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NNLO QCD ANALYSIS OF CCFR DATA ON xF_3 STRUCTURE FUNCTION AND GROSS—LLEWELLYN SMITH SUM RULE WITH HIGHER TWIST AND NUCLEAR CORRECTIONS

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1 Introduction

The progress of perturbative Quantum ChromoDynamics (QCD) in the description of the high energy physics of strong interactions is considerable [1]. Recent experimental data on the structure function (SF) of neutrino deep-inelastic scattering obtained at the Fermilab Tevatron [2] provide a good possibility to precisely verify the QCD predictions for scaling violation with experiment data. The QCD predictions for SF evolution of deep-inelastic scattering (DIS) are calculated now up to the next-to-next-to-leading order (NNLO) of a perturbative theory [3, 4, 5, 6]. The method of comparison of 3-loop QCD predictions with SF experimental data has been developed in [7, 8, 9] based on the Jacobi polynomial structure function expansion [10]. It is well known that beyond the perturbative QCD there are other effects (higher twist effects, nuclear corrections, target mass corrections, etc.) that should be included in the joint QCD analysis of SF.

In the paper, we present the results of NNLO QCD analysis of the data on the xF_3 structure function obtained by the CCFR Collaboration with taking in account of the target mass, higher twist and nuclear corrections. The Q^2-evolution of SF is studied and parametrizations of perturbative (leading twist) and power terms of the structure function are constructed. The results of our NNLO QCD analysis of the structure function are in good qualitative agreement with NNLO theoretical predictions for the Q^2-evolution of the Gross–Llewellyn Smith sum rule [11]: $S_{GLS}^{theor}(Q^2) = 3\cdot[1-\alpha_S/\pi - 3.25\cdot(\alpha_S/\pi)^2]$ [12].

In Section 2, the method of NNLO QCD analysis of SF based on the SF Jacobi polynomial expansion including the target mass, higher twist and nuclear corrections is described. The results of NNLO QCD analysis of SF are presented in Section 3. In Section 4, the α_S/π-expansion of the Gross-Llewellyn Smith sum rule is considered and expansion parameters are found.

2 Method of QCD Analysis

2.1 Jacobi Polynomial Expansion Method

We use, for the QCD analysis, the Jacobi polynomial expansion method proposed in [10]. It was developed in [13] and applied for the 3-loop order of perturbative QCD (pQCD) to fit F_2 [7] and xF_3 data [8, 9, 14, 15].

Following the method [13], we can write the structure function xF_3 in the form

$$xF_3^{pQCD}(x,Q^2) = x^{\alpha}(1-x)^{\beta}\sum_{n=0}^{N_{max}}\Theta_n^{\alpha,\beta}(x)\sum_{j=0}^{n}c_j^{(n)}(\alpha,\beta)M_3^{pQCD}\left(j+2,Q^2\right), \qquad (1)$$

where $\Theta_n^{\alpha\beta}(x)$ is a set of Jacobi polynomials and $c_j^n(\alpha,\beta)$ are coefficients of the series of $\Theta_n^{\alpha,\beta}(x)$ in powers of x :

$$\Theta_n^{\alpha,\beta}(x) = \sum_{j=0}^{n}c_j^{(n)}(\alpha,\beta)x^j. \qquad (2)$$

The Q^2-evolution of the moments $M_3^{pQCD}(N,Q^2)$ is given by the well-known perturbative QCD [16, 17] formula

$$M_3^{pQCD}(N,Q^2) = \left[\frac{\alpha_S(Q_0^2)}{\alpha_S(Q^2)}\right]^{d_N} H_N\left(Q_0^2,Q^2\right) M_3^{pQCD}(N,Q_0^2), \quad N=2,3,... \tag{3}$$
$$d_N = \gamma^{(0),N}/2\beta_0.$$

Here $\alpha_s(Q^2)$ is the next-to-next-to-leading order strong interaction constant, $\gamma_N^{(0)NS}$ are the nonsinglet leading order anomalous dimensions, and the factor $H_N\left(Q_0^2,Q^2\right)$ contains next- and next-to-next-to-leading order QCD corrections to the coefficient functions and anomalous dimensions [1] and is constructed in accordance with [8, 9] based on theoretical results of [3]-[6].

The unknown coefficients $M_3(N,Q_0^2)$ in (3) could be parametrized as Mellin moments of some function:

$$M_3^{pQCD}(N,Q_0^2) = \int_0^1 dx x^{N-2} a_1 x^{a_2}(1-x)^{a_3}(1+a_4 x), \quad N=2,3,... \tag{4}$$

Here coefficients a_i should be found by the QCD fit of experimental data.

The target mass corrections (TMC) are included into our fits through the Nachtmann moments [19] of the SFs:

$$M_N^{pQCD}(Q^2) = \int_0^1 dx \xi^{N+1}/(x^3) F_3(x,Q^2)(1+(N+1)V)/(N+2), \tag{5}$$

where $\xi = 2x/(1+V)$, $V=\sqrt{1+4M_{nucl}^2x^2/Q^2}$ and M_{nucl} is the mass of a nucleon. We are taking into account the order $O(M_{nucl}^2/Q^2)$ corrections:

$$M_N(Q^2) = M_N^{pQCD}(Q^2) + \frac{N(N+1)}{N+2}\frac{M_{nucl.}^2}{Q^2} M_{N+2}^{pQCD}(Q^2) \tag{6}$$

where M_N are the Mellin moments of measured xF_3 SF.

2.2 Higher Twist Correction

We take into account the higher twist (HT) contribution following the method of [20] and [14, 15, 9]. To extract the HT contribution, the SF is parametrized as follows:

$$xF_3(x,Q^2) = xF_3^{pQCD}(x,Q^2) + \frac{h(x)}{Q^2}, \tag{7}$$

where the Q^2 dependence of the first term in the r.h.s is determined by perturbative QCD.

The function $h(x)$ as well as the parameters $a_1 - a_4$ and scale parameter Λ should be determined by fitting the experimental data.

2.3 Nuclear Correction

We have used the covariant approach in the light-cone variables [21, 22] to estimate the ratio of structure functions

$$R_F^{D/N} = \frac{F_3^D(x,Q^2)}{F_3^N(x,Q^2)} \tag{8}$$

[1] For reviews and references on higher order QCD results, see[18].

and to perform the joint NNLO QCD analysis of the data [2] on the structure function F_3 (for the NLO analysis, see Ref.[23]).

We would like to remind that the ratio $R_F^{A/N}$ describes the influence of nuclear medium on the structure of a free nucleon in the process. We use the approximation that $R_F^{D/N} = R_F^{Fe/N}$. It gives lower estimation on the effect of nucleon Fermi motion in a heavy nucleus.

The covariant approach in the light-cone variables is based on the relativistic deuteron wave function (RDWF) with one nucleon on mass shell. The RDWF depending on one variable, the virtuality of nucleon $k^2(x,k_\perp)$, can be expressed via the Dpn vertex function $\Gamma_\alpha(x,k_\perp)$. The deep-inelastic structure function of neutrino-deuteron scattering F_3^D in the approach can be written as follows:

$$F_3^D(\alpha,Q^2) = \int_\alpha^1 dx\, d^2k_\perp\, \Delta(x,k_\perp)\cdot F_3^N(\alpha/x,Q^2). \tag{9}$$

The nucleon SF is defined as $F_3^N = (F_3^{\nu N} + F_3^{\bar\nu N})/2$, $\alpha = -q^2/2(pq)$. The function $\Delta(x,k_\perp)$ describes the left (right)-helicity distribution for an active nucleon (antinucleon) that carries away the fraction of deuteron momentum $x = k_{1+}/p_+$ and transverse momentum $k_\perp$. It is expressed via the RDWF $\psi_\alpha(k_1)$ as follows:

$$\Delta(x,k_\perp) \propto Sp\{\bar\psi^\alpha(k_1)\cdot(m+\hat k)\cdot\psi^\beta(k_1)\cdot\hat q\cdot\sigma^{\mu\nu}\cdot\gamma_5\cdot\rho_{\alpha\beta}^{(S)}\cdot\epsilon_{\mu\nu\gamma\delta}q^\gamma p^\delta\}, \tag{10}$$

where $\rho_{\alpha\beta}^{(S)}$ is the polarization density matrix for an unpolarized deuteron. Note that in the approach used the distribution function $\Delta(x,k_\perp)$ includes not only usual S- and D-wave components of the deuteron but also a P-wave component. The latter describes the contribution of $N\bar N$-pair production. The contribution of this mechanism is small over a momentum range $(x<1)$.

The nuclear effect in a deuteron for the $\nu + D \to \mu^- + X$ process has been estimated in [23]. It has been found that the ratio $R_F^{D/N}$ is practically independent of the parametrization of parton distributions [24, 25, 26] and the nucleon SF [27] over a wide kinematic range of $x = 10^{-3} - 0.7$, $Q^2 = 1 - 500\,(GeV/c)^2$. The curve has an oscillatory feature and cross-over point $x_0 : R_F^{D/N}(x_0) = 1$, $x_0 \simeq 0.03$.

Thus, the obtained results give evidence that the function $R_F^{D/N}$ is defined by the structure of the relativistic deuteron wave function and can be used to extract the nucleon SF F_3^N from the experimentally known deuteron one:

$$F_3^N(x,Q^2) = \left[R_F^{D/N}(x)\right]^{-1}\cdot F_3^D(x,Q^2). \tag{11}$$

The performed analysis of the nuclear correction for the nucleon SF also allows one to consider the influence of the nuclear effect on the GLS sum rule [11]:

$$S_{GLS} = \int_0^1 F_3^N(x)dx. \tag{12}$$

We have used the result on the $R_F^{D/N}$ ratio to study the x- and Q^2-dependences of the GLS integral

$$S_{GLS}(x,Q^2) = \int_x^1 F_3^N(y,Q^2)dy. \tag{13}$$

It has been shown in [23] that the nuclear effect of Fermi motion is very important for the NLO QCD analysis of the structure function F_3 and verification of the Gross–Llewellyn

Smith sum rule over a wide region of $Q^2 = 3 - -500$ $(GeV/c)^2$. Therefore in the paper, we include the nuclear effect in the NNLO QCD analysis of both the structure function and the Gross–Llewellyn Smith sum rule.

3 NNLO QCD Analysis of Structure Function xF_3^N

In this section, we perform the QCD analysis of the xF_3^N experimental data [2] taking into account the 3-loop QCD, higher twist and nuclear corrections. We consider as a first approximation that $R_F^{Fe/N} = R_F^{D/N} \equiv R$:

$$xF_3^N(x,Q^2) = a_1(Q^2)x^{a_2(Q^2)}(1-x)^{a_3(Q^2)}(1+a_4(Q^2)x) + \frac{h(x)}{Q^2}. \quad (14)$$

The constants $h(x_i)$ (one per x–bin) parametrize the HT x–dependence. The points x_i are chosen in accordance with [2]: 0.0075 - 0.75.

The values of the parameters $a_i(Q_0^2)$ $(i = 1--4)$, scale parameter Λ and constants $h(x_i)$ have been determined by fitting the whole set of xF_3 data [2] (116 experimental points) for different values of Q_0^2 in the kinematic region: $2\ (GeV/c)^2 \le Q^2 \le 200\ (GeV/c)^2$ [2]. Only statistical errors are taken into account. The χ^2 parameter is found to be about 105 for 116 experimental points.

Figure 1(a–d) shows the parameters of SF a_i at the points $Q^2 = 1.5 - 200$ $(GeV/c)^2$. The $\Lambda_{\overline{MS}}$ parameter was found in the interval $(210 - -250)$ MeV with statistical error about ± 20 MeV.

Figure 2 demonstrates the x–dependence of the HT contribution $h(x_i)$.

We use the result of our NNLO QCD analysis presented in Figs.1 and 2 to obtain the parametrisation of xF_3 in form (14).

The Q^2–dependence of the coefficients a_i is parametrized as follows:

$$a_i(Q^2) = \sum_{j=0}^{2} c_j \cdot z^j, \quad z = \log(Q^2) \quad (15)$$

and is shown in Fig.1 by solid lines. The coefficients c_i are presented in Table 1.

Table 1. Coefficients c_i for the parametrization of the function $xF_3(x,Q^2)$

	a_1	a_2	a_3	a_4
c_0	3.1430	$6.6704e-1$	$3.2753e+0$	$2.92880e+0$
c_1	1.6794	$4.0720e-3$	$8.2294e-1$	$-1.65755e+0$
c_2	-0.3896	$-7.7966e-3$	$-1.3154e-1$	$3.09344e-1$

Figure 1 shows the dependence of coefficients a_i on Q^2. The black circles are experimental points, the lines are results of the fit.

[2] The number of flavors is taken to be equal to 4.

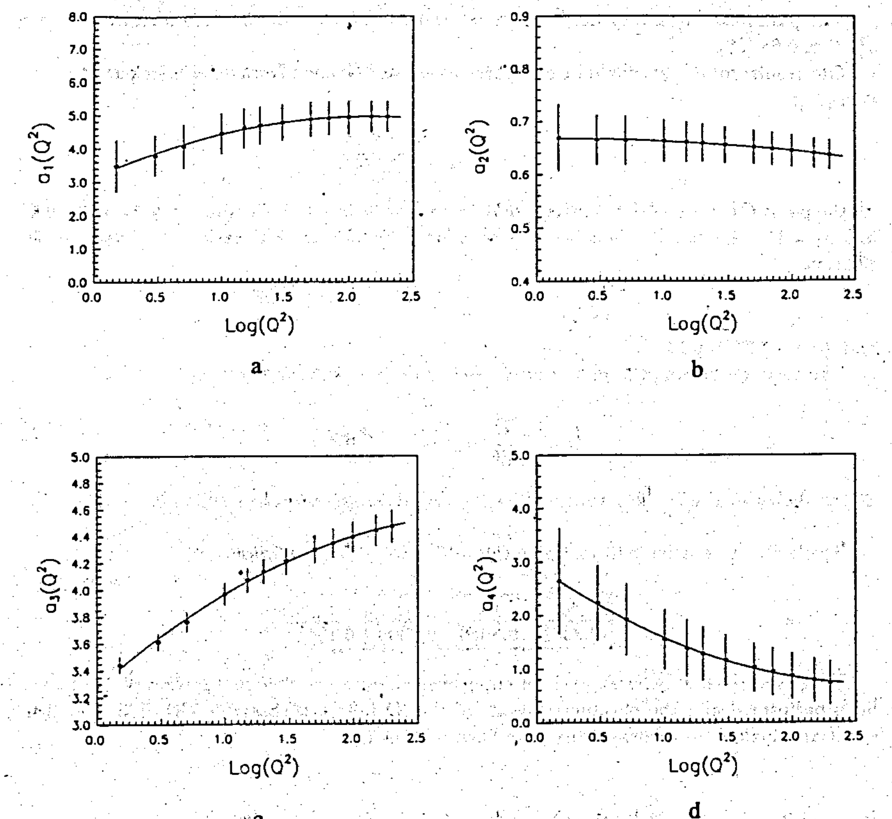

Fig. 1. Dependence of the coefficients a_1-a_4 on Q^2. The black circles are experimental points, the lines are results of the fit

The parameter a_2 slowly depends on Q^2 and corresponds to the theoretical estimation $a_2^{theor} \simeq 0.68$ [28].

Our results for $a_3(Q^2)$ could be compared to the well-known Buras-Gaemers parametrization [29]:

$$a_3 = \eta_1 + \eta_2 \cdot \frac{ln(Q^2/\Lambda^2)}{ln(Q_0^2/\Lambda^2)} \tag{16}$$

At the point $Q_0^2 = 1.8\ GeV^2$ it gives [16] $\eta_1 = 2.6$ and $\eta_2 = 0.8$. Our result is $\eta_1 = 3.48 \pm 0.03$ and $\eta_2 = 1.18 \pm 0.09$. It is not far from the pQCD theoretical estimation [30] based on the relation

$$\frac{d}{dln(Q^2)} a_3(Q^2) = \frac{4}{3\pi}\alpha_S(Q^2) + O(\alpha_S^2(Q^2)) \tag{17}$$

and gives $\eta_2^{theor} = 0.64$.

The term describing the higher twist correction is written in the form

$$h(x) = \sum_{i=0}^{3} d_i \cdot z^i, \quad z = \log(x) \tag{18}$$

and is shown in Fig.2 . The values of coefficients d_i are presented in Table 2.

Table 2. Coefficients d_i for the parametrization of the function $h(x)$

d_0	d_1	d_2	d_3
0.2329	0.8060	0.7308	0.1842

Thus, the NNLO QCD analysis of experimental data of the structure function xF_3 has been performed and the parametrizations of the NNLO perturbative QCD, TMC, nuclear effect and higher twist corrections have been obtained.

4 Experimental Constraints on Coefficients of α_S–Expansion of Gross–Llewellyn Smith Sum Rule

In this section, we would like to show the status of the NNLO QCD analysis of the experimental data [2].

The Q^2–dependence of the parameters $a_i(Q^2)$ allows us to study the behavior of $S_{GLS}(Q^2)$ in a wide kinematic region of the momentum transfer squared.

Figure 3 shows the dependence of $S_{GLS}(Q^2)$ on α_S/π in thw 3-loop approximation. The black and open circles correspond to the NNLO QCD analysis with and without nuclear corrections, respectively. The statistical errors are about ± 0.4 and are not presented at the figure.

Figure 3 demonstrates the increase of $S_{GLS}(Q^2)$ with decreasing α_S/π. The result is in qualitative agreement with the Q^2-dependence of the sum rule found in [31] in the NLO QCD analysis without taken into account target mass corrections, higher twist and nuclear effect. A similar tendency in the NLO QCD approximation, with target mass corrections and nuclear effect was found in [23]. For the estimation of the order $O(\alpha_s^4)$ and power

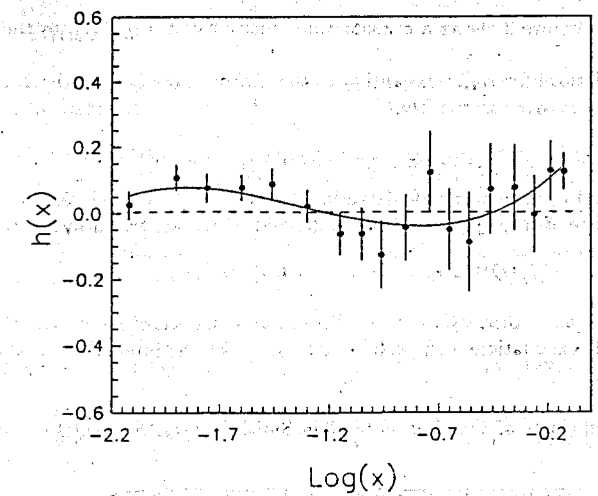

Fig. 2. The function $h(x)$ vs $log(x)$ describing a higher twist correction. The black circles are experimental points, the line is the result of the fit

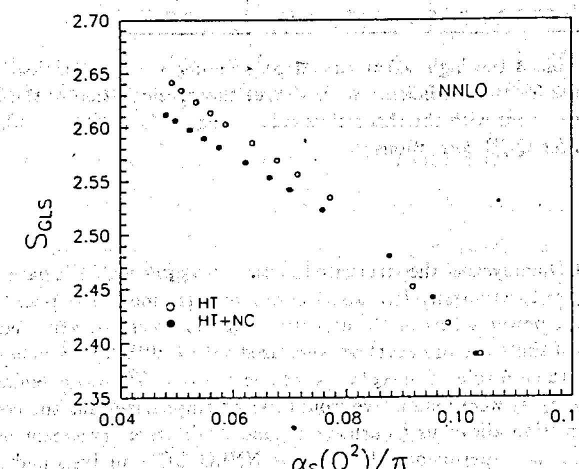

Fig. 3. The α_S/π-expansion of the Gross-Llewellyn Smith sum rule $S_{GLS}(Q^2)$. The black and open circles are results of NNLO QCD analysis obtained with and without nuclear correction, respectively

corrections, see [32]. Figure 3 shows a considerable sensitivity of $S_{GLS}(Q^2)$ to the nuclear correction.

The pQCD predictions for α_S/π–expansion of the Gross–Llewellyn Smith sum rule up to $(\alpha_S/\pi)^2$ could be presented in the form

$$S_{GLS}^{theor}(Q^2) = c_0 \cdot (1 + c_1 \cdot (\alpha_S/\pi) + c_2 \cdot (\alpha_S/\pi)^2). \tag{19}$$

The coefficients c_1 and c_2 have been calculated in [12]: $c_1 = -1$, $c_2 = -3.25$.

The α_S-dependence of S_{GLS} presented in Fig.3 could be parametrized by the parabola:

$$S_{GLS}^{exp}(Q^2) = s_0 + s_1 \cdot (\alpha_S/\pi) + s_2 \cdot (\alpha_S/\pi)^2. \tag{20}$$

This expansion allows us to directly compare the results of the experimental data analysis with the NNLO QCD calculations. The values of s_0, s_1, s_2 for the interval $\alpha_S/\pi < 0.09$ are presented in Table 3.

Table 3 The coefficients of the Gross-Llewellyn Smith integral $S_{GLS}(Q^2)$ expansion in α_S/π.

	NNLO+HT	NNLO+HT+NC	Theor. [12]
s_0	2.86 ± 0.01	2.74 ± 0.01	3.00
s_1	-4.93 ± 0.32	-2.22 ± 0.23	-3.00
s_2	0.98 ± 2.54	-7.86 ± 1.74	-9.75

One can see from Table 3 the high sensitivity of parameters s_i to the nuclear correction. The obtained results for the coefficients s_i show that the consideration of the nuclear correction gives good agreement with the theoretical calculation of the next–to–leading and next–to–next–leading order QCD corrections.

5 Conclusion

A detailed NNLO QCD analysis of the structure function xF_3 of new CCFR data including the target mass, higher twist and nuclear corrections was performed. The parametrizations of perturbative and power terms of the structure function were constructed. The results of QCD analysis of the structure function were used to study the Q^2–dependence of the Gross–Llewellyn Smith sum rule. The α_S/π–expansion of $S_{GLS}(Q^2)$ was exemined and expansion parameters s_1, s_2, s_3 were found. We would like to emphasize that the consideration of the nuclear correction allows us to achieve a good qualitative agreement between the results obtained from the experimental data by the NNLO QCD analysis and perturbative QCD predictions for the Gross–Llewellyn Smith sum rule in next–to–leading and next–to–next–to–leading order.

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