СООБЩЕНИЯ
ОБЪЕДИНЕННОГО
ИНСТИТУТА
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

Дубна

E2-97-210

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DYNAMICAL ANALOGY OF THE CABIBBO–KOBAYASHI–MASKAWA MATRICES (WITH *CP*-VIOLATION)

1997

1 INTRODUCTION

At the present time the theory of electroweak interactions has the status of a theory which is confirmed with a high degree of precision. However, some experimental results (the existence of quark and of lepton families, etc.) did not get any explanation in the framework of the theory. One such part of the electroweak theory is the existence of quark mixing which is introduced by the Cabibbo-Kobayashi-Maskawa matrices (i.e., these matrices are used for parametrization of the quark mixing).

In previous works [1] a dynamical mechanism of quark mixing by the use of four doublets of massive vector carriers of weak interaction $B^{\pm}, C^{\pm}, D^{\pm}, E^{\pm}$ was proposed.

In this work in continuation of [1] a dynamical mechanism of quark and lepton mixing including CP-violation by using four doublets of massive vector carriers of the weak interaction $B^{\pm}, C^{\pm}, D^{\pm}, E^{\pm}$ in the quark sector and $X_1^{\pm}, X_2^{\pm}, X_3^{\pm}, X_4^{\pm}$ in the lepton sector are suggested, i.e., expansion of the electroweak theory at a tree level is proposed. Then together with the $W^{\pm}, Z^0$ bosons there arise a pair of four doublets of massive vector carriers of the weak interaction $B^{\pm}, C^{\pm}, D^{\pm}, E^{\pm}$, $X_1^{\pm}, X_2^{\pm}, X_3^{\pm}, X_4^{\pm}$ leading to the quark families and lepton families mixing. An estimation of the boson masses is performed. The quasi-elastic processes proceeding through an exchange of the bosons and the production cross sections are given.

2 QUARK SECTOR

In this section the transitions between the quark families are considered.

a) Dynamical Analogy of the Cabibbo-Kobayashi-Maskawa Matrices

Now let us go on to generalization of the work [1], which includes CP-violation.

In the case of three families of quarks the current J^μ has the following form:

$$J^\mu = (\bar{u}\bar{c}\bar{t})_L \gamma^\mu V \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L , \qquad (1)$$

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix},$$

where V is the Kobayashi-Maskawa matrix [2].

We shall choose a parametrization of the matrix V in the form offered by Maiani [3]:

$$V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\gamma & s_\gamma \\ 0 & -s_\gamma & c_\gamma \end{pmatrix} \begin{pmatrix} c_\beta & 0 & s_\beta \exp(-i\delta) \\ 0 & 1 & 0 \\ -s_\beta \exp(i\delta) & 0 & c_\beta \end{pmatrix} \begin{pmatrix} c_\theta & s_\theta & 0 \\ -s_\theta & c_\theta & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$c_\theta = \cos\theta, s_\theta = \sin\theta, \exp(i\delta) = \cos\delta + i\sin\delta. \qquad (2)$$

To the nondiagonal terms (which are responsible for mixing of the d, s, b quarks and CP-violation in the three matrices) in (2) we shall make correspond four doublets of vector bosons $B^\pm, C^\pm, D^\pm, E^\pm$ (we will suppose that the real part $Re(s_\beta \exp(i\delta)) = s_\beta \cos\delta$ corresponds to the vector boson $C^\pm$, and the imaginary part $Im(s_\beta \exp(i\delta)) = s_\beta \sin\delta$ corresponds to the vector boson $E^\pm$(the couple constant of E is an imaginary value !)) whose contributions are parametrized by four angles $-\theta, \beta, \gamma, \delta$. Then, when $q^2 << m_W^2$, we get

$$\tan\theta \cong \frac{m_W^2 g_B^2}{m_B^2 g_W^2},$$

$$\tan\beta \cong \frac{m_W^2 g_C^2}{m_C^2 g_W^2},$$

$$\tan\gamma \cong \frac{m_W^2 g_D^2}{m_D^2 g_W^2},$$

$$\tan\delta \cong \frac{m_W^2 g_E^2}{m_E^2 g_W^2}. \qquad (3)$$

If $g_{B^\pm} \cong g_{C^\pm} \cong g_{D^\pm} \cong g_{E^\pm} \cong g_{W^\pm}$, then

$$\tan\theta \cong \frac{m_W^2}{m_B^2},$$

$$\tan\beta \cong \frac{m_W^2}{m_C^2},$$

$$\tan\gamma \cong \frac{m_W^2}{m_D^2},$$

$$\tan\delta \cong \frac{m_W^2}{m_E^2}. \qquad (4)$$

Concerning the neutral vector bosons B^0, C^0, D^0, E^0, the neutral scalar bosons B'^0, C'^0, D'^0, E'^0 and the GIM mechanism [4] we can repeat the same arguments as were given in the previous work [1].

The proposed Lagrangian for expansion of the weak interaction theory (without CP-violation) has the following form:

$$L_{int} = i \sum_i g_i (J^{i,\alpha} A^i_\alpha + c.c.). \qquad (5)$$

where $J^{i,\alpha} = \bar{\psi}_{i,L} \gamma^\alpha T \varphi_{i,L}$,

$$T = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

$$\begin{matrix} & i=1 & i=2 & i=3 \\ \psi_{i,L} = & \begin{pmatrix} u \\ c \end{pmatrix}_L, & \begin{pmatrix} u \\ t \end{pmatrix}_L, & \begin{pmatrix} c \\ t \end{pmatrix}_L; \end{matrix}$$

$$\begin{matrix} & i=1 & i=2 & i=3 \\ \varphi_{i,L} = & \begin{pmatrix} d \\ s \end{pmatrix}_L, & \begin{pmatrix} d \\ b \end{pmatrix}_L, & \begin{pmatrix} s \\ b \end{pmatrix}_L. \end{matrix}$$

The weak interaction carriers A^i_α, which are responsible for the weak transitions between different quark families, are connected with the B, C, D bosons in the following manner:

$$A^1_\alpha \to B^\pm_\alpha, A^2_\alpha \to C^\pm_\alpha, A^3_\alpha \to D^\pm_\alpha. \qquad (6)$$

b) Reactions Proceeding through the $B^\pm, C^\pm, D^\pm, E^\pm$ Bosons and Estimation of their Masses

The reactions with substitutions $d \leftrightarrow s, d \leftrightarrow b, s \leftrightarrow b$ proceed via the $B^\pm, C^\pm, D^\pm, E^\pm$ bosons. Besides the decays of leptons and hadrons, the following quasi-elastic reactions proceed through exchange of the bosons:

a) $u + d \xrightarrow{B} u + s, \ p + p \xrightarrow{B} p + \Sigma^+,$

$$u(\bar{u}) + \bar{s}(s) \xrightarrow{B} \nu_i(\bar{\nu}_i) + \bar{e}_i(e_i), \qquad i = e, \mu, \tau,$$

$$\nu_i(\bar{\nu}_i) + \bar{e}_i(e_i) \xrightarrow{B} u(\bar{u}) + \bar{s}(s);$$

b) $u + d \xrightarrow{C} u + b, \ p + p \xrightarrow{C} p + \Sigma^+(q_b), \qquad (7)$

$$u(\bar{u}) + \bar{b}(b) \xrightarrow{C} \nu_i(\bar{\nu}_i) + \bar{e}_i(e_i),$$

$$\nu_i(\bar{\nu}_i) + \bar{e}_i(e_i) \xrightarrow{C} u(\bar{u}) + \bar{b}(b);$$

c) $c + s \xrightarrow{D} c + b,$

$$c(\bar{c}) + \bar{b}(b) \xrightarrow{D} \nu_i(\bar{\nu}_i) + \bar{e}_i(e_i),$$

$$\nu_i(\bar{\nu}_i) + \bar{e}_i(e_i) \xrightarrow{D} c(\bar{c}) + \bar{b}(b);$$

d) through E boson will go the following reactions with CP-violation: $K^o_1 \leftrightarrow K^o_2, B^o_1 \leftrightarrow B^o_2 \cdots$ etc.

Let us estimate the masses of the $B^\pm, C^\pm, D^\pm, E^\pm$ bosons. For this

purpose we shall use the data from [5] and equation (4):

1) $tg\theta \cong 0.218 \div 0.224,$
$m_{B^\pm} \cong 169.5 \div 171.8$ GeV;

2) $tg\beta \cong 0.032 \div 0.054,$ (8)
$m_{C^\pm} \cong 345.2 \div 448.4$ GeV;

3) $tg\gamma \cong 0.002 \div 0.007,$
$m_{D^\pm} \cong 958.8 \div 1794$ GeV;

4) $tg\delta \cong 0.00036 \div 0.00037,$
$m_{E^\pm} \cong 4170 \div 4230$ GeV.

c) An Estimation of the Production Cross Sections of the Vector Bosons

For estimation of the production cross sections of the vector bosons $B^\pm, C^\pm, D^\pm, E^\pm$ at resonance in $p\bar{p}$ reaction we can use a standard formula [6] ($g_W = g_X$):

$$\sigma_X = \frac{4\pi^2}{3s}\left(\frac{\Gamma_X}{m_X}\right) Br(X \longrightarrow q\bar{q}) Br(X \longrightarrow f)$$
$$\int\int dx_1 dx_2 u_i(x_1) \bar{u}_k(x_2) \delta(x_1 x_2 - M_X^2/s), \qquad (9)$$

where $s = (p_p + p_{\bar{p}})^2$, Br(...) is branching mode probability, $u_i(x)$ are quark structure functions, $i = u, d, s$ quarks, m_X is the mass of the vector boson X, $X = B, C, D, E$ is boson, Γ_X is total width of the vector boson X.

Using expression (9) at energy $E = 1.8$ TeV (FNAL), we obtain the following values for production cross sections of $B^\pm$ boson at resonance:

$$\sigma(u\bar{d} \xrightarrow{B} all) \cong 245 pb,$$
$$\sigma(u\bar{d} \xrightarrow{B} e\bar{\nu}) \cong 20 pb, \qquad (10)$$

in this case we have used quark structure functions of u, d quarks from [7];

$$\sigma(u\bar{s} \xrightarrow{B} all) \cong 7.6pb,$$
$$\sigma(u\bar{s} \xrightarrow{B} e\bar{\nu}) \cong 0.64pb, \qquad (11)$$

in this case we have used the u, s quark structure functions from [8].

3 THE LEPTON SECTOR

Weak interactions of quarks and leptons have essential distinctions. At the weak interaction of the quarks from one and the same family, the aromatic numbers are changed, while at the weak interaction of the leptons from the same family the lepton number is conserved. In spite of such distinctions we can use the above-suggested approach to describe transitions between different lepton families without conservations of lepton numbers.
Let us pass on to description of these transitions.

a) Dynamical Analogy of the Cabibbo-Kobayashi-Maskawa Matrices

In this section transitions between different lepton families are considered . In the case of three families of leptons the current j^μ has the following form:

$$j^\mu = (\bar{\nu}_e\bar{\nu}_\mu\bar{\nu}_\tau)_L \gamma^\mu V' \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}_L . \qquad (12)$$

$$V' = \begin{pmatrix} V_{ee} & V_{e\mu} & V_{e\tau} \\ V_{\mu e} & V_{\mu\mu} & V_{\mu\tau} \\ V_{\tau e} & V_{\tau\mu} & V_{\tau\tau} \end{pmatrix},$$

where V' is an analogy of the Cabibbo-Kobayashi-Maskawa matrices for leptons. For this case we also choose a parametrization of the matrix V' in the form offered by Maiani (2):

$$V' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{\gamma'} & s_{\gamma'} \\ 0 & -s_{\gamma'} & c_{\gamma'} \end{pmatrix} \begin{pmatrix} c_{\beta'} & 0 & s_{\beta'}\exp(-i\delta') \\ 0 & 1 & 0 \\ -s_{\beta'}\exp(i\delta') & 0 & c_{\beta'} \end{pmatrix} \begin{pmatrix} c_{\theta'} & s_{\theta'} & 0 \\ -s_{\theta'} & c_{\theta'} & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$c_{\theta'} = \cos\theta', s_{\theta'} = \sin\theta', \exp(i\delta') = \cos\delta' + i\sin\delta'. \qquad (13)$$

To the nondiagonal terms (which are responsible for mixing of the ν_e, ν_μ, ν_τ leptons and CP-violation in the three matrices) in (13) we shall make correspond four doublets of the vector bosons $X_1^\pm, X_2^\pm, X_3^\pm, X_4^\pm$ (as in the above-considered quark case, we will suppose that the real part $Re(s_{\beta'}\exp(i\delta')) = s_{\beta'}\cos\delta'$ corresponds to the vector boson $X_2^\pm$, and the imaginary part $Im(s_{\beta'}\exp(i\delta')) = s_{\beta'}\sin\delta'$ corresponds to the vector boson $X_4^\pm$ (the couple constant of X_4 is an imaginary value !)) whose contributions are parametrized by four angles $\theta', \beta', \gamma', \delta'$. Then, when $q^2 << m_W^2$, we get(in this case the angles $\theta', \beta', \gamma', \delta'$ are very small and therefore we will change the function tg on sin:

$$\sin\theta' \cong \frac{m_W^2 g_{X_1}^2}{m_{X_1}^2 g_W^2},$$
$$\sin\beta' \cong \frac{m_W^2 g_{X_2}^2}{m_{X_2}^2 g_W^2},$$
$$\sin\gamma' \cong \frac{m_W^2 g_{X_3}^2}{m_{X_3}^2 g_W^2},$$
$$\sin\delta' \cong \frac{m_W^2 g_{X_4}^2}{m_{X_4}^2 g_W^2}. \qquad (14)$$

If $g_{X_1^\pm} \cong g_{X_2^\pm} \cong g_{X_3^\pm} \cong g_{X4^\pm} \cong g_{W^\pm}$, then

$$\sin\theta' \cong \frac{m_W^2}{m_{X1}^2},$$
$$\sin\beta' \cong \frac{m_W^2}{m_{X_2}^2},$$
$$\sin\gamma' \cong \frac{m_W^2}{m_{X_3}^2},$$

$$\sin\delta' \cong \frac{m_W^2}{m_{X_4}^2}. \quad (15)$$

In the considered approach the neutral bosons $X_1^0, X_2^0, X_3^0, X_4^0$ may appear together with $X_1^\mp, X_2^\mp, X_3^\mp, X_4^\mp$. It is clear that (by analogy with the $W^\pm, Z^0$ bosons) masses of $X_1^0, X_2^0, X_3^0, X_4^0$ must be bigger than the corresponding masses of $X_1^\mp, X_2^\mp, X_3^\mp, X_4^\mp$.

The Lagrangian of the expanded theory of the weak interaction (without CP-violation) for the lepton sector has the same form as for the quark Lagrangian L_{int} (5), where the following substitution is made:

$$u \to \nu_e, \; c \to \nu_\mu, \; t \to \nu_\tau,$$

$$d \to e^-, \; s \to \mu^-, \; b \to \tau^-. \quad (16)$$

b) Reactions Proceeding through the Bosons X_1, X_2, X_3, X_4 and Estimation of their Masses

The transitions between $l_e \leftrightarrow l_\mu, l_\mu \leftrightarrow l_\tau, l_e \leftrightarrow l_\tau$ (l is ν or charged lepton) with violations of lepton numbers will proceed via the bosons $X_1^\mp, X_2^\mp, X_3^\mp, X_4^\mp$. Besides the decays of the leptons $(\mu, \tau, \nu_\mu, \nu_\tau)$

$$\mu^- \xrightarrow{X_1} \nu_e(e^-\bar{\nu}_e),$$

$$\tau^- \xrightarrow{X_2} \nu_\tau(\mu^-\bar{\nu}_\mu), \quad (17)$$

$$\tau^- \xrightarrow{X_3} \nu_\tau(e^-\bar{\nu}_e),$$

the following reactions will take place through the neutral bosons:

$$\mu^- \xrightarrow{X_1^0} e^-(e^+e^-, \bar{\nu}_e\nu_e),$$

$$\tau^- \xrightarrow{X_2^0} \mu^-(e^+e^-),$$

$$\tau^- \xrightarrow{X_3^0} e^-(e^-e^+).$$

Besides the decays of the leptons $\mu, \tau, \nu_\mu, \nu_\tau$ the reverse reactions will also take place through these bosons:

$$\bar{\nu}_e + e^- \xrightarrow{X_1} \bar{\nu}^\mu + e^-, \; \nu_e + p \xrightarrow{X_1} \nu_\mu + p,$$

$$\bar{\nu}_e + e^- \xrightarrow{X_3} \bar{\nu}_\tau + e^-, \; \nu_e + p \xrightarrow{X_3} \nu_\tau + p,$$

$$\bar{\nu}_\mu + \mu^- \xrightarrow{X_2} \bar{\nu}_\tau + \mu^-,$$

$$\bar{\nu}_e + e^- \xrightarrow{X_1} \bar{\nu}_e + \mu^-, \quad (18)$$

$$\bar{\nu}_e + e^- \xrightarrow{X_3} \bar{\nu}_e + \tau^-,$$

$$\bar{\nu}_\mu + \mu^- \xrightarrow{X_2} \bar{\nu}_\mu + \tau^-.$$

The reactions with CP-violation will go through the bosons X_4 :

$$\nu_i \leftrightarrow \nu_j (i \neq j; \; i, j = e, \mu, \tau). \quad (19)$$

Let us estimate the masses of the X bosons. For this purpose we will use data from [5] and equation (15).

a) From the upper limit for the process $\mu^+ \longrightarrow e^+e^-e^+$ we get

$$\frac{m_W^4}{m_{X_1^0}^4} < 2.4\ 10^{-12} \quad or \quad m_{X_1^0} > 10^3 m_W; \quad (20)$$

b) from the upper limit for the process $\tau^- \longrightarrow \mu^-\mu^+\mu^-$ we get

$$\frac{m_W^4}{m_{X_2^0}^4} < 2.7\ 10^{-5} \quad or \quad m_{X_2^0} > 26 m_W. \quad (21)$$

It is obvious that the masses of X_1, X_2 are not bigger than the masses of X_1^0, X_2^0, if they exist.

4 CONCLUSION

A dynamical analogy of the Cabibbo-Kobayashi-Maskawa matrices (including CP-violation), i.e. a phenomenological expansion of the weak interaction theory working at a tree level, is proposed. For this purpose four doublets of vector bosons $B^\pm, C^\pm, D^\pm, E^\pm$ for the quark sector and four doublets of vector bosons $X_1^{\pm}, X_2^{\pm}, X_3^{\pm}, X_4^{\pm}$ for the lepton sector are introduced. Lagrangians of these expansions are given. Estimation of their masses ($m_B \cong 170$ GeV, $m_C \cong 345$ GeV, $m_D \cong 1000$ GeV, $m_E \cong 4200$ GeV) is calculated. The quasi-elastic reactions proceeding through the exchange of these bosons are listed. The estimation of the production cross section for energy $E = 1.8$ TeV (FNAL) and decay widths of B boson is given.

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Received by Publishing Department
on July 8, 1997.