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NEW ASPECTS OF THE VAVILOV—CHERENKOV RADIATION

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1 Introduction

Although the Vavilov-Cherenkov effect is a well established phenomenon widely used in physics and technology [1], many its aspects remain uninvestigated up to now. In particular, it is not clear how takes place a transition from the sub-light velocity regime to the the super-light one. Some time ago [2,3] it was suggested that side by side with the usual Cherenkov and bremsstrahlung shock waves, the shock wave associated with the charged particle passing the medium light velocity c_n should exist. The consideration presented there was pure qualitative without any formulae and numerical results. It was grounded on the analogy with the phenomena occurring in acoustics and hydrodynamics. It seems to us that this analogy is not complete. In fact, the electromagnetic waves are pure transversal, while acoustic and hydrodynamic waves contain longitudinal components. Further, the analogy itself cannot be considered as a final proof.

Usually, treating the Vavilov-Cherenkov effect, one considers the charge motion in an infinite medium with a constant velocity $v > c_n$. In the absence of c_n dispersion, there is no electromagnetic field (EMF) before the Mach cone accompanying the charge, an infinite EMF on the Mach cone itself and finite values of EMF strengths behind it (see, e.g., [4]). In this case an information concerning the transition effects arising when charge velocity coincides with c_n is lost (except for the existence of the Cherenkov shock wave itself). Thus, there is a necessity to consider accelerated and decelerated charge motions and to study above mentioned effects.

In Ref. [5] it was considered the straight-line motion of the charged particle with a constant acceleration: $z = at^2$. It is easy to check that this motion law is obtained from the relativistic equation

$$m\frac{d}{dt}\frac{v}{\sqrt{1-\beta^2}} = eE, \quad v = \frac{dz}{dt}, \quad \beta = v/c,$$

(m is the rest mass) for the following electric field directed along the z axis

$$E_z = \frac{2ma}{e(1-4az/c^2)^{3/2}}. \tag{1.1}$$

For the case of accelerated motion it was found there that two shock waves arise when charge velocity coincides with c_n. First of them is a well-known Cherenkov shock wave C_M having the form of the Mach cone and propagating with the velocity of charge. The second of these waves (C_L) closing the Mach cone and propagating with the velocity c_n is just the shock wave existence of which was predicted in Refs. [2,3]. These two waves form an indivisible entity. As time goes, the dimensions of this complex grow, but its form remains essentially the same. The singularities of of C_L and C_M shock waves are the same and much stronger than the singularity of the bremsstrahlung shock wave arising from the beginning of charge motion.

For the case of decelerated motion it was found in the same reference that an additional shock wave arises at the moment when charge velocity coincides with

c_n. This wave being detached from the charge exists even after termination of the charge motion. It propagates with the velocity c_n and has the same singularity as the Cherenkov shock wave.

The drawback of Ref. [5] is that the electric field (1.1) maintaining charge motion tends to ∞ as z approaches $c^2/4a$. This singularity makes the creation of electric field (1.1) to be rather problematic. This, in turn, complicates the experimental verification of the shock waves existence predicted in [2,3].

Here we consider the straight-line motion of a point charge in a constant uniform electric field and evaluate EMF arising from such a motion. Obviously, it is much easier to create the constant uniform electric field than the singular electric field (1.1). Qualitatively, we confirm the results obtained in [5] concerning the existence of new shock waves associated with the passing the light medium velocity.

Previously, the accelerated motion of the point charge in a vacuum was considered by Schott [6]. Yet, his qualitative consideration was pure geometrical, not allowing the numerical investigations.

2 Statement of the physical problem

Let a charged particle moves inside the medium with the polarizabilities ϵ and μ along the given trajectory $\vec{\xi}(t)$. Then, its EMF at the observation point (ρ, z) is given by the Lienard-Wiechert potentials

$$\Phi(\vec{r},t) = \frac{e}{\epsilon}\sum\frac{1}{|R_i|}, \quad \vec{A}(\vec{r},t) = \frac{e\mu}{c}\sum\frac{\vec{v}_i}{|R_i|}, \quad div\vec{A} + \frac{\epsilon\mu}{c}\dot{\Phi} = 0 \tag{2.1}$$

Here

$$\vec{v}_i = (\frac{d\vec{\xi}}{dt})|_{t=t_i}, \quad R_i = |\vec{r} - \vec{\xi}(t_i)| - \vec{v}_i(\vec{r} - \vec{\xi}(t_i))/c_n$$

and c_n is the light velocity inside the medium ($c_n = c/\sqrt{\epsilon\mu}$). The summing in (2.1) is performed over all physical roots of the equation

$$c_n(t - t') = |\vec{r} - \vec{\xi}(t')| \tag{2.2}$$

To preserve the causality, the time of radiation t' should be smaller than the observation time t. Obviously, t' depends on the coordinates $\vec{r}, t$ of the point P at which the EMF is observed. With the account of (2.2) one gets for R_i

$$R_i = c_n(t - t_i) - \vec{v}_i(\vec{r} - \vec{\xi}(t_i))/c_n \tag{2.3}$$

Consider the motion of the charged point-like particle of the rest mass m inside the medium in a constant electric field E along the Z axis. The motion law is given by (see, e.g.,[7])

$$z(t) = \sqrt{z_0^2 + c^2t^2} - z_0, \quad z_0 = mc^2/E > 0. \tag{2.4}$$

The charge velocity is given by

$$v = \frac{dz}{dt} = c^2t(z_0^2 + c^2t^2)^{-1/2}.$$

Clearly, it tends to the light velocity in vacuum for $t \to \infty$. The retarded times t' satisfy the following equation

$$c_n(t - t') = [\rho^2 + (z + z_0 - \sqrt{z_0^2 + c^2t'^2}\,)^2]^{1/2} \tag{2.5}$$

It is convenient to introduce the dimensionless variables

$$\tilde{t} = ct/z_0, \quad \tilde{z} = z/z_0, \quad \tilde{\rho} = \rho/z_0 \tag{2.6}$$

Then,

$$\alpha(\tilde{t} - \tilde{t}') = [\tilde{\rho}^2 + (\tilde{z} + 1 - \sqrt{1 + \tilde{t}'^2}\,)^2]^{1/2}, \quad \alpha = c_n/c \tag{2.7}$$

In order not to overload exposition we drop the tilda signs:

$$\alpha(t - t') = [\rho^2 + (z + 1 - \sqrt{1 + t'^2}\,)^2]^{1/2} \tag{2.8}$$

For the treated case of one-dimensional motion the denominators R_i are given by:

$$R_i = \frac{z_0}{\alpha\sqrt{1 + t_i^2}}[\alpha^2(t - t_i)\sqrt{1 + t_i^2} - t_i(z + 1 - \sqrt{1 + t_i^2}\,)] \tag{2.9}$$

We consider the following two problems :

I. A charged particle rests at the origin up to a moment $t' = 0$. After that it is uniformly accelerated in the positive direction of the Z axis.

II. A charged particle decelerates uniformly moving from $z = \infty$ to the origin. After the moment $t' = 0$ it rests there.

It is easy to check that the moving charge acquires the light velocity c_n at the moments $t_l = \pm\alpha/\sqrt{1 - \alpha^2}$ for the accelerated and decelerated motion, resp. The position of a charge at those moments is $z_l = 1/\sqrt{1 - \alpha^2} - 1$.

It is our aim to investigate space-time distribution of EMF arising from such particle motions. For this we should solve Eq.(2.8). Squaring it we obtain the fourth degree algebraic equation relative to t'. It is just this way of finding shock waves positions which was adopted in [5]. It was shown in the same reference that there is another, much simpler approach for recovering of these singularities (it was extensively used by Schott [6]). We seek zeros of denominators R_i entering into the definition of electromagnetic potentials (2.1). They are obtained from the equation

$$\alpha^2(t - t')\sqrt{1 + t'^2} - t'(z + 1 - \sqrt{1 + t'^2}\,) = 0 \tag{2.10}$$

We rewrite (2.8) in the form

$$\rho^2 = \alpha^2(t - t')^2 - (z + 1 - \sqrt{1 + t'^2})^2. \tag{2.11}$$

Recovering t' from (2.10) and substituting it into (2.11) we find the surfaces $\rho(z,t)$ carrying the singularities of electromagnetic potentials. They are just shock waves which we seek for. It turns out that the bremsstrahlung shock waves (i.e., moving singularities arising from the beginning or termination of a charge motion) are not described by Eqs. (2.10) and (2.11). The reason for this is that on these surfaces the electromagnetic strengths, not potentials, are singular [4]. The mentioned above simplified procedure for recovering of EMF singularities is to find solutions of (2.10) and (2.11) and add to them "by hands" the positions of the bremsstrahlung shock waves defined by the equation $r = \alpha t$, $r = \sqrt{\rho^2 + z^2}$. The equivalence of this approach to the complete solution of (2.8) was proved in [5].

3 Numerical results

Accelerated motion.

For the first of the treated problems (uniform acceleration of the charge initially resting at the origin) the resulted configurations of the shock waves are shown in Fig.1 ($\alpha = 1/2$) and Fig. 2 ($\alpha = 1/4$). We see on them the Cherenkov shock wave C_M having the form of the Mach cone and the surface C_L closing the Mach cone. It turns out that the surface C_L with a good accuracy is approximated by the part of the sphere $\rho^2 + (z - z_l)^2 = (t - t_l)^2$ (shown by the short-dash curve C) describing the shock wave emitted from the point $z_l = (1 - \alpha^2)^{-1/2} - 1$ at the moment $t_l = \alpha(1 - \alpha^2)^{-1/2}$ when the velocity of the charged particle coincides with the velocity of light in the medium. On the internal sides of the surfaces C_L and C_M electromagnetic potentials acquire the infinite values. On the external side of C_M lying outside of C_0 the electromagnetic potentials are zero (as there are no solutions there). On the external sides of C_L and on the part of the C_M surface lying inside C_0 the electromagnetic potentials have finite values.

The positions of the shock waves for different observation times are shown in Fig. 3 ($\alpha = 1/2$) and Fig. 4 ($\alpha = 1/4$). The dimension of the Mach cone is zero for $t < t_l$ and continuously rises with time $t > t_l$. The physical reason for this is that C_L shock wave closing the Mach cone propagates with the light velocity c_n, while the head part of the Mach cone C_M attached to the charged particle propagates with the velocity $v > c_n$.

In the negative z semi-space the experimentalist will detect only the bremsstrahlung shock wave. In the positive z semi-space the observer placed not not very far from the z axis will detect Cherenkov shock wave first, bremsstrahlung shock wave later and, finally, shock wave originating from charge exceeding the medium light velocity. The observer at the larger distance from the z axis will see only the bremsstrahlung shock wave.

In the gasdynamics the existence of at least two shock waves attached to the finite body moving with a supersonic velocity was proved on the very general grounds by Landau and Lifshitz ([8] , Chapter 13). In the present context we associate them with C_L and C_M shock waves.

Decelerated motion.

Now we turn to the second problem (uniform deceleration of the charged particle along the positive z semi-axis up to a moment $t = 0$ after which it rests at the origin). In this case only negative retarded times t_i have a physical meaning.

For the observation time $t > 0$ the resulting configuration of the shock waves is shown in Fig.5 for $\alpha = 1/2$ and Fig.6 for $\alpha = 1/4$. On them we see the bremsstrahlung shock wave C_0 arising from the termination of the charge motion and the blunt shock wave C_M. Its head part with a good accuracy is described by the sphere $\rho^2 + (z - z_l)^2 = (t + t_l)^2$ (shown by the short-dash curve) corresponding to the shock wave emitted from the point $z_l = (1 - \alpha^2)^{-1/2} - 1$ at the moment $t_l = -\alpha(1 - \alpha^2)^{-1/2}$ when the velocity of the decelerated charged paricle coincides with the velocity of light in the medium. The electromagnetic potentials vanish outside of C_M (as no solutions exist there) and acquire the infinite values on the internal part of C_M. Therefore, the surface C_M represents the shock wave. As a result, for the decelerated motion after termination of the particle motion ($t > 0$) one has the shock wave C_M and the bremsstrahlung shock wave C_0 arising from the termination of the particle motion.

The positions of the shock waves for different observation times are shown in Fig. 7 ($\alpha = 1/2$) and Fig. 8 ($\alpha = 1/4$). One sees on them how the acute Cherenkov shock wave attached to the moving charge transforms into the blunt shock wave detached from the charge.

For the decelerated motion and $t < 0$ (i.e., before termination of the charge motion) the physical solutions exist only inside the Mach cone C_M ($t = -2$ on Figs. 7 and 8). On the internal boundaries of the Mach cones the electromagnetic potentials acquire the infinite values. On their external boundaries the electromagnetic potentials are zero (as no solutions exist there). When the charge velocity coincides with c_n the Cherenkov shock wave leaves the charge and expands with the velocity c_n ($t = 2$, 4 and 8 on Figs. 7 anf 8). As it was mentioned the blunt head parts of these waves with a good accuracy is approximated by the equation $\rho^2 + (z - z_l)^2 = (t + t_l)^2$ (short-dash curves).

In the negative z semi-space the experimentalist will detect the blunt shock wave first and bremsstrahlung shock wave later. In the positive z semi-space for the observation point not very far from the z axis the observer will see Cherenkov shock wave first and bremsstrahlung shock wave later. For larger distances from the z axis he will see the blunt shock associated with the passing of the medium light velocity first and bremsstrahlung shock wave later .

In order not to hamper the exposition we did not mention on the continuous radiation which reaches the observer between the arrival of two shock waves or after the arrival of the last shock wave. It is easily restored from the complete exposition presented in Ref. [5] for the $z = at^2$ motion law. Also, in order not to overload Figs. 3,4,7 and 8 we did not show on them the bremsstralung shock waves . Their positions is restored by using the equation $r = \alpha t$, $r = \sqrt{\rho^2 + z^2}$, $t > 0$.

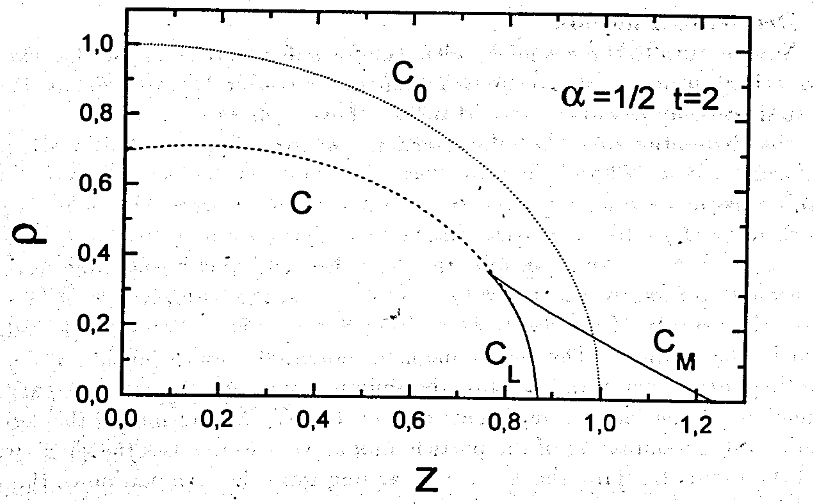

Fig.1: Distribution of the shock waves for the uniformly accelerated charge and $\alpha = 1/2$, $t = 2$ (α is the ratio of the light velocity in medium to that of in vacuum). C_M is the Cherenkov shock wave, C_L is the shock wave emitted from the point $z_l = (1 - \alpha^2)^{-1/2} - 1$ at the moment $t_l = \alpha(1 - \alpha^2)^{-1/2}$ when the charge velocity coincides with the medium light velocity. With a good accuracy it is described by the spherical surface $\rho^2 + (z - z_l)^2 = (t - t_l)^2$ (shown by the short-dash curve). C_0 is the bremsstrahlung shoch wave originating from the beginning (at the moment $t = 0$) of the charge motion.

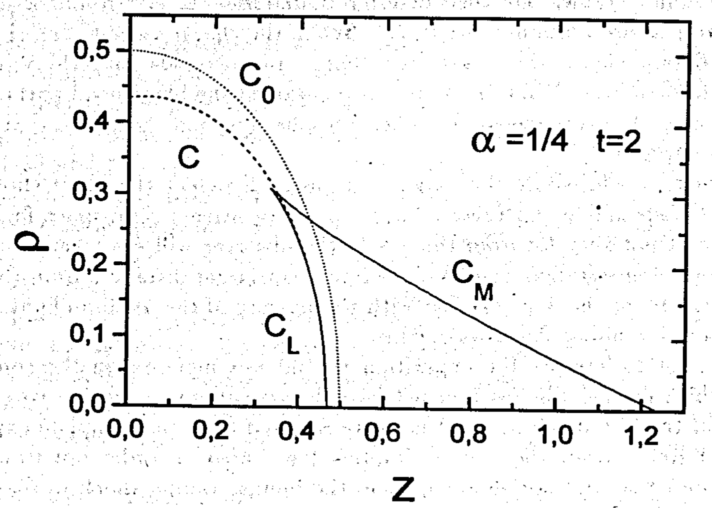

Fig.2: The same as in Fig.1 but for $\alpha = 1/4$.

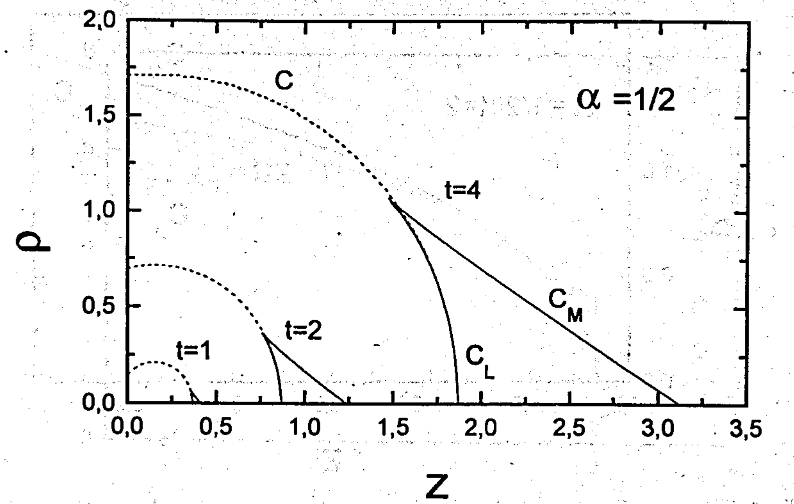

Fig.3: Time evolution of the shock waves for the uniformly accelerated charge and $\alpha = 1/2$. C_M is the usual Cherenkov shock wave, C_L is the shock wave emitted at the moment when the charge velocity coincides with the medium light velocity. Short dash curves C are the same as in Fig. 1.

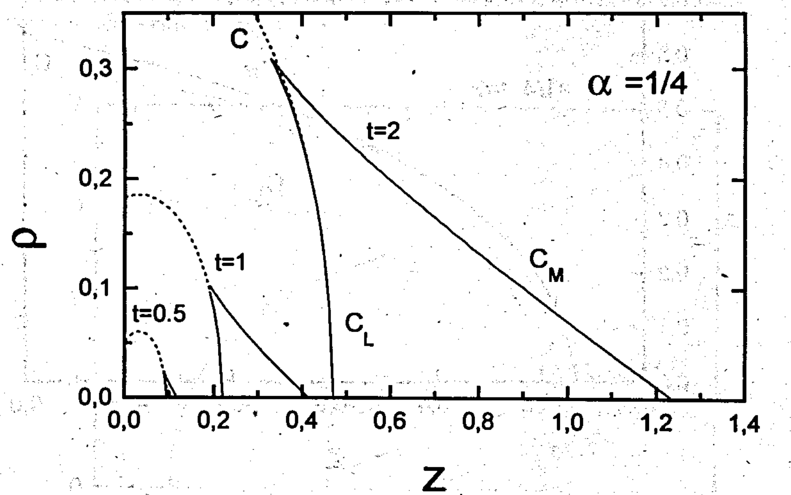

Fig.4: The same as in Fig.3 but for $\alpha = 1/4$.

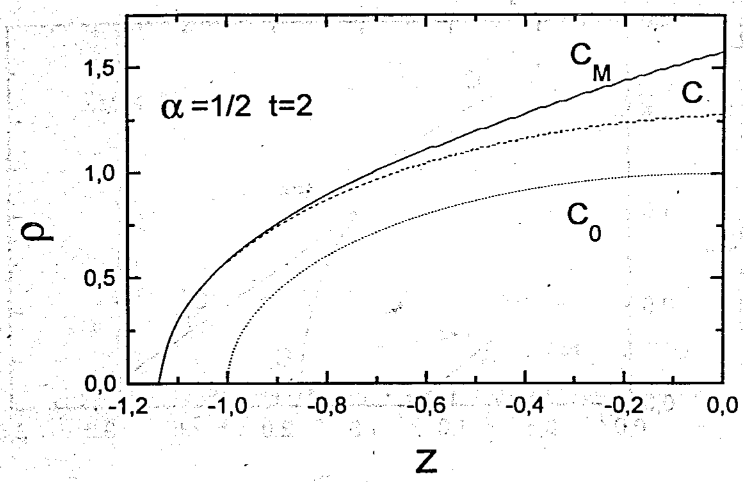

Fig.5: The distribution of the shock waves for the uniformly decelerated charge ($\alpha = 1/2$, $t = 2$). C_M is the blunt shock wave. Part of it with a good accuracy it is approximated by the spherical surface $\rho^2 + (z - z_l)^2 = (t + t_l)^2$ (it is shown by short dash curve C). C_0 is the bremsstrahlung shock wave originating from the termination (at the moment $t = 0$) of the charge motion.

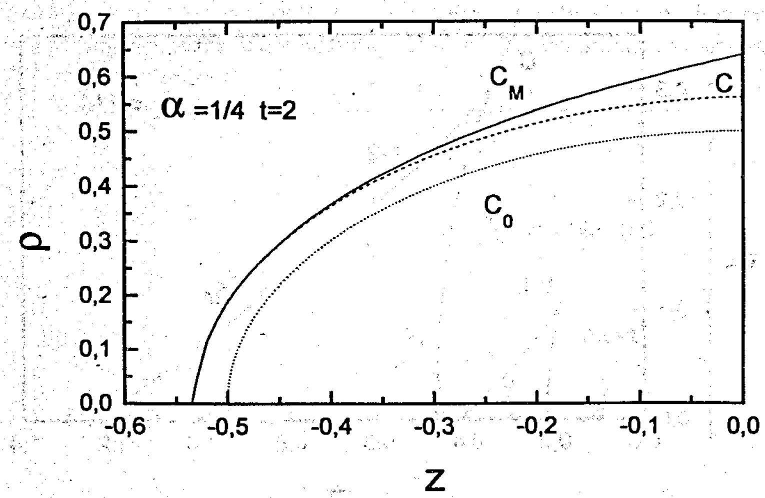

Fig.6: The same as in Fig.5 but for $\alpha = 1/4$.

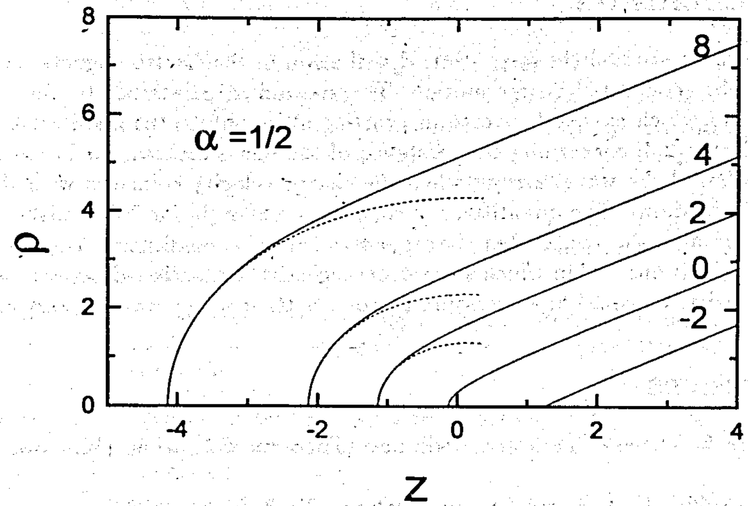

Fig.7: Continuous transformation of the Cherenkov shock wave attached to a moving charge ($t = -2$) into the blunt shock waves ($t = 2$, 4, 8) detached from a charge for the decelerated motion and $\alpha = 1/2$. The charge motion terminates at the moment $t = 0$. The numbers near the curves mean the observation times. Short dash curves are the same as in Fig. 5.

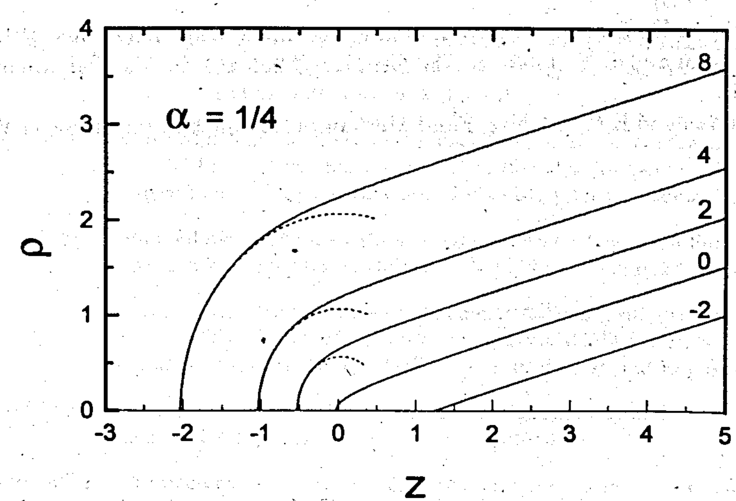

Fig.8: The same as in Fig.7 but for $\alpha = 1/4$.

4 Conclusion

To the end, we studied the space-time distribution of the electromagnetic field arising from the accelerated charge motion. This motion is maintained by the constant electric field which is easy to create in practice. We confirm the qualitative predictions of Refs. [2,3] concerning the existence of the new shock wave (in addition to the Cerenkov shock wave) arising when the charge velocity coincides with the light velocity in medium. The quantitive conclusions made in [5] for less realistic electric field maintaining the accelerated charge motion are also confirmed. We specify under what conditions and in which space-time regions the mentioned above new shock waves do exist. It would be interesting to observe these shock waves experimentally.

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