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V.S.Barashenkov*

MULTI TIME MECHANICS

*E-mail: barashenkov@lcta30.jinr.dubna.su

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Introduction

As it was shown in our paper [1] the hypothesis that world time, as a matter of fact, is a three-dimensional vector $\hat{t} = (t_1, t_2, t_3)$ geometrically analogous to a space co-ordinate $\mathbf{x} = (x_1, x_2, x_3)$ does not contradict any known today experimental data. Our persuasion in the time one-dimensionality is only a reflection of the fact that in surrounding as part of Universe all observed macroscopic bodies move along parallel time trajectories, therefore only one scalar co-ordinate , namely, a length along these trajectories, is enough to describe all processes taking place in our world. Possible, that is stipulated by peculiarities of the evolution of our Universe in the first instants after its beginning when space regions scattered away on huge distances have lost any mutual correlation and each of them has now its own time direction.

It is not out of the question also that our Universe was created possessing already a difinite time direction.

Why, however, interacting bodies don't change their time trajectories in interactions? What prevents that? Is it possible under any conditions? The answer to these questions can be found in the bounds of mechanics. The mechanical motion is the simplest type of physical phenomena and by this pattern one can become aware of the peculiarities of multi-temporal events.

In the next sect. a formulation of a multi-time variation principle and deriving of equations of motion are considered. Sect. 3 discusses the six-dimensional energy-momentum vector of a moving body and the corresponding conservation laws. Sect. 4 is devoted to particular examples of the multi-time motion.

In what follows the tree-dimensional vectors in x- and t-subspaces will be denoted, respectively, by bold symbols and by a hat, six-dimensional vectors will be marked by bold symbols with a hat. The "six-dimensional nabla" $\hat{\boldsymbol{\nabla}} = (\nabla, \hat{\nabla})$, where time operator $\hat{\nabla} = (-\partial/\partial t_1, -\partial/\partial t_2, -\partial/\partial t_3)$. We also suppose that co- and contravariant vectors are distinguished by the sign of their space components, e. g., $(\hat{\mathbf{x}})_\mu = (-\mathbf{x}, c\hat{t})_\mu$, $(\hat{\mathbf{x}})^\mu = (\mathbf{x}, c\hat{t})^{T\mu}$.

Equations of motion

We begin to study multi-time mechanics by considering the simplest case of a slow particle in a given field. To derive the corresponding equations of motion, we use the action principle

$$\delta S = \int \delta \mathcal{L}\, dt = 0 \qquad (2.1)$$

with the lagrangian $\mathcal{L}(\hat{\mathbf{x}}, \hat{\mathbf{u}}, t)$ where the six-dimension velocity vector

$$\hat{\mathbf{u}} = d\hat{\mathbf{x}}/ds = \gamma c^{-1}(\mathbf{v}, c\hat{\tau})^T = \gamma c^{-1}\hat{\mathbf{V}}, \tag{2.2}$$

and

$$\gamma = (1 - v^2/c^2)^{1/2}, \quad ds = cdt/\gamma, \quad \hat{\tau}^2 \equiv (\sum_i dt_i/dt)^2 = 1. \tag{2.3}$$

(Properties of the multi-dimensional velocity are discussed in more detail in a paper [2]).

The scalar time t is read along the trajectory $\hat{t}$. The variation is performed around the space-time trajectory $\hat{\mathbf{x}}(t)$ of the considered body taking into account that the scalar time t has to remain constant:

$$\delta S = \int \left[\frac{\partial L}{\partial \hat{\mathbf{x}}} \delta\hat{\mathbf{x}} + \frac{\partial L}{\partial \hat{\mathbf{u}}} \delta\hat{\mathbf{u}} \right] ds = 0. \tag{2.4}$$

Here L is the covariant Lagrangian $\gamma\mathcal{L}$. The corresponding Lagrange equations

$$\frac{\partial L}{\partial \hat{\mathbf{x}}} - \frac{d}{ds}\left(\frac{\partial L}{\partial \hat{\mathbf{u}}}\right) = 0, \tag{2.5}$$

where

$$\frac{\partial}{\partial \hat{\mathbf{x}}} = \left(\frac{\partial}{\partial \mathbf{x}}, \frac{\partial}{\partial \hat{x}}\right), \quad \frac{\partial}{\partial \mathbf{x}} = \nabla, \quad \frac{\partial}{\partial \hat{x}} \equiv \frac{\partial}{c\partial \hat{t}} = c^{-1}\hat{\nabla} \tag{2.6}$$

and the time differentiation is performed if one consids the velocity $\hat{\mathbf{u}}$ asindependent variable, i. e. $\hat{\boldsymbol{\nabla}}\hat{\mathbf{u}} = \hat{\mathbf{u}}\hat{\boldsymbol{\nabla}}$.

The operator d/ds can be replaced by $-(\hat{\mathbf{u}}\hat{\boldsymbol{\nabla}})$, if we take into account that for a function $f(\hat{\mathbf{x}})$

$$df(\hat{\mathbf{x}})/ds = \gamma c^{-1} df(\hat{\mathbf{x}})/dt = \gamma c^{-1}\{\lim_{\Delta t \to 0}[f(\hat{\mathbf{x}} + \hat{\mathbf{v}}\Delta t) - f(\hat{\mathbf{x}})]/\Delta t\} =$$

$$\frac{\partial f(\hat{\mathbf{x}})}{\partial \mathbf{u}} + \frac{\partial f(\hat{\mathbf{x}})}{\partial \hat{u}} = -(\hat{\mathbf{u}}\hat{\boldsymbol{\nabla}})f(\hat{\mathbf{x}}), \tag{2.7}$$

Particularly, when a body with the mass $m = \gamma m_o$ is moving in a given field $\hat{\mathbf{A}} = (\mathbf{A}, \hat{A})$, the Lagrangian

$$L = \frac{m_o c}{2}\hat{\mathbf{u}}^2 - \frac{q}{c}\hat{\mathbf{u}}\hat{\mathbf{A}}, \tag{2.8}$$

The equation of motion (2.5) gets then the form

$$m_o c^2 d\hat{\mathbf{u}}/ds = q(\hat{\mathbf{u}}\hat{\boldsymbol{\nabla}})\hat{\mathbf{A}} - q\hat{\boldsymbol{\nabla}}(\hat{\mathbf{u}}\hat{\mathbf{A}}). \tag{2.9}$$

the right part of which can be written as

$$q\{(\hat{u}\hat{\nabla})\hat{\mathbf{A}} - \hat{\boldsymbol{\nabla}}(\hat{u}\hat{A})\} - \{(\mathbf{u}\nabla)\hat{\mathbf{A}} - \hat{\boldsymbol{\nabla}}(\mathbf{u}\mathbf{A})\} \tag{2.10}$$

The space components of the first and the second braces can be expressed, respectively, through a (3×3) electric field tensor

$$\hat{\mathbf{E}} = \mathbf{A}\hat{\nabla} - \nabla\hat{A} \tag{2.11}$$

and a magnetic field vector

$$\mathbf{H} = \nabla \times \mathbf{A}. \tag{2.12}$$

The time components of two braces in (2.10) can be transformed by means of a "temporal-magnetic" field

$$\hat{G} = -\hat{\nabla} \times \hat{A} \tag{2.13}$$

and the tensor $\hat{\mathbf{E}}$.

So, we have

$$m_o c^2 d\mathbf{u}/ds = q\hat{\mathbf{E}}\hat{u} + q\mathbf{u} \times \mathbf{H} \tag{2.14a}$$

$$m_o c^2 d\hat{u}/ds = q\hat{\mathbf{E}}^{\mathbf{T}}\mathbf{u} + q\hat{u} \times \hat{G}, \tag{2.14b}$$

where $\hat{\mathbf{E}}^T$ is the transposed matrix [1].

Using the three-dimensional velocities we get

$$m d\mathbf{v}/dt = q\hat{\mathbf{E}}\hat{\tau} + qc^{-1}\mathbf{v} \times \mathbf{H} - \mathbf{v}dm/dt \tag{2.15a}$$

$$m d\hat{\tau}/dt = qc^{-2}\hat{\mathbf{E}}^T\mathbf{v} + qc^{-1}\hat{\tau} \times \hat{G} - \hat{\tau}dm/dt \tag{2.15b}$$

$$dm/dt = qc^{-2}\mathbf{v}\left(\hat{\mathbf{E}}\hat{\tau}\right). \tag{2.15c}$$

The latter relation is derived by means of the scalar multiplication of the equation (2.14b) by the vector $\hat{\tau}$ taking into account that $\hat{\tau}^2 = 1$, i. e. $\hat{\tau}d\hat{\tau}/dt = 0$.

In the one-dimensional world where all time trajectories are parallel to a given direction $\hat{\tau}$ the potential $\hat{A} \to \varphi\hat{\tau}$, $c\hat{\nabla} \to -\hat{\tau}\partial/\partial t$, the temporal-magnetic field $\hat{G}$ is equal to zero,

$$\hat{G} = -\hat{\nabla} \times \hat{A} = c^{-1}(d\varphi/dt)\hat{\tau} \times \hat{\tau} = 0. \tag{2.16}$$

[1] One must note that according to its meaning in the equation (2.5) the operator $\hat{\nabla} = -\partial/\partial\hat{t}$ does not act on the velocity $\hat{\mathbf{u}}$.

and the electric field tensor

$$\hat{\mathbf{E}}_{ik} = -\tau_k(dA_i/dt + \partial\varphi/\partial x^i) = E_i\tau_k \tag{2.17}$$

where $\mathbf{E}$ is the electric field vector of the one-dimensional theory [2]. Respectively, the equation (2.15a) turns into the Neutonean equation with the Lorentz force in its right part, and the equation (2.15b) takes the form $d\hat{\tau}/dt = 0$.

Energy - momentum vector

Let us define a six-dimensional energy-momentum by the relation

$$\hat{\mathbf{P}} = \partial L/\partial\hat{\mathbf{u}}. \tag{3.1}$$

If the lagrangian L is defined by the eq. (2.8), then

$$\hat{P} = m_o c\hat{\mathbf{u}} + qc^{-1}\hat{\mathbf{A}}. \tag{3.2}$$

In this case the energy is a vector quantity

$$\hat{E} \equiv \hat{P}c = mc^2\hat{\tau} + q\hat{A} \tag{3.3}$$

depending on the time directions of the particle and the field acting on it. Henceforth an important conclusion follows: every variation of a particle time trajectory is accompanied always by a respective change of the particle energy

$$\Delta E = \mid \hat{E}' - \hat{E} \mid = E \mid \hat{\tau}' - \hat{\tau} \mid = 2E\sin\theta/2 \tag{3.4}$$

where θ is the angle between the old and the new time trajectories. The amount of energy consumed to perform the "time turn" of a macroscopic body is of the some order as its rest mass. For example, in order to turn one kg of matter through the angle $\theta = 1^{o}$, one needs to consume about $1.610^{15}J$, i. e, as many as it is produced by an explosion of several hundred tons of nitrotoluol [3, 4].

Figuratively speaking, an encounter with an invisible multi-time cosmic "mine" promises nothing good!

One more interesting consequence of the vectoriality of energy is a light frequency change when light is emitted by a body moving at an

[2] The physical meaning of the tensor $\hat{\mathbf{E}}$ becomes more clear if we note that its value on a fixed time trajectory $\hat{\tau}$: $\hat{\mathbf{E}}\hat{\tau} = -\nabla\varphi - \partial\mathbf{A}/\partial\hat{\tau}$, where $\varphi = \hat{A}\hat{\tau}$. This is analogous to the Maxwell expression $\mathbf{E} = -\nabla\varphi - \partial\mathbf{A}/\partial t$.

angle to the observer's time trajectory. Indeed, an absorption of an photon with an frequency ν changes the detector energy $\hat{E} = E\hat{\tau}$:

$$\hat{E}' = \hat{E} + \hbar\hat{\tau}'. \tag{3.5}$$

The energy measured by the observer is equal to the projection of the vector $\hat{E}'$ on the direction of the observer's motion in t-subspace, i. e.

$$E_{obs} = E + \hbar\nu\cos\theta \tag{3.6}$$

where $\cos\theta = \hat{\tau}\hat{\tau}'$. This is equivalent to the decrease of the photon frequency

$$\Delta\nu = \nu(1 - \cos\theta). \tag{3.7}$$

One should note, however, that the body motion along the trajectory $\hat{t}(t)$ is accompanied by a change of the angle θ from the value $\theta = \pi/2$ at the moment when the radiation arrives for the first time the detector up to the value $\theta = 0$ at the moment when the emitter leaves the space region where it can be observed [2, 5]. Therefore, the detector fixes an appearance "from nowhere" a source of radiation with spectrum displacing gradually to a region of infrared rays.

From the viewpoint of observers some emitters in multi-dimension world flash suddenly, change their color and die down also unexpectedly as ghosts.

If a system of N particles is isolated from outer fields, then using the condition of a space-time homogeneity when the lagrangian

$$\mathcal{L}(\hat{\mathbf{x}}_1, ..., \hat{\mathbf{x}}_N, \hat{\mathbf{v}}_1, ..., \hat{\mathbf{v}}_N, \hat{\mathbf{x}}) = \sum_{n=1}^{N}\left\{m_{on}\gamma^{-1}c\hat{\mathbf{v}}_n^2 + q_n c^{-1}\hat{\mathbf{v}}_n^2\hat{\mathbf{A}}(\hat{\mathbf{x}})\right\} + \mathcal{L}_{em}(\hat{\mathbf{x}}) \tag{3.8}$$

is invariant with respect to equal displacements of all space-time coordinates $\delta\hat{\mathbf{x}}_n = \delta\hat{\mathbf{x}}$ ($\mathcal{L}_{em}$ is a Lagrangian of an internal field), one may prove a conservation law for each of the six components of the total energy-momentum vector:

$$\delta\mathcal{L} = \delta\hat{\mathbf{x}}\sum_{n=1}^{N}\partial\mathcal{L}/\partial\hat{\mathbf{x}}_n = \delta\hat{\mathbf{x}}\sum_{n=1}^{N}\frac{d}{dt}(\partial\mathcal{L}/\partial\hat{\mathbf{v}}_n) + \delta\hat{\mathbf{x}}d\hat{\mathbf{P}}_{em}, \tag{3.9}$$

where $\hat{\mathbf{P}}_{em}$ is the energy-momentum vector of the internal electromagnetic field [11, 16]. Deriving this relation we have used the Lagrange equations which arise from the action principle with the Lagrangian (3.8).

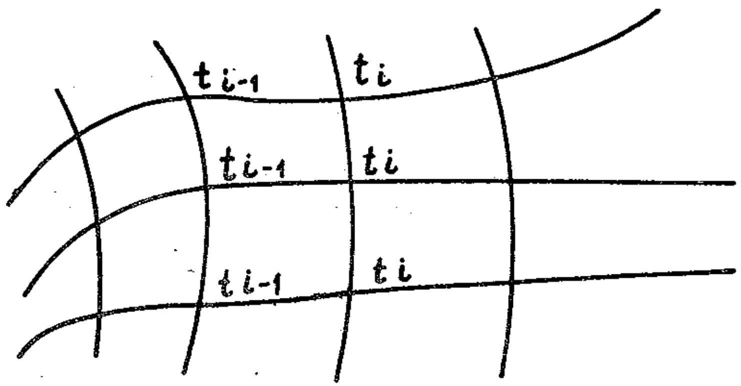

Figure 1: Six-dimensional momentum-energy vector remaines constant on surfaces which are devided by equal intervals $\Delta t = t_i - t_{i-1}$ counted along the time trajectories.

So, the total energy-momentum vector of the system

$$\hat{\mathbf{P}} = \sum_{n=1}^{N} \partial\mathcal{L}/\partial\hat{\mathbf{v}}_n + \hat{\mathbf{P}}_{em} = \text{const}, \tag{3.10}$$

where the time t is read from arbitrary chosen points along the particle trajectories $\hat{t}_n(t)$ but under an condition of a synchronization with the observer's time (e. g. by means of light signals). Particularly, in case of radial time trajectories the momentum and energy have constant values on spheres $\hat{t}^2 = R(t)^2$. For arbitrary directions of trajectories $\hat{t}(t)$ the surfaces where one needs to compare the values of $\hat{\mathbf{P}}(t)$, are more complicated (Fig. 1). The relation (2.10) demonstrates that matter and motion are conserved in the multi-time world. One can show that the six-dimensional momentum $\hat{\mathbf{P}}$ is conserved even in a more general case when the considered physical system is under an action of an outer field, if this field is time-independent. In this case $d\hat{\mathbf{P}}/dt = \partial L/\partial\hat{\mathbf{x}} = 0$, i. e. the vector $\hat{\mathbf{P}}$ remains constant. However, all these conclusions don't take into account that some bodies can leave or, conversely, get into regions of their visibility and the conservatiuon laws can be violated from an observer's viewpoint.

There we encounter the problem of vacuum nonstability discussed in Dorling and Demer's papers [5, 6]. By virtue of the vector nature of energy in multi-time world the conservation laws allow a creation of particle groups with zero summary energy. The simplest example is a production of a pair of particles having opposite energy vectors $\hat{E}_1$ and $\hat{E}_2 = -\hat{E}_1$. Also decays of particles into more heavy ones become possible: $e^- \to \mu^- + p + \bar{p}$, $\pi^o \to n + \bar{n}$ etc. Since such processes are never observed, this circumstance is considered usually as one of the main objections against the time multi-dimensionality. One has to note, however, that a vector sum is equal to zero only in the case when several its vector-components have negative projections, i. e. if the corresponding bodies move back in time what is strictly forbidden by the principle of time non-irreversibility [3]. In our papers [1, 11, 16] it was already stressed that the time irreversibility of events in the multi-dimensional world supposes the existance of some preferred ("relict") reference frame. If the time trajectory of f djdy is declined with respect to the axes of this frame, the decay products can move along distinct t-trajectories (Fig. 2) because in such cases all projections of the time vectors $\hat{\tau}$ are positive. As we, however, do not observe any multi-iime effects [1], we must conclude that our own t-trajectory is close to the time axes of the mentioned preferred reference frame.

In other words, due to the time irreversibility principole and the energy-momentum conservation law we cannot change our time direction with the help of surrounding us macroscopic bodies. Time vectors can be turned only in processes with a participation of bodies whose time trajectories are inclined with respect to the axes of "relict" reference frame. The turn of $\hat{t}$ can occur in regions where the energy conservation law $\sum \hat{E}_n = 0$ is violated and the time homogeneity is absent. Particularly, it could take place during the Big Bang and an primary exponential expansion of our Universe. Possible, a radiation with the "turned time" is created in very strong gravitation fieldaboratoruy ofs where the notion of the local energy looses its sense by itself. Besides, though the time-turned "T-matter" is absent in the surrounding us part of Universe, one cannot exclude that analogously to that what has happened with antimatter it would be produced artificiality (for example, in

[3]Formally, the laws of the usual one-time mechanics allow an existence of bodies both with a positive and a negative masses. Inside this theory a question why the latter don't appear in Nature is a puzzle [8, 9, 10]. In the multi-dimension theory where the sign of mass (energy) is associated with time direction (E is directed along the body trajectory $\hat{t}$) the negative masses are expelled by the time non-irreversibility.

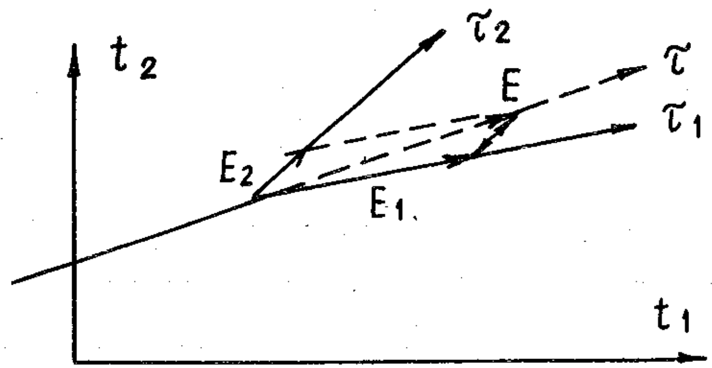

Figure 2: A body moving along an inclined time trajectory $\hat{\tau}(t)$ can decay into parts with diverse trajectories $\hat{\tau}_n(t)$. However, the postulate of time non-irreversibility demands that all projections of energy vectors $\hat{E}_n$ must be possitive, i. e. $\sum \hat{E}_n > 0$, therefore, in particular, any particle creation from vacuum (vacuum decay) is impossible.

any "gravitation reactor"). When a question is a problem far exceeding the bounds of the known, it is difficult to draw a distinction between the science and a fancy! There is another region where one would wait for a creation of the T-matter — interactions of high-energy elementary particles. Owing to the uncertainty relation the classical conservation laws are violated in microscopic space-time intervals where a creation of particles with various t-trajectories becomes possible on a background of virtual processes, especially if we take into account that the energy expenditure accompanying changes of particle time trajectories is of the same order of value as an energy realizing in usual nuclear reactions. One may think that the time multi-dimensionality manifests itself essentially on the properties of quantum-mechanical processes in small intervals $\Delta\mathbf{x}$ and $\Delta\hat{t}$.

Several simple illustrations

To clarify the peculiarities of the multi-time motion, let us consider solutions of the equations (2.15) for some particular cases.

First of all, we note that two members of the right part of the equation (2.15a) represent a generalized Lorentz force which differs from the corresponding fopce of the usual electrodynamics by an replacement of the electric field $\mathbf{E}$ by the three-dimension projection of the tensor $\hat{\mathbf{E}}^T$ on the body time trajectory. The right part of the equation (2.15b) is constructed in the symmetrical way: the vector $\hat{G}$ takes the part of a magnetic field in t-subspace and the projection $\hat{\mathbf{E}}^T$ on the body x-trajectory plays the role of an electric field. The equations (14) can be transformed one into the other by the transposition of space and time components:

$$\mathbf{v} \leftrightarrow c\hat{\tau}, \quad \hat{\mathbf{E}} \leftrightarrow \hat{\mathbf{E}}^T, \quad \mathbf{H} \leftrightarrow \hat{G} \tag{4.1}$$

One can see, when an outer field doesn't act on the moving bogy the latter moves along rectilinear space-time trajectory

$$\hat{\mathbf{x}} = \hat{\mathbf{a}}t + \hat{\mathbf{b}} \tag{4.2}$$

In the case when variations of the time vector $\hat{\tau}$ influence the velocity $\mathbf{v}$ and inversely a connection of the space and time body trajectories is realized by the field $\hat{\mathbf{E}}$. If itjs absent, then the vectors $\mathbf{v}$ and $\hat{\tau}$ become independent and the body trajectories are spiral lines in both x- and t-subspaces.

In a region of small velocities $v \ll c$ time vector alterations are stipulated only by the field $\hat{G}$, and $\hat{t}$ remains constant without it. The calculated expressions for x-space co-ordinates and the velocity $\mathbf{v}$ differ in this case from the respective one-time quantities only by the effective decrease of the charge $q \to q\chi$ where $\chi = \hat{\tau}\hat{\tau}'$ is the cosine of the angle between the time trajectories of the considered body and the acting field $\mathbf{E} = \hat{\mathbf{E}}\hat{\tau}'$. The more the body trajectory $\hat{t}(t)$ deviates from the field trajectory $\hat{t}'(t)$, the slower this body is accelerated and the more it lags behind a body obeying the one-time theory.

In the multi-time world every body undergoes an influence of an outer field only in a time interval ΔT the duration of which depends on the inclination of the time vectors $\hat{\tau}$ and $\hat{\tau}'$ and on a distance up to the field source [2, 5, 12, 13]. If the interval ΔT is sufficiently large, we can consider the outer field as time independent: $\hat{A} = \varphi(\mathbf{x})\hat{\tau}'$. The equation describing the body motion can be written then as

$$d(m\boldsymbol{\beta})/dt = qc^{-1}\mathbf{E}(\mathbf{x})\chi \tag{4.3a}$$

$$d(m\hat{\tau})/dt = qc^{-1}\boldsymbol{\beta}\mathbf{E}(\mathbf{x})\hat{\tau}' \tag{4.3b}$$

$$dm/dt = qc_{-1}\boldsymbol{\beta}\mathbf{E}(\mathbf{x})\chi \tag{4.3c}$$

where $\hat{\boldsymbol{\beta}} = \mathbf{v}/c$ and $\mathbf{E} = -\nabla\varphi$.

Let us multiply these expressions respectively by the vectors $\boldsymbol{\beta}$ and $\hat{\tau}'$. At the next step we multiply cross-wise the left and the right parts of the obtained relations and get, as a result, the symmetrical equation

$$\boldsymbol{\beta} d(\gamma\boldsymbol{\beta})/dt = \chi d(\gamma\chi)/dt \tag{4.4}$$

which can be represented in the form

$$d\beta^2/(1-\beta^2) = d\chi^2/(1-\chi^2), \tag{4.5}$$

if we take into account the relations $d\gamma/dt = \gamma^3\beta d\beta/dt = \gamma^3\beta d\beta/dt$.

An integral of this equation

$$1-\chi^2 = \alpha(1-\beta^2) \tag{4.6}$$

or

$$\gamma^2 = \gamma_o^2(1-\chi_o^2)/(1-\chi^2) \tag{4.7}$$

with the integration constant α defined at the point $\mathbf{x}_o = 0$:

$$\alpha = \gamma_o^2(1-\chi_o^2), \quad \gamma_o = \gamma(\mathbf{x}_o), \quad \chi_o = chi(\mathbf{x}_o). \tag{4.8}$$

The $\chi(t)$ conserves its initial value χ_o if this value is equal to unit. When the body speed is large ($\beta \simeq 1$), then independently of an initial direction of the body time vector $\hat{\tau}$ the cosine $\chi \to 1$, i. e. the outer field forces the body to move along the field direction $\hat{\tau}'$.

Now, substituting γ from (4.7) into the equation (4.3b) we get

$$\frac{d\chi}{(1-\chi^2)^{3/2}} = \frac{qE}{mc^2}\frac{dx}{(1-\chi_o^2)^{1/2}}. \tag{4.9}$$

A solution of this equation

$$\chi[(1-\chi_o^2)/(1-\chi^2)]^{1/2} = \omega x + C \tag{4.10}$$

with $\omega = qE/mc^2$ and an integration constant $C = \chi_o/(1-\chi_o^2)^{1/2}$. Hence

$$\chi = (\chi_o + \omega x)/[1+\omega x(\omega x + 2\chi_o)]^{1/2}. \tag{4.11}$$

Insertung the expression (4.11) into (4.9) and the obtained expression of γ inverse into (4.11) we get

$$\gamma^2 = \gamma_o^2[1+\omega x(\omega x + 2\chi_o)], \tag{4.12}$$

$$\chi = (\gamma_o/\gamma)(\omega x + \chi_o). \tag{4.13}$$

If $\omega x \ll \chi_o$ we obtain a non-relativistic approximation

$$\gamma \simeq \gamma_o + \chi_o\omega x. \tag{4.14}$$

$$\chi(x) \simeq \chi_o + \omega x(1-\chi_o^2) \tag{4.15}$$

In an ultra-relativistic region when $\omega x \gg 1$, i. e. $\omega x \gg 1$,

$$\gamma \simeq \gamma_o\omega x, \tag{4.16}$$

$$\chi \simeq (1-\chi_o^2)/2\omega^2x^2. \tag{4.17}$$

The expressions (4.12) — (4.17) determine the body speed v and the cosine χ as functions of a space point x and the quantities q, E, χ_o. When $\chi_o \to 1$ we have the known expression of the one-time theory [14]. In the both limiting cases (4.14) and (4.16) the expressions for γ differ from the respective one-time expressions only by a decrease of the charge: $q \to q\chi_o$, i. e. by a smaller acceleration rete of the moving body.

A time dependency $x(t)$ can be calculated if the kinematic relation

$$dx/dt = c[\gamma(x)^2 - 1]^{1/2}/\gamma(x) \tag{4.18}$$

is considered as a differential equation for the time derivative dx/dt, with an integral

$$ct(\chi, x) = \int_0^x \gamma(x)[\gamma(x)^2-1]^{-1/2}\,dx. \tag{4.19}$$

A delay of the accelerated body appearance at a point x in comparison with the one-time theory

$$\Delta t/t(1,x) = [t(\chi,x) - t(1,x)]/t(1,x) \tag{4.20}$$

is shown in Fig. 3. As we see, a significant delay takes place only for large deviations of the time vectors $\hat{\tau}$ and $\hat{\tau}'$ ($\chi \lesssim 0.5$) and for small distances x. In all other cases the time trajectories of accelerated bodies are parallel to the field vector $\hat{\tau}'$. True, the distance x in the expressions for χ and γ occurs always as the combination Ex/m, therefore the region where multi-time effects become essential increases by an acceleration of more heavy particles. For example, in a proton beam the scale of distances x in Fig. 3 must be up in two thousand times and a significant delay can be observed up to several hundred meters.

It's not difficult also to find trajectories of bodies moving in a time independent central symmetrical field

$$\hat{\varphi}(\mathbf{x}) = \varphi(\mathbf{x})q_c\hat{\tau}'/r \tag{4.21}$$

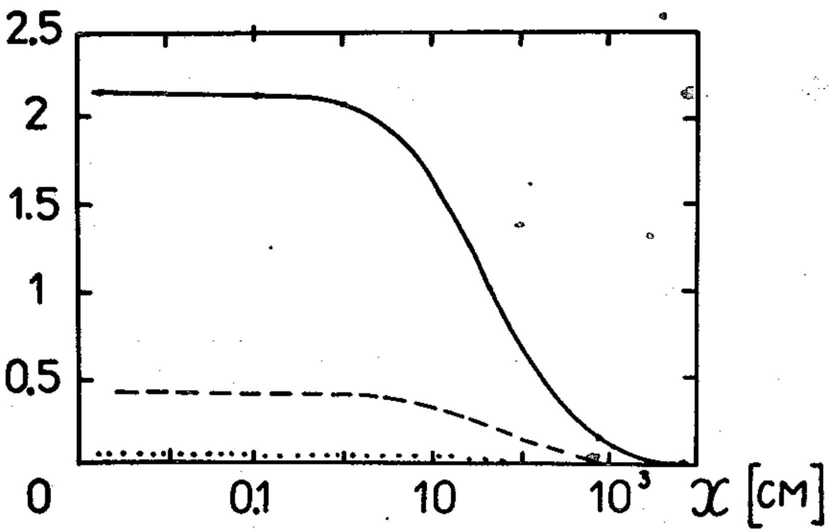

Figure 3: The relative time delay $\Delta t/t(1,x)$ of an electron at the point x. The solid, dotted and pointwise curves correspond to $\chi_o = 0.1, 0.5, 0.9$. The accelerating field $E = 10^4$ v/cm, the initial speed $v_o = 0$.

which is a solution of the Poisson equation

$$\nabla^2\varphi = -4\pi q_c \delta(r)\hat{\tau}'. \tag{4.22}$$

In general form such a problem is considered by Cole [15]. In this case the angle between time trajectories of the scattering body $\hat{\tau}$ and the field $\hat{\tau}'$ is in this case a function of a reciprocal distance again but with equal asymptotic values at the beginning and the end of the interaction. By that some parts of the space trajectories $\mathbf{x}(t)$ find themselves in the regions which are kinematically forbidden in one-time theory. However, if we bear in mind, as it is proposed, particularly, in the paper [15], an observation of a time multi-dimensionality possibly hidden in our world, then an interaction duration ΔT (a time of a "mutual visibility" of the body and a field source) is very short for small impact parameters for which the interaction could distort noticeable the trajectory of the scattering body while in a region of large impact parameters the trajectories remain practically rectlinear.

A finite motion in the potential (4.21) seems more interesting as it is associated with expectations to discover the multi-time effects among

cosmic phenomena. In this case a solution of the equations of motion can be represented again in the form (4.7) where $\mathbf{E}$ is now the known gravitation force

$$\mathbf{E} = \kappa M\mathbf{r}/r^2, \tag{4.23}$$

and the quantities γ_o and χ_o have to be taken at any fixed point of the planet orbit, for instance, at the perihelion $r = r_o$.

To calculate the cosine χ of an angle between the constant time vector of the central body and a rotating planet time vector $\hat{\tau}$, we take into account that the defined by eq. (3.3) the total planet energy

$$\hat{E}mc^2\hat{\tau} - \kappa mM\hat{\tau}'/r \tag{4.24}$$

is conserved. Hence, comparing the value of scalar product $\hat{E}\hat{\tau}'$ with its value at the perihelion we get

$$\chi = (\gamma_o/\gamma)\chi_o + Q(r), \tag{4.25}$$

where

$$Q(r) = \kappa c^{-2}M(1/r - \gamma/\gamma_o r_o) \simeq \kappa c^{-2}M(1/r - 1/r_o), \tag{4.26}$$

since for all planets of solar system $\gamma/\gamma_o \simeq 1$.

The dependencies $\chi(r)$ and $\gamma(r)$ are determined now by the relations (4.14) and (4.15) where we must only change $\omega x \to Q(r)$.

One can show [1] that a deviation of planet time trajectories from the Sun time trajectory results, particularly, in an additional perihelion precession $\Delta\theta \sim (1 - \chi_o^2)$.

Conclusions

We see that by using the variation principle for a generalized action S one can construct a consistent multi-time theory of mechanical motions in diverse fields. The six-dimensional energy-momentum vector of every closed (isolated) system satisfies a conservation law. An additional demand of the time irreversibility ensures vacuum stability and eliminates a possibility of an anomalous reactions when, for example, the mass of particles created in a decay exceeds the decaying particle mass.

At the same time in the multi-time world some bodies can become invisible or can appear suddenly for the observer who would interpret such phenomena as obvious violations of the conservation laws.

The multi-time theory can be formulated in a relativistic invariant way, however, the possibility of a consitancy of the time irreversibility far all time vector components t_i with the generalized Lorentz transformations [3, 17] deserves a more detailed analuze.

One may wait for multi-time effects somewhere in very strong gravitation fields where a local energy is not defined and in quantum processes where the uncertainty relation allows a creation of particles with various directions of time vectors.

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