/M^2} = \frac{1}{4\pi^2}\int_0^{s^L_0} \rho^{pert}_L(x;s)e^{-s/M^2}ds \quad (6)$$
$$+\Delta\Phi_G(x;M^2) - \Delta\Phi_S(x;M^2) + \Delta\Phi_V(x;M^2) + \Delta\Phi_{T_1}(x;M^2) + \Delta\Phi_{T_2}(x;M^2) + \Delta\Phi_{T_3}(x;M^2);$$

$$\left(f^T_\rho\right)^2 \varphi^T_\rho(x)e^{-m^2_\rho/M^2} + \left(f^T_{b_1}\right)^2 \varphi^T_{b_1}(x)e^{-m^2_{b_1}/M^2} = \frac{1}{4\pi^2}\int_0^{s^T_0} \rho^{pert}_T(x;s)e^{-s/M^2}ds \quad (7)$$
$$+\Delta\Phi_G(x;M^2) - \Delta\Phi'_G(x;M^2) + \Delta\Phi_V(x;M^2) + \Delta\Phi_{T_1}(x;M^2) + \Delta\Phi_{T_2}(x;M^2) - \Delta\Phi_{T_3}(x;M^2),$$

where $s^{L,T}_0$ are the effective continuum thresholds in the $L$ and $T$ cases, respectively. Perturbative spectral densities $\rho^{pert}_{L,T}(x;s)$ have been presented in an order of $O(\alpha_s)$ in [8, 9] for the $L$ case and in [6] for the $T$ case (see Appendix B). Radiative corrections amount to 10 % of $\rho^{pert}_{L,T}$. Contributions $\Delta\Phi_\Gamma(x;M^2)$ depend on a specific form of NLC's $M(z^2)$, ..., *etc.* .



**Table 1.**

| meson WF | The contributions to SR from different types of condensates | | | | | |
|---|---|---|---|---|---|---|
| | $\Delta\Phi_S(x)$ | $\Delta\Phi_V(x)$ | $\Delta\Phi_{T_1}(x)$ | $\Delta\Phi_{T_2}(x)$ | $\Delta\Phi_{T_3}(x)$ | $\Delta\Phi_G(x)$ |
| $(f_{\pi,A_1})^2\varphi_{\pi,A_1}(x)$ | 1 | 1 | 1 | 1 | 1 | 1 |
| $(f^L_{\rho,\rho'})^2\varphi^L_{\rho,\rho'}(x)$ | −1 | 1 | 1 | 1 | 1 | 1 |
| $(f^T_{\rho,b_1})^2\varphi^T_{\rho,b_1}(x)$ | 0 | 1 | 1 | 1 | −1 | $1-\frac{\Delta\Phi'_G}{\Delta\Phi_G}$ |

To construct SR for WF's, it is useful to parameterize these NLC behaviours by the "distribution functions" [8, 9, 14] *a'la* $\alpha$-representation of propagators, *e.g.* , $f_S(\nu)$ for the scalar condensate $M(z^2)$ [2]

$$M\left(z^2\right) = \langle\bar{q}(0)q(0)\rangle \int_0^\infty e^{\nu z^2/4} f_S(\nu)\, d\nu, \text{ where } \int_0^\infty f_S(\nu)\, d\nu = 1,\ \int_0^\infty \nu f_S(\nu)d\nu = \frac{\lambda^2_q}{2}. \quad (8)$$

The function $f_S(\nu)$ and other similar functions $f_\Gamma(\nu)$ describe distributions of vacuum fields in virtuality $\nu$ for every type of NLC. They completely determine the RHS of SR's in (6) and (7). The general forms of elements $\Delta\Phi_\Gamma(x; M^2)$ as functionals of $f_\Gamma(\nu)$ will be published in a separate paper. For the standard (constant) condensates $\langle G(0)G(0)\rangle$ and $\langle q(0)q(0)\rangle$ these distributions are of a trivial form, *e.g.* , $f_S(\nu) = \delta(\nu)$ (Appendix A). To involve the condensates of higher dimensions into consideration, one should use the contributions to $f_S(\nu)$ proportional to the derivatives of $\delta$-functions – $\delta(\nu)'$, $\delta(\nu)''$, .... It is clear that in the absence of the QCD vacuum theory merely models of real distributions may be suggested for these distribution functions $f_\Gamma(\nu)$. However, for the purpose of QCD SR's for moments $\langle\xi^N\rangle$ we need a rather rough information about the $f_\Gamma(\nu)$ behaviour. Therefore, we apply here the simplest ansatz [8, 9], like $f_S(\nu) = \delta(\nu - \lambda^2_q/2)$, to take into account only the main effect – the non-zero average virtuality of vacuum fields. One may consider such a form of $f_S(\nu)$ as the result of resummation of the subset of the above mentioned contributions $\sim \delta(\nu)^{\{n\}}$ connected with the single scale $\lambda^2_q$ [8, 9]. The corresponding expressions for $\Delta\Phi_{G,S,V,T_i}(x; M^2)$ are collected in Appendix A.

Now let us take the limits $\lambda^2_q \to 0$, $\Delta\Phi_\Gamma(x, M^2) \to \Delta\varphi_\Gamma(x, M^2)$ and $\rho^{pert}_L(x, s) \to \rho^{Born}_L(x, s)$ for SR in eq.(6) to return to the standard approach (see these reduced elements in Appendix). We try to inspect the subtle points and the range of validity of C&Z SR. These authors extracted $\langle\xi^{2N}\rangle$ exactly in the same way as the $f_\rho$ value (and B&B limit themselves to extraction of only $\langle\xi^2\rangle$). However, the nonperturbative terms in their sum rule ($\rho'$-contribution is omitted for simplicity) have a completely different $N$-dependence compared to the perturbative one and, *a priori*, it is not clear whether a straightforward use of the $N = 0$ technology can be justified

[2] In deriving these sum rules we can always make a Wick rotation, i.e., we assume that all coordinates are Euclidean, $z^2 < 0$.

for higher $N$ (for definiteness, we consider here only the $\rho$-meson (longitudinal) case; the same arguments work also in the pion case, see criticism in [9, 10, 13]).

$$\left(f_\rho^L\right)^2 \langle\xi^N\rangle_\rho^L e^{-m_\rho^2/M^2} + \frac{3M^2}{4\pi^2(N+1)(N+3)} e^{-s_N/M^2} =$$
$$= \frac{3M^2}{4\pi^2(N+1)(N+3)} + \frac{\langle(\alpha_s/\pi)GG\rangle}{12M^2} + \frac{16}{81}\pi(4N-7)\frac{\langle\sqrt{\alpha_s}\bar{q}q\rangle^2}{M^4} \qquad (9)$$

The scale determining the magnitude of all hadronic parameters including $s_N$ (the "continuum threshold" [7]) is eventually settled by the ratios of condensate contributions to the perturbative term. If the condensate contributions in the C&Z sum rule (9) had the same $N$-behavior as the perturbative term, the $N$-dependence of $\langle\xi^N\rangle$ would be determined by the overall factor $3/(N+1)(N+3)$ and the resulting WF $\varphi(x)$ would coincide with the "asymptotic" form $\varphi^{as}(x)$.

However, the ratios of the $\langle\bar{q}q\rangle$- and $\langle GG\rangle$-corrections to the perturbative term in eq.(9) are growing functions of $N$. This results in reducing the predictable power of the local QCD SR's with the growth of $N$. In order to reveal consequences of this effect more clearly, let us consider the so-called SR fidelity windows, i.e. regions of the Borel parameter $M^2$ where one should obtain valid SR predictions. In accord with the QCD practice [7] these fidelity windows are determined by two conditions: the lower bound $M_-^2$ – by demanding that relative value of the $\langle GG\rangle$- and $\langle\bar{q}q\rangle$-contributions to OPE series shouldn't be larger than 30%, the upper one $M_+^2$ – by demanding that relative contribution of higher states in the phenomenological part of SR shouldn't be larger than 30%. Suggesting independence of the threshold of $N$ ($s_N \approx s_0 \approx 1.5$ GeV$^2$) we have in the case of $N=0$: $M_-^2 = 0.4$ GeV$^2$, $M_+^2 = 1.34$ GeV$^2$. But for $N=2$ we have $M_-^2 = 0.73$ GeV$^2$, $M_+^2 = 1.34$ GeV$^2$, and for $N=4$ – even $M_-^2 = 1.5$ GeV$^2$, $M_+^2 = 1.34$ GeV$^2$. That is, the fidelity window shrinks to empty set in the last case. C&Z suggest that $s_2 \approx 1.9$ GeV$^2$ and $s_4 \approx 2.2$ GeV$^2$. It is hard to imagine such a strange type of a spectral model, but there are no principal objections.

In our opinion, there is no need to propose such an exotic spectral model ($s_N = s_0 + \text{const}\cdot N$), because the reason for this "exploding" behaviour of Local SR is quite evident, namely, a completely different dependence on $N$ of the perturbative (the first term in the second line of Eq.(9)) contribution and of condensate ones. And the origin of this difference is also clear: as was explained in a series of papers [9, 10, 13] this is due to Taylor expansion of initial nonlocal objects like $\langle\bar{q}(0)E(0,z)q(z)\rangle$ in powers $z^k$. The first constant term of this expansion, $\langle\bar{q}q\rangle$, produces an $(N)^0$-dependent term in SR (9); the next term, an $N$-dependent and so on.

On the contrary, the NLC terms $\Delta\Phi_\Gamma(x;M^2)$ in (6) and (7) lead to the moments $\langle\xi^N\rangle$ which well decay with $N$-growth; so physically motivated $N$-independent continuum threshold $s_0^L$ naturally appears in the SR processing.

## 3 The moments and the models of the wave functions

Before analyzing the results of processing of SR (6) and (7) for the moments $\langle\xi^N\rangle_M$, let us consider the peculiarity of the QCD SR structure, represented in Table 1. Opposite to the $\pi$-meson case the contribution of the numerically most significant "four-quark condensate" $\Delta\Phi_S(x, M^2)$ [8] is equal to zero (for the $T$ case, see sign at $\Delta\Phi_{T3}$) or even has the opposite sign (for the $L$ case). For this reason the role of a vacuum interaction for the $\rho$-meson is weaker than for the pion. As a consequence of such an SR structure the nonlocal effects partially compensate themselves. Therefore, the extracted values of $\langle\xi^2\rangle_M$ in the framework of NLC SR don't drastically differ from the results of B&B, obtained in the standard way [3]. However, the sensitivity and

[3]Note here, that results of $T$ case in original CZ work shouldn't be taken as a pattern for standard SR – there is an error in sign of quark condensate contribution, see [18, 6].



stability of NLC SR are much better than for the standard one, compare the accuracy for the first moments in Table 2 (errors are depicted in brackets following a standard manner). This allows us to estimate the first ten moments in both channels for $\rho$-, $\rho'$- and $b_1$-mesons. We have determined the following values for practically $N$-independent continuum thresholds: $s_0^L \approx 2.4$ GeV$^2$ and $s_0^T \approx 2.3$ GeV$^2$. Fidelity windows for the $L$ case are: 0.6 GeV$^2 \le M^2 \le 2.1$ GeV$^2$ for all $N = 0, 2, \ldots, 10$. And for the $T$ case they are: 0.5 GeV$^2 \le M^2 \le 2.0$ GeV$^2$ for $N=0$ and 1.0 GeV$^2 \le M^2 \le 1.9$ GeV$^2$ for $N=10$.

**Table 2.** [4]

| Type of SR | The moments $\langle\xi^N\rangle_M(\mu^2)$ at $\mu^2 \sim 1$ GeV$^2$ | | | | | |
|---|---|---|---|---|---|---|
| | $f_M$ (GeV$^2$) | $N=2$ | $N=4$ | $N=6$ | $N=8$ | $N=10$ |
| NLC : $\rho^L$ | 0.206(3) | 0.218(5) | 0.093(4) | 0.049(3) | 0.029(1) | 0.019(1) |
| B&B : $\rho^L$ | 0.205(10) | 0.26(4) | – | – | – | – |
| C&Z : $\rho^L$ | 0.194 | 0.26 | 0.15 | – | – | – |
| NLC : $\rho^T$ | 0.176(4) | 0.225(5) | 0.079(4) | 0.031(3) | 0.009(1) | 0.0020(3) |
| B&B : $\rho^T$ | 0.160(10) | 0.27(4) | – | – | – | – |
| C&Z : $\rho^T$ | 0.200 | 0.15 | $\le 0.06$ | – | – | – |
| NLC : $b_1^T$ | 0.160(5) | 0.27(1) | 0.185(5) | 0.140(5) | 0.116(4) | 0.090(5) |
| B&B : $b_1^T$ | 0.180-0.170 | – | – | – | – | – |
| NLC : $\rho'^L$ | 0.145(5) | 0.330(16) | 0.215(10) | 0.158(7) | 0.118(6) | 0.094(5) |

The ranges of stability within these fidelity windows almost coincide with these windows, starting at higher values of $M^2$. For example, in the $L$ case these ranges start for all $N$ at $M_{-,L}^2 + 0.2$ GeV$^2$, and in the $T$ case they start at $M_{-,T}^2(N) + 0.1$ GeV$^2$.

Another evidence of the efficiency of NLC SR is the estimate of the $\rho'$-meson mass. First, the $\rho'$-resonance with tabular mass $m_{\rho'} = 1465$ MeV was inserted in SR (6) to improve the stability. But at the second step $m_{\rho'}$ was estimated from our SR, see Fig.1. It appears to be rather close to the experimental value [19]:

$$m_{\rho'}^{theor} = 1524 \pm 54 \text{ MeV}, \qquad m_{\rho'}^{exp} = 1465 \pm 22 \text{ MeV}. \qquad (10)$$

Possible models of WF's corresponding to the moments in Table 2 have the form:

$$\varphi_\rho^{L,mod}(x,\mu^2) = \varphi^{as}(x)\left(1 + 0.043\cdot C_2^{3/2}(\xi) - 0.027\cdot C_4^{3/2}(\xi) - 0.055\cdot C_6^{3/2}(\xi)\right), \qquad (11)$$
$$\varphi_\rho^{T,mod}(x,\mu^2) = \varphi^{as}(x)\left(1 + 0.054\cdot C_2^{3/2}(\xi) - 0.20\cdot C_4^{3/2}(\xi) - 0.070\cdot C_6^{3/2}(\xi)\right), \qquad (12)$$

[4]C&Z give all moments normalized to the normalization point $\mu_0 = 500$ MeV, here we present these moments normalized to the normalization point $\mu = 1$ GeV.

$$\varphi_{\rho'}^{L,mod}(x,\mu^2) = \varphi^{as}(x)\left(1+0.38\cdot C_2^{3/2}(\xi)+0.414\cdot C_4^{3/2}(\xi)\right), \quad (13)$$
$$\varphi_{b_1}^{T,mod}(x,\mu^2) = \varphi^{as}(x)\left(1-0.206\cdot C_2^{3/2}(\xi)+0.528\cdot C_4^{3/2}(\xi)\right), \quad (14)$$

where $\xi \equiv 1-2x$ and $\mu^2 \simeq 1$ GeV$^2$ corresponds to an average value of $M^2$. To check the reliability of these models, let us estimate the functional $I[\varphi_M] = \int_0^1 \frac{\varphi_M(x)}{x} dx$ that often appears in the calculations of different form factors, see *e.g.* [20]. It is clear that $I[\varphi_M]$ is a new independent (of moments $\langle \xi^N \rangle_M$) quantity. Besides, the values of $I[\varphi_M]$ allow us to discriminate better different models for the same $\varphi_M$. The $I[\varphi_M]$ can be obtained in two different ways: (i) from QCD SR (6) adapted to $I[\varphi^L]$ by integration with weight $1/x$; (ii) by direct integration of the WF models (11, 13). As it is seen from Table 3, the agreement of both kinds of the results for $I[\varphi^L]$ is rather good for the $\rho_L$-case (the discrepancy is smaller than 7%) and worse for the $\rho'_L$-case.

**Table 3.**

| | Asymp.WF | SR [here] | WF [here] | WF [B&B] | WF [C&Z] |
|---|---|---|---|---|---|
| $I[\varphi_\rho^L]$ | 3 | $3.1 \pm 0.1$ | 2.9 | 3.54 | 4.38 |
| $I[\varphi_{\rho'}^L]$ | 3 | $4.7 \pm 0.2$ | 5.38 | — | — |
| $I[\varphi_\rho^T]$ | 3 | — | 2.35 | 3.6 | — |

These models confirm the property of NLC SR about closeness of WF's of a meson in the "ground state" to asymptotical WF (due to nonlocality effects), proposed by Radyushkin [13], in respect of $\varphi^L_{\rho,mod}(x)$ (see Fig.2) and WF $\varphi^T_{\rho,mod}(x)$ (see Fig.3). The curve $\varphi^L_{\rho,mod}(x)$ oscillates around the asymptotic WF curve, so one may conclude that they practically coincide in an order of the uncertainty of SR. The function $\varphi^T_{\rho,mod}(x)$ is close to the B&B model and is not too far from the asymptotic one as well. One may expect that WF's of resonances would oscillate by analogy with the pion resonance $\pi'$ case [14]. Indeed, the shapes of WF's $\varphi_{\rho'}^{L,mod}(x)$ and $\varphi_{b_1}^{T,mod}(x)$ for resonances look similar to the WF $\pi'$.

## 4 Conclusion

Our basic interest in the present paper is to explore the well-working method of NLC QCD SR in analysis of WF's in the $\rho$-meson vector and tensor channels. As was noted in the previous papers [8, 9, 10, 14, 13] just in problems of nonlocal characteristics of hadrons, such as wave functions, form factors,... one should use the formalism of nonlocal condensates. Let us summarize the main results of this paper:

1. The generalized sum rules for WF's of the $\rho$-meson and related resonances with nonlocal condensates are constructed. With using the simplest ansatz [9] for nonlocal quark condensates we obtain new estimates for the first ten moments of the $\rho$-meson and its resonance WF's. It should be emphasized that analogous evaluation within the standard QCD SR approach is impossible.



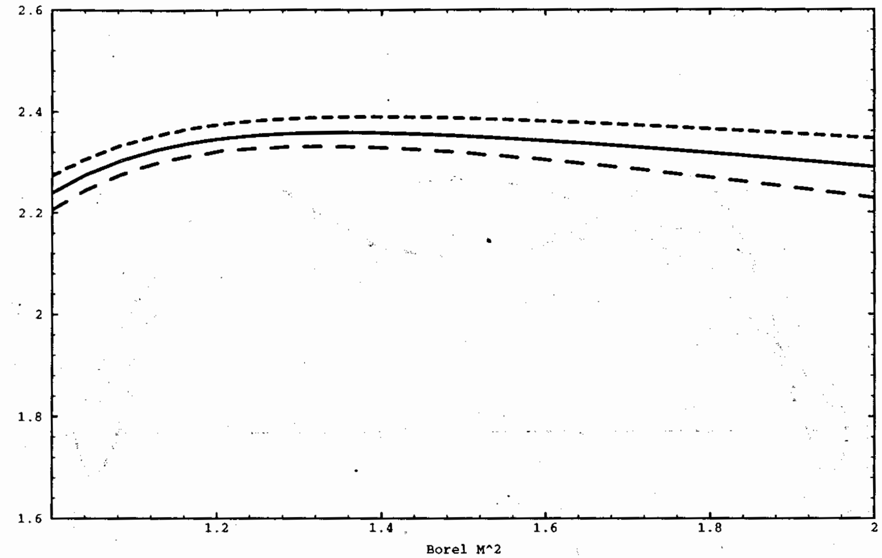


Fig. 1: Extracted squared mass of the $\rho'$-meson (in GeV$^2$): dashed line – $s_0 = 2.4$ GeV$^2$, solid line – $s_0 = 2.5$ GeV$^2$, dotted line – $s_0 = 2.6$ GeV$^2$.

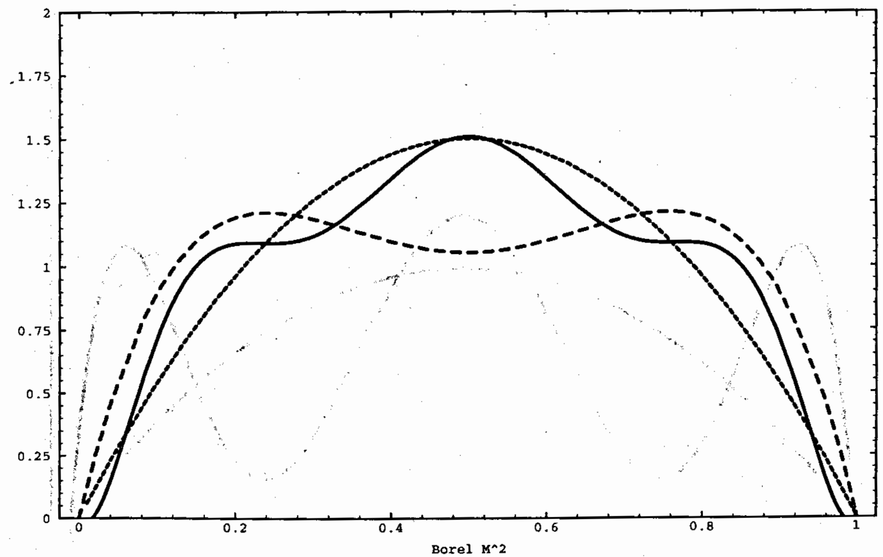


Fig. 2: Longitudinal wave function of the $\rho$-meson: solid line – from NL QCD SR, dotted line – asymptotic WF, dashed line – C&Z WF.

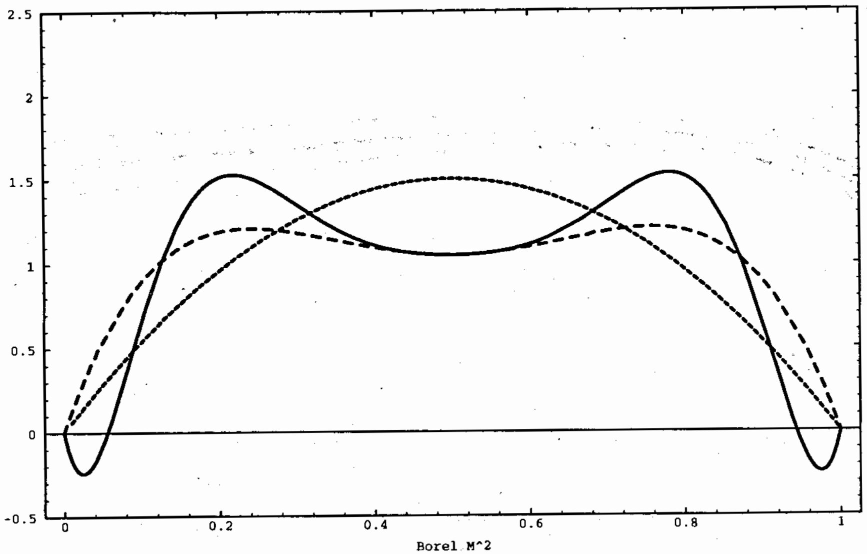


Fig. 3: Transverse wave function of the $\rho$-meson: solid line – from NL QCD SR, dotted line – asymptotic WF, dashed line – B&B WF.

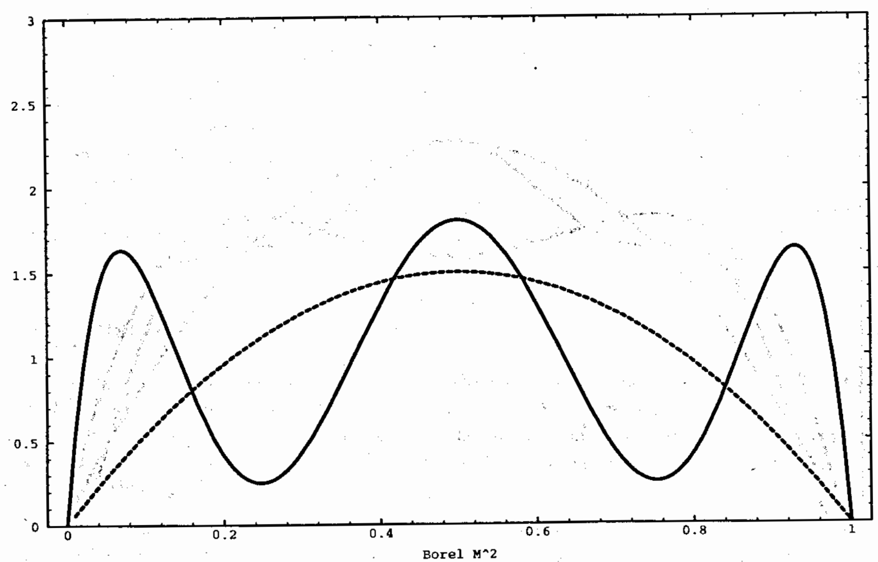


Fig. 4: Longitudinal wave function of the $\rho'$-meson: solid line – from NL QCD SR, dotted line – asymptotic WF.



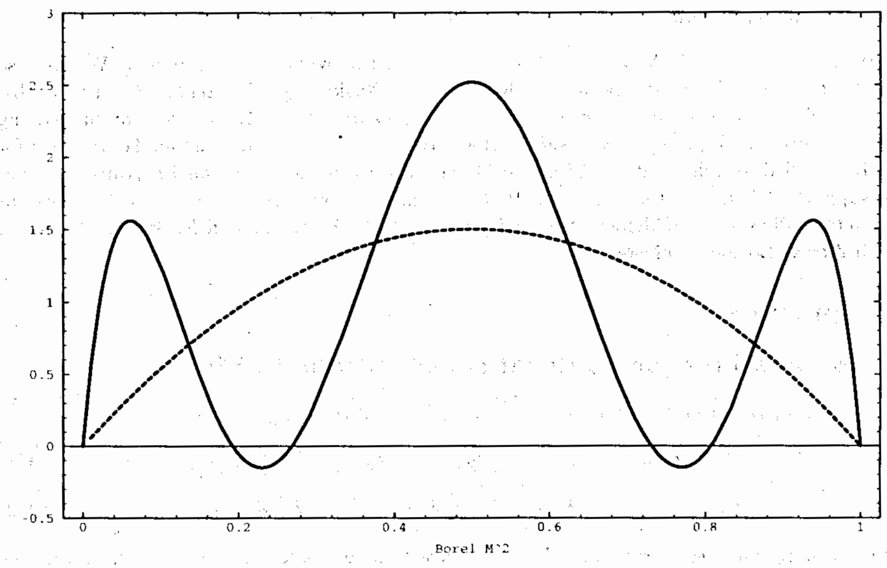


Fig. 5: Transverse wave function of the $b_1$-meson: solid line – from NL QCD SR, dashed line – asymptotic WF.

2. We suggest the models for (see Figs.2–5) WF's of $\rho$-, $\rho'$- and $b_1$-mesons. The form of the obtained longitudinal $\rho$-meson WF is not far from asymptotic WF (this conclusion noticeably differs from the results of the Local QCD SR [5]), while the tensor $\rho$-meson WF is similar to the naive model of Ball–Braun [6].

3. As a by-product we predict the lepton decay constant $f_{\rho'} = 0.145 \pm 0.005$ GeV$^2$ and estimate the mass of the $\rho'$-meson, $m_{\rho'} = 1524 \pm 54$ MeV, which is now under experimental investigation [19].

The wave functions obtained here are crucially important for calculations of semileptonic form factors in heavy-light decays of mesons in the framework of the light-cone sum rule approach, see *e.g.* [20].

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# Appendix

## A Expressions for nonlocal contributions to SR

For vacuum distribution functions $f_\Gamma(\nu)$ we use the set of the simplest ansatzes:

$$f_S(\nu) = \delta\left(\nu - \lambda_q^2/2\right) \; ; \qquad f_V(\nu) = \delta'\left(\nu - \lambda_q^2/2\right) \; ; \tag{A.1}$$

$$f_{T_i}(\alpha_1, \alpha_2, \alpha_3) = \delta\left(\alpha_1 - \lambda_q^2/2\right)\delta\left(\alpha_2 - \lambda_q^2/2\right)\delta\left(\alpha_3 - \lambda_q^2/2\right) . \tag{A.2}$$

Their meaning and relation to initial NLC's have been discussed in detail in [8, 9, 15] The contributions of NLC's $\Delta\Phi_\Gamma(x, M^2)$ corresponding to these ansatzes are shown below; the limit of these expressions to the standard (local) contributions $\varphi_\Gamma(x, M^2)$ – $\lambda_q^2 \to 0$, $\Delta\Phi_\Gamma(x, M^2) \to \Delta\varphi_\Gamma(x, M^2)$ are also written for comparison. Here and in what follows $\Delta \equiv \lambda_q^2/(2M^2)$, $\bar{\Delta} \equiv 1 - \Delta$ :

$$\begin{aligned}\Delta\Phi_S\left(x, M^2\right) &= \frac{A_S}{M^4}\frac{18}{\bar{\Delta}\Delta^2}\{\theta\left(\bar{x} > \Delta > x\right)\bar{x}\left[x + (\Delta - x)\ln\left(\bar{x}\right)\right] + \left(\bar{x} \to x\right) + \\ &\quad +\theta(1 > \Delta)\theta\left(\Delta > x > \bar{\Delta}\right)\left[\bar{\Delta} + (\Delta - 2\bar{x}x)\ln(\Delta)\right]\} ,\end{aligned} \tag{A.3}$$

$$\Delta\varphi_S\left(x, M^2\right) = \frac{A_S}{M^4}9\left(\delta(x) + \left(\bar{x} \to x\right)\right);$$

$$\Delta\Phi_V\left(x, M^2\right) = \frac{A_S}{M^4}\left(x\delta'\left(\bar{x} - \Delta\right) + \left(\bar{x} \to x\right)\right), \tag{A.4}$$

$$\Delta\varphi_V\left(x, M^2\right) = \frac{A_S}{M^4}\left(x\delta'\left(\bar{x}\right) + \left(\bar{x} \to x\right)\right); \tag{A.5}$$

$$\begin{aligned}\Delta\Phi_{T_1}\left(x, M^2\right) &= -\frac{3A_S}{M^4}\left\{\left[\delta(x - 2\Delta) - \delta(x - \Delta)\right]\left(\frac{1}{\Delta} - 2\right)\theta(1 > 2\Delta) + \theta(2\Delta > x)\cdot\right. \\ &\quad \left.\theta(x > \Delta)\theta(x > 3\Delta - 1)\frac{\bar{x}}{\bar{\Delta}}\left[\frac{3x}{\Delta} - 6 - \frac{1 + \bar{x}}{\bar{\Delta}}\right]\right\} + \left(\bar{x} \to x\right),\end{aligned} \tag{A.6}$$



$$\Delta\varphi_{T_1}\left(x, M^2\right) = \frac{3A_S}{M^4}\left(\delta'\left(\bar{x}\right) + \left(\bar{x} \to x\right)\right);$$

$$\begin{aligned}\Delta\Phi_{T_2}\left(x, M^2\right) &= \frac{4A_S}{M^4}\bar{x}\left\{\frac{\delta(x - 2\Delta)}{\Delta}\theta(1 > 2\Delta) - \theta(2\Delta > x)\theta(x > \Delta)\theta(x > 3\Delta - 1)\cdot\right. \\ &\quad \left.\frac{1 + 2x - 4\Delta}{\bar{\Delta}\Delta^2}\right\} + \left(\bar{x} \to x\right),\end{aligned} \tag{A.7}$$

$$\Delta\varphi_{T_2}\left(x, M^2\right) = -\frac{2A_S}{M^4}\left(x\delta'\left(\bar{x}\right) + \left(\bar{x} \to x\right)\right);$$

$$\begin{aligned}\Delta\Phi_{T_3}\left(x, M^2\right) &= \frac{3A_S\bar{x}}{M^4\bar{\Delta}\Delta}\left\{\theta(2\Delta > x)\theta(x > \Delta)\theta(x > 3\Delta - 1)\left[2 - \frac{\bar{x}}{\bar{\Delta}} - \frac{\Delta}{\bar{\Delta}}\right]\right\} \\ &\quad + \left(\bar{x} \to x\right),\end{aligned} \tag{A.8}$$

$$\Delta\varphi_{T_3}\left(x, M^2\right) = \frac{3A_S}{M^4}\left(\delta\left(\bar{x}\right) + \left(\bar{x} \to x\right)\right);$$

$$\Delta\Phi_G\left(x, M^2\right) = \frac{\langle\alpha_s GG\rangle}{24\pi M^2}\left(\delta\left(x - \Delta\right) + \left(\bar{x} \to x\right)\right), \tag{A.9}$$

$$\Delta\varphi_G\left(x, M^2\right) = \frac{\langle\alpha_s GG\rangle}{24\pi M^2}\left(\delta\left(\bar{x}\right) + \left(\bar{x} \to x\right)\right);$$

$$\Delta\Phi'_G\left(x, M^2\right) = \frac{\langle\alpha_s GG\rangle}{6\pi M^2}, \tag{A.10}$$

Here $A_S = \frac{8\pi}{81}\langle\sqrt{\alpha_s}\bar{q}(0)q(0)\rangle^2$, for quark and gluon condensate we use the standard estimates $\langle\sqrt{\alpha_s}\bar{q}(0)q(0)\rangle \approx (0.238\ \text{GeV})^3$, $\frac{\langle\alpha_s GG\rangle}{12\pi} \approx 0.001$ GeV$^4$ [7] and $\lambda_q^2 = \frac{\langle\bar{q}\left(\sigma_{\mu\nu}G^{\mu\nu}\right)q\rangle}{2\langle\bar{q}q\rangle} = 0.4 \pm 0.1$ GeV$^2$, normalized at $\mu^2 \approx 1$ GeV$^2$ [8].

## B Expressions for perturbative spectral densities

First, $\rho(x, s)_L^{pert}$ in an order of $O(\alpha_s)$ was calculated in [8, 9], but there was missed the trivial term $2\ln\left[\frac{s}{\mu^2}\right]$; here we have restored it. The corresponding term for the $T$ case $\rho(x, s)_T^{pert}$ has recently been presented in [6]. We have recalculated it and confirmed this answer:

$$\rho_L^{Born}(x, s) = \frac{3}{2\pi^2}x\bar{x}, \tag{B.1}$$

$$\rho_L^{pert}(x, s) = \frac{3}{2\pi^2}x\bar{x}\left\{1 + \frac{\alpha_s(\mu^2)C_F}{4\pi}\left(2\ln\left[\frac{s}{\mu^2}\right] + 5 - \frac{\pi^2}{3} + \ln^2(\bar{x}/x)\right)\right\}, \tag{B.2}$$

$$\rho_T^{pert}(x, s) = \frac{3}{2\pi^2}x\bar{x}\left\{1 + \frac{\alpha_s(\mu^2)C_F}{4\pi}\left(2\ln\left[\frac{s}{\mu^2}\right] + 6 - \frac{\pi^2}{3} + \ln^2(\bar{x}/x) + \ln(x\bar{x})\right)\right\}. \tag{B.3}$$

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Волновые функции ρ-мезона и связанных с ним мезонов
в правилах сумм КХД с нелокальными конденсатами

Формализм нелокальных конденсатов применяется для построения обобщенных правил сумм КХД (с учетом радиационных $O(\alpha_s)$-поправок) для волновых функций твиста 2ρ-, ρ′- и $b_1$-мезонов. При обработке правил сумм векторного канала оценена масса ρ′-мезона. Для всех указанных мезонов получены первые 10 моментов волновых функций и предложены соответствующие модели для этих функций. Результаты сравниваются с результатами Черняка — Житницкого и Баль — Брауна.

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Bakulev A.P., Mikhailov S.V. E2-97-419

The ρ-Meson and Related Meson Wave Functions
in QCD Sum Rules with Nonlocal Condensates

We apply the nonlocal condensate formalism and construct generalized QCD sum rules (including $O(\alpha_s)$-radiative corrections) for ρ-, ρ′- and $b_1$-meson wave functions of twist 2. As a byproduct we predict the lepton decay constant $f_{\rho'}$ and estimate the mass of ρ′-meson. For all these mesons we obtain the first 10 moments of wave functions and suggest the models for the last. These results are compared with those of Chernyak and Zhitnitsky and of Ball and Braun.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

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