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THE VAVILOV–CERENKOV RADIATION IN A MEDIUM WITH A NONZERO ABSORPTION COEFFICIENT

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1 Introduction

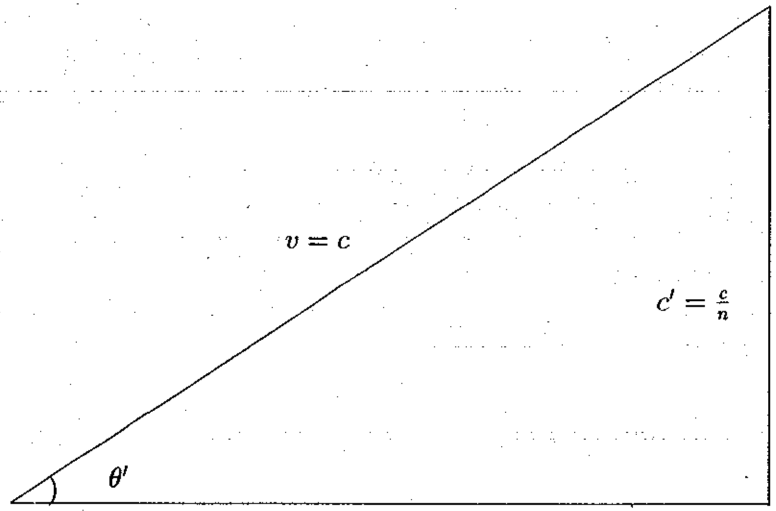
In the standard theory of Vavilov- Cerenkov radiation created by I.E. Tamm and I.M. Frank [1], a relativistic particle moving with a velocity β through a medium with a coefficient refraction $n > 1$, radiates photons in the direction θ ($\cos\theta = \frac{1}{\beta n}$). If the velocity of this particle is equal to $\beta = \frac{1}{n}$, this radiation disappears. However, it is obvious, due to common reasons, that at the velocity of the particle which is equal to the velocity of the light in the medium $\beta = \frac{1}{n}$, the radiation (caused by polarization of the medium) should exist [2]. This radiation must be strictly in the direction $\theta = 0$, i.e., in the direction of the velocity vector of the particle. And, probably, when the velocity of the particle is less than that of the light in this medium, this radiation is expected to disappear.

This work deals with the formal physical description of passing of the relativistic particle through the medium. Besides, to expand the Tamm- Cerenkov theory, it is necessary to include the radiation of the relativistic particle at its velocity equal (or near) to the light velocity in the medium ($\beta \simeq \frac{1}{n}$). The refraction coefficient of the medium is supposed to be a complex value.

2 Field around the charged particle in the medium

The field of the charged relativistic particle is given by the Lienard-Wiechert potential [3]. If we fasten the center of the counting system to the charged particle in a certain direction, then the physical picture of the processes becomes more clear. For example, the field of this particle is not central symmetric and the characteristics of the field of the particle in the transverse direction change by the factor $\gamma = \frac{1}{\sqrt{1-\beta^2}}$. If we consider the field of this particle over the distance R_0, it is clear that when the velocity of this ralativistic particle is β, the field around this particle acquires an ellipsoid form (or the form of a thin disk) with $R_{tran} = \frac{R_0}{\gamma}$ in the transverse direction.

It is interesting to know the form of the relativistic particle field in the medium when its velocity β is bigger than the velocity of the light in the medium ($n > 1$). The velocity of the light in the medium is $c' = \frac{c}{n}$, therefore the field around the particle will spread out in this medium with the velocity c' (which is less than c) and the field will remain behind this relativistic particle. The triangle of the velocities of this particle in the medium has the following form:



$$\sin\theta' = \frac{c}{vn} = \frac{1}{\beta n},$$
$$\theta' = \frac{\pi}{2} - \theta.$$

So, we can see that the field around the relativistic particle ($v > c'$) has the form of a thin surface film of a rotation cone with the slope angle θ' to the direction of the moving particle, and θ' is determined by the value $\sin\theta' = \frac{1}{\beta n}$. Then, the larger the refraction coefficient n

is, the smaller the scope angle of the surface of the rotation cone is obtained (see work [4]).

When the charged particle is moving through the medium, this medium is polarized. The direction of polarization must be transverse to the surface of the rotation cone. If the velocity of the charged particle is higher (or equal) than the light velocity in the medium, then this medium remains polarized after passing of this charged particle through it. This polarization will be taken off by radiation of photons in the direction of this polarization.

3 Vavilov-Cerenkov radiation of a relativistic particle in a medium (expansion of the Theory)

The classic theory of Vavilov -Cerenkov radiation was created by I.E. Tamm and I.M. Frank [1]. The equation for the charged particle moving through the medium in the direction z (in the cylindrical coordinates ρ, ϕ, z) has the form:

$$\frac{\partial^2 F}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial F}{\partial \rho} + s^2 F = -\frac{4}{\pi\rho}\delta(\rho). \tag{1}$$

where

$$s^2 = \frac{\omega^2}{v^2}(\beta^2 n^2 - 1) \tag{2}$$

(n is a real value). Equation (1) out of the region $\rho = 0$ has the following form:

$$\frac{\partial^2 F}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial F}{\partial \rho} + s^2 F = 0. \tag{3}$$

The solution of equation (3) has the form:

$$F = c_1 H_0^{(1)}(s\rho) + c_2 H_0^{(2)}(s\rho), \tag{4}$$

if s is an imaginary value, then $s \to is$ and

$$F = c_1 H_0^{(1)}(-is\rho) + c_2 H_0^{(2)}(-is\rho). \tag{5}$$

In case when the particle velocity is small, i.e. $\beta n < 1$, s in (5) is a real value, then the physical solution of (5) has the form:

$$F = iH_0^{(1)}(is\rho), \tag{6}$$

and asymptotic expression of F when $s\rho \gg 1$ is

$$F = \sqrt{\tfrac{2}{\pi s\rho}} \exp(-s\rho).$$

From this equation we can see that in this case the radiation does not take place.

If the velocity of the charged particle in the medium is $\beta n > 1$, then for the diveregence wave we have

$$F = -iH_0^{(2)}(s\rho), \tag{7}$$

where $s = \frac{|\omega|}{v}\sqrt{\beta^2 n^2 - 1}$.
The asymptotic expression of F in (7) has the following form:

$$F = \sqrt{\frac{2}{\pi s\rho}} \exp(i\omega(t - \frac{z}{v}) - is\rho + i\frac{\pi}{4}), \tag{8}$$

$$\omega > 0,$$

or

$$F = \sqrt{\frac{2}{\pi s\rho}} \exp(i\omega(t - \frac{z\cos\theta + \rho\sin\theta}{u}) + i\frac{\pi}{4}), \tag{9}$$

where $u = \frac{c}{n}$.

The full energy of the Vavilov- Cerenkov radiation in the medium is

$$\frac{dW}{dt} = \frac{e^2}{c^2}\int \omega d\omega (1 - \frac{1}{\beta^2 n^2}). \tag{10}$$

So, this radiation exists only if $\beta^2 n^2 > 1$. When $\beta^2 n^2 = 1$, this radiation disappears.

As we have already stressed above, it is difficult to understand why the cases $\beta^2 n^2 > 1$ and $\beta^2 n^2 = 1$ are broken. In the latter case the particle moves through the medium with the light velocity and the medium is polarized and therefore radiation in the forward direction ($\cos\theta = 1$) should take place. This result is a consequence of the supposition that n is a real value in equation (1). In the real case, in general, n is a complex value and

$$\epsilon = \epsilon_1 + i\epsilon_2. \tag{11}$$

If $\epsilon_2 = 0$, then

$$\epsilon = \epsilon_1 = n^2. \tag{12}$$

So, in the common case, the expession for s in (1) gets the following form:

$$s^2 = \frac{\omega^2}{v^2}((\beta^2\epsilon_1 - 1) + i\beta^2\epsilon_2) = \frac{\omega^2}{v^2}(a + ib), \tag{13}$$

where $a = \beta^2\epsilon_1 - 1$, $b = \beta^2\epsilon_2$.
Then in equations (4) or (5) the term $(s\rho)$ must be a complex value.

From (13) we obtain

$$s = \frac{\omega}{v}(s' + id), \tag{14}$$

where $s' = \frac{\sqrt{a+\sqrt{a^2+b^2}}}{2}$,
$d = \frac{b}{\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}}$.

The full energy of the Vavilov- Cerenkov radiation is

$$\frac{dW}{dt} = \frac{e^2}{c^2}\int \omega d\omega (1 - \frac{1}{\beta^2 n'^2}), \tag{15}$$

where

$$\frac{1}{n'^2} = Re\frac{1}{\epsilon_1 + i\epsilon_2} = \frac{\epsilon_1}{\epsilon_1^2 + \epsilon_2^2}.$$

In case $\beta\sqrt{\epsilon_1} = 1$ (i.e. $a = 0$) we obtain

$$\frac{dW}{dt} = \frac{e^2}{c^2}\int \omega d\omega (1 - \frac{\epsilon_1}{\beta^2(\epsilon_1^2 + \epsilon_2^2)}) = \frac{e^2}{c^2}\int \omega d\omega (\frac{\beta^2\epsilon_2^2}{(1 + \beta^4\epsilon_2^2)}). \tag{16}$$

So, we see that if $\beta = \frac{1}{n}$, the Vavilov- Cerenkov radiation is defined by expression (16) and differs from zero (in this equation the absorption part is not taken into account). The probability of this radiation is proportional to ϵ_2^2. The direction of this radiation is $\theta = 0$.

We should stress that, except the absorption coefficient , the imaginary part of Im $n'^2 = \epsilon_2$ also includes the term responsible for the energy losses at $\beta \geq \frac{1}{n}$, i.e., losses at the light barrier.

4 Conclusion

In the Vavilov-Cerenkov theory the radiation goes in the direction θ ($\cos\theta = \frac{1}{\beta n}$) and the full energy of radiation per one length is given by equation (10). According to equation (10), at the point $\theta = 0$ ($\beta = \frac{1}{n}$) the energy radiation is zero. However, it is obvious that the radiation at this point cannot be equal to zero (i.e. at the particle velocity equal to the light velocity in the medium).There is a strict indication of the existence of this radiation [2].

We have taken into account the fact that in any real medium the refraction coefficient is a complex value $\epsilon = \epsilon_1 + i\epsilon_2$, and in this case we have obtained that in the extended Vavilov- Cerenkov theory the charged particle also radiates photons when $\beta = \frac{1}{n}$ in the forward direction $\theta = 0$. The probability of this radiation is proportional to ϵ_2^2.

We would like to emphasize one possibility to solve the problem which appears in the Electrodynamics of the medium— the problem of infinity of the values at the point $\beta = \frac{1}{n}$. For example, the energy of the relativistic particle in the medium (n)

$$E = \frac{m}{\sqrt{1 - \beta^2 n^2}}$$

has singularity at $\beta = \frac{1}{n}$.
It is possible to avoid the singularity: if we take a real medium with $\epsilon = \epsilon_1 + i\epsilon_2$ and

$$E = \frac{m}{\sqrt{1 - \beta^2\epsilon}},$$

then $ReE = E'$ has the following form:

$$E' = \frac{mf}{\sqrt{(1 - \beta^2\epsilon_1)^2 + (\beta^2\epsilon_2)^2}},$$

where

$$f = \frac{\sqrt{2(1 - \beta^2\epsilon_1) + \sqrt{(1 - \beta^2\epsilon_1)^2 + (\beta^2\epsilon_2)^2}}}{\sqrt{2}}.$$

at $\beta^2\epsilon_1 = 1$

$$E' = \frac{m}{\beta\sqrt{2\epsilon_2}}.$$

Now E' has no singularity.

An analogous example exists in the Particle Physics where the cross section of the particle production does not have infinity at the point of resonance, for the particle has a width of decay Γ ($m = m_2 + i\Gamma$) (see the Breit- Wigner equation for the cross section).

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