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SOME REMARKS ON THE WOLFENSTEIN EQUATION OF NEUTRINO OSCILLATIONS IN MATTER. ELASTIC INTERACTIONS OF NEUTRINOS IN MATTER AND POLARIZATION OF MATTER BY NEUTRINOS

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Некоторые замечания относительно уравнения Вольфенштейна
по осцилляции нейтрино в веществе.
Упругие взаимодействия нейтрино в веществе и поляризация вещества нейтрино

Анализируется физика, лежащая в основе уравнения Вольфенштейна. Для этой цели изучается прохождение фотона, массивной заряженной частицы и массивного нейтрино через вещество. Показывается, что поскольку слабое взаимодействие не может генерировать массу, то энергия (W) упругого взаимодействия нейтрино в веществе равна нулю. Это приводит к заключению: резонансное усиление осцилляции нейтрино в веществе не должно происходить.

Показывается, что в слабых взаимодействиях черенковское излучение не существует.

Гипотетическое лево-правосимметричное слабое взаимодействие, которое используется в уравнении Вольфенштейна, может генерировать резонансное усиление осцилляции нейтрино в веществе, но (как показано в работе [1]) условие для существования такого резонанса в звездах не реализуется. Более того, показывается, что при выходе нейтрино из вещества (Солнца) в вакуум резонансное усиление осцилляции нейтрино должно исчезнуть бесследно.

Работа выполнена в Лаборатории сверхвысоких энергий ОИЯИ.

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Beshtoev Kh.M. E2-97-360
Some Remarks on the Wolfenstein Equation of Neutrino Oscillations in Matter.
Elastic Interactions of Neutrinos in Matter and Polarization of Matter by Neutrinos

The physics laying on the basis of the Wolfenstein equation, is analysed. For this purpose a photon, a massive charged particle and a massive neutrino passing through matter are studied. It is also shown that since the standard weak interaction cannot generate masses, it cannot generate permanent polarization of matter either, due to this reason the energy (W) of neutrino elastic interaction in matter in the Wolfenstein equation is zero. It leads to the conclusion: resonance enhancement of neutrino oscillations in matter does not exist.

It is shown that in the standard weak interaction the Cherenkov radiation cannot exist.

The hypothetical left-right symmetrical weak interaction, which is used in the Wolfenstein equation, can generate the resonance enhancement of neutrino oscillation in matter, but for stars the condition for realization of the resonance is not fulfilled. Moreover, it is shown that the neutrino resonance enhancement in matter must disappear without leaving a trace when neutrinos go out into vacuum from matter (the Sun).

The investigation has been performed at the Laboratory of Particle Physics, JINR.

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1 Introduction

In the previous work [1] the physics laying on the basis of the Wolfenstein equation was analysed. There was drawn a conclusion: in the Wolfenstein equation [2] a hypothetical left-right symmetrical weak interaction is used but not the standard (left-side) weak interaction. Therefore the conclusions (resonance enhancement of neutrino oscillations in matter) obtained there, have no connection with the weak interaction physics.

This work is to continue the discussion on the problem of neutrinos passing through matter.

2 The Wolfenstein Equation for Neutrino in Matter

In the ultrarelativistic limit the evolution equation for the neutrino wave function ν_Φ in matter has the form [2]

$$i\frac{d\nu_\Phi}{dt} = (p\hat{I} + \frac{\hat{M}^2}{2p} + \hat{W})\nu_\Phi, \tag{1}$$

where $p, \hat{M}^2, \hat{W}$ are, respectively, the momentum, the square mass matrix in vacuum (it is nondiagonal) and the matrix taking into account neutrino interactions in matter,

$$\nu_\Phi = \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix},$$

$$\hat{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

The matrix $\hat{M}^2$ is diagonalized by rotation through the angle θ (vacuum oscillation):

$$\hat{M}^2 = \begin{pmatrix} m^2_{\nu_e\nu_e} & m^2_{\nu_e\nu_\mu} \\ m^2_{\nu_\mu\nu_e} & m^2_{\nu_\mu\nu_\mu} \end{pmatrix},$$

$$\tan(2\theta) = \frac{2m^2_{\nu_e\nu_\mu}}{\mid m^2_{\nu_\mu\nu_\mu} - m^2_{\nu_e\nu_e} \mid}, \qquad \hat{M}^2_{diag} = \begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix}, \tag{2}$$

$$m^2_{1,2} = \frac{1}{2}[(m^2_{\nu_e\nu_e} + m^2_{\nu_\mu\nu_\mu}) \pm \sqrt{(m^2_{\nu_e\nu_e} - m^2_{\nu_\mu\nu_\mu})^2 + 4m^4_{\nu_e\nu_\mu}}],$$

$$\Delta m^2 = \sqrt{(m^2_{\nu_e\nu_e} - m^2_{\nu_\mu\nu_\mu})^2 + 4m^4_{\nu_e\nu_\mu}},$$

and the length of oscillation L_0 in this case is

$$L_0 = \frac{4\pi p}{\mid m_1^2 - m_2^2 \mid}, \qquad E \cong pc. \tag{3}$$

Since $\hat{M}^2$ is a nondiagonal matrix (evidently, $\hat{M}^2$ appears at very short distances r_P, $r_P >> \frac{1}{m_W}$), this vacuum oscillation of neutrinos will take place at any energies with the length of oscillation L_0. The solution of equation (1) is considered in detail in [3] and here the main results will be shown in the reduced form.

When neutrinos are passing through matter, their influence (see equation (1)) leads to changes of the rotation angle θ for diagonalizing the mass matrix $\hat{M}^2$, if diagonal matrix $\hat{W}$, responsible for the difference between the interactions of the neutrinos (ν_e, ν_μ), is added to the mass term $\hat{M}^2/2p$, and then θ becomes $\theta'(\theta' \neq \theta)$.

Thus, neutrino mixing in matter is determined by $\sin^2(2\theta')$:

$$\sin^2(2\theta') = \sin^2(2\theta)/[(\cos(2\theta) - \frac{L_0}{L^0})^2 + \sin^2(2\theta)], \tag{4}$$

$$m'^2_{1,2} = \frac{1}{2}[(m'^2_{\nu_e\nu_e} + m'^2_{\nu_\mu\nu_\mu}) \pm \sqrt{(m'^2_{\nu_e\nu_e} - m'^2_{\nu_\mu\nu_\mu}) + 4m^4_{\nu_e\nu_\mu}}],$$

$$\Delta m'^2 = \sqrt{(m'^2_{\nu_e\nu_e} - m'^2_{\nu_\mu\nu_\mu})^2 + 4m^4_{\nu_e\nu_\mu}},$$

$$L'_0 = \frac{L_0}{\sin(2\theta)},$$

where $m'_{\nu_e\nu_e}, m'_{\nu\mu\nu_\mu}$ are masses of ν_e, ν_μ in matter, L^0 is a diffraction length (i.e., length of formation),

$$L^0 = 2\pi m_p(2^{0.5}G_F\rho Y_e)^{-1} =$$

$$310^7(m)(\rho(g/cm^3)2Y_e)^{-1}, \tag{5}$$

Y_e is the number of electrons per nucleon.

In the common case θ' depends on the difference of masses m_1, m_2, density ρ of matter and the neutrino momentum.

At $L'_0 \cong L^0$ the resonant neutrino oscillations take place, i.e., $\sin^2(2\theta') \cong 1$ or $\theta' \cong \frac{\pi}{4}$. But, since $\Delta m'^2 \cong m^2_{\nu_e\nu_\mu}$, the length of oscillations in matter L'_0 is defined by $m^2_{\nu_e\nu_\mu}$

$$L'_0 = \frac{4\pi p}{| m^2_{\nu_e\nu_\mu} |}$$

and increases relatively to the vacuum oscillations length L_0.

We remind that at resonance (if $\nu_\mu(0) = 0$) the oscillations are defined by the following expression:

$$\nu_e(t) \cong \frac{1}{2}[\exp(-iE'_1 t) + \exp(-iE'_2 t)]\, \nu_e(0)$$

$$\nu_\mu(t) \cong \frac{1}{2}[\exp(-iE'_1 t) - \exp(-iE'_2 t)]\, \nu_e(0), \qquad (6)$$

or

$$P(\nu_e \to \nu_e, t) = 1 - \sin^2(2\theta'(r)) \sin^2[\frac{\pi r}{L'_0(r)}], r = ct, \qquad (7)$$

(where E'_1, E'_2 are energies of ν_1, ν_2 neutrinos), there neutrino resonance mixings take place. It is necessary to notice that the position of the angle mixing maximum at resonance in matter ($\sin^2(2\theta'(r))$) may not coincide with the position of the maximum of neutrino oscillations in matter ($\sin^2[\frac{\pi r}{L'_0(r)}]$), therefore the picture of neutrino oscillations in matter is complicated.

If all $\nu_e(t)$ neutrinos are transformed to $\nu_\mu(t)$, then $\sin(\theta') = 1$ ($\cos(\theta') = 0$), $\theta' = \frac{\pi}{2}$ and $\sin^2(2\theta') = 0$. It is obvious that the considered mechanism cannot lead to such transitions.

The equation (1) was obtained at supposition that the neutrino behaviour in matter is analogous to a photon behaviour in matter with refraction coefficient-n.

The photon velocity c' in matter with refraction index n is

$$c' = \frac{c}{n} \qquad (8)$$

and depends on characteristics of matter.

The laws of conservation of the energy and the momentum of the photon in matter have the following form:

$$E_0 = E + E_{matt} = \frac{E_0}{n} + \frac{E_0(n-1)}{n}$$

$$p_0 = p + p_{matt} = \frac{p_0}{n} + \frac{p_0(n-1)}{n} \qquad (9)$$

$$E_0 = \hbar w_0, w = \frac{w_0}{n}, E = pc',$$

where E_0, p_0 are primary energy and momentum of the photon, E, p, E_{matt}, p_{matt} are, respectively, energy and momentum of the photon in matter and energy and momentum of the matter polarization of the passing photon (matter response).

If we suppose that the neutrinos in matter behave in analogy with the photon in matter and the neutrino refraction indices are defined by the expression

$$n_i = 1 + \frac{2\pi N}{p^2} f_i(0), \qquad (10)$$

(where i is type of neutrinos (e, μ, τ), N is density of matter, $f_i(0)$ are a real part of the forward scattering amplitude), then the velocity of neutrinos in matter is determined by n_i.

The electron neutrino (ν_e) in matter interacts via $W^\pm, Z^0$ bosons and ν_μ, ν_τ interact only via Z^0 boson. These differences in interactions lead to the following differences on the refraction coefficients of ν_e and ν_μ, ν_τ

$$\Delta n = \frac{2\pi N}{p^2} \Delta f(0) \qquad (11)$$

$$\Delta f(0) = -\sqrt{2}\frac{Gp}{2\pi},$$

where G is Fermi constant.

Therefore the velocities of ν_e and ν_μ, ν_τ in matter are different. And at the suitable density of matter this difference can lead to a resonance enhancement of neutrino oscillation in matter [3,1].

3 Analysis of the Wolfenstein Equation

In previous works [1] it was shown that a hypothetical left-right symmetrical weak interaction is used in the Wolfenstein equation therefore we can use some analogy with the Electrodynamics.

Then arises the question: can neutrinos in matter behave as the photon in matter?

This question arises since the neutrinos are supposed to be massive particles but not massless ones. And the massive charged particle behaves unlike the photon in matter. Velocity v can be higher than velocity c' of the photon in matter and $v < c$.

Elastic and inelastic interactions can take place in matter. Here we are interested only in elastic interactions, namely potential interactions of the charged particle in matter. These interactions lead to polarization of the matter, in the result of it a definite part of energy and momentum of the massive charged particle go for polarization of matter. The laws of conservation of the energy and the momentum of the charged particle in matter have the following form:

$$E_0 = E + E_{matt}$$

$$p_0 = p + p_{matt}, \qquad (12)$$

where E_0, p_0 are primary energy and momentum of the charged particle; E, p, E_{matt}, p_{matt} are, respectively, energy and momentum of the charged particle in matter and energy and momentum of the matter polarization.

It is clear that the matter polarization moves with the velocity which is equal to the velocity of the charged particle in matter, v, if this velocity v is less than the velocity of light in matter c'. If v is equal or larger than c', the energy and the momentum of polarization will go for the Cherenkov radiation [4], i.e., the energy and the momentum losses will take place.

It is interesting to know distributions of the energy and momentum between the charged particle and the matter polarization or which part of the energy goes for mass alteration of the charged massive particle (the field of this particle is left-right symmetrical and can be changed

the effective mass of the particle). To solve this problem, it is necessary to do a detailed computing of this interaction (connection between m_0, m can be obtained using equations (12)). Since it is not of our interest, we do not do this computing. The interest is connected with the problem of resonance neutrino oscillations in the hypothetical left-right symmetrical interaction which is used in the Wolfenstein equation (see also expressions (3), (4) and part 3 in [1]).

Work [1] estimates the deposit of the hypothetical left-right interaction in the mass of ν_e neutrino in the regions where enhancement of neutrino oscillations in matter can appear. As in the previous case (electromagnetic interaction) we do not fulfil a detailed computing since it is not of our practical interest.

One needs to notice, from the analogy of the electrodynamics, that the considered process is an elastic one, therefore and if even the neutrino resonance enhancement arises inside the matter (the Sun), when these neutrinos go out from matter (the Sun) into vacuum, this enhancement disappears without leaving a trace and the oscillations transit to vacuum neutrino oscillations (in vacuum the masses, energies and momenta of the neutrinos are restored). The same result is obtained from eq. (6) since in vacuum $\frac{L'_0}{L_0} \to 0$ and $\sin^2(2\theta') \to \sin^2(2\theta)$.

It is very interesting to notice that in the considered case (the hypothetical left-right symmetrical weak interaction) if

$$n_i - 1 > 0, \qquad (13)$$

then, in analogy with the Electrodynamics, when $v_i > \frac{c}{n_i}$, the Cherenkov radiation will take place.

4 Neutrinos in Matter

Let us pass to a more detailed discussion of the problem of resonance enhancement of neutrino oscillations in matter in the weak interaction.

As is known [5,1], this interaction cannot generate the masses. Therefore when the massive neutrino is passing through mater, its mass does not change.

The laws of conservation of the neutrino energy and the momentum have the following form (we do not take into account inelastic processes):

$$a)\ E_0 = E + W,$$

$$b)\ p_0 = p + W\beta, \tag{14}$$

where E_0, p_0, E, p are, respectively, energy and momentum neutrino in vacuum and in matter, W is elastic energy of neutrino interaction in matter, $\beta = \frac{v}{c}$.

It is obvious that the response of matter moves with the velocity $c(\beta = 1)$ since the weak interaction cannot generate a mass (the mass of system is not changed). If expression b) is substituted into expression a), we obtain the following expression:

$$\sqrt{p_0^2 + m_0^2} = \sqrt{(p_0 - W\beta)^2 + m_0^2} + W, \tag{15}$$

which is solved only if:

$$m_0 = 0,$$

or

$$W = 0. \tag{16}$$

The requirement of conservations of energy and momentum for the weak interacting particle in matter leads to a conclusion that or

$$m_0 = 0,$$

or if

$$m_0 \neq 0,$$

then the elastic energy of neutrino interaction with matter (or the energy of the matter response W) must be zero $W = 0$, i.e., there take place only virtual interactions of the neutrino in matter, which is admitted by nonidentity relations, and there is no permanent response of matter (in comparision with the electrodynamics).

As soon as the neutrinos are massive particles in the Wolfenstein equation, the W must be equal to zero, and, this is why there are no any changes of neutrino oscillations in matter.

In the reverse cases (when $m_{0i} = 0$) the W can differ from zero, but in this case, as is well known, the vacuum oscillation of neutrinos cannot take place.

So, we come to a conclusion: no resonance enhancement of neutrino oscillations in matter arises through the standard weak interaction

It is interesting to remark that, when the neutrinos are passing through matter, there is no permanent polarization ($n_i = 1$) of the matter , that is why the Cherenkov radiation cannot arise there.

5 Conclusion

The physics laying on the basis of the Wolfenstein equation, is analysed. For this purpose a photon, a massive charged particle and a massive neutrino passing through matter are studied. It is also shown, that since the standard weak interaction cannot generate masses, it cannot generate permanent polarization of matter either, due to this reason the energy (W) of neutrino elastic interaction in matter in the Wolfenstein equation is zero. It leads to the conclusion: resonance enhancement of neutrino oscillations in matter does not exist.

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Work [6] suggested the nonresonance mechanism of the neutrino oscillations enhancement in matter through the quasi-elastic weak interactions.

In conclusion we would like to stress that in the experimental data from [7] there is no visible change in the spectrum of the B^8 Sun neutrinos. The measured spectrum of neutrinos coincides with the computed spectrum of the B^8 neutrinos [8].

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