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ON THE COSMOLOGICAL ORIGIN
OF THE HOMOGENEOUS SCALAR FIELD
IN UNIFIED THEORIES

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1. Introduction

The homogeneous scalar field, generating elementary particle masses in unified theories, is based on the Higgs potential. The physical motivation for this potential as a consequence of the first symmetry principles is unclear and the existence of this potential leads to a number of difficulties in cosmology. Among them are the great vacuum density [1], monopole creation [2], domain walls [3]. These difficulties are overcome in the inflationary models based on the assumption of minimal scalar - gravity coupling [4]. The Higgs potential, as well as the minimal coupling, break the principle about conformal invariance of gravity - matter interaction [5]. If we take into consideration this conformal symmetry principle, it is necessary to switch off the Higgs potential and to introduce a conformally invariant scalar field-gravity interaction [5]. As a result, the only source of conformal symmetry breaking is the Einstein gravity itself.

In the present paper, we shall show that the Einstein gravity theory with a conformally coupled scalar field can replace the Higgs potential under the assumption of homogeneous matter distribution. We shall also find the scalar field value from the Friedmann equations for the homogeneous Universe with a Friedmann-Robertson-Walker metric (FRW).

2. The Higgs effect without the Higgs potential in the FRW metric

We begin from the $U(1)$ theory with the Lagrangian

$$L(A,\phi) = |(\partial_\mu + ieA_\mu)\phi|^2 - \frac{1}{4}F_{\mu\nu}(A)F^{\mu\nu}(A) - V_{Higgs}(|\phi|) \tag{1}$$

with a complex scalar field $\phi = |\phi|\exp(ie\chi)$. The Higgs effect consists in i) the absorption of the angular component of the scalar field by the transformation

$$A'_\mu = \partial_\mu\chi + A_\mu \tag{2}$$

$$L(A',\phi) = \partial_\mu|\phi|\partial^\mu|\phi| + e^2|\phi|^2 A'_\mu A'^\mu - \frac{1}{4}F_{\mu\nu}(A')F^{\mu\nu}(A') - V_{Higgs}(|\phi|) \tag{3}$$

and in ii) the nonzero vacuum value of the scalar field module which follows from the Higgs potential.

For computing this vacuum value one commonly minimizes the Lagrangian (3) without interaction
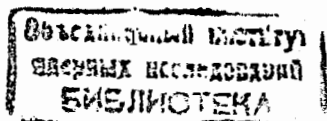
$$L_o(A',\phi) = \partial_\mu|\phi|\partial^\mu|\phi| - \frac{1}{4}F_{\mu\nu}(A')F^{\mu\nu}(A') - V_{Higgs}(|\phi|). \tag{4}$$

Our idea is to consider just this theory (4) without the Higgs potential ($V_{Higgs}(|\phi|) = 0$) but in the presence of the Einstein gravity theory with a conformally coupled scalar field

$$L_o^G(g,A',|\phi|) = \sqrt{-g}\left[-\frac{{}^{(4)}R(g)}{6}\left(\frac{3M_{Pl}^2}{8\pi} - |\phi|^2\right) + \partial_\mu|\phi|\partial^\mu|\phi| - \frac{1}{4}F_{\mu\nu}(A')F^{\mu\nu}(A')\right]. \tag{5}$$

To find minimum of the action with the Lagrangian (5), we express it in terms of the conformal invariant variables (marked by (c))

$$g_{\mu\nu} = a^2 g_{\mu\nu}^{(c)} \; ; A_\mu = A_\mu^c; \;\; |\phi| = \frac{\phi_c}{a}, \tag{6}$$

extracting the space-scale factor [6]

$$a = [^{(3)}g]^{1/6}. \tag{7}$$

The space components of the new metric $g_{(c)}$ satisfy the constraint $\sqrt{^{(3)}g^{(c)}} = 1$ by the definitions (6) and (7). Lagrangian (5) has the symmetric form with respect to a and ϕ_c with $\sqrt{-g^{(c)}} = N_c$:

$$L_o^G = N_c \left[-\frac{^{(4)}R(g^{(c)})}{6} \left(\frac{3M_{Pl}^2}{8\pi}a^2 - \phi_c^2 \right) + \partial_\mu \phi_c \partial^\mu \phi_c - \frac{3M_{Pl}^2}{8\pi}\partial_\mu a \partial^\mu a - \frac{1}{4}F_{\mu\nu}(A')F^{\mu\nu}(A') \right]. \tag{8}$$

The supposition about the homogeneous distribution of the field A' can lead to homogeneity of both the scale factor a and scalar field ϕ_c as the motion equation for scalar field repeats the one for the scale factor.

To get the homogeneous sector of the theory (8), it is sufficient to use the isotropic version of the FRW metric

$$(ds)^2 = a^2(x,t)[N_c^2(x,t)dt^2 - dx_i^2], \tag{9}$$

$$a(x,t) = a_0(t)S(x) ; \quad N_c(x,t) = \frac{N_c^0(t)}{S(x)} ; \quad S = (1 - \frac{k|x|^2}{4r_0^2})^{-1}; \quad (k = +1;0;-1). \tag{10}$$

In order to calculate the homogeneous scalar field ϕ_c we restrict ourselves to the case of constant positive space curvature (1) $k = +1$ with a volume $V_{(3)} = \int d^3 x S^3(x) = 2\pi^2 r_0^3$ and consider harmonic excitations of the vector field A', in this space, with harmonics ω_l described by the action

$$W_R^H(A) = \int dt d^3 x N_c \left[-\frac{1}{4}F_{\mu\nu}(A')F^{\mu\nu}(A') \right] = \frac{V_{(3)}}{2}\int_0^{t_1} dt \sum_l \left[\frac{\dot{A}_l^2}{N_c^0} - N_c^0 \omega_l^2 A_l^2 \right] \quad (\dot{A} = \frac{dA}{dt}). \tag{11}$$

The substitution of (10) and (11) into the Lagrangian (8) leads to the action

$$W^H = V_{(3)} \int_0^{t_1} dt \left[-\frac{3M_{Pl}^2}{8\pi}\left(\frac{\dot{a}_0^2}{N_c^0} - N_c^0 \frac{a_0^2}{r_0^2} \right) + \left(\frac{\dot{\phi}_c^2}{N_c^0} - N_c^0 \frac{\phi_c^2}{r_0^2} \right) \right] + W_R^H. \tag{12}$$

This action describes a set of oscillators evaluated in respect to the invariant conformal time $d\eta = N_c^0 dt$ (see Appendix) with conserved energy densities

$$\rho_{cr} = \frac{3M_{Pl}^2}{8\pi}\left(\frac{\dot{a}_0^2}{N_c^{02}} + \frac{a_0^2}{r_0^2} \right); \quad \rho_\phi^0 = \left(\frac{\dot{\phi}_c^2}{N_c^{02}} + \frac{\phi_c^2}{r_0^2} \right); \quad \rho_R = \sum_l \left(\frac{\dot{A}_l^2}{N_c^{02}} + \omega_l^2 A_l^2 \right). \tag{13}$$

connected by the constraint (Einstein - Friedmann equation $\delta W^H / \delta N_c^0 = 0$)

$$- \rho_{cr} + \rho_\phi^0 + \rho_R = 0. \tag{14}$$

The geometrical observables of the Friedmann Universe, in comoving frame of reference, are constructed by using the inverse conformal transformation (7) of the dynamical variables and coordinates, including the Friedmann time interval $dt_F = a_0 d\eta$ and distance $D_F = a_0 D_c$ [6, 7]. The evolution of the cosmic scale $a(t_F)$, in the considered case, coincides with the one of the Friedmann Universe filled by radiation

$$t_F^{(\eta)} = r_0 \Omega_0^{-\frac{1}{2}}(1 - \cos\frac{\eta}{r_0}) ; \quad \Omega_0 = \frac{3M_{Pl}^2}{8\pi r_0^2 \rho_{cr}} \tag{15}$$

$$a(\eta) = \Omega_0^{-\frac{1}{2}}\sin\frac{\eta}{r_0} = \left[\frac{t_F}{r_0}(2\Omega_0^{\frac{1}{2}} - \frac{t_F}{r_0}) \right]^{1/2} \tag{16}$$

$$\eta = r_0 \, arccos[1 - \Omega_0^{\frac{1}{2}}t_F/r_0]. \tag{17}$$

with the Hubble constant

$$H_0 = \frac{da}{adt_F} \equiv \frac{d o_c}{o_c dt_F} = \frac{1}{t_F}\left[\frac{1 - (t_F/r_0)\Omega_0^{1/2}}{2 - (t_F/r_0)\Omega_0^{1/2}} \right]\Bigg|_{r_0 \to \infty} \simeq \frac{1}{2t_F}. \tag{18}$$

The scalar field o_c repeats this evolution (16)

$$o_c(t_F) = M_{Pl}\left(\frac{3\rho_o^0}{8\pi\rho_{cr}} \right)^{\frac{1}{2}} a(t_F). \tag{19}$$

While the initial scalar field $|\phi|$ defined by eq.(7) is equal to a constant

$$|\phi| = \frac{o_c}{a} = M_{Pl}\left(\frac{3\rho_o^0}{8\pi\rho_{cr}} \right)^{\frac{1}{2}}. \tag{20}$$

The value of this scalar-field, which follows from the Weinberg-Salam theory $|o| \sim 10^2 \text{GeV}$. allows us to estimate the value of the relation of energy densities of the scalar field (ρ_o^0) and the expansion of the Universe (ρ_{cr}):

$$\rho_\phi^{Cosmic} = 10^{-34}\rho_{cr} \tag{21}$$

Recall, that the Higgs potential leads to the opposite situation (see [1])

$$\rho_\phi^{Higgs} = 10^{54}\rho_{cr}. \tag{22}$$

The homogeneity of the scalar field (as the consequence of the homogeneous distribution of matter) excludes monopoles [2] and domain walls [3].

3. Weinberg-Salam model cosmology without Higgs potential

We consider the Einstein theory supplemented by the conformal invariant part of the Weinberg-Salam theory

$$W_{tot} = \int dt dx^3 \sqrt{-g}\left[-\frac{^{(4)}R(g)}{6}(\frac{3M_{Pl}^2}{8\pi} - |\phi|^2) + \partial_\mu |\phi| \partial^\mu |\phi| \right] + W_{WS}^c(\{^{(n)}F\}). \tag{23}$$

where W_{WS}^c depends on the entirety of fields $\{^{(n)}F\}$ with conformal weights (n): gravitational (g) $n_g = 2$; , vector (A, B) $n_{A,B} = 0$; spinor (e,ν) $n_{\nu,e} = -\frac{3}{2}$; the doublet of the scalar fields (ϕ) $n_\phi = -1$ with a module $|\phi|$

$$\phi = \begin{vmatrix} \phi_1 \\ \phi_2 \end{vmatrix} = |\phi|\begin{pmatrix} n_1 \\ n_2 \end{pmatrix} ; \quad n_1 \bar{n}_1 + n_2 \bar{n}_2 = 1.$$

the angular components of which (n_1, n_2) are absorbed by the vector field (W, Z):

$$W_{WS}^c = \int d^4x \sqrt{-g} \left[|\phi|^2 \left(W_\mu W_\nu \frac{g^2}{2} + Z_\mu Z_\nu \frac{g^2+g_1^2}{2} \right) g^{\mu\nu} - |\phi| \frac{G_e}{2} \bar{e}e \right.$$

$$+ i\nu_e \hat{\partial}_{Fock} \nu_e + i\bar{e} \hat{\partial}_{Fock} e - \frac{1}{2} G_{\mu\nu}^a(A^a) G^{a\mu\nu}(A^a) - \frac{1}{4} F_{\mu\nu}(B) F^{\mu\nu}(B)$$

$$+ \frac{gg_1}{(g^2+g_1^2)^{1/2}} \bar{e}\gamma^\mu e A^\mu - \frac{g}{2\sqrt{2}} [\bar{\nu}_e \gamma^\mu (1+\gamma^5) e W_\mu + \bar{e}\gamma^\mu(1+\gamma^5)\nu W_\mu^*]$$

$$\left. + \frac{(g^2+g_1^2)^{1/2}}{4} [\bar{\nu}_e \gamma^\mu (1+\gamma^5)\nu_e - 2\bar{e}\gamma^\mu \left(\gamma^5 + \frac{g^2-3g_1^2}{g^2+g_1^2} \right) e] Z_\mu \right], \qquad (24)$$

$$Z_\mu = (g^3+g_1^2)^{-1/2}(-gA_\mu^3 + g_1 B_\mu) \; ; \quad A_\mu = (g^2+g_1^2)^{-1/2}(g_1 A_\mu^3 + g B_\mu) \qquad (25)$$

$$W_\mu(\pm) = \frac{A_\mu^1 \pm A_\mu^2}{\sqrt{2}}; \qquad (26)$$

where $(\hat{\partial}_{Fock})$ is the Fock covariant derivation in metric $g_{\mu\nu}$, $G_{\mu\nu}^a$ is the Yang-Mills tension for A_μ^a, $F_{\mu\nu}$ is $U(1)$-tension for B_μ.

As we have seen in the previous section, the central point in the derivation of the dynamics of the cosmological model from field theory is the relation between the geometric observables in the Einstein theory (23)$\{^{(n)}F\}$ and the dynamical observables $\{^{(n)}F_c\}$ of the Lagrangian (or Hamiltonian) approach to the cosmological model of the expanding Universe

$$^{(n)}F = {}^{(n)}F_c a^{+n}, \qquad (27)$$

for which the integrals of motion are found [6, 7]. This relation can be formulated in the form of a principle about the conformal invariance of dynamical variables $\{^{(n)}F_c\}$. In the same way the dust mass in the Friedman Universe M_D appears from the electrodynamical action

$$M_D = m \int d^3x \sqrt{g^{(3)}} \bar{\psi}\psi = m \int d^3x a^3 \bar{\psi}\psi = m < n_f > V_{(3)}, \qquad (28)$$

as an integral of motion ($\dot{M}_D = 0$), if $\psi = a^{-3/2}\psi_c$.

Due to the conformal invariance the action W_{WS}^c does not depend on the scale variable (a).

Let us calculate the homogeneous scalar field in this theory (23) in the supposition about the homogeneous distribution of all matter fields in the Universe. The action W_{WS}^c does not change the evolution of the cosmic scale factor (16) and can lead only to the additional terms in the energy density of the scalar field of the type of (28),

$$\rho_\phi = \rho_\phi^0 - \phi_c < n_f > + \phi_c^2 < n_b^2 >, \qquad (29)$$

associated with the fermion and boson "dusts" at rest, the masses of which are formed by the homogeneous scalar field itself. Here $\rho_\phi, < n_f >, < n_b^2 >$ are phenomenological parameters which determine the solution to the homogeneous scalar field equation. For the case considered we have

$$\phi_c(\eta) = \rho_\phi^{1/2} \omega_\phi^{-1} \sin\omega_\phi\eta + \frac{1}{2} < n_f > \omega_\phi^{-2}(1 - \cos\omega_\phi\eta), \qquad (30)$$

where η is defined by eq.(17), $\omega_\phi^2 = 1/r_0^2 + < n_b^2 >$.

If the dust term dominates and $\omega_\phi \neq 1/r_0$, the WS-particle masses (ϕ_c/a) become dependent on time. A photon radiated by an atom on an astronomical object (with a distance D to the

Earth) at the time $t_F - D$ remembers the value of this mass at this time. As the result, the red shift and the Hubble law, in the comoving frame of reference, is defined by the product of two factors: the expansion of the Universe space (a) and the alteration of the elementary particle masses (ϕ_c/a)

$$a \left(\frac{\phi_c}{a} \right) = \phi_c. \qquad (31)$$

Finally, for the unified theory version of the homogeneous Universe we got the red shift Z and the Hubble law

$$Z(D) = \frac{\phi_c(t_F)}{\phi_c(t_F - D)} - 1 \; ; \quad H_0 = \frac{d\phi_c}{\phi_c dt_F}. \qquad (32)$$

If $< n_b^2 >= 0$ and dust dominates, $H_0 = t_F^{-1}$. If $< n_b^2 >\gg 1/r_0^2$ we got the oscillator-like behavior of the red shift which can immitate the large scale structure of the Universe [8].

4. Conclusion

We tried to describe a homogeneous scalar field in the Weinberg-Salam theory, unified with Einstein gravity, starting from the assumption that the only source of conformal invariance breaking (on a classical level) is the Einstein gravity. This means that the investigated version of unified theory does not contain the Higgs potential and is based on the conformal invariant scalar field - gravity coupling. In such a theory the homogeneous scalar field is calculated from the Friedmann equations for the homogeneous Universe. As a result, one of the versions of the physical realizations of Mach's principle appears, namely that the mass of elementary particle forming the matter are determined by the distribution of this matter in the Universe.

Some new consequences of the investigated version of the unified theory are a somewhat different Hubble law for the dust Universe and the cosmological evolution of elementary particle masses in the comoving reference frame. Such a mass evolution can lead to a gradual (in a cosmological scale) decrease of the relative distance between the gravitating objects. The principle of conformal invariant interaction of matter fields with the Einstein gravity and the conventional Friedmann assumption of homogeneous matter distribution lead to very small energy density of scalar field and exclude the monopole creation and domain walls.

Appendix

The actions of homogeneous models considered here can be represented in the Hamiltonian form

$$W^H(p_a, a, F) = \int_0^{t_1} dt \left[-P_a \frac{da}{dt} + \sum_F P_F \frac{dF}{dt} - N_c^0(-H_a + H_F) \right] \qquad (A.1)$$

where

$$H_a = \frac{P_a^2}{V_{(P)}} + \frac{a_0^2}{r_0^2} V_{(P)}; \quad V_{(P)} = V_{(3)} \frac{3M_{Pl}^2}{8\pi}, \qquad (A.2)$$

H_F depends only on a set of the field harmonic excitations in the FRW metric space, and it does not depend on the cosmic scale a.

The reduction of the constrained (A.1) to an equivalent unconstrained one can be fulfilled by the canonical transformation to the new cosmic variables [6, 7, 9]:

$$(P_a, a) \Longrightarrow (\Pi, \eta) ; \quad \{P_a, a\}_{(\Pi, \eta)} = 1; \quad \{P_a, a\}_{(\Pi, \eta)} = 1, \qquad (A.3)$$

so that the cosmic part of the constraint $-H_a + H_F = 0$ converts into a new momentum Π: $H_a = \Pi$. This equation represents a map of the circle (A.2), in the old phase space, into a line, in the new phase space. There are two maps of this type

$$P_a = \pm\sqrt{V_{(P)}\Pi} \cos\frac{\eta}{r_0} ; \quad a_0 = \pm\sqrt{V_{(P)}^{-1}\Pi} \sin\frac{\eta}{r_0}; \qquad (A.4)$$

Thus, we got two actions instead of (A.1):

$$W_\pm^H = \int_0^{t_1} dt \left[\pm\Pi\frac{d\eta}{dt} + \sum_F P_F \frac{dF}{dt} - N_c^0(-\Pi + H_F)) \right]. \qquad (A.5)$$

In this version of the theory the equations of motion for Π

$$\frac{\delta W_\pm}{\delta \Pi} = 0 \implies \pm d\eta = N_c^0 dt \qquad (A.6)$$

determine the invariant parameter of the dynamical evolution of the variables in the theory in the sector of the Dirac observables F and the equation for N_c^0 gives the Hamiltonian for such an evolution.

$$\frac{\delta W_\pm}{\delta N_c} = 0 \implies \Pi = H_F \qquad (A.7)$$

As a result we receive the equivalent (A.5) dynamical system without constraints

$$W_\pm^{Red} = \int_0^{\eta(t)} d\eta \left[\sum_F P_F \frac{dF}{d\eta} \mp H_F \right]. \qquad (A.8)$$

In this approach, we can see that the new cosmic variable becomes invariant time of evolution in the sector of the Dirac observables.

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О космологическом происхождении однородного скалярного поля в единых теориях

Мы рассматриваем возможность описания эффекта Хиггса в единых теориях без потенциала Хиггса в присутствии эйнштейновской гравитации с конформно-скалярной связью в предположении однородного распределения материи.

Значение скалярного поля может быть найдено из фридмановских уравнений для однородной Вселенной. Рассмотренный космологический механизм решает проблему вакуумной плотности (мы получили $\rho_\Phi^{Cosmic} = 10^{-34} \rho_{cr}$ вместо $\rho_\Phi^{Higgs} = 10^{54} \rho_{cr}$) и исключает образование монополей и доменных стенок.

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On the Cosmological Origin of the Homogeneous Scalar Field in Unified Theories

We consider the possibility of describing the Higgs effect in unified theories without the Higgs potential in the presence of the Einstein gravity with the conformal gravity-scalar coupling inder the assumption of homogeneous matter distribution.

The scalar field values can be found from the Friedmann equations for the homogeneous Universe. The considered cosmological mechanism solves the vacuum density problem (we got $\rho_\Phi^{Cosmic} = 10^{-34} \rho_{cr}$ instead of $\rho_\Phi^{Higgs} = 10^{54} \rho_{cr}$), and excludes the monopole creation and the domain walls.