ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

Дубна

E2-97-227

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SOME REMARKS ON THE WOLFENSTEIN EQUATION OF NEUTRINO OSCILLATIONS IN MATTER

Submitted to «Europhysics Letters»

1997

1 Introduction

The solution of the Wolfenstein equation for neutrinos passing in matter [1] (for simplicity we shall consider the mixing of only two sorts of neutrinos, ν_e, ν_μ) results in the conclusion that at certain values of the difference between the square neutrino masses, the mixing angle and density of the matter there arise resonance neutrino oscillations [2]. This phenomenon is known as the MSW effect.

The present paper is devoted to the analysis of the Wolfenstein equation describing the neutrinos passing through matter.

2 The Wolfenstein equation for neutrino in Matter

In the ultrarelativistic limit the evolution equation for the neutrino wave function ν_Φ in matter has the form [1]

$$i\frac{d\nu_\Phi}{dt} = (p\hat{I} + \frac{\hat{M}^2}{2p} + \hat{W})\nu_\Phi, \tag{1}$$

where $p, \hat{M}^2, \hat{W}$ are, respectively, the momentum, the square mass matrix in vacuum (it is nondiagonal) and the matrix taking into account neutrino interactions in matter,

$$\nu_\Phi = \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix},$$

$$\hat{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

The matrix $\hat{M}^2$ is diagonalized by rotation through the angle θ (vacuum oscillation):

$$\hat{M}^2 = \begin{pmatrix} m^2_{\nu_e\nu_e} & m^2_{\nu_e\nu_\mu} \\ m^2_{\nu_\mu\nu_e} & m^2_{\nu_\mu\nu_\mu} \end{pmatrix},$$

$$\tan(2\theta) = \frac{2m^2_{\nu_e\nu_\mu}}{\mid m^2_{\nu_\mu\nu_\mu} - m^2_{\nu_e\nu_e} \mid}, \qquad \hat{M}^2_{diag} = \begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix}, \tag{2}$$

and the length of oscillation L_0 in this case is

$$L_0 = \frac{4\pi p}{\mid m_1^2 - m_2^2 \mid}, \qquad E \cong pc. \qquad (3)$$

Since $\hat{M}^2$ is a nondiagonal matrix (evidently, $\hat{M}^2$ appears at very short distances $r_P, r_P >> \frac{1}{m_W}$), this vacuum oscillation of neutrinos will take place at any energies with the length of oscillation L_0. The solution of equation (1) is considered in detail in refs. [2, 3] and here the main results will be shown in the reduced form.

When neutrinos pass through matter, its influence (see equation (1)) leads to changes of the rotation angle θ for diagonalizing the mass matrix $\hat{M}^2$, if diagonal matrix $\hat{W}$, responsible for the difference between the interactions of the neutrinos (ν_e, ν_μ), is added to the mass term $\hat{M}^2/2p$, and then θ becomes $\theta'(\theta' \neq \theta)$.

Thus, neutrino mixing in matter is determined by $\sin^2(2\theta')$:

$$\sin^2(2\theta') = \sin^2(2\theta)/[(\cos(2\theta) - \frac{L_0}{L^0})^2 + \sin^2(2\theta)], \qquad (4)$$

where L^0 is a diffraction length (i.e. length of formation):

$$L^0 = 2\pi m_p (2^{0.5} G_F \rho Y_e)^{-1} =$$

$$310^7 (m)(\rho(g/cm^3) 2Y_e)^{-1}, \qquad (5)$$

Y_e is the number of electrons per nucleon.

In the common case θ' depends on the difference of masses m_1, m_2, density ρ of matter and the neutrino momentum.

At $L_0 \cong L^0$ the resonant neutrino oscillations take place, i.e. $\sin^2(2\theta') \cong 1$.

Now, the following question arises: although equation (1) yields the above results, is this equation formulated correctly from a physical point of view? Or, in other words, is it possible to write the equation for the neutrino wave function ν_Φ in matter in the form of equation (1) without violating the necessary physical requirements?

In previous papers [4] we discussed various aspects of equation (1). Now we shall go on discussing equation (1).

3 Analysis of the Wolfenstein equation

1. This is a Schrödinger type equation for the function

$$\nu_\phi = \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix},$$

where $\nu_e = \nu_{eL} + \nu_{eR}, \nu_\mu = \nu_{\mu L} + \nu_{\mu R}$. Thus, this is a left-right symmetric equation for the spinor function. This equation contains the term $\hat{W}$,which arises from the weak interaction (contribution of W bosons) and contains only the left interaction of the neutrinos, and is substituted into the left-right symmetric equation (1) without indication of its left origin, i.e., this equation contains superfluous components $\hat{W}\nu_{eR}, \hat{W}\nu_{\mu R}$. Then we see that Eq. (1) is the equation including the term $\hat{W}$ that arises not from the weak interaction but a hypothetical left-right symmetric one. Moreover, the obtained effect of resonant enhancement of neutrino oscillations in matter is caused by this hypothetical left-right symmetric interaction.

As can be seen from Eqs. (1)-(4), if enhancement of neutrino oscillations is to occur in matter (see Eqs. (1)-(4)), the result of the interaction of the neutrinos with matter must reduce to a change in the effective masses of ν_e, ν_μ (W is a diagonal matrix). If the effective masses of ν_e, ν_μ become near $(m_{\nu_e} \cong m_{\nu_\mu})$ in matter as a result of the contribution of the hypothetical interaction , then the mixing angle θ' will be of order $\theta' \cong \frac{\pi}{4}$.

The standard weak interaction takes part only in the left-side interaction. But to generate masses of fermions, it is necessary to have a left-right symmetric interaction [5]; that is why the standard weak interaction cannot result in enhancement of neutrino oscillations in matter. So, we see that Eq.(1) is the equation including a hypothetical left-right interaction and having no connection with the standard weak interaction.

2. Let us consider whether this equation, with the hypothetical interaction, can lead to the existence of resonant enhancement in matter. The answer is clear. In such a form this equation leads to resonant enhancement of the oscillations in matter. However, a question again

arises regarding the equation itself: is it possible to substitute simultaneously the nondiagonal mass term $\frac{M^2}{2p}$ responsible for the neutrino mixing and the term $\hat{W}$ associated with the hypothetical left-right symmetric interaction? This question appears because in the region of the hypothetical interaction (an action identical to the weak interaction), i.e., at distances of order $\frac{1}{m_W}$, no significant nonconservation of lepton numbers is found (in the weak interaction the lepton numbers are conserved). Then it may be concluded that the nonconservation of the lepton numbers occurs at shorter distances, i.e., the nondiagonal mass term $\hat{M}^2$ must take place at shorter distances $r_P \cong \frac{1}{m_P}$ (m_P - characteristic mass where the lepton numbers are violated). So, we have the following question: Can this mass term arising at shorter distances be sensitive to a mass arising at distances of order $\frac{1}{m_W}$? The answer to this question is again clear. The mass term $\hat{M}^2$ can be sensitive only to the masses that arise at distances comparable with the distance at which this term appears itself. In other words, if we wish to take into account the contribution of the hypothetical interaction in Eq.(1) to the neutrino mass, we must take into account only the part of the mass that arises from this hypothetical interaction at distances of the order of the distances where the nondiagonal mass term $\hat{M}^2$ appears, i.e., at $r \cong r_P$. Thus, we have arrived at the conclusion that Eq.(1) in such a form does not take into account the fact that the terms $\hat{M}^2$ and $\hat{W}$ arise at different distances and to formulate the equation correctly, they should be combined.

Let us combine these terms in Eq.(1). For this purpose we estimate the deposit of the hypothetical interaction on the mass of ν_e at distances $r \cong r_P$. In the expression for L^0 there is a term G_F which has the following form:

$$G_F = \sqrt{2}\frac{g_W^2}{8m_W^2},$$

and includes a vector boson propagator $\frac{1}{m_W^2 - q^2}$ and when $m_W^2 >> q^2$ this propagator transforms into the expression $\frac{1}{m_W^2}$. If q^2 is a very large value, i.e., $q^2 >> m_W^2$ and $q^2 \cong m_P^2$, then the propagator will have the following form: $\frac{1}{m_P^2}$ (the sign is not taken into account for it characterizes the sign of its addition to the diagonal term of mass matrix $\hat{M}^2$).

So, the length formation $L^{0'}$ for the hypothetical left-right interaction at $q^2 \cong m_P^2$ (or at distances of the order of the lepton numbers violation) is

$$L^{0'} = (\frac{m_P^2}{m_W^2})L^0. \qquad (6)$$

The value of m_P can be estimated from limitation of the $\mu^- \to e^-(e^+e^-)$ [6] decays

$$\frac{W(\mu^- \to e^-e^+e^-)}{W(all)} \leq 2.710^{-12},$$

$$i.e.(\frac{m_W}{m_l}) \leq 2.710^{-12} \quad or \quad m_l^2 > 10^6 m_W^2.$$

If $m_P \cong m_l$ then

$$L^{0'} \cong 10^6 L^0.$$

The value of $L^{0'}$ is very large and is much bigger than the dimension of stars (or the Sun).

From the expression (6) at $\frac{L_0}{L^{0'}} \to 0$ we also obtain $\sin^2(2\theta_m) = \sin^2(2\theta)$, i.e., there is no resonance neutrino oscillation enhancement in matter connected with the hypothetical left-right symmetric interaction.

4 Neutrinos in Matter

What happens when the oscillating neutrinos (with $E_\nu \leq 15MeV$) are passing through matter (the Sun)? The oscillating neutrinos take part in the weak interactions in matter and these interactions are at low energies with low momentum transfer (i.e., deposit from the tail of the W bosons exchange, or of interactions at long distances). It is clear that these interactions cannot give contribution to the nondiagonal matrix $\hat{M}^2$ arising from interactions at very high momentum transfer (or at very small distances), moreover, the weak interaction is left-side one and cannot generate the masses (see refs. [4,5]). Then the

Wolfenstein equation (1) can be rewritten in the following form:

$$i\frac{d\nu_\Phi}{dt} = (\hat{P} + \frac{\hat{M}^2}{2p})\nu_\Phi,$$

where $P = \begin{pmatrix} p + W_{\nu_e} & 0 \\ 0 & p + W_{\nu_\mu} \end{pmatrix}$ (here P is taken in nonoperator form).

The values W_{ν_e}, W_{ν_μ} are very small ones and therefore the momenta P_{ν_e} and P_{ν_μ} slightly differ (these values must be added to momenta of the left-side neutrinos, but not right-side ones). To consider this question more strictly we need to use the Dirac equation rather than the Wolfenstein one and then we will obtain the correct picture of neutrinos passing through matter. But we do not realize such a procedure here for it does not lead to any interesting consequences.

It is possible to put it in a more understandable form. If E_ν is the neutrino energy and $E_\nu^2 = p^2 + m_\nu^2$, there arises the following question: where will the deposit go from the weak neutrino interaction, to p or m_ν? From the above discussion it is clear that the weak interaction gives the deposit into p but m_ν remains without change. As a result, there is no enhancement of neutrino oscillations in matter.

5 Conclusion

The Wolfenstein equation for neutrinos passing through matter has been considered. Although from this equation we can obtain enhancement of neutrino oscillations in matter, in deriving this equation some physical requirements have not been taken into account:

1. The Wolfenstein equation is left-right-side symmetric but not the equation with a left-side (the standard weak) interaction.

2. In this equation we must put only the terms which appear at the same distances, namely $\hat{M}^2, \hat{W}$ (i.e., their contributions must be combined).

Taking into account these physical requirements results in this equation including a hypothetical interaction but not the weak one. Besides, this equation does not result in enhancement of neutrino oscillations in matter.

In conclusion we would like to stress that in the experimental data from [7] there is no visible change in the spectrum of the B^8 Sun neutrinos. The measured spectrum of neutrinos coincides with the computed spectrum of the B^8 neutrinos [8].

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Received by Publishing Department on July 22, 1997.