ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

Дубна

E2-97-322

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MODIFIED $N=2$ SUPERSYMMETRY AND FAYET–ILIOPOULOS TERMS

Submitted to «Ядерная физика»

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*On leave of absence from: Institute of Applied Physics, Tashkent State University, Uzbekistan

1997

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Модифицированная $N=2$ суперсимметрия и члены Файе–Илиопулоса

Изучаются особенности реализации $N=2$ суперсимметрии в $N=2$ абелевой калибровочной теории с двумя типами *FI*-членов, электрическим и магнитным, в явно суперсимметричных формализмах с препотенциалом Мезинческу и гармоническо-аналитическим препотенциалом. Получена «магнитная», дуально-преобразованная суперполевая форма $N=2$ максвелловского эффективного голоморфного действия со стандартным электрическим *FI*-членом и показано, что в этой системе вне массовой поверхности $N=2$ суперсимметрия реализуется в необычной голдстоуновской моде, соответствующей частичному спонтанному нарушению до $N=1$. На массовой поверхности возникает стандартное полное нарушение. В системе с двумя типами *FI*-членов внемассовая $N=2$ суперсимметрия реализуется в частично нарушенной моде в электрическом и магнитном представлениях. Этот режим сохраняется на массовой поверхности благодаря механизму Антониадиса—Партуша—Тэйлора. Показано, что алгебра $N=2$ суперсимметрии в частично нарушенной реализации модифицируется на калибровочно-преобразующихся потенциалах и препотенциалах. Замыкание спинорных зарядов включает некоторые калибровочные преобразования независимо от какой-либо фиксации калибровки.

Работа выполнена в Лаборатории теоретической физики им.Н.Н.Боголюбова ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна, 1997

Ivanov E.A., Zupnik B.M. E2-97-322
Modified $N=2$ Supersymmetry and Fayet–Iliopoulos Terms

We study peculiarities of realization of $N=2$ supersymmetry in $N=2$ abelian gauge theory with two sorts of *FI* terms, electric and magnetic ones, within manifestly supersymmetric formulations via the Mezincescu and harmonic-analytic prepotentials. We obtain a «magnetic», duality-transformed superfield form of the $N=2$ Maxwell effective holomorphic action with standard electric *FI* term and demonstrate that in such a system off-shell $N=2$ supersymmetry is inevitably realized in an unusual Goldstone mode corresponding to the *partial* spontaneous breaking down to $N=1$. On shell, the standard total breaking occurs. In a system with the two sorts of *FI* terms, off-shell $N=2$ supersymmetry is realized in the partial breaking mode both in the electric and magnetic representations. This regime is retained on shell due to the Antoniadis—Partouche—Taylor mechanism. We show that the off-shell algebra of $N=2$ supersymmetry in the partial breaking realization is modified on gauge-variant objects like potentials and prepotentials. The closure of spinor charges involves some special gauge transformations before any gauge-fixing.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna, 1997

1 Introduction

A celebrated mechanism of spontaneous breakdown of rigid $N = 2$ supersymmetry consists in adding a Fayet-Iliopoulos (FI) term to the action of $N = 2$ gauge theory. Recently, Antoniadis, Partouche and Taylor (APT) [1] have found that the dual formulation of $N = 2$ abelian gauge theory (inspired by Seiberg-Witten duality conjecture) provides a more general framework for such a spontaneous breaking due to the possibility to define two kinds of the FI terms (see also [2]). One of them ('electric') is standard, while another ('magnetic') is related to a dual $U(1)$ gauge supermultiplet. APT show that a partial spontaneous breakdown of $N = 2$ supersymmetry to $N = 1$ becomes possible, if one starts with an effective $N = 2$ Maxwell action (with some holomorphic function of $N = 2$ superfield strength W as a superfield Lagrangian) and simultaneously includes two such FI terms.

In this paper we study $N = 2$ Maxwell action with the two types of FI-terms and its invariance properties in the framework of manifestly off-shell supersymmetric $N = 2$ superfield formalism, using both the formulation via the Mezincescu prepotential [3] and the harmonic superspace formulation [4]. Our basic observation is that after duality transformation of a system with even one sort of the FI term, the electric one, off-shell $N = 2$ supersymmetry is inevitably modified, it starts to be realized in a mode with *partial* spontaneous breaking. The dual $N = 2$ superfield covariant strength acquires an unavoidable inhomogeneous term in its supersymmetry transformation and it can naturally be called $N = 2$ *Goldstone - Maxwell superfield* (by analogy with $N = 1$ Goldstone - Maxwell superfield introduced in [5, 6] in the nonlinear realizations approach). One of the dual gaugino is the relevant off-shell Goldstone fermion. On shell such a system is equivalent to the original system with the standard 'electric' form of the FI term, so after passing on shell the total breaking of $N = 2$ supersymmetry occurs (under some restrictions on the holomorphic Lagrangian function). The situation is radically changed after including both types of the FI terms. We show that in this case off-shell $N = 2$ supersymmetry is realized in a partial breaking fashion in both duality-related formulations, 'electric' and 'magnetic' ones, with the electric and magnetic $N = 2$ superfield strengths as the relevant Goldstone-Maxwell superfields. This partial breaking regime is preserved on shell due to the APT mechanism. We demonstrate how simple the latter is when using a manifestly $N = 2$ supersymmetric formalism. We study the realization of modified $N = 2$ supersymmetry transformations on the gauge-variant objects ($N = 2$ harmonic-analytic prepotential and $N = 1$ gauge prepotential) and find that the $N = 2$ supersymmetry algebra itself is also necessarily modified in this case. Namely, the closure of $N = 2$ supercharges contains, besides translations, some special gauge transformations before any gauge-fixing.

In Sect. 2 we give a brief account of the standard $N=2$ superfield formulation of abelian $N=2$ gauge theory with the electric FI term, where off-shell $N=2$ supersymmetry is realized in a customary way. In Sect. 3 we present a duality-transformed 'magnetic' superfield form of the action of such a theory and demonstrate that $N=2$ supersymmetry in this representation is necessarily realized off-shell in a partial breaking mode. In Sect. 4 we discuss a general situation with the two sorts of the FI terms added and show that the regime of off-shell partial breaking of $N=2$ supersymmetry in this case is stable against duality transformation and is preserved on shell. In Sect. 5 we briefly discuss how our observations look in the $N=1$ superfield formulation. In Sect. 6 we pass to the formulation via the harmonic-analytic $N=2$ prepotential V^{++} and study the modified $N=2$ supersymmetry transformations and their closure on this fundamental object of $N=2$ gauge theory. We discuss difficulties of constructing minimal couplings of V^{++} to the matter q^+ hypermultiplets in the framework of such a modified $N=2$ supersymmetry.

2 $N=2$ gauge theory in ordinary $N=2$ superspace

Superfield constraints of $N=2, D=4$ supersymmetric gauge theory were given for the first time in ref.[7]. For the abelian case they read

$$F^{kl}_{\alpha\beta} = D^k_\alpha A^l_\beta + D^l_\beta A^k_\alpha = i\epsilon^{kl}\varepsilon_{\alpha\beta}\overline{W} \tag{2.1}$$

$$F^{kl}_{\dot\alpha\dot\beta} = \bar D^k_{\dot\alpha}\bar A^l_{\dot\beta} + \bar D^l_{\dot\beta}\bar A^k_{\dot\alpha} = -i\epsilon^{kl}\varepsilon_{\dot\alpha\dot\beta}W \tag{2.2}$$

$$F^{kl}_{\alpha\dot\beta} = D^k_\alpha \bar A^l_{\dot\beta} + \bar D^l_{\dot\beta} A^k_\alpha - i\epsilon^{kl}A_{\alpha\dot\beta} = 0\,. \tag{2.3}$$

Here $A_M = (A^i_\alpha,\ \bar A_{i\dot\alpha},\ A_{\alpha\dot\beta})$ are gauge superfield potentials in the real $N=2$ superspace with the coordinates $z^M = (x^m, \theta^\alpha_i, \bar\theta^{\dot\alpha i})$. They are usually assumed to possess the $SU(2)$-covariant standard off-shell $N=2$ supersymmetry transformation laws

$$\delta_\epsilon A_M = i(\epsilon^\alpha_k Q^k_\alpha + \bar\epsilon^{\dot\alpha}_k \bar Q^k_{\dot\alpha})A_M\,. \tag{2.4}$$

The constraints (2.1) - (2.3) can be solved either in terms of unconstrained real prepotential V^{ik} of dimension 2 (the Mezincescu prepotential [3]) or in terms of dimensionless analytic harmonic prepotential V^{++} in the framework of the harmonic superspace approach [4]. We postpone a discussion of the harmonic-superspace formulation to Sect. 6, and will firstly deal with the formulation via V^{ik}.

As a consequence of the above constraints and Bianchi identities the gauge invariant $N=2$ superfield strength W is chiral

$$\bar D_{i\dot\alpha} W = 0 \tag{2.5}$$

and satisfies the additional constraint

$$D^{ik}W - \bar D^{ik}\overline{W} = 0\,, \tag{2.6}$$

where the standard notation for bilinear combinations of the spinor derivatives D^i_α and $\bar D_{i\dot\alpha}$ is used, $D^{ik} = D^{i\alpha}D^k_\alpha$ and $\bar D^{ik} = \bar D^i_{\dot\alpha}\bar D^{k\dot\alpha}$. The constraint (2.6) is the reality condition implying the auxiliary component of $N=2$ Maxwell multiplet,

$$X^{ik} \equiv -\frac{1}{4}D^{ik}W|_0\,, \tag{2.7}$$

to be real

$$(X^{ik})^\dagger = \epsilon_{il}\epsilon_{km}X^{lm} = X_{ik} \tag{2.8}$$

(the symbol $|_0$ means restriction to the lowest, θ, $\bar\theta$ -independent component of $N=2$ superfield).

Both these constraints on W can be solved through the Mezincescu prepotential [3]

$$W_V = (\bar D)^4 D_{ik}V^{ik}\,. \tag{2.9}$$

It should be emphasized that $N=2$ gauge theory can be fully specified by the covariant strength superfield W subjected to the constraints (2.5), (2.6) (or a generalization of the latter, see next Sections). So we can deal entirely with W and V^{ik} as the basic objects of the theory and not care about their geometric origin.

A holomorphic effective action for the abelian (electric) prepotential V^{ik} has the following form

$$S(V) = \frac{i}{4}\int d^4x d^4\theta \mathcal{F}(W_V) + \text{c.c.}\,. \tag{2.10}$$

Here, $\mathcal{F}(W_V)$ is some holomorphic function and $d^4\theta = (D)^4$. The prepotential V^{ik} can be also used to construct a gauge-invariant FI term which breaks the $SU(2)$-authomorphism symmetry and is capable to induce a spontaneous breakdown of $N=2$ supersymmetry

$$S_{FI}(V) = \int d^{12}z E_{ik}V^{ik}\,, \quad S(V) \rightarrow S_E(V) = S(V) + S_{FI}(V)\,. \tag{2.11}$$

Here, $E^{ik} = i\vec{E}(\vec\sigma)^{ik}$ is a $SU(2)$ triplet of constants satisfying the same reality condition (2.8) as the auxiliary field X^{ik}:

$$(E^{ik})^\dagger \equiv E^\dagger_{ik} = \varepsilon_{il}\varepsilon_{kn}E^{ln} = E_{ik}\,, \quad \text{or} \quad \vec{E}^\dagger = \vec{E}\,. \tag{2.12}$$

Note that for any real vector $\vec{E} \neq 0$ the matrix E^{ik} is non-degenerate

$$\text{Det}E^{ik} \sim \vec{E}^2 \neq 0\,. \tag{2.13}$$

The superfield equation of motion following from the action $S_B(V)$ by varying V^{ik} reads

$$D^{kl}\mathcal{F}_W(W_V) - \text{c.c.} = \left[\tau(W_V)D^{kl}W_V + \tau'(W_V)D^{k\alpha}W_V D^l_\alpha W_V\right] - \text{c.c.} = 4iE^{kl}\,, \qquad (2.14)$$

where $\mathcal{F}_W = \partial\mathcal{F}/\partial W$ and the standard notation for the effective coupling constant and its derivative is used

$$\tau(W) = \frac{\partial^2\mathcal{F}}{\partial W^2} = \tau_1 + i\tau_2 \;\; (\tau_2 > 0)\,, \quad \tau'(W) = \frac{\partial^3\mathcal{F}}{\partial W^3}\,. \qquad (2.15)$$

Hereafter, it is assumed that the $SU(2)$ indices in the c.c. pieces are put in a proper position with the help of skew-symmetric tensors, e.g.

$$\overline{X^{ik}} = \epsilon^{ij}\epsilon^{kl}X^\dagger_{jl}\,.$$

A possibility of spontaneous breakdown of $N=2$ supersymmetry by the FI term is related to the possibility to have a non-zero vacuum solution for the auxiliary component X^{ik} in this case

$$< X^{ik} > \equiv x^{ik} \sim E^{ik}\,. \qquad (2.16)$$

Provided that such a solution exists and corresponds to a stable classical vacuum, there appears an inhomogeneous term in the on-shell supersymmetric transformation law of the $N=2$ gaugino doublet $\lambda^{i\alpha}$

$$\delta\lambda^{i\alpha} \sim \epsilon^\alpha_k E^{ik}\,, \qquad (2.17)$$

ϵ^α_k being the transformation parameter. Thus there are Goldstone fermions in the theory, which is a standard signal of spontaneous breaking of $N=2$ supersymmetry.

It is easy to see that for any non-degenerate matrix E^{ik} *both* $\lambda^{1\alpha}, \lambda^{2\alpha}$ are shifted by independent parameters, and so they both are Goldstone fermions in this case. Thus, with the standard FI term, only *total* spontaneous breaking of $N=2$ supersymmetry can occur. Recall that the inhomogeneous pieces in the transformation laws of $\lambda^{\alpha i}$ appear as a result of solving the equation of motion for X^{ik}, so it is natural to assign the term '*on-shell* Goldstone fermions' to these fermionic fields.

In order to get a feeling in which cases the FI term indeed generates a spontaneous breaking of $N=2$ supersymmetry, let us examine whether a non-trivial vacuum background solution with constant values of the auxiliary component $< X^{ik} >= x^{ik}$ and the scalar field $< \Phi > \equiv < W_V > |_0 = a$ exists. So, we choose the ansatz

$$< W_V >_0 = a + (\theta_i\theta_k)\, x^{ik}\,, \qquad (2.18)$$

where $(\theta_i\theta_k) = \varepsilon_{\alpha\beta}\theta^\alpha_i\theta^\beta_k$, and substitute it into the equation of motion (2.14). Using the identity $D^{ik}(\theta_j\theta_l) = -2(\delta^i_j\delta^k_l + \delta^k_j\delta^i_l)$ we get two independent equations

$$x^{ik}\,\tau_2(a) = -\tfrac{1}{2}E^{ik}\,, \qquad (2.19)$$

$$\tau'(a)\, x^{ik}x_{ik} = \tau'(a)\,|x|^2 = 0 \qquad (2.20)$$

(the second one most directly follows from the equation $(D)^4\mathcal{F}_W \sim \Box\overline{\mathcal{F}}_W$ which can be obtained by applying D_{kl} to eq. (2.14)).

A constant solution $x^{ik} \sim E^{ik}$ to eqs. (2.19), (2.20) evidently exists only if $\tau' = 0$, that corresponds to the quadratic Lagrange function $\mathcal{F}(W_V) \sim W_V^2$, i.e. to the free $N=2$ Maxwell theory. Thus for non-trivial functions $\mathcal{F}$ in the action $S(W) + S_{FI}$, the coupled set of equations of motion for physical and auxiliary bosonic fields admits no constant regular solutions which could trigger a spontaneous breaking of $N=2$ supersymmetry. This fact was firstly noticed in ref. [8]. In the same reference, it was also shown that a stable vacuum with a constant nonvanishing X^{ik} and, hence, spontaneously broken $N=2$ supersymmetry exists in a system of at least two $N=2$, $U(1)$ gauge superfields with the FI term for one of them. As is discussed in the next Section, a spontaneous breakdown with a non-trivial function $\mathcal{F}$ and yet one gauge superfield becomes possible when choosing a more general $N=2$ Maxwell action with two different sorts of FI terms, 'electric' and 'magnetic' [1, 2]. Moreover, in this case a *partial* breaking of $N=2$ supersymmetry down to $N=1$ can occur.

3 Dual form of FI term and modification of $N=2$ supersymmetry

Now we turn to discussing the spontaneous breakdown of $N=2$ supersymmetry within dual formulations of the $N=2$ Maxwell effective action. In constructing such formulations we follow the lines of refs. [1, 9, 10].

The passing to the dual description goes through some intermediate 'master' action with an enlarged set of superfields. It involves a chiral and otherwise unconstrained 'electric' superfield strength W and some constrained 'magnetic' superfield strength . Both the original and dual formulations follow from this 'master' action upon varying it with respect to proper superfields.

To get the 'master' action, let us add the constraint (2.6) to the action (2.10) with the help of an unconstrained $N=2$ superfield Lagrange multiplier L_{ik}

$$S(V) \to S(W,L) = S(W) + \frac{i}{4}\int d^{12}z\, L_{ik}(\bar{D}^{ik}\overline{W} - D^{ik}W) \equiv S(W) + S_L\,, \qquad (3.1)$$

where $S(W)$ is obtained via the substitution $W_V \to W$ in (2.10). Thus, the action $S(W,L)$ includes an unconstrained real superfield L_{ik} and a chiral superfield W that is otherwise arbitrary.

Varying L^{ik} yields the constraint (2.6) and hence leads us back to the 'electric' action (2.10) written in terms of W_V, eq. (2.9). On the other hand, one can rewrite (3.1) as an integral over the chiral subspace [1, 10]

$$S(W,L) = \frac{i}{4}\int d^4xd^4\theta[\mathcal{F}(W) - WW_L] + \text{c.c.}\,, \tag{3.2}$$

$$W_L = (\bar{D})^4 D_{ik} L^{ik}\,. \tag{3.3}$$

The newly introduced chiral object W_L by construction satisfies the same constraint (2.6) as W_V, i.e.

$$D^{ik}W_L - \bar{D}^{ik}\overline{W}_L = 0\,, \tag{3.4}$$

and is expressed via L^{ik} just in the same fashion as W_V via the Mezincescu prepotential V^{ik}. Therefore it is natural to think of W_L as the dual or 'magnetic' $N = 2$ Maxwell superfield strength, and the Lagrange multiplier L^{ik} as the dual or 'magnetic' prepotential.

In order to obtain a 'magnetic' representation of the $N = 2$ Maxwell action, one should eliminate W from the 'master' action (3.1) by varying the latter with respect to this superfield. As a result one gets an algebraic equation

$$\mathcal{F}_W = W_L \tag{3.5}$$

that allows one to express W in terms of W_L

$$W = W(W_L)\,, \tag{3.6}$$

$$\partial W/\partial W_L = [\partial W_L/\partial W]^{-1} = (\tau(W))^{-1} \equiv -\hat{\tau}(W_L)\,. \tag{3.7}$$

After this one arrives at the magnetic representation of the $N = 2$ Maxwell action

$$S(L) = \frac{i}{4}\int d^4xd^4\theta\hat{\mathcal{F}}(W_L) + \text{c.c.}\,, \tag{3.8}$$

with the new dual holomorphic Lagrangian function

$$\hat{\mathcal{F}}(W_L) \equiv \mathcal{F}[W(W_L)] - W_L\, W(W_L)\,. \tag{3.9}$$

The 'magnetic' equation of motion has the following simple form:

$$D^{ik}\hat{\mathcal{F}}' - \text{c.c.} = (\hat{\tau}D^{ik}W_L + \hat{\tau}' D^{k\alpha}W_L D^l_\alpha W_L) - \text{c.c.} = 0\,. \tag{3.10}$$

Thus, the functional $S(W,L)$ (3.1) defines the duality transformation between the 'electric' and 'magnetic' forms of the $N = 2$ gauge theory action

$$S(V) \leftrightarrow S(W,L) \leftrightarrow S(L)\,. \tag{3.11}$$

How to get the dual form of the FI-term (2.11)? Recall that in the original 'electric' representation it is constructed using the prepotential V_{ik}, the object which appears as

the solution to the constraint (2.6) and which is certainly lacking in the formalism with the chiral and otherwise unconstrained superfield W and the dual superfield strength W_L.

To answer this question, let us come back to the 'master' action (3.1) and extend it by the term

$$S_e = -\frac{1}{8}\int d^4xd^4\theta E^{ik}(\theta_i\theta_k)W + \text{c.c.}\,, \quad S(W,L) \to S_E(W,L) = S(W,L) + S_e\,. \tag{3.12}$$

Note that the constants E^{ik} in S_e, without loss of generality, can be chosen real; their possible imaginary parts can always be absorbed into a redefinition of W_L or L^{ik} without affecting the reality properties of these superfields. The term S_e becomes just (2.11) after the substitution $W \to W_V$, i.e. after passing to the 'electric' representation, and hence it can be regarded as a 'disguised' form of the standard electric FI term. Its dual 'magnetic' form can now be obtained by passing to the 'magnetic' representation of the extended action $S_E(W,L)$ by eliminating W from it, like this was done for the action $S(W,L)$.

However, at this step one encounters a trouble. We observe that S_e is not invariant under the standard $N = 2$ supersymmetry transformations unless W is subjected to the constraint (2.6). The invariance of the full action can be restored (before imposing (2.6), i.e., varying with respect to L^{ik}) by means of the following redefinition of the off-shell transformation law of the dual superfield strength

$$\delta_\epsilon W_L = i(\epsilon_k\theta_l)E^{kl} + i(\epsilon Q + \bar{\epsilon}\bar{Q})W_L\,, \tag{3.13}$$

where $Q^i_\alpha, \bar{Q}^i_{\dot\alpha}$ are standard $N = 2$ supersymmetry generators. Note that the appearance of the $SU(2)$-breaking shift in eq.(3.13) is still compatible with the constraint (3.4) for W_L, thanks to the relation $D^{ij}(\epsilon_k\theta_l) = 0$.

This modified transformation law still has the space-time translations as the off-shell closure, but implies a Goldstone-type transformation for the fermionic component $D^{\alpha i}W_L \equiv \lambda^{\alpha i}_L$ (i.e., the 'magnetic' photino)

$$\delta\lambda^{\alpha i}_L \sim i\epsilon^\alpha_k E^{ik}\,. \tag{3.14}$$

This inhomogeneous transformation is valid off-shell, before using the equations of motion, therefore $\lambda^{\alpha i}_L$ can be called *off-shell* Goldstone fermions.

With the definition (3.13), inhomogeneous pieces are present in *both* supersymmetry transformations, so at first sight we are facing the phenomenon of total off-shell spontaneous breaking of $N = 2$ supersymmetry in this case. It is not so, however. Namely, let us show that by a proper shift of the real auxiliary field of W_L

$$W_L \to \widetilde{W}_L = W_L + \frac{1}{2}(\theta_i\theta_k)C^{ik}, \tag{3.15}$$

one can restore a homogeneous transformation law with respect to *one* of two $N = 1$ supersymmetries present in $N = 2$ supersymmetry (it is easy to find the appropriate redefinition of L^{ik}). The newly defined object $\widetilde{W}_L$ transforms as follows

$$\delta_\epsilon \widetilde{W}_L = (\epsilon_k \theta_l)(C^{kl} + iE^{kl}) + i(\epsilon Q + \bar{\epsilon}\bar{Q})\widetilde{W}_L \,. \tag{3.16}$$

One can always choose C^{ik} so that

$$\det\,(C + iE) = 0 \,. \tag{3.17}$$

Indeed, this condition amounts to requiring C^{ik} to be orthogonal to E^{ik} and to have the same norm

$$(a)\; E^{ik}C_{ik} = 0, \quad (b)\; |E| = |C| \,. \tag{3.18}$$

It is easy to find a general solution to these equations. E.g., for the two different choices of the $SU(2)$ frame:

$$(i)\; E^{12} \neq 0,\; E^{11} = E^{22} = 0\,; \quad (ii)\; E^{12} = 0,\; E^{11} = E^{22} \tag{3.19}$$

we have

$$(i)\; C^{12} = 0,\; C^{11}C^{22} = |E^{12}|^2\,; \quad (ii)\; C^{12} = 0,\; C^{11} = \pm iE^{11},\; C^{22} = \mp iE^{11} \,. \tag{3.20}$$

Note that in the second case we have fixed the residual $U(1)$ freedom up to a reflection.

Eq.(3.17) means that $C^{ik} + iE^{ik}$ is a degenerate symmetric 2×2 matrix, so it can be brought to the form with only one non-zero entry (ii). As a result, $\widetilde{W}_L$ is actually shifted under the action of only *one* linear combination of the modified $N = 2$ supersymmetry generators $\hat{Q}^{1,2}_\alpha$, while under the orthogonal combination it is transformed homogeneously. The same is true of course for the physical fermionic components: only *one* their combination is the genuine off-shell Goldstone fermion. All these options are related by some $SU(2)$ transformations (they can be continuous or discrete). For instance, in the case (ii) in eq. (3.20) the ϵ^α_2 or ϵ^α_1 supersymmetries are broken for the first or second choices of the sign, respectively.

Thus we arrive at the important conclusion: in the dual, 'magnetic' representation of $N = 2$ Maxwell theory with FI term $N = 2$ supersymmetry is realized *off-shell* in a mode with *partial spontaneous breaking*, so that some $N = 1$ supersymmetry remains unbroken.

It should be specially pointed out that, contrary to the original, electric representation of the FI term, in the dual representation the phenomenon of spontaneous breaking of $N = 2$ supersymmetry occurs already *off shell*, irrespective of the form of the holomorphic Lagrangian function $\hat{\mathcal{F}}(W_L)$. This situation resembles the nonlinear realization approach [11, 12, 5, 6] where one introduces from the very beginning, according to some prescription,

inhomogeneously transforming Goldstone fields or superfields which express in a most pure way an effect of off-shell spontaneous breaking of some symmetry or supersymmetry.

Partial spontaneous breaking of $N = 2$ supersymmetry in the framework of the nonlinear realizations approach, with the relevant Coldstone fermion placed into chiral or vector $N = 1$ multiplets, was considered in ref. [12, 5, 6]. Using the terminology of [5, 6] and taking into account that W_L contains Goldstone fermion components, it is natural to call this inhomogeneously transforming superfield the $N = 2$ *Goldstone-Maxwell* (GM) superfield. In what follows we will deal with this superfield, keeping in mind that one can always pass to $\widetilde{W}_L$, eq. (3.16), in terms of which the phenomenon of the *off-shell partial $N = 2$ supersymmetry breaking* becomes manifest.

One can go a step further and wonder how the modified transformations are realized on the gauge-variant objects: gauge potentials and prepotentials. In Sect. 5 we will present the modified supersymmetry transformation of the $N = 1$ GM prepotential V. In Sect. 6, we will also give how the modified $N = 2$ supersymmetry is realized on the harmonic prepotential and study a modified supersymmetry algebra in this realization. As we will see, the GM mechanism of spontaneous breaking implies an essential modification of the algebra of $N = 2$ supersymmetry transformations on gauge superfields. Note that the transformation properties of the magnetic gauge connections A_M are also appropriately modified by inhomogeneous terms containing E^{ik}, so as to preserve the covariance of the magnetic analogs of the constraints (2.4).

As was already mentioned, after passing to the 'electric' representation of $S_E(W, L)$ by varying it with respect to L^{ik} one gets the action (2.10) accompanied by the standard FI term (2.11), both being written in terms of the 'electric' prepotential V^{ik}. This prepotential and its covariant chiral strength W_V possess standard $N = 2$ supersymmetry transformation properties, so the modification of $N = 2$ supersymmetry in the action $S_E(W, L)$ is to some extent an artifact related to the insertion of the constraint (2.6) into the action and the appearance of additional superfield L^{ik} there. However, this modified $N = 2$ supersymmetry is retained after putting $S_E(W, L)$ into the pure 'magnetic' representation by eliminating the superfield W. Thus it is a characteristic unavoidable feature of the dual ('magnetic') off-shell formulation of $N = 2$ Maxwell action with the FI term.

To get the precise form of such a dual action, let us combine the L^{ik} (or W_L) piece S_L in the action $S_E(W, L)$, eq. (3.12), together with S_e into the following mixed term

$$\hat{S}_L = S_L + S_e = -\frac{i}{4}\int d^4x d^4\theta\, W\hat{W}_L + \text{c.c.}\,, \tag{3.21}$$

where a shifted GM-superfield $\hat{W}_L$ with a homogeneous $N = 2$ supersymmetry transformation law was introduced

$$\hat{W}_L = -(i/2)(\theta_k\theta_l)E^{kl} + W_L\,, \tag{3.22}$$
$$\delta_\epsilon \hat{W}_L = i(\epsilon Q + \bar{\epsilon}\bar{Q})\hat{W}_L\,. \tag{3.23}$$

The net difference between $\hat{W}_L$ and W_L is that the auxiliary scalar field component of the former,

$$\hat{Y}^{ik} \equiv -\frac{1}{4}D^{ik}\hat{W}_L|_0\,,$$

contains a constant imaginary part proportional to E^{ik}, while the auxiliary component Y^{ik} of W_L is real by construction (eq. (3.3)). One has

$$\hat{Y}^{kl} = Y^{kl} - (i/2)E^{kl}\,. \tag{3.24}$$

By its definition, the quantity $\hat{W}_L$ obeys the modified constraint

$$D^{kl}\hat{W}_L - \text{c.c.} = 4iE^{kl}\,, \tag{3.25}$$

demonstrating the presence of the imaginary constant part just mentioned.

Clearly, passing to $\hat{W}_L$ does not redefine the 'magnetic' gaugini and does not affect the interpretation of their one combination as a Goldstone fermion with an inhomogeneous piece in the off-shell transformation law.

It is easy to get a purely 'magnetic' form of the modified 'master' action $S_E(W,L)$, eq. (3.12). Varying W now yields the modified equation

$$\frac{\partial\mathcal{F}}{\partial W} = \hat{W}_L\,, \tag{3.26}$$

which leads to the following θ-dependent modification of the magnetic action (3.8)

$$\tilde{S}(L) = \frac{i}{4}\int d^4x d^4\theta \hat{\mathcal{F}}(\hat{W}_L) + \text{c.c.}\,, \tag{3.27}$$

with $\hat{\mathcal{F}}$ defined by eq. (3.9).

Thus in the dual 'magnetic' formulation the whole effect of the original 'electric' FI term (2.11) is the appearance of θ-dependent terms in the action, with coefficients proportional to the $SU(2)$ breaking constants E^{ik}. Despite this explicit θ dependence, the action is still $N=2$ super-invariant due to the modification (3.13) of the transformation law of W_L.

The appropriate equation of motion follows from (3.10) by substituting there $W_L \to \hat{W}_L$ and taking account of the modified constraint (3.25). Analyzing this equation, it is straightforward to show that the dual action (3.27) leads to the same vacuum structure as the original 'electric' action with the FI term (2.11). In particular, a Poincaré-invariant

vacuum with spontaneously broken $N=2$ supersymmetry exists only for $\hat{\tau}'=0$, i.e. for a free theory, like in the electric representation (this restriction is equaivalent to $\tau'=0$, of course).

Thus on shell in the 'magnetic' representation we again face the total spontaneous breaking of $N=2$ supersymmetry, despite the fact that off-shell $N=2$ supersymmetry is realized in the mode with partial spontaneous breaking.

To summarize, in the 'magnetic' representation of $N=2$ Maxwell theory with a single FI term there occurs a partial spontaneous breaking of $N=2$ supersymmetry off shell which goes over into the total one after passing on shell. Thus, a crucial difference from the original, 'electric' representation of the same theory is that in the 'magnetic' case the effect of spontaneous supersymmetry breaking has an unavoidable 'inborn' off-shell constituent corresponding to the partial breaking mode. This new phenomenon is related to the modification of off-shell realization of $N=2$ supersymmetry after performing a duality transform of the 'electric' FI term. Passing on shell changes the type of spontaneous breakdown, promoting it to the total one (simultaneously imposing a severe constraint on the admissible form of the holomorphic Lagrangian).

In the next Section we will see that adding of some different kind of the FI term radically changes this interplay between the off-shell and on-shell constituents of spontaneous $N=2$ supersymetry breaking. Namely, there appears a stable vacuum ensuring the phenomenon of partial supersymmetry breaking to retain on shell, too.

4 $N=2$ supersymmetry in the presence of electric and magnetic FI terms

New possibilities come out if we simultaneously include two alternative mechanisms of the spontaneous supersymmetry breaking, the previously discussed one and a new one discovered in [1]. It consists in extending the intermediate action $S_E(W,L)$ (3.12) by a superfield 'magnetic' FI-term

$$S_m = \frac{1}{8}\int d^4x d^4\theta M^{kl}(\theta_k\theta_l)W_L + \text{c.c.} = -\int d^{12}z M^{kl}L_{kl}\,, \tag{4.1}$$

with M^{ik} being another triplet of real constants. The W_L-form of S_m is invariant under the Goldstone-type transformation (3.13) since $\int d^4\theta(\theta)^3 = 0$ (a contribution from the shift of explicit θs in (4.1) disappears by rewriting W_L through L^{ik}, restoring the full integration measure and integrating by parts). One can show that the inhomogeneous piece in the transformation of L_{ik} does not contain terms higher than those of 7th order in the spinor coordinates, so the invariance of the second representaton of S_m in (4.1) is also guaranteed, $\int d^8\theta\delta_\epsilon L_{ik} = 0$.

As in the previous consideration, one can pass from this modified intermediate action to its either 'electric' or 'magnetic' representations, varying it with respect to L^{ik} or W, respectively.

When one descends to the 'electric' representation (by varying L^{ik}), the only effect of magnetic FI term is the modification of the constraint (2.6) on W. The modified constraint reads

$$D^{ik}W - \bar{D}^{ik}\overline{W} = 4iM^{ik} \ . \qquad (4.2)$$

It suggests the redefinition

$$W = W_V - \frac{i}{2}(\theta_i\theta_k)M^{ik} \ , \qquad (4.3)$$

with W_V satisfying eq. (2.6) and hence given by eq. (2.9) [1], and means the appearance of the constant imaginary part $-(i/2)M^{ik}$ in the auxiliary field of W,

$$\hat{X}^{ik} \equiv -\frac{1}{4}D^{ik}W|_0 = X^{ik} - \frac{i}{2}M^{ik} \ , \quad X^{ik} \equiv -\frac{1}{4}D^{ik}W_V|_0 \ .$$

The inclusion of magnetic FI term cannot change the transformation properties of W under $N = 2$ supersymmetry which are standard. Then the relation (4.3) requires modifying the transformation law of W_V, and, respectively, V^{ik} on the pattern of eq. (3.13)

$$\delta_\epsilon W_V = i(\epsilon_k\theta_l)M^{kl} + i(\epsilon Q + \bar{\epsilon}\bar{Q})W_V \qquad (4.4)$$

(we do not give how the transformation law of V^{ik} is changed, it is easy to find this modification). In other words, $N = 2$ supersymmetry is now realized in a Goldstone-type fashion in the 'electric' representation as well, but with M^{ik} instead of E^{ik} as the 'structure' constants. Thus we see that the adding of magnetic FI term modifies $N = 2$ supersymmetry in the electric representation quite similarly to what happens in the magnetic representation after adding the standard 'electric' FI term. So, when both FI terms are present, $N = 2$ supersymmetry is realized in the Goldstone mode in both representations, 'electric' and 'magnetic', and there is no way to restore the standard $N = 2$ supersymmetry off shell. In particular, $N = 2$ transformations of the standard 'electric' gauge connections introduced by eqs. (2.1) - (2.4) acquire inhomogeneous $SU(2)$ breaking terms proportional to M^{ik} (note that in the r.h.s. of the constraints (2.1) -(2.2), by definition, just the covariant strength W_V appears). The same arguments as in the previous Section show that $N = 2$ supersymmetry in both representations is realized off shell in *the partial breaking mode*.

An effect of the magnetic FI term in the magnetic representation is a further modification of the equation of motion for W_L: the term $4iM^{ik}$ appears in its r.h.s. Of course, this is related to the fact that the reality constraint for W_V is the equation of motion for the dual superfield strength, and vice versa.

The issue of on-shell spontaneous breaking of $N = 2$ supersymmetry in the presence of electric and magnetic FI terms can be analyzed both in the electric and magnetic versions of the full intermediate action

$$S_{(E,M)} = S_E(W, L) + S_m \ , \qquad (4.5)$$

with the same final conclusions. In order to be closer to the original paper [1], we prefer to do this in the electric representation, with the prepotential V^{ik} and its covariant strength W_V as the basic entities.

A general electric effective action of the abelian gauge model with the (E, M)- mechanism of the spontaneous breaking can be obtained by substituting the expression for W, eq.(4.3), into the action (2.10) and supplying the latter with the electric FI term (2.11):

$$S_{(E,M)} = \left[\frac{i}{4}\int d^4xd^4\theta \mathcal{F}(W) + \text{c.c.}\right] + \int d^{12}zE_{ik}V^{ik} \ , \quad W = W_V - (i/2)(\theta_i\theta_k)M^{ik} \ . \ (4.6)$$

The superfield equation of motion in the electric representation of the (E, M)-model reads

$$[\tau D^{kl}W + \tau' D^{k\alpha}WD^l_\alpha W] - \text{c.c.} = 4iE^{kl} \ . \qquad (4.7)$$

The corresponding equation for the auxiliary component is as follows:

$$2\tau_2(a)X^{kl} = \tau_1(a)M^{kl} - E^{kl} \ . \qquad (4.8)$$

Then we take for $< W_V >$ the ansatz (2.18) and substitute it into (4.7) with taking account of the relation (4.3). Besides the expression for the vacuum value of auxiliary field $x^{kl} =< X^{kl} >$

$$x^{kl} = \frac{1}{2\tau_2(a)}\left(\tau_1(a)M^{kl} - E^{kl}\right) \ , \qquad (4.9)$$

one gets, from vanishing of the coefficient before $(\theta_i\theta_k)$, the following generalization of eq. (2.20)

$$\tau' \ \hat{x}^{ik}\hat{x}_{ik} = 0 \ , \quad \hat{x}^{ik} =< \hat{X}^{ik} >= x^{ik} - (i/2)M^{ik} \ . \qquad (4.10)$$

A crucial new point compared to (2.20) is that the vector $\hat{x}^{ik} = x^{ik} - (i/2)M^{ik}$ is *complex*, so the vanishing of its square does not imply it to vanish. As a result, besides the trivial solution $\tau' = 0$, eq. (4.10) possesses the nontrivial one

$$\tau' \neq 0 \ , \quad \hat{x}^{ik}\hat{x}_{ik} = 0 \quad (\text{or} \quad \det \hat{x} = 0) \ . \qquad (4.11)$$

[1] Note that the $SU(2)$-noninvariant constraint (4.2) with an arbitrary M^{ik} (equally as its 'magnetic' analog (3.25)) is consistent with the constraint of ref. [1] which can be written in the following form:

$$D^{ik}D_{ik}W = D^{ik}\bar{D}_{ik}\overline{W} \sim \Box\overline{W} \ .$$

With using this constraint (or its magnetic analog), the constants M^{ik} or E^{ik} appear as integration constants, while in the approach we keep to both these sets are present in the action from the beginning as some moduli of the theory.

This solution, after substitution of the expression (4.9), amounts to the following relations between $\tau_{1,2}(a)$ and the $SU(2)$ breaking parameters E^{ik}, M^{ik}

$$\tau_1(a) = \frac{\vec{E}\vec{M}}{\vec{M}^2}\ , \quad |\tau_2(a)| = \frac{\sqrt{\vec{E}^2\vec{M}^2 - (\vec{E}\vec{M})^2}}{\vec{M}^2} = \frac{|\vec{E}\times\vec{M}|}{\vec{M}^2}\ . \tag{4.12}$$

This is just the vacuum solution that was found in [1] by minimazing the scalar field potential in the component version of the action (4.6). It triggers a *partial* spontaneous breaking of $N=2$ supersymmetry down to $N=1$.

Let us show in two equivalent ways that at this point of moduli space the half of supersymmetry indeed continues to be unbroken on shell.

It follows already from the nilpotency of the complex matrix $\hat{x}^{ik}$ (eq. (4.11)) that by a proper rotation it can be brought into the form with only one nonvanishing element. As a result, only one linear combination of the inhomogeneously transforming Goldstone fermions

$$\delta\lambda^{\alpha i} = \epsilon^{\alpha}_{k}\hat{x}^{ik}$$

retains an inhomogeneous piece in its transformation.

To be more explicit, let us choose the $SU(2)$ frame so as to leave in E^{ik}, M^{ik} only three independent components, e.g.

$$\begin{aligned} M^{12} &= E^{12} = 0, \quad M^{11} = M^{22} \equiv m, \quad \mathrm{Re}\, E^{11} \equiv -e, \quad \mathrm{Im}\, E^{11} \equiv \xi \Rightarrow \\ \tau_1 &= -\frac{e}{m}, \quad |\tau_2| = \frac{|\xi|}{|m|}\ . \end{aligned} \tag{4.13}$$

In this frame

$$\hat{x}^{12} = 0, \quad \hat{x}^{11} = \frac{i}{2\tau_2}\left(\xi - m\frac{|\xi|}{|m|}\right), \quad \hat{x}^{22} = -\frac{i}{2\tau_2}\left(\xi + m\frac{|\xi|}{|m|}\right)\ . \tag{4.14}$$

One observes that either $\hat{x}^{11}$ or $\hat{x}^{22}$ is zero, depending on the relative sign of moduli m, ξ. Choosing, for definiteness, $m > 0, \xi > 0$, one finds

$$\hat{x}^{12} = \hat{x}^{11} = 0, \quad \hat{x}^{22} = -im\ . \tag{4.15}$$

As a result, only $\lambda^{\alpha 2}$ contains an inhomogeneous term $\sim m\,\epsilon^{\alpha}_2$ in its transformation, and it is the only genuine on-shell goldstino. So, the ϵ^{α}_2 supersymmetry is broken, while the ϵ^{α}_1 one is not.

An equivalent way to reach the same conclusions is to study the action of the modified $N=2$ supersymmetry generators $\hat{Q}^i_\alpha$ on the vacuum superfield

$$< W_V >= a + (\theta_i\theta_k)x^{ik}\ . \tag{4.16}$$

In accord with eqs. (4.4), (4.3) one gets

$$\hat{Q}^i_\alpha < W_V >= Q^i_\alpha < W_V > + \theta_{\alpha l}M^{il} = Q^i_\alpha < W >\ , \tag{4.17}$$

where

$$< W >= a + (\theta_i\theta_k)\hat{x}^{ik}\ . \tag{4.18}$$

Substituting the entries (4.15) (we are always at freedom to choose this special $SU(2)$ frame) into the last relation in the chain (4.17), one observes that $< W >$ depends only on θ^{α}_2. As a result

$$\hat{Q}^1_\alpha < W_V >= 0, \quad \hat{Q}^2_\alpha < W_V > \neq 0\ , \tag{4.19}$$

i.e. the generator $\hat{Q}^1_\alpha$ annihilates the vacuum, while $\hat{Q}^2_\alpha$ does not. Hence, $N=1$ supersymmetry generated by $\hat{Q}^1_\alpha$ is unbroken, and we indeed deal with the partial breaking $N=2 \rightarrow N=1$ in this case, both off and on shell.

In this connection, let us recall that the partial breaking nature of the off-shell $N=2$ supersymmetry Goldstone-type transformation of W_V (4.4) can be revealed (prior to any on-shell analysis) by shifting the real auxiliary field X^{ik} of W_V by a constant triplet c^{ik} such that the matrix $c^{ik} - (i/2)\ M^{ik}$ is degenerate (in a full analogy with the discussion after eq. (3.15)). It will be natural to choose $c^{ik} - (i/2)\ M^{ik} = \hat{x}^{ik}$, so as to have a one-to-one correspondence between the off- and on-shell partial breaking regimes. In terms of superfields this amounts to the special choice of $V^{ik} = V^{ik}_s$ and $W_{V_s} \equiv W_s$, such that the vacuum value of X^{ik}_s vanishes:

$$\begin{aligned} W &= W_s + \hat{x}^{ik}(\theta_i\theta_k)\ , && (4.20) \\ \delta_\epsilon W_s &= -2(\epsilon_k\theta_l)\hat{x}^{kl} + i(\epsilon Q + \bar{\epsilon}\bar{Q})W_s\ . && (4.21) \end{aligned}$$

Finally, we note that the transformation properties of the (E,M)-model equations of motion under the duality group $SL(2,\mathbf{Z})$ become transparent in terms of the dual pair of superfields W and $\hat{W}_L = \mathcal{F}_W$

$$\begin{aligned} D^{kl}\hat{W}_L - \bar{D}^{kl}\bar{\hat{W}}_L &= 4iE^{kl}\ , && (4.22) \\ D^{kl}W - \bar{D}^{kl}\overline{W} &= 4iM^{kl}\ . && (4.23) \end{aligned}$$

This pair of equations is covariant under the duality group, provided that the latter properly rotates E^{kl}, M^{kl} through each other.

5 Passing to $N=1$ superfields

The properties of the (E,M)-mechanism of the $N=2$ supersymmetry spontaneous breaking were studied in [1] basically in the $N=1$ superfield formalism. For completeness, we will briefly discuss how the above consideration can be translated to this language, with focusing on the realizations of $N=2$ supersymmetry.

One can pass to the $N=1$ superfield description, expanding $N=2$ superfields in powers of the coordinate θ_2. The coordinate θ_1 is assumed to parametrize $N=1$ superspace, the generators Q^1_α, $\bar{Q}_{\dot{\alpha}1}=\bar{Q}^2_{\dot{\alpha}}$ forming the appropriate $N=1$ subalgebra of $N=2$ supersymmetry.

We will use for W the natural decomposition (4.20). The $N=1$ superfield expansion of W reads

$$W=\hat{\varphi}(x,\theta_1)+i\theta_2^\alpha\hat{W}_\alpha(x,\theta_1)+(\theta_2)^2A(x,\theta_1)\ . \tag{5.1}$$

Then the $N=1$ superfield form of the relation (4.20) is given by

$$\hat{\varphi}=\varphi_s+\hat{x}^{11}(\theta_1)^2\ , \tag{5.2}$$

$$\hat{W}^\alpha=W_s^\alpha-2i\hat{x}^{12}\theta_1^\alpha\ , \tag{5.3}$$

$$A=(1/4)(\bar{D}_1)^2\bar{\varphi}_s+\hat{x}^{22}\ , \tag{5.4}$$

where the $N=1$ chiral superfield φ_s and the $N=1$ Maxwell chiral superfield strength W_s^α with a real auxiliary component are coefficients in the $N=1$ superfield expansion of W_s. The superfield W_s^α satisfies the standard Bianchi identity that can be solved in terms of the real $N=1$ prepotential V_s

$$W_s^\alpha=(\bar{D}_1)^2D^{\alpha 1}V_s\ . \tag{5.5}$$

Integrating over $\theta_2^\alpha,\bar{\theta}^2_{\dot{\alpha}}$, in the $N=2$ superfield action of the (E,M)-model, one arrives at the $N=1$ superfield action of ref.[1]. The analysis of vacuum solutions of the relevant equations of motion has been already made in ref. [1], so we will limit ourselves to discussing transformation properties of the involved $N=1$ superfields under the modified $N=2$ supersymmetry in the electric representation.

The $N=1$ superfield components of W possess the standard homogeneous supersymmetry transformation law

$$\delta\hat{\varphi}=-i\epsilon_2^\alpha\hat{W}_\alpha+i(\epsilon_1Q^1+\bar{\epsilon}^1\bar{Q}_1)\hat{\varphi}\ ,\quad \delta\hat{W}_\alpha=2i\epsilon_{\alpha 2}A+2\bar{\epsilon}^{\dot{\beta}2}\partial_{\alpha\dot{\beta}}\hat{\varphi}+i(\epsilon_1Q^1+\bar{\epsilon}^1\bar{Q}_1)\hat{W}_\alpha\ . \tag{5.6}$$

These transformations produce Goldstone-type transformations of the $N=1$ superfields φ_s and W_s^α

$$\delta\varphi_s=-2(\hat{x}^{11}\epsilon_1^\alpha+\hat{x}^{12}\epsilon_2^\alpha)\theta_{\alpha 1}-i\epsilon_2^\alpha W_{s\,\alpha}+i(\epsilon_1Q^1+\bar{\epsilon}^1\bar{Q}_1)\varphi_s \tag{5.7}$$

$$\delta W_s^\alpha=2i(\hat{x}^{22}\epsilon_2^\alpha+\hat{x}^{12}\epsilon_1^\alpha)+(i/2)\epsilon_2^\alpha[(\bar{D}_1)^2\bar{\varphi}_s]+2\bar{\epsilon}^{\dot{\beta}2}\partial_{\alpha\dot{\beta}}\varphi_s+i(\epsilon_1Q^1+\bar{\epsilon}^1\bar{Q}_1)W_s^\alpha\ . \tag{5.8}$$

The transformation of the Goldstone scalar superfield φ_s and the GM superfield W_s^α contains inhomogeneous terms which correspond to the constant translations of spinor

fields. To be convinced in this language that just the *partial* breaking is realized off shell, let us choose the $SU(2)$ frame (4.13). It is easy to observe that in this frame, with $\hat{x}^{22}\neq 0\,,\hat{x}^{12}=\hat{x}^{11}=0$, only W_s^α undergoes an inhomogeneous shift with the parameter ϵ_2^α while $N=1$ supersymmetry is realized on φ_s and W_s^α linearly and homogeneously. Thus W_s^α is the Goldstone $N=1$ superfield associated with the partial spontaneous breaking $N=2\rightarrow N=1$. This off-shell modification of $N=2$ supersymmetry in the $N=1$ superfield formalism of the APT model was previously noticed in [5]. Our consideration in the previous Sections shows that this phenomenon arises already after dualization of one of FI terms (this time, the magnetic one) and actually does not require, on its own range, the addition of another type of such a term.

Note that the equally admissible choice $\hat{x}^{12}=\hat{x}^{22}=0\,,\hat{x}^{11}\neq 0$ leads to the vanishing of inhomogeneous terms in the supersymmetry transformation of W_s^α and the appearance of such term $\sim\epsilon_1^\alpha$ in the transformation of φ_s. This means that with this choice the ϵ_1^α $N=1$ supersymmetry is broken, while the ϵ_2^α one is not. But this does not mean that φ_s can be treated as the corresponding Goldstone superfield in the spirit of ref. [12]. In order to reveal which kind of $N=1$ Goldstone superfield is actually relevant to one or another pattern of the partial breaking $N=2\rightarrow N=1$ in the APT model, one should always decompose $N=2$ superfields over those $N=1$ superfields associated with the *unbroken* $N=1$ supersymmetry. For instance, in the case of the just mentioned choice of $\hat{x}^{ik}$ a natural decomposition would be one with respect to the coordinates $\theta_1,\bar{\theta}^1$, so that the corresponding $N=1$ superfields live on the $N=1$ superspace $(x,\theta_2,\bar{\theta}^2)$. Once again, it is the appropriate $N=1$ gauge superfield which plays the role of $N=1$ GM superfield in this case. The same is true for any other choice of $\hat{x}^{ik}$. Thus, the model under consideration is a kind of 'non-minimal' variant of the nonlinear realization of the partial breaking with $N=1$ Maxwell superfield as the Goldstone superfield [5]. Another version of such a nonlinear realization, with chiral $N=1$ superfield as the Goldstone one [12], seems to bear no direct relation to the present model despite the presence of chiral $N=1$ superfield φ_s in $N=2$ GM supermultiplet (actually, φ_s is massive at the points of moduli space corresponding to the partial breaking).

A modification of $N=2$ supersymmetry can be studied on the $N=1$ prepotential V_s. For the choice (4.15), $N=2$ transformation of V_s is as follows

$$\delta_\epsilon V_s=m(\bar{\theta}^1)^2\theta_1^\alpha\epsilon_{\alpha 2}+(i/2)\theta_1^\alpha\epsilon_{\alpha 2}\bar{\varphi}_s+\text{c.c}+i(\epsilon_1Q^1+\bar{\epsilon}^1\bar{Q}_1)V_s\ . \tag{5.9}$$

The Lie bracket of these Goldstone supersymmetry transformations contains pure gauge terms with $(\theta_1)^2$ and $(\bar{\theta}^1)^2$, similarly to the transformation of the vector field in ref. [2]. We will discuss the modification of $N=2$ supersymmetry algebra in more detail in the next Section.

It would be interesting to study the relation of this approach to the formalism of nonlinear realization [6, 5, 12]. One can expect that both approaches are related by an $N=2$ analog of the nonlinear transformation constructed in [13] for the case of $N=1$ supersymmetry.

6 Consideration in harmonic superspace

Harmonic superspace was introduced in ref. [4] for off-shell description of the gauge, supergravity and matter $N=2$ supermultiplets. In ref. [8] this approach was applied for analysis of spontaneous breaking of $N=2$ supersymmetry in a general abelian $N=2$ gauge theory. We will show that it is also helpful for studying dual formulations of $N=2$ gauge theory and partial $N=2$ supersymmetry breaking. We use the $SU(2)/U(1)$ harmonics $u_i^\pm$ and the notation of refs.[4, 14] for the harmonic and spinor derivatives in the harmonic superspace, for instance

$$D^{++} = \partial^{++} - 2i\theta^{\alpha+}\bar\theta^{\dot\beta+}\partial^{A}_{\alpha\dot\beta} + \theta^{\alpha+}\partial^{+}_{\alpha} + \bar\theta^{\dot\beta+}\bar\partial^{+}_{\dot\beta}\,, \tag{6.1}$$

$$D^{+}_{\alpha} = \partial/\partial\theta^{\alpha-} \equiv \partial^{+}_{\alpha}\,,\quad \bar D^{+}_{\dot\beta} = \partial/\partial\bar\theta^{\dot\beta-} \equiv \bar\partial^{+}_{\dot\beta}\,, \tag{6.2}$$

$$D^{-}_{\alpha} = -\partial/\partial\theta^{\alpha+} + 2i\bar\theta^{\dot\beta-}\partial^{A}_{\alpha\dot\beta}\,, \tag{6.3}$$

$$\bar D^{-}_{\dot\beta} = -\partial/\partial\bar\theta^{\dot\beta+} - 2i\theta^{\alpha-}\partial^{A}_{\alpha\dot\beta}\,. \tag{6.4}$$

Here

$$\begin{aligned} u^{+i}u^{-}_{i} &= 1\,;\quad \partial^{++} = u^{+i}\frac{\partial}{\partial u^{-i}}\,,\quad \theta^{\pm\alpha} = \theta^{i\alpha}u^{\pm}_{i}\,,\quad \bar\theta^{\pm\dot\alpha} = \bar\theta^{i\dot\alpha}u^{\pm}_{i}\,,\\ x^{m}_{A} &= x^{m} - i\theta^{k}\sigma^{m}\bar\theta^{l}(u^{+}_{k}u^{-}_{l} + u^{+}_{l}u^{-}_{k})\,,\quad \partial^{A}_{\alpha\dot\beta} = \partial/\partial x^{\alpha\dot\beta}_{A}\,. \end{aligned} \tag{6.5}$$

The harmonic-superspace solution of the constraint (2.9) is given by [4, 15]

$$W_{V} \equiv W(V^{++}) = (\bar D^{+})^{2}\int du_{1}\frac{V^{++}(z,u_{1})}{(u^{+}u^{+}_{1})^{2}} = \int du(\bar D^{-})^{2}V^{++}(z,u)\,, \tag{6.6}$$

where a harmonic Green function [14] is used. The superfield

$$V^{++}(z,u) = V^{++}(x^{m}_{A},\theta^{+\alpha},\theta^{+\dot\alpha},u^{\pm i}) \equiv V^{++}(\zeta,u) \tag{6.7}$$

is the abelian analytic prepotential,

$$D^{+}_{\alpha}V^{++} = \bar D^{+}_{\dot\beta}V^{++} = 0,$$

a fundamental geometric object of $N=2$ gauge theory. It contains $N=2$ vector multiplet and an infinite tower of pure gauge harmonic components. The gauge freedom of V^{++} is given by the transformations

$$V^{++} \to V^{++} + D^{++}\lambda\,, \tag{6.8}$$

$\lambda = \lambda(\zeta,u)$ being an arbitrary analytic gauge function. Both V^{++} and λ, as well as the analytic subspace $(\zeta^{M},u) \equiv (x^{A\,\alpha\dot\alpha},\theta^{+\alpha},\bar\theta^{+\dot\alpha},u^{\pm}_{i})$ are real with respect to some generalized conjugation [4] (it becomes the ordinary complex conjugation when applied to conventional $N=2$ superfields).

All the properties of W, eqs. (2.6), (2.7), follow in the harmonic-superspace approach from the analyticity of V^{++}. The harmonic superspace form of (2.6) is as follows

$$(D^{+})^{2}W_{V} - (\bar D^{+})^{2}\overline{W}_{V} = 0\,. \tag{6.9}$$

The fact that W does not depend on harmonics is expressed as the condition

$$D^{++}W = 0\,.$$

The prepotential V^{++} is related to the Mezincescu prepotential as

$$V^{++} = (D^{+})^{2}(\bar D^{+})^{2}[u^{-}_{i}u^{-}_{k}V^{ik} + \text{pure gauge terms}]\,. \tag{6.10}$$

Various representations of the $N=2$ Maxwell action with and without FI terms discussed in the previous Sections can be re-formulated in terms of harmonic superfields. One can define two sorts of the analytic prepotentials, 'electric' V^{++} and 'magnetic' L^{++}, the latter naturally arising as an analytic Lagrange multiplier for the constraint (6.9) in an analog of the intermediate 'master' action introduced in Sect. 3. Below we present a brief account of the harmonic-superspace version of the APT model considered in Sect. 4.

The basic, holomorphic part of the 'master' action has the previous form (2.11), with W_V being replaced by an arbitrary chiral superfield W. To appropriately rewrite the Lagrange multiplier term S_L in (3.1), (3.2) we define the dual magnetic chiral superfield strength W_L on the pattern of eq. (6.6)

$$W_{L} = (\bar D^{+})^{2}\int du_{1}\frac{L^{++}(z,u_{1})}{(u^{+}u^{+}_{1})^{2}} = \int du(\bar D^{-})^{2}L^{++}(z,u)\,. \tag{6.11}$$

Then this term in the harmonic formalism can be written as

$$\begin{aligned} S_{L} &= -(i/4)\textstyle\int d^{4}x d^{4}\theta du\, W(\bar D^{-})^{2}L^{++} + \text{c.c.}\\ &= (i/4)\textstyle\int d\zeta^{(-4)}du\, L^{++}\,[(\bar D^{+})^{2}\overline{W} - (D^{+})^{2}W]\,, \end{aligned} \tag{6.12}$$

where

$$d\zeta^{(-4)}du = d^{4}x(D^{-})^{2}(\bar D^{-})^{2}du$$

is the measure of integration over the analytic subspace $(\zeta^{M},u^{\pm}_{i})$. Note that the expression in the square brackets in the second line of (6.12) is analytic in virtue of the property $(D^{+})^{3} = 0$ and chirality of W. Varying L^{++} as an unconstrained analytic superfeild immdeiately yields the condition (6.9) which, together with the chirality of W, imply the representation (6.6).

The 'electric' FI term in the electric representation (2.11) can also be written as an integral over the analytic subspace

$$S_{FI}(V) = \int d\zeta^{(-4)} du \; E^{++} V^{++} \, , \tag{6.13}$$

where $E^{++} = E^{ik} u^+_i u^+_k$. Its 'disguised' form pertinent to the master action and defined in eq. (3.12) becomes just (6.13) after using (6.6) and decomposing the explicit θs in (3.12) in their $\pm$ harmonic projections. The equation of motion of the abelian model with such a term is equivalent to the following analytic equation of motion:

$$(D^+)^2 \mathcal{F}_W(W_V) - \text{c.c.} = \left[\tau (D^+)^2 W_V + \tau' D^{+\alpha} W_V D^+_\alpha W_V\right] - \text{c.c.} = 4i E^{++} \, . \tag{6.14}$$

It can be obtained by varying the sum of actions (2.10) (with W_V given by (6.6)) and (6.13) with respect to V^{++} (one should beforehand rewrite the whole action as an integral over the analytic subspace).

The constraint (6.9) is modified if we add to the master action the harmonic version of the magnetic FI-term [4]

$$S_m = -\int d\zeta^{(-4)} du \; M^{++} L^{++} \, , \quad M^{++} = M^{ik} u^+_i u^+_k \, , \tag{6.15}$$

which is an analog of the term (4.1) (and goes into it after making use of the magnetic counterpart of the relation (6.10)). The modified form of the constraint contains the $SU(2)$-breaking constant term (cf. eq.(4.2))

$$(D^+)^2 W - (\bar{D}^+)^2 \overline{W} = 4i M^{++} \, . \tag{6.16}$$

At last, the modified magnetic chiral superfield strength $\hat{W}_L$ defined by eq. (3.22) and dual to W obeys the following harmonic-superspace form of the constraint (3.25)

$$(D^+)^2 \hat{W}_L - (\bar{D}^+)^2 \hat{\overline{W}}_L = 4i E^{++} \tag{6.17}$$

(it amounts to eq. (6.14) because of the relation $\hat{W}_L = \mathcal{F}_W(W)$ that follows from the master action by varying it with respect to W). In the rest of this Section we consider how the magnetic-FI term induced modification of $N = 2$ supersymmetry in the electric representation affects the transformation properties of V^{++}. This modification was already discussed in Sect. 4 in terms of ordinary $N = 2$ superfields and in Sect. 5 in terms of $N = 1$ superfields.

Like in Sect. 5, it will be convenient to use the decomposition (4.20) to single out the superfield strength W_V subjected to the standard constraint (6.9) from the object W satisfying the M^{++}-modified constraint (6.16)

$$W = W_s + (\theta_k \theta_l) \hat{x}^{kl} \, . \tag{6.18}$$

The first term is expressed according to eq.(6.6) through the real analytic prepotential V^{++}_s with a zero vacuum expectation value, while the second term contains the constant vacuum auxiliary field $\hat{x}^{kl}$ defined by eqs. (4.9) - (4.12). By the modified $N = 2$ supersymmetry transformation (4.21) of W_s we can restore, modulo the analytic gauge transformations (6.8), the modified transformation of V^{++}_s. As is expected, it is of the Goldstone type

$$\delta_\epsilon V^{++}_s = -\hat{x}^{kl} \epsilon^\alpha_k \theta^+_\alpha (\bar{\theta}^+)^2 u^-_l + \text{c.c.} + i(\epsilon Q + \bar{\epsilon}\bar{Q}) V^{++}_s \, . \tag{6.19}$$

For the choice of $SU(2)$ frame as in (4.15), the inhomogeneous term of this transformation is reduced to

$$im \left[\, \epsilon^\alpha_2 \theta^+_\alpha (\bar{\theta}^+)^2 u^-_2 - \bar{\epsilon}^{\dot{\alpha}2} \bar{\theta}^+_{\dot{\alpha}} (\theta^+)^2 u^-_1 \, \right] . \tag{6.20}$$

It does not contain the parameters of the first supersymmetry ϵ_1 and $\bar{\epsilon}^1$, reflecting the fact that the latter is unbroken under this choice.

It is straightforward to calculate the Lie bracket of the modified supersymmetry transformations (6.19)

$$[\delta_\eta, \delta_\epsilon] V^{++}_s = \delta^{stan}_{(\eta,\epsilon)} V^{++}_s + \delta^{mod}_{(\eta,\epsilon)} V^{++}_s \, . \tag{6.21}$$

Besides ordinary x-translations and central charge transformations $\delta^{stan}_{(\eta,\epsilon)} V^{++}_s$, it contains an extra term which is a special case of the analytic gauge transformation (6.8)

$$\begin{aligned} \delta^{mod}_{(\eta,\epsilon)} V^{++}_s \;\; = \;\; & im[\epsilon^\alpha_2 \eta_{\alpha 1} - (\eta \leftrightarrow \epsilon)](\bar{\theta}^+)^2 u^+_2 u^-_2 + 2im[\eta^\alpha_2 \bar{\epsilon}^{\dot{\alpha}2} - (\eta \leftrightarrow \epsilon)]\theta^+_\alpha \bar{\theta}^+_{\dot{\alpha}} u^+_2 u^-_2 \\ & + 2im[\eta^\alpha_2 \bar{\epsilon}^{\dot{\alpha}1} - (\eta \leftrightarrow \epsilon)]\theta^+_\alpha \bar{\theta}^+_{\dot{\alpha}} u^+_1 u^-_2 + \text{c.c.} \, . \end{aligned} \tag{6.22}$$

Defining the appropriate dimension 1 gauge generators

$$\begin{aligned} & G \, V^{++}_s \equiv im \, (\bar{\theta}^+)^2 u^+_2 u^-_2 \, , \quad G_{\alpha\dot{\alpha}} \, V^{++}_s \equiv 2im \, \theta_\alpha \theta_{\dot{\alpha}} (u^+_2 u^-_2 - u^+_1 u^-_1) \, , \\ & \tilde{G}_{\alpha\dot{\alpha}} \, V^{++}_s \equiv 2im \, \theta_\alpha \theta_{\dot{\alpha}} u^+_1 u^-_2 \, , \end{aligned} \tag{6.23}$$

one can write the modification of $N = 2$ superalgebra on V^{++}_s as follows (omitting the standard pieces with the 4-translation and central charges generators)

$$\{Q^2_\alpha, Q^1_\beta\}^{mod} = \varepsilon_{\alpha\beta} G \, , \quad \{Q^2_\alpha, \bar{Q}_{\dot{\beta}2}\}^{mod} = G_{\alpha\dot{\beta}} \, , \quad \{Q^2_\alpha, \bar{Q}_{\dot{\beta}1}\}^{mod} = \tilde{G}_{\alpha\dot{\beta}} \, . \tag{6.24}$$

Having found the explicit form of the modified generators $Q, \bar{Q}$, one can deduce the full modified $N = 2$ superalgebra: commuting the dimension 1 generators with $Q, \bar{Q}$ produces gauge generators of dimension 1/2, etc. In this way one can single out the whole finite set of the mutually (anti)commuting gauge generators which together with $Q, \bar{Q}$ form a closed superalgebra. We do not quote it here explicitly, limiting ourselves to a few remarks concerning its structure.

(i) The standard Poincaré or super-Poincaré algebras form a semi-direct product with the corresponding gauge groups treated, in the spirit of ref. [16], as groups with an infinite number of generators of the type (6.23). In the present case some of these generators appear on the r.h.s. of the basic anticommutators of $N = 2$ superalgebra, indicating that we are dealing with a non-trivial unification of $N = 2$ supersymmetry and $N = 2$ gauge group. The existence of such an extended algebraic structure does not contradict the famous Coleman-Mandula theorem (or any its supersymmetric generalization), since the two important assumptions of this theorem, manifest Lorentz invariance and positive definiteness of the metric in the space of states, cannot be simultaneously satisfied for gauge theories.

(ii) The above modification should be distinguished from the well-known modification of $N = 2$ current algebra by a constant 'central charge' in the case of partial spontaneous breaking [17]. The latter modification takes place in the APT model too [2, 18], and it does not influence transformation properties of the involved $N = 2$ superfields, irrespective of whether they are gauge-invariant or not.

(iii) Gauge transformations in the r.h.s. of the anticommutator of spinor charges appear also in the standard supersymmetric gauge theories (without FI terms) as the result of fixing a gauge. In our case such transformations are present before any gauge-fixing.

(iv) Let us emphasize that the modification of $N = 2$ transformation law (6.19) and, correspondingly, the modification of $N = 2$ supersymmetry algebra (6.24) are unavoidable, they cannot be removed by any redefinition of the GM-prepotential V_s^{++}. However, using the freedom of adding some special gauge transformations to the 'minimal' transformation law (6.19), one can change the structure of the anticommutators (6.24) by gaining some additional dimension 1 gauge generators in their r.h.s.

(v) In the modified $N = 2$ superalgebra the automorphism $SU(2)$ symmetry is explicitly broken down to some $U(1)$ due to the presence of the $SU(2)$-breaking parameters M^{ik} (or E^{ik} in the magnetic representation) as structure constants.

(vi) Due to a non-zero vacuum value of $W|_0 = a$ at the minimum of the scalar potential corresponding to the partial breaking of $N = 2$ supersymmetry in the APT model, modified $N = 2$ superalgebra necessarily contains, in the anticommutator $\{Q^1_\alpha, Q^2_\beta\}$ and its conjugate, a central charge proportional to the global $U(1)$ generator of the gauge group. It still commutes with all gauge generators, is vanishing on V^{++} itself, but possesses a

non-trivial action on any charged matter multiplet.

(vii) Obviously, the above modification of $N = 2$ superalgebra, in analogy with the central charge deformation just mentioned, should manifests itself on any charged matter hypermultiplet. This entails problems with constructing invariant minimal couplings of V^{++} to the analytic q^+ hypermultiplets. The standard gauge invariant q^+ Lagrangian

$$\bar{q}^+(D^{++} + iV_s^{++})q^+ \qquad (6.25)$$

is not invariant under the modified transformations (6.19) and we do not know how to modify the $N = 2$ transformation properties of q^+ (and/or the coupling (6.25)) in order to achieve such an invariance. Difficulties with coupling of the APT model to an extra charged matter were earlier noticed in [19] at the component level. It is an interesting open problem how to couple q^+ to V_s^{++}, and a careful analysis of the representations of the modified $N = 2$ superalgebra should be made in order to solve it.

Finally, we would like to point out that a consistent interpretation of the partial supersymmetry breaking in the APT model along the lines of the nonlinear realization approach of refs. [5], [12] will require constructing such a realization for the modified $N = 2$ superalgebra. One can hope that on this way some unsolved questions raised in ref. [5] can be answered. It seems interesting to seek any other realization of such supersymmetry-gauge algebras and to reveal their possible stringy origin.

Acknowledgements

We are grateful to J. Bagger, A. Galperin, S. Krivonos, O. Lechtenfeld, D. Lüst, A. Pashnev and M. Vasiliev for useful discussions. This work was partially supported by Russian Foundation of Basic Research (grants RFBR-96-02-17634 and RFBR-DFG-96-02-00180), by INTAS-grants 93-127, 93-633, 94-2317 and the grant of the Dutch NWO organization. B.Z. is grateful to Uzbek Foundation of Basic Research for the partial support (grant N 11/97).

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Received by Publishing Department
on November 12, 1997.