A.V.Smirnov[1], Yu.N.Uzikov[2]

DEUTERON DISINTEGRATION BY PROTONS IN KINEMATICS OF THE QUASI-ELASTIC BACKWARD pd-SCATTERING

[1]P.N.Lebedev Physical Institute RAS, Moscow, Russia;
E-mail address: asmirnov@sci.lpi.msk.su
[2]E-mail address: uzikov@nusun.jinr.dubna.su

1997

1 Introduction

The main goal of experiments on deuteron break-up by electrons [1] or protons [2] at high transferred momenta consists in getting information about nucleon interaction at small distances or, in other words, high-momentum components of the NN-wave functions (w.f's). From this point of view the pole one-nucleon-exchange (ONE) mechanism (Fig. 1) seems mostly interesting, since in this case the break-up cross section is proportional to the product of the modules squared of the deuteron w.f., $|\Psi_d(\mathbf{q})|^2$, where $\mathbf{q}$ is the relative pn-momentum, and the half-off-shell NN-scattering amplitude, $|T_{NN}(\mathbf{q}',\mathbf{k})|^2$ ($\mathbf{q}'$ and $\mathbf{k}$ are the corresponding relative momenta in the initial and final NN-states, respectively). For exact definition of $\mathbf{q}$, $\mathbf{q}'$ and $\mathbf{k}$ see Eqs. (5)–(7) below.

In previous theoretical approaches to the reactions $p + d \rightarrow p + n + p$ [3] and $d + p \rightarrow p(0°) + X$ (see, for instance, [4], [5] and references therein) in the framework of the same pole mechanism the on-shell amplitude of the pn-interaction has been used for the upper vertex, that corresponds to the so-called impulse approximation. At small values of nucleon-spectator momenta $p_3' < 50$ MeV/c this approximation looks quite justified, since the departure of the amplitude $T_{NN}(\mathbf{q}',\mathbf{k})$ from the energy shell is negligible: $q'^2 - k^2 \approx p_3'^2 + m|\varepsilon| \ll q'^2$, k^2 (ε is the deuteron binding energy and m denotes the nucleon mass). However, when the momentum p_3' is large, validity of the impulse approximation becomes doubtful and requires special consideration. The influence of the off-energy-shell effects on the observables for the reaction of deuteron disintegration by protons $p + d \rightarrow N + (NN)$ in the region of incident proton kinetic energies in the laboratory frame $T_p = 0.5$–2 GeV was investigated for the first time in Ref. [6]. Kinematical conditions of this reaction were chosen in Ref. [6] to be similar to those for quasielastic backward pd-scattering, namely, the final nucleon pair (NN) moved forward with small relative energy of nucleons, $E_{NN} \leq 3$ MeV, while the nucleon-spectator flew in the back half-sphere with almost the same momentum, as for the initial proton in the $p + d$ center-of-mass frame (CMF). In Ref. [6] it was found, that if the final NN-pair is in the singlet $^1S_0$ state (this takes place for the reactions $p + d \rightarrow n + (pp)$ or $n + d \rightarrow p + (nn)$ at E_{pp} and E_{nn} of the order of a few MeV), the contribution of Δ-isobar mechanism is considerably suppressed by isotopic factors, whereas the ONE-mechanism dominates. It was established also, that the off-energy-shell effects were of great importance in the kinematics considered. In particular, the differential cross section of this reaction, calculated in the framework of the ONE mechanism, must possess by distinct minimum at the incident proton energy $T_p \sim 0.7$ GeV. The occurence of such a minimum is tightly connected with the node of the half-off-shell $^1S_0$-partial amplitude of NN-scattering, $t(q',k)$ ($q'^2/m \neq k^2/m = E_{NN}$), at the point $q' \approx 0.4$ GeV/c. This node, in its turn, arises due to repulsive core in the $^1S_0$-potential of NN-interaction, like the node in the S-wave component of the deuteron w.f. in the momentum space is caused by analogous core in the
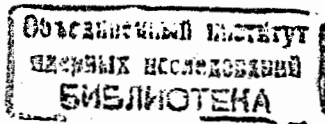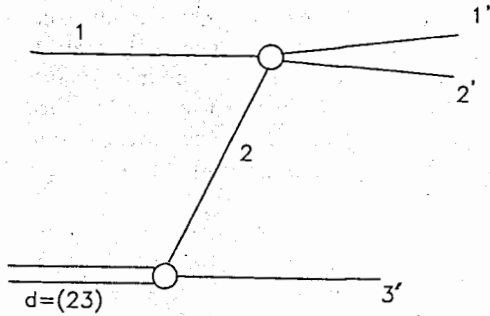
Figure 1: One-nucleon-exchange (ONE) mechanism for the reaction $p + d \rightarrow N + (NN)$.

$^3S_1$–$^3D_1$ state. So, we may conclude that the upper vertex of the pole diagram of Fig. 1 is not a bit less interesting, than its lower $d \rightarrow p + n$ vertex, since both of them directly relate to the properties of the NN-interaction potential at short interparticle distances.

In this paper the approach of Ref. [6] is extended for including higher partial waves in the amplitude of pN-scattering within the ONE mechanism. This allows to study the dependence of the deuteron break-up cross section on the final NN-pair relative energy in the range $E_{NN} = 0$–20 MeV, corresponding to conditions of the planned experiment at COSY [7]. Besides that, we investigate the relations between differential cross sections of the elastic backward pd-scattering and triplet pn-pair formation $p + d \rightarrow p + (pn)_t$ on the basis of theorems concerning analytic continuation of pn-scattering amplitude [8] and pn-scattering w.f. [9] to the bound state pole.

2 Differential cross section of deuteron disintegration in the framework of the ONE mechanism

We start from the general formula for the invariant cross section of a process

$$p(\mathbf{p}_1, \sigma_1) + d(\mathbf{P}_d, \lambda_d) \rightarrow N(\mathbf{p}'_1, \sigma'_1) + N(\mathbf{p}'_2, \sigma'_2) + N(\mathbf{p}'_3, \sigma'_3) \qquad (1)$$

(in round brackets the three-momenta and polarizations of the particles are indicated):

$$d\sigma = (2\pi)^4 \delta^{(4)}(P_i - P_f) \frac{1}{4I_0} \overline{|A_{fi}|^2} \frac{d^3 p'_1}{2E'_1 (2\pi)^3} \frac{d^3 p'_2}{2E'_2 (2\pi)^3} \frac{d^3 p'_3}{2E'_3 (2\pi)^3}, \qquad (2)$$

where $\overline{|A_{fi}|^2}$ is the spin-averaged amplitude squared of the reaction given, P_i, P_f are the total four-momenta of the initial and final systems of particles, respectively, $I_0 = \sqrt{(E_1 E_d - \mathbf{p}_1 \mathbf{P}_d)^2 - m^2 M_d^2}$, M_d is the deuteron mass, $E_i = \sqrt{\mathbf{p}_i^2 + m^2}$, $E'_i = \sqrt{\mathbf{p}_i'^2 + m^2}$, $E_d = \sqrt{\mathbf{P}_d^2 + M_d^2}$. Performing the integration over the three-momentum $\mathbf{p}'_2$ and over the energy E'_3, we get:

$$E'_1 \frac{d\sigma}{d^3 p'_1 \, d\Omega'_3} = \frac{1}{(4\pi)^5} \frac{p'^3_3 \overline{|A_{fi}|^2}}{I_0 \left| E'_2 p'^2_3 - (\mathbf{p}'_2 \mathbf{p}'_3) E'_3 \right|}. \qquad (3)$$

The amplitude A_{fi} is calculated on the basis of concrete assumptions about dynamics of the process. ONE is the simplest possible mechanism of the deuteron break-up reaction. However, as has been mentioned above, it contains a nontrivial physical information about NN-interaction and, therefore, must be taken into account as a first approximation.

A correct calculation of the amplitude of the ONE diagram (Fig. 1) requires incorporation of relativistic effects. Unfortunately, at present the problem of describing reativistic processes with hadronic compound systems is not completely solved yet. Different approaches give different approximate solutions coinciding only in the nonrelativistic limit. For instance, a widely spread method based on the noncovariant light-front dynamics or equivalent to it dynamics in the infinite momentum frame (see Ref. [4] and references therein) suffers from explicit violation of the rotational invariance, that becomes a definite flaw at intermediate energies. A more general approach — the covariant light-front dynamics [10] — turns out to be very effective for solving the problems of electromagnetic interactions with relativistic bound systems, including deuteron [11]. However, its practical application to the processes of hadron-deuteron interactions still encounters some difficulties connected with dependence of approximately calculated amplitudes on the position of the light front surface. Using the covariant Feynman technique requires extremely complicated calculations of the deuteron Bethe–Salpeter function

and the half-off-mass-shell NN-amplitude. Although such an algorithm has been partially realized in Ref. [12] for the inclusive reaction $p + d \to p(0°) + X$, the simplifying approximations made in that paper (neglect of the spin structure and off-shell effects in the NN-scattering amplitude) are evidently not applicable for our purposes.

Keeping all these in mind, we apply here for constructing the amplitudes $p + d \to N + N + N$ and $p + d \to p + d$ the same method, as was used in Ref. [6], namely, the relativistic quantum mechanics of three-body system [13], based on the construction of the full set of the Póincare group generators. In this approach the equation for two-body mass operator eigenfunctions and eigenvalues coincides in form with the Schrödinger equation [14]. In spite of restriction by the sector with fixed number of particles (that is, in fact, a minimal relativisation only), such an approach allows to incorporate in a self-consistent way a rather rich nonrelativistic phenomenology of NN-interactions.

In the framework of the formalism [13] the expression for the ONE-amplitude of the process (1) has the simplest form in the CMF of the system $p + d$ [15]

$$A_{fi} = \frac{\sqrt{E_d(E_2 + E_3')\,\varepsilon_p(q)}}{E_2} \sum_{\sigma_2} \Psi^{\lambda_d}_{\sigma_3'\sigma_2}(\mathbf{q}) T_{NN\,\sigma_1\sigma_2}^{\sigma_1'\sigma_2'}(\mathbf{q}', \mathbf{k}). \tag{4}$$

Here $\varepsilon_p(q) = \sqrt{\mathbf{q}^2 + m^2}$ and the relativistic relative momenta $\mathbf{q}$, $\mathbf{q}'$ and $\mathbf{k}$ are defined as

$$\mathbf{q} = L^{-1}\left(\frac{m(\mathbf{p}_2 + \mathbf{p}_3')}{\sqrt{(p_2 + p_3')_\mu^2}}\right) \mathbf{p}_3', \tag{5}$$

$$\mathbf{q}' = L^{-1}\left(\frac{m(\mathbf{p}_1 + \mathbf{p}_2)}{\sqrt{(p_1 + p_2)_\mu^2}}\right) \mathbf{p}_1, \tag{6}$$

$$\mathbf{k} = L^{-1}\left(\frac{m(\mathbf{p}_1' + \mathbf{p}_2')}{\sqrt{(p_1' + p_2')_\mu^2}}\right) \mathbf{p}_1', \tag{7}$$

where $\mathbf{p}_1$, $\mathbf{P}_d$, $\mathbf{p}_1'$, $\mathbf{p}_2'$, $\mathbf{p}_3'$ are taken in the CMF of $p + d$, where, according to Ref. [13], the conservation of 3-momenta in the vertices $d \to 2 + 3'$ and $1 + 2 \to 1' + 2'$ takes place. By this reason we use in Eqs. (5), (6) the relations $\mathbf{p}_2 = \mathbf{P}_d - \mathbf{p}_3'$ and $(p_2)_0 \equiv E_2 = \sqrt{\mathbf{p}_2^2 + m^2}$. $(p_i)_\mu$ is the four-momentum of the particle i and $L^{-1}(\mathbf{Q})$ is the Lorentz boost acting on some four-momentum $a_\mu = (a_0, \mathbf{a})$, $a_\mu^2 = m^2$ as follows:

$$L^{-1}(\mathbf{Q})\mathbf{a} = \mathbf{a} - \frac{\mathbf{Q}}{m}\left[a_0 - \frac{\mathbf{a}\mathbf{Q}}{m + \sqrt{\mathbf{Q}^2 + m^2}}\right]. \tag{8}$$

By its physical meaning, the relative momentum of a pair of nucleons is nothing else, than the momentum of one of the nucleons in their common CMF. The momenta (5) and (6) are completely identical to the corresponding momenta $\mathbf{q}$ and

$\mathbf{q}'$ defined by the formulas (17), (18) in Ref. [6]. In the nonrelativistic limit we get $\mathbf{q} = (\mathbf{p}_3' - \mathbf{p}_2)/2$, $\mathbf{q}' = (\mathbf{p}_1 - \mathbf{p}_2)/2$, $\mathbf{k} = (\mathbf{p}_1' - \mathbf{p}_2')/2$, and the kinematical factor $\sqrt{E_d(E_2 + E_3')\,\varepsilon_p(q)}/E_2$ in Eq. (4) turns to $2\sqrt{m}$, as it ought to be.

The half-off-shell NN-scattering amplitude $T_{NN\,\sigma_1\sigma_2}^{\sigma_1'\sigma_2'}(\mathbf{q}', \mathbf{k})$ in Eq. (4) satisfies the Lippman–Schwinger equation [1]

$$T_{NN\,\sigma_1\sigma_2}^{\sigma_1'\sigma_2'}(\mathbf{q}', \mathbf{k}) = -4m^2 U_{\sigma_1\sigma_2}^{\sigma_1'\sigma_2'}(\mathbf{q}' - \mathbf{k})$$

$$- m \sum_{\rho_1\rho_2} \int \frac{d^3p}{(2\pi)^3} \frac{U_{\rho_1\rho_2}^{\sigma_1'\sigma_2'}(\mathbf{q}' - \mathbf{p}) T_{NN\,\sigma_1\sigma_2}^{\rho_1\rho_2}(\mathbf{p}, \mathbf{k})}{p^2 - k^2 - i0}, \tag{9}$$

where U is the Fourier transform of the interaction potential. Being taken on the energy shell $q' = k$, the amplitude T_{NN} relates to the CMF differential cross section of unpolarized nucleons as

$$\frac{d\sigma_{12}}{d\Omega} = \frac{1}{64\pi^2 s_{12}} \frac{1}{4} \sum_{\substack{\sigma_1'\sigma_2' \\ \sigma_1\sigma_2}} \left| T_{NN\,\sigma_1\sigma_2}^{\sigma_1'\sigma_2'}(\mathbf{q}', \mathbf{k}) \right|^2. \tag{10}$$

Here $s_{12} = 4(\mathbf{q}'^2 + m^2) = 4(\mathbf{k}^2 + m^2)$. The factor $1/4$ in Eq. (10) appeared because of averaging over initial particle polarizations.

The w.f. $\Psi^{\lambda_d}_{\sigma_3'\sigma_2}(\mathbf{q})$ is the eigenfunction of the mass squared operator of two interacting nucleons, corresponding to the eigenvalue M_d^2. It satisfies the following equation [14]:

$$\left(\frac{q^2}{m} + m - \frac{M_d^2}{4m}\right) \Psi^{\lambda_d}_{\sigma_3'\sigma_2}(\mathbf{q}) + \sum_{\rho_1\rho_2} \int \frac{d^3p}{(2\pi)^3} U_{\rho_1\rho_2}^{\sigma_3'\sigma_2}(\mathbf{q} - \mathbf{p}) \Psi^{\lambda_d}_{\rho_1\rho_2}(\mathbf{p}) = 0 \tag{11}$$

and the normalization condition

$$\sum_{\sigma_2\sigma_3'} \int \frac{d^3q}{(2\pi)^3} \Psi^{\lambda_d'*}_{\sigma_3'\sigma_2}(\mathbf{q}) \Psi^{\lambda_d}_{\sigma_3'\sigma_2}(\mathbf{q}) = \delta_{\lambda_d'\lambda_d}. \tag{12}$$

The quantity $(M_d^2/4m) - m$ in Eq. (11) is very close to the deuteron binding energy $\varepsilon = M_d - 2m$ and, consequently, the w.f. $\Psi^{\lambda_d}_{\sigma_3'\sigma_2}(\mathbf{q})$ practically coincides with the solution of the Schrödinger equation in the momentum space.

For further analysis it is convenient to introduce partial NN-scattering amplitudes $t_{LL'}^{JS}(q', k)$ by means of the following decomposition:

$$T_{NN\,\sigma_1\sigma_2}^{\sigma_1'\sigma_2'}(\mathbf{q}', \mathbf{k}) = 4\pi \sum_{\substack{JM_JLL'S \\ m_Lm_L'm_Sm_S'}} \mathcal{N}_{NN}(L, S)\, C_{Lm_L Sm_S}^{JM_J} C_{L'm_L'Sm_S'}^{JM_J}$$

---

[1] According to Ref. [13] the amplitude T_{NN} coming into the formula (4) is the two-body amplitude of the process $1 + 2 \to 1' + 2'$ in the three-body space. In contrast to the nonrelativistic approach, it does not coincide with the two-body amplitude for the purely two-body problem. The coincidence can be reached, strictly speaking, only in the limit $(p_1 + P_d)_\mu^2 \to \infty$ (see Ref. [15]). We will neglect this circumstance, considering the amplitude T_{NN} as obeying the two-body Lippman-Schwinger equation.

$$\times\, C^{Sm_S}_{\frac{1}{2}\sigma_1 \frac{1}{2}\sigma_2} C^{Sm'_S}_{\frac{1}{2}\sigma'_1 \frac{1}{2}\sigma'_2} Y^*_{Lm_L}(\hat{\mathbf{q}}')Y_{L'm'_L}(\hat{\mathbf{k}})\, t^{JS}_{LL'}(q',k), \tag{13}$$

where $C^{j_1 m_1}_{j_2 m_2\, j_3 m_3}$ are Clebsch–Gordan coefficients and the combinatorial factor

$$\mathcal{N}_{NN}(L,S) = \begin{cases} 1 + (-1)^{L+S}, & \text{if } NN = pp \text{ or } nn, \\ 1, & \text{if } NN = pn \end{cases} \tag{14}$$

is introduced to take into account antisymmetric properties of the amplitude for the case of identical nucleons. Indeed, for pp- or nn-scattering the states with even $L + S$ only are admittable, that follows from the Pauli principle, while for nonidentical nucleons the sum $L + S$ can be arbitrary, and no restrictions on the quantum numbers are imposed.

We represent the deuteron w.f. analogously to Eq. (13):

$$\Psi^{\lambda_d}_{\sigma_3\sigma_2}(\mathbf{q}) = \sum_{\substack{L=0,2 \\ \mu_d\, m_L}} C^{1\lambda_d}_{Lm_L\, 1\mu_d} C^{1\mu_d}_{\frac{1}{2}\sigma_3 \frac{1}{2}\sigma_2} Y_{Lm_L}(\hat{\mathbf{q}})\, u_L(q). \tag{15}$$

Here $u_0(q)$ and $u_2(q)$ are the usual S- and D-waves in deuteron, normalized as

$$\int_0^\infty \frac{dq\, q^2}{(2\pi)^3}\left[u_0^2(q) + u_2^2(q)\right] = 1. \tag{16}$$

Substituting Eqs. (13), (15) into Eq. (4), we get after squaring, averaging over initial and summation over final polarizations

$$\overline{|A_{fi}|^2} = \frac{1}{6}\sum_{\lambda_d\sigma_1\sigma'_1\sigma'_2\sigma_3}|A_{fi}|^2$$

$$= \frac{E_d(E_2 + E'_3)\,\varepsilon_p(q)}{16\pi E_2^2}\left[u_0^2(q) + u_2^2(q)\right] F(\mathbf{q}',\mathbf{k}). \tag{17}$$

The function $F(\mathbf{q}',\mathbf{k})$ can be written in the following form:

$$F(\mathbf{q}',\mathbf{k}) = \sum_{\substack{\sigma'_1\sigma'_2 \\ \sigma_1\sigma_2}}\left|T_{NN}{}^{\sigma'_1\sigma'_2}_{\sigma_1\sigma_2}(\mathbf{q}',\mathbf{k})\right|^2$$

$$= \sum_{JLL'\tilde{J}\tilde{L}\tilde{L}'Sl} \mathcal{N}^2_{NN}(L,S)\, t^{JS}_{LL'}(q',k)\left(t^{\tilde{J}S}_{\tilde{L}\tilde{L}'}(q',k)\right)^* C^{\tilde{L}0}_{L0\,l0} C^{\tilde{L}'0}_{L'0\,l0}(2J+1)(2\tilde{J}+1)$$

$$\times (2l+1)\sqrt{(2L+1)(2L'+1)}\left\{\begin{matrix}\tilde{L} & L & l \\ J & \tilde{J} & S\end{matrix}\right\}\left\{\begin{matrix}\tilde{L}' & L' & l \\ J & \tilde{J} & S\end{matrix}\right\} P_l(\mathbf{q}'\mathbf{k}/q'k), \tag{18}$$

where the figured brackets stand for usual notation of $6j$-symbols. Note that, according to Eq. (10), the function (18) at $q' = k$ describes the differential cross sections of pn- and pp-scattering. As is seen from Eq. (18), the states with different spins S do not interfere in the cross section of the reaction, that is a consequence of

the total angular momentum J and parity conservation. Therefore, it makes sense to separate transitions in singlet ($S = 0$) and triplet ($S = 1$) channels.

The partial amplitudes $t^{JS}_{LL'}(q',k)$ are found by means of numerical solving the Lippman–Schwinger equation (9), which after separation of angular variables reduces to the system of coupled one-dimensional integral equations

$$t^{JS}_{LL'}(q',k) = v^{JS}_{LL'}(q',k) + \frac{2}{(4\pi)^2}\sum_{L''}\int_0^\infty \frac{dp\, p^2}{m}\frac{v^{JS}_{LL''}(q',p)t^{JS}_{L''L'}(p,k)}{p^2 - k^2 - i0}, \tag{19}$$

where

$$v^{JS}_{LL'}(q',k) = -\frac{m^2}{\pi}\sum_{\substack{\sigma_1,\sigma_2,\sigma'_1,\sigma'_2 \\ m_S,m'_S,m_L,m'_L}} \int d\Omega_{\mathbf{q}'}d\Omega_{\mathbf{k}}\, C^{JM_J}_{Lm_L\, Sm_S} C^{JM_J}_{L'm'_L\, Sm'_S} C^{Sm_S}_{\frac{1}{2}\sigma_1 \frac{1}{2}\sigma_2} C^{Sm'_S}_{\frac{1}{2}\sigma'_1 \frac{1}{2}\sigma'_2}$$

$$\times\, U^{\sigma'_1\sigma'_2}_{\sigma_1\sigma_2}(\mathbf{q}' - \mathbf{k})\, Y_{Lm_L}(\hat{\mathbf{q}}')Y^*_{L'm'_L}(\hat{\mathbf{k}}). \tag{20}$$

Concerning the reaction $p + d \to n + p + p$ at small relative energy of the final protons the question about the role of the Coulomb effects in pp-scattering arises. To take the Coulomb interaction into account, it is necessary to replace in Eqs. (9), (20) the function U by the sum $U + U_C$, where U_C is the Fourier transform of the Coulomb potential. However, as was found in Ref. [6], for a rather big departure from the energy shell, $q' \gg k$, the influence of the Coulomb forces becomes negligible. Since this just takes place for our case, we will exclude the Coulomb effects from consideration. Let us emphasize, that on the energy shell $q' = k$ such an approximation would be unacceptable.

If the kinematics is collinear (the scattering angles in the laboratory frame are $\theta'_1 = \theta'_2 = 0°$ for nucleons in the NN-pair, $\theta'_3 = 180°$ for the nucleon-spectator), then $\mathbf{q}' \parallel \mathbf{k}$, and Eq. (18) can be simplified:

$$F(\mathbf{q}',\mathbf{k}) = |A_0|^2 + \sum_{\mu=-1,0,1}|B_\mu|^2 \tag{21}$$

with

$$A_0 = \sum_J \mathcal{N}_{NN}(J,0)\,\xi_J\,(2J+1)t^{J0}_{JJ}(q',k), \tag{22}$$

$$B_\mu = \sum_{JLL'} \mathcal{N}_{NN}(L,1)\,\xi_L\,(2J+1)C^{L0}_{J\mu\, 1-\mu}C^{L'0}_{J\mu\, 1-\mu}\, t^{J1}_{LL'}(q',k) \tag{23}$$

and

$$\xi_l = \begin{cases} 1, & \text{if } (\mathbf{q}'\mathbf{k}/q'k) = 1, \\ (-1)^l, & \text{if } (\mathbf{q}'\mathbf{k}/q'k) = -1. \end{cases} \tag{24}$$

The terms $|A_0|^2$ and $|B_\mu|^2$ in Eq. (21) represent the contributions from singlet ($S = 0$) and triplet ($S = 1$, $m_S = \mu$) states of the NN-pair, respectively. Note that $B_{-1} = B_1$.

Provided the relative energy of nucleons in the pair is small enough, we have a quasi two-body kinematics. However, the state of this pair is characterized by spin projections σ_1' and σ_2' of individual nucleons, rather than by definite common quantum numbers like the total angular momentum J, its projection M_J, total spin S and orbital momentum L (as it will be discussed in the next section.) Then, below in this section we will denote the reaction considered as $p + d \rightarrow$
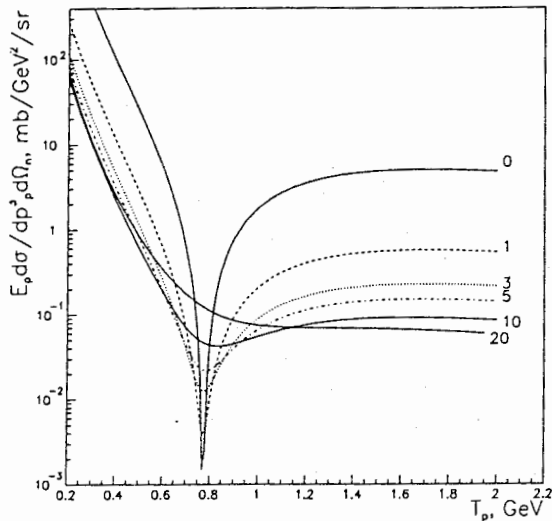

Figure 2: The CMF differential cross section of the reaction $p + d \rightarrow p(0°) + p + n(180°)$ as a function of the initial proton kinetic energy in the laboratory frame, calculated in the framework of the ONE mechanism by Eqs. (3), (17), (21), (22) with S-, P-, D-, F- and G-waves in the state of the forward going pp-pair taken into account. The numbers at the curves denote the energy of relative motion of nucleons in the pair, E_{pp}, in MeV.

$N_1'(0°) + N_2' + N_3'(180°)$ to indicate explicitly, which nucleons are registered in the experiment (in brackets the corresponding scattering angles of registered nucleons in the laboratory frame are pointed out). At the same time, one should remember that the kinematics corresponds to the situation, when the nonregistered nucleon N_2' moves at the angle 0° in the laboratory system, and the relative energy of the forward going pair of nucleons $N_1'N_2'$ is small: $E_{NN} = 0$–20 MeV. As has been noticed above, in the collinear kinematics discussed the angle between the relative momenta q' and k can be either 0°, or 180°. For the case of pp-pair formation the result for the cross section does not depend on which sign of the scalar product

($q'k$) in Eq. (24) we take. Indeed, since $\xi_J = 1$ for $S = 0$ and $\xi_L = -1$ for $S = 1$, under the substitution $(q'k) \rightarrow -(q'k)$ one gets $A_0 \rightarrow A_0$, $B_\mu \rightarrow -B_\mu$, and the function $F(q', k)$ remains unchanged. But the function $F(q', k)$ for pn-pair does not possess by this property. In principle, the cases $(q'k/q'k) = -1$ and $(q'k/q'k) = 1$ can be experimentally separated. Our calculations showed that the replacement of $(q'k/q'k) = -1$ to $(q'k/q'k) = 1$ at such low relative energies of pn-pair as 0–20 MeV results in some quantitative difference in the cross sections obtained, but, nevertheless, does not lead to appearance of any new qualitative features in the cross section behaviour. By this reason, below we consider in detail the case of zero angle between q' and k.

The numerically calculated differential cross sections of the reactions $p + d \rightarrow p(0°) + p + n(180°)$ and $p + d \rightarrow p(0°) + n + p(180°)$ as functions of the initial proton kinetic energy in the laboratory frame T_p ($T_p = 0.2$–2 GeV) are shown in Figs. 2, 3, respectively. The calculations were carried out by the formulas (3), (17), (21)–(24). For finding the deuteron w.f. and NN-scattering amplitudes the Paris potential [16] was used. We found that it was enough to restrict ourselves by taking into account the partial amplitudes $t_{LL'}^{JS}(q', k)$ describing transitions to the S-, P-, D-, F- and G-states of the final nucleon pair (i. e. those with $L' = 0, 1, 2, 3, 4$), the relative contribution of the amplitudes with higher L' being less than 1% in all the range of T_p and E_{NN} considered.
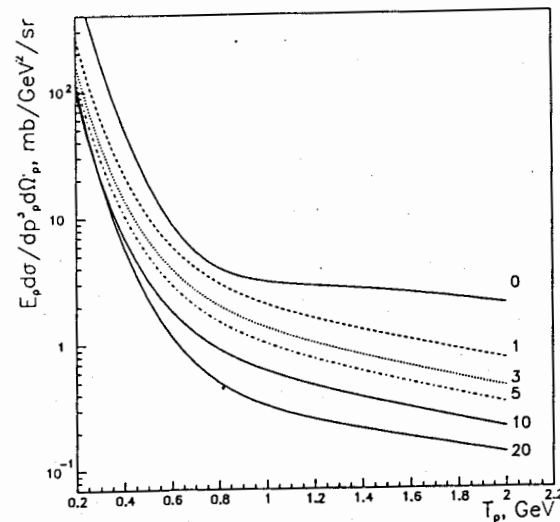

Figure 3: The same as in Fig. 2, but for the reaction $p + d \rightarrow p(0°) + n + p(180°)$.

The first qualitative feature, which draws attention, is a distinct minimum of the cross section for pp-pair formation at $T_p = 0.77$ GeV (see Fig. 2), occuring for relative energies $E_{pp} \sim 0-5$ MeV. This minimum is caused by the node of the half-off-shell amplitude $t_{00}^{00}(q',k)$ corresponding to the $^1S_0 \to {}^1S_0$ transition. In paper [6], where the Reid-soft-core NN-potential was exploited, the similar minimum took place at another energy $T_p \sim 0.7$ GeV, that was connected with the difference between the Reid-soft-core and Paris potentials. At $E_{pp} > 5$ MeV the minimum rapidly fills up due to the contributions from the highest partial waves, and at $E_{pp} > 10$ MeV it practically disappears. For the pn-pair formation the behaviour of the corresponding cross section (Fig. 3) is, on the contrary, monotonous for all values of E_{pn}. The reason for it consists in the following. At small enough E_{NN} the behaviour of the cross sections discussed is determined, in the main, by the NN-amplitudes $t_{L0}^{JS}(q',k)$ describing transitions to the S-states of the final NN-pair. Three such amplitudes exist, namely, $t_{00}^{00}(q',k)$ ($^1S_0 \to {}^1S_0$), $t_{00}^{11}(q',k)$ ($^3S_1 \to {}^3S_1$) and $t_{20}^{11}(q',k)$ ($^3D_1 \to {}^3S_1$). For the case of pp-pair formation the two latter amplitudes are absent (since for them $L+S$ is odd), the only remaining $t_{00}^{00}(q',k)$-amplitude has a node leading to the minimum in the cross section, as has been already pointed out above. For the case of pn-pair there are no any restrictions on quantum numbers, and all the three transitions are admittable. The amplitude $t_{00}^{11}(q',k)$, like $t_{00}^{00}(q',k)$, possesses by a node, while $t_{20}^{11}(q',k)$ does not. This situation is illustrated by Fig. 4, where the modules of the three amplitudes considered are shown as functions of q' at $k = \sqrt{mE_{NN}} = 0$. We see that $|t_{20}^{11}(q',k)|$ has a broad maximum just near the points of the nodes of $|t_{00}^{00}(q',k)|$ and $|t_{00}^{11}(q',k)|$ (the positions of these nodes are very close to each other). Namely the contribution of the "nodeless" amplitude $t_{20}^{11}(q',k)$ leads to disappearance of the minimum in the cross section of pn-pair production.

Both pp- and pn-pair formation cross sections fall sharply with increase of T_p up to the value $T_p \simeq 0.8$ GeV. This is caused by the simultaneous decrease of the deuteron w.f. squared $u_0^2(q) + u_2^2(q)$ and the function $F(\mathbf{q}',\mathbf{k})$ (Eq. (18)) in that region. The nodes of the $t_{00}^{00}(q',k)$- and $t_{00}^{11}(q',k)$-amplitudes lead to a minimum of $F(\mathbf{q}',\mathbf{k})$ at $T_p \sim 0.8$ GeV; with further increase of T_p the function $F(\mathbf{q}',\mathbf{k})$ starts growing. For the case of pp-pair with small relative energy, $E_{pp} < 10$ MeV, the rate of this growth is enough to suppress the decrease of the factor $u_0^2(q) + u_2^2(q)$. As a result, the corresponding cross section in the region 0.8 GeV $< T_p <$ 1.4 GeV increases considerably. It is interesting to note that at higher T_p the pp-pair formation cross section becomes practically a constant. For the pn-pair the growth of the function $F(\mathbf{q}',\mathbf{k})$ is not so fast and can only reduce the slope of the cross sections.

As is seen from Figs. 2, 3, the absolute values of the cross sections of both pp- and pn-pair formation are very sensitive to variations of E_{NN}, especially at E_{NN} close to zero. The latter circumstance is stipulated mainly by the virtual state pole of the $t_{00}^{00}(q',k)$-amplitude at $k^2/m \sim -0.1$ MeV. On the whole, as E_{NN} increases

from 0 to 20 MeV, both pp- (excepting for narrow region near the point of the minimum) and pn-pair formation cross sections reduce almost by two orders of magnitude.
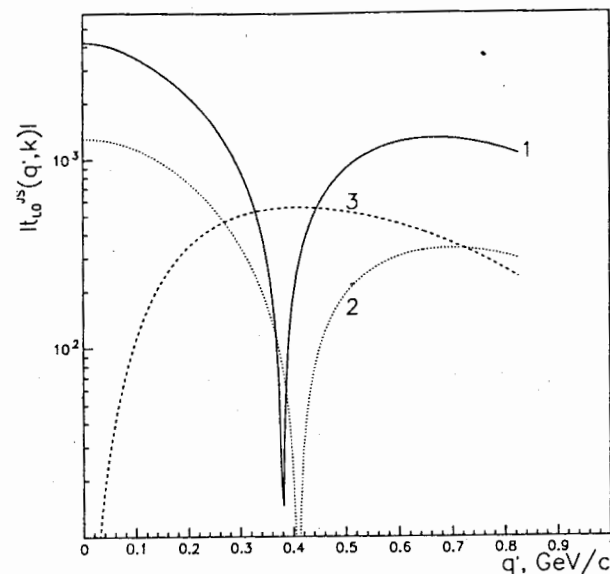

Figure 4: The modules of the half-off-shell partial amplitudes of NN-scattering, $|t_{00}^{00}(q',k)|$ (the solid curve 1), $t_{00}^{11}(q',k)$ (the dotted curve, 2), $|t_{20}^{11}(q',k)|$ (the dashed curve 3) as functions of q' at $k = 0$.

At $E_{NN} \to 0$ the partial amplitude $t_{00}^{00}(q',k)$ becomes dominant (again, unless q' is in the vicinity of the minimum point) due to its nearthreshold pole mentioned above, which is much closer to the NN-threshold, than the pole of the $t_{00}^{11}(q',k)$- and $t_{20}^{11}(q',k)$-amplitudes, occuring at $k^2/m = -2.23$ MeV and corresponding to deuteron. If we retained on the right-hand-side of Eq. (18) the term with the $t_{00}^{00}(q',k)$-amplitude only, then the ratio of the pp- and pn-pair formation cross sections would be exactly equal to $\mathcal{N}_{pp}^2(0,0)/\mathcal{N}_{pn}^2(0,0) = 4$ (note that if the final pair is in the antisymmetric state with quantum numbers J, M_J, L, S then this ratio equals 2 [6]). In Fig. 5 the cross sections for pp- and pn-pair formation (solid lines) are shown together at $E_{pp} = E_{pn} = 0$. As is seen, the cross section for pp-pair production is indeed greater than the one for pn-pair anywhere beyond the region close to the minimum of the former. However, at $T_p = 0.2$-2 GeV their ratio does not exceed 2.77. This indicates, that the contribution of the $^3S_1$-state of the pn-

pair (shown in Fig. 5 by the dotted line) is not negligible under these conditions even for $E_{pn} = 0$. Moreover, it is easy to get from Eqs. (3), (17), (21), (22) that at $E_{pn} = 0$ the pn-pair production cross section is proportional to the sum

$$|t_{00}^{00}(q',k)|^2 + 3\left(|t_{00}^{11}(q',k)|^2 + |t_{20}^{11}(q',k)|^2\right),$$

so the contribution of the triplet state of the pair is additionally amplified by the spin statistical factor of 3.
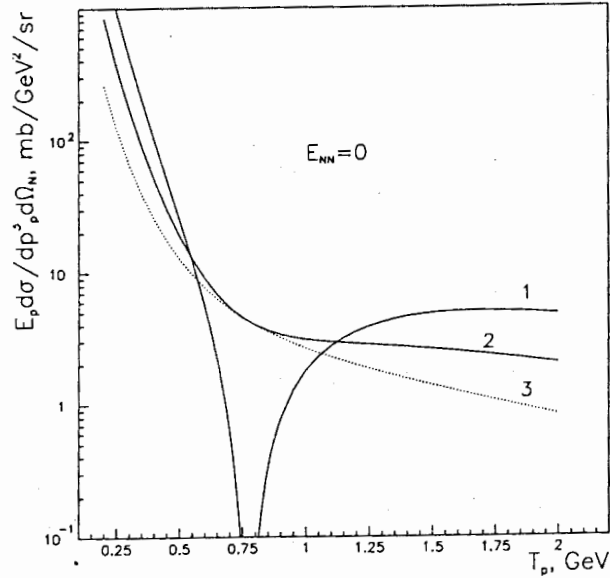

Figure 5: Comparison of contributions of different S-states of the final forward going nucleon pair to the CMF differential cross sections of the reaction $p + d \rightarrow p(0°) + N + N(180°)$ at $E_{pN} = 0$: the contribution of the $^1S_0$-state for pp-pair (the solid curve 1), total contribution of the $^1S_0$- and $^3S_1$-states for pn-pair (the solid curve 2), the single $^3S_1$-state contribution for pn-pair (the dotted curve 3).

To analyse the contribution of the highest partial waves (i. e. those with $L' \neq 0$) in the state of the final NN-pair, we plotted in Fig. 6 the cross sections for pp- and pn-pair formation, calculated by the same formulas (3), (17), (21)–(24) with all the important transition amplitudes $t_{LL'}^{JS}(q',k)$ included (the solid lines 3 and 5) and with transition amplitudes $t_{L0}^{JS}(q',k)$ only kept (the dashed lines 2 and 4). All the curves mentioned correspond to $E_{NN} = 20$ MeV. The influence of the partial

waves with $L' \neq 0$ in the pp-pair state is seen to be very important, especially near the minimum of the dashed curve for pp-pair. For the case of pn-pair the states with $L' \neq 0$ contribute significantly only at rather low energies $T_p \sim 0.2$–0.5 GeV, increasing the cross section in this region by a factor of 1.25–1.5. At higher values of T_p the total contribution of the $^1S_0$- and $^3S_1$-states of the pn-pair practically exhausts the cross section due to destructive interference of the amplitudes with $L' \neq 0$ in Eqs. (22), (23). For the pn-pair case the role of higher partial waves rapidly reduces with decrease of E_{pn}. For instance, already at $E_{pn} = 10$ MeV the relative error of neglect of the pn-pair states with $L' \neq 0$ is less than 5% at any T_p. For pp-pair taking into account the highest partial waves near the minimum point of the $^1S_0$-state contribution remains important even for much smaller values of E_{pp}.
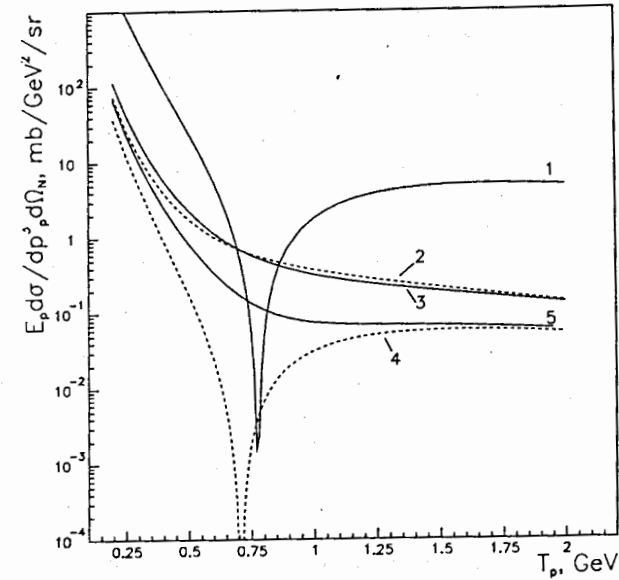

Figure 6: The CMF differential cross sections of the reaction $p + d \rightarrow p(0°) + N + N(180°)$ for pp- and pn-forward going pairs formation. The curves show the results of calculations taking into account all the important partial waves (i. e. S-, P-, D-, F-, G-) in the state of the pair (solid lines) and S-waves only (dashed lines): 1 — for pp-pair at $E_{pp} = 0$; 2 and 3 — for pn-pair at $E_{pn} = 20$ MeV; 4 and 5 — for pp-pair at $E_{pp} = 20$ MeV

One more circumstance is also worth to emphasize. It is known [17] that the

position of the node of the amplitude $t_{00}^{00}(q', k)$ as a function of q' practically does not depend on k in a rather wide interval of $E_{NN} = 0$–100 MeV. However, when E_{NN} varies from от 0 to 20 MeV, the node shifts approximately by 60 MeV towards the region of smaller T_p, as is seen from the comparison of the curves 1 and 4 in Fig. 6, describing the $^1S_0$-state contributions to the pp-pair formation cross sections for $E_{pp} = 0$ and $E_{pp} = 20$ MeV, respectively. This is a purely kinematical phenomenon connected with increase of q' (at fixed T_p), as E_{NN} is growing.

3 Relation between the cross sections of the processes of a triplet $(pn)_t$-pair formation and elastic backward pd-scattering

The kinematics of the reaction $p + d \to p + (pn)$ we deal with is very close to that one of the elastic backward pd-scattering. If the final pn-pair is formed in the triplet state (presumably in $^3S_1$ at low relative energies), then it is possible to get some approximate relations between the cross sections of the processes $p + d \to p + d$ and $p + d \to p + (pn)_t$ at high momentum transfers. One of such relations was obtained rather long ago by Anisovich, Dakhno and Makarov (ADM) [8] from consideration of the analytical properties of the corresponding scattering amplitudes. Recently, Boudard, Fäldt and Wilkin (BFW) derived another relation [9], based on analytical continuation of $^3S_1$-scattering w.f. to the bound state pole. Both ADM and BFW relations were formulated for a single-channel problem (i. e. as if the interaction did not mix channels with different values of orbital momenta). One can show, however, that such a restriction is not necessary, and the similar result holds for a two-channel system as well, e. g. for deuteron (see Appendix and Ref. [18]). By this reason, below we will use the subscript "t" in the process $p + d \to p + (pn)_t$ as a short notation of the $^3S_1$–$^3D_1$-state of the final pn-pair.

The ADM and BFW formulas outwardly look quite different, so, it is interesting to compare the quality of approximation, which can be reached by using them. The importance of these relations may become still greater, if the contribution of the $^3S_1$–$^3D_1$ final state can be experimentally separated anyhow from the total contribution of the singlet and triplet pair formation (see, for instance, Ref. [19]). In the latter case one can make some estimations of the cross section of the process $p + d \to p + (pn)_t$ directly from the data on the well-studied elastic backward pd-scattering.

In an explicit form the ADM and BFW formulas can be written as

$$\frac{d\sigma}{dk^2 d\Omega_f}(p + d \to p + (pn)_t) = \frac{p'_3}{p_1} f(k) \frac{d\sigma}{d\Omega_f}(p + d \to p + d), \qquad (25)$$

where $d\Omega_f$ is the solid angle element of the backward going proton in the CMF of

the system $p + d$. According to Ref. [8], the function $f(k)$ is

$$f_{ADM}(k) = r_t \sqrt{1 - \frac{2r_t}{a_t}} \left[2\pi \left(1 - \sqrt{1 - \frac{2r_t}{a_t}} \right) \right]^{-1} k \left[k^2 + \left(-\frac{1}{a_t} + \frac{1}{2} r_t k^2 \right)^2 \right]^{-1}. \qquad (26)$$

In Eq. (26) $a_t = 5.41$ fm and $r_t = 1.75$ fm are the triplet scattering length and triplet effective radius, respectively. The BFW prescription for $f(k)$ looks much simpler:

$$f_{BFW}(k) = \frac{k}{2\pi(k^2 + \alpha_t^2)\alpha_t} \qquad (27)$$

with $\alpha_t = \sqrt{m|\varepsilon|} = 0.232$ fm^{-1}. It is very important to emphasize that the relations adduced above practically do not depend on reaction mechanism, provided the momentum transfer to the final nucleon pair and to the final deuteron is large enough, but k is, on the contrary, small. However, at $k^2 > 0$ both ADM and BFW relations are approximate. The reason of it consists in that the exact connection between the bound state (deuteron) and pn-scattering w.f.'s holds only at $k^2 = k_b^2 = -m|\varepsilon|$, separated from the real relative momentum of the pn-pair $k^2 = m E_{pn}$ by a finite interval. Our aim here is to check the validity of Eq. (25) in concrete kinematical conditions for the case of the ONE-mechanism.

In order to bring the deuteron break-up cross section into the form of the left-hand-side of Eq. (25), we must, first of all, go over from $d^3 p'_1$ in Eq. (2) to $d^3 k$, the relativistic relative momentum k being defined by Eq. (7). The phase space volume element is

$$d\Phi = \frac{d^3 p'_3}{2E'_3(2\pi)^3} \left(\frac{d^3 p'_1}{2E'_1(2\pi)^3} \frac{d^3 p'_2}{2E'_2(2\pi)^3} \delta^{(4)}(P_i - P_f) \right) \equiv \frac{d^3 p'_3}{2E'_3(2\pi)^3} d\Phi_2. \qquad (28)$$

The two-body phase space volume element $d\Phi_2$ is a relativistic invariant and can be calculated in any convenient reference frame. We will choose for finding $d\Phi_2$ the system, where $\mathbf{p}_1 + \mathbf{P}_d - \mathbf{p}'_3 = 0$. Evidently, this system coincides with the CMF of the final nucleon pair, where $\mathbf{p}'_1 = -\mathbf{p}'_2 = \mathbf{k}$. Removing the δ-function, we get

$$d\Phi_2 = \frac{1}{(2\pi)^6} \frac{k \, d\Omega_{\mathbf{k}}}{8 E_k}, \qquad (29)$$

where $E_k = \sqrt{\mathbf{k}^2 + m^2}$.

Now we find the connection between p'_3 and k. On the one hand,

$$k^2 = \frac{1}{4} \left[(E'_1 + E'_2)^2 - (\mathbf{p}'_1 + \mathbf{p}'_2)^2 \right] - m^2. \qquad (30)$$

Using conservation laws for the energy and 3-momentum, we can rewrite Eq. (30) in the CMF of the system $p + d$ as follows:

$$k^2 = \frac{1}{4} \left[s - 3m^2 - 2\sqrt{s} E'_3 \right]. \qquad (31)$$

14

15

Here $\sqrt{s}$ is the invariant mass of the system $p+d$. »From Eq. (31) we easily obtain

$$dk^2 = \frac{\sqrt{s}}{2} \frac{p_3' dp_3'}{E_3'}. \qquad (32)$$

Using Eqs. (2), (28), (29) and (32), we arrive at the following formula for the cross section[2]:

$$\frac{d\sigma}{dk^2 d\Omega_f} = \frac{p_3'}{p_1} \frac{k}{(4\pi)^5 s E_k} \int d\Omega_{\mathbf{k}} \overline{|A_{fi}|^2}. \qquad (33)$$

We have denoted $d\Omega_3' = d\Omega_f$ for matching with the corresponding solid angle element for the case of elastic pd-scattering. For identical particles in the forward going NN-pair one has to multiply the right-hand-side of Eq. (33) by the factor $\frac{1}{2}$. Now, substituting the expression (17) for $\overline{|A_{fi}|^2}$ into Eq. (33) and applying the partial wave decomposition (18), we can easily perform integration over $d\Omega_{\mathbf{k}}$ due to the orthogonality of Legendre polynomials. The result is

$$\frac{d\sigma}{dk^2 d\Omega_f}(p+d \to N+(NN)) = \frac{p_3'}{p_1} \frac{k}{(4\pi)^5 s E_k} \mathcal{K} \left[u_0^2(q) + u_2^2(q) \right]$$

$$\times \sum_{JSLL'} \mathcal{N}_{NN}(L,S)(2J+1) |t_{LL'}^{JS}(q',k)|^2, \qquad (34)$$

where the kinematical factor $\mathcal{K}$ has the form

$$\mathcal{K} = \frac{E_d (E_2 + E_3') \varepsilon_p(q)}{4 E_2^2}. \qquad (35)$$

Note that the formula (34) can be derived not only by using the amplitude of the reaction (1), but also directly from the amplitude of the process $p+d \to N+(NN)_{JM_J LS}$, where the state of the final NN-pair is determined by the quantum numbers J, M_J, L, S and by the module of the relative momentum k.

It is worth mentioning that for the reaction $p+d \to p(180°) + (pn)$, when only the momentum of the backward going proton is registered, the kinematics is not collinear and admits any angle between the vectors $\mathbf{q'}$ and $\mathbf{k}$. At the same time, for the reaction $p+d \to p(0°)+n+p(180°)$ discussed in the previous section the vector $\mathbf{k}$ can be directed either along, or oppositely to the momentum $\mathbf{q'}$. Therefore, the role of the highest partial waves in the cross sections of these reactions may be different.

Since Eq. (25), strictly speaking, is valid for the triplet $^3S_1$-$^3D_1$-state of the nucleon pair, we should retain in the sum over J, S, L, L' only the terms

---

[2]In Ref. [6] the phase space volume was slightly underestimated by the additional factor $m/(\sqrt{s} - E_3')$ appeared because of using the nonrelativistic expression for the relative momentum in the corresponding formula.
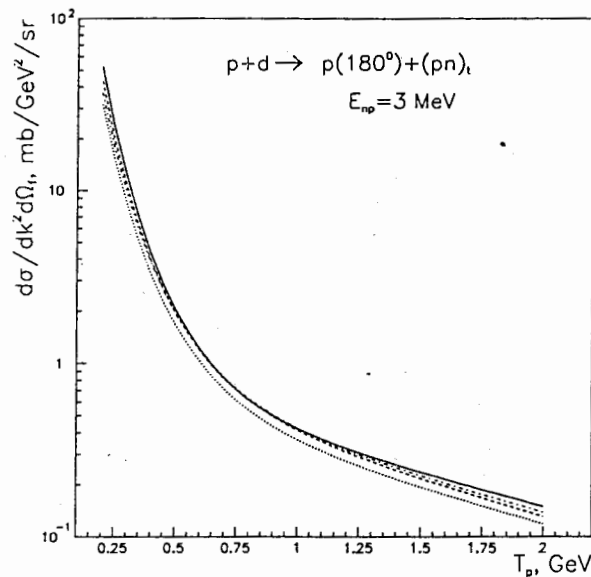
Figure 7: The CMF differential cross section of the quasi two-body reaction $p + d \to p(180°) + (pn)$ as a function of the initial proton kinetic energy in the laboratory frame. The solid line represents the cross section calculated according to Eq. (34) taking into account all the important states of the final pn-pair. The dashed line describes the cross section obtained by Eq. (36) for the triplet final state $^3S_1$-$^3D_1$ only. The dotted and dash-dotted lines show the right-hand-side of Eq. (25), obtained by using the functions $f_{ADM}(k)$ (Eq. (26)) and $f_{BFW}(k)$ (Eq. (27)), respectively. All the curves correspond to the relative energy of nucleons in the pn-pair $E_{pn} = 3$ MeV.

corresponding to $J = 1$, $S = 1$, $L = 0, 2$ and $L' = 0, 2$:

$$\frac{d\sigma}{dk^2 d\Omega_f}(p+d \to p+(pn)_t) = \frac{p_3'}{p_1} \frac{3k}{(4\pi)^5 s E_k} \mathcal{K} \left[u_0^2(q) + u_2^2(q) \right]$$

$$\times \left[|t_{00}^{11}(q',k)|^2 + |t_{20}^{11}(q',k)|^2 + |t_{02}^{11}(q',k)|^2 + |t_{22}^{11}(q',k)|^2 \right]. \qquad (36)$$

The CMF cross section of the elastic pd-scattering for the ONE mechanism is well known in the relativistic quantum mechanics [20] and can be written as

$$\frac{d\sigma}{d\Omega_f}(p+d \to p+d) = \frac{3}{64\pi^4 s} \Pi^2 \left[u_0^2(q) + u_2^2(q) \right] \left[u_0^2(q') + u_2^2(q') \right], \qquad (37)$$

where

$$\Pi = E_d\,(E_p + E_n)\,\varepsilon_p(q)\,\frac{\sqrt{s} - M_0}{4\,E_n}, \qquad (38)$$

$E_d = E_d'$ is the energy of the deuteron, $E_p = E_p'$ is the energy of the proton, E_n is the energy of the intermediate neutron, $M_0 = 2E_p + E_n$. The relativistic relative momenta $\mathbf{q}$ and $\mathbf{q}'$ are determined by the same formulas (5) and (6), respectively, keeping in mind the difference between the invariant mass of the $N_1'N_2'$-pair and the deuteron. Note that the equality $q = q'$ takes place in the CMF of the system $p + d$.
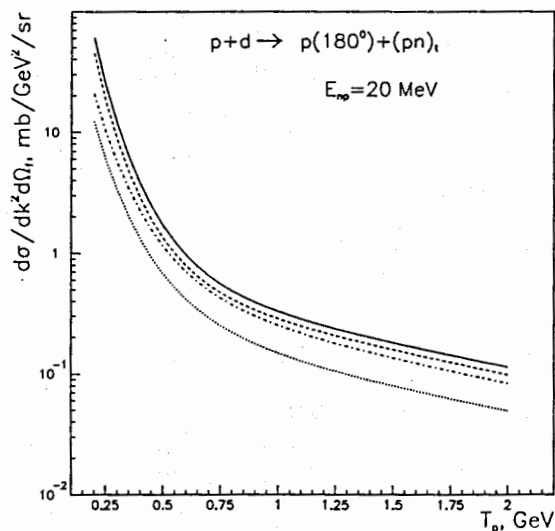

Figure 8: The same as in Fig. 7, but for $E_{pn} = 20$ MeV

The calculated numerically differential cross sections $d\sigma/(dk^2\,d\Omega_f)$ of the reaction $p + d \rightarrow p(180°) + (pn)$ as functions of the initial proton kinetic energy T_p are shown in Figs. 7 and 8 for $E_{pn} = 3$ MeV and $E_{pn} = 20$ MeV, respectively. The solid line corresponds to the cross section found by the formula (34) including all the important partial waves (both singlet and triplet) in the state of the final pn-pair. The dashed line describes the cross section calculated by Eq. (36), for only the $^3S_1$-$^3D_1$-state of the pn-pair taken into account. For comparison we showed in the same figures the calculated right-hand-side of Eq. (25), obtained by means of Eqs. (26), (37), (38) (the dotted line) and Eqs. (27), (37), (38) (the dash-dotted line). »From the figures presented it is seen that Eq. (25) reproduces fairly well the

shape of the cross section in the whole range of initial energies $T_p = 0.2$–2 GeV, regardless to the choice of the function $f(k)$ (ADM or BFW) and to the pn-pair excitation energy E_{pn} in the interval 0–20 MeV. However, the magnitude of the cross section of $(pn)_t$-pair formation (the dashed line) can be obtained from Eq. (25) with an acceptable precision only at rather small excitation energies. Note that the BFW relation holds with better accuracy than the ADM one: at $E_{pn} = 3$ MeV (Fig. 7) and $T_p = 0.5$–2 GeV the relative error of the former is about 5%, while the latter systematically underestimates the cross section by 10–20%. The accuracy of Eq. (25) worsens at lower values of T_p, because in this region one of the applicability conditions — large value of the momentum transfer — violates. When the relative energy E_{pn} increases, the departure from the bound state (deuteron) pole also increases, and the accuracy of Eq. (25) falls off. For instance, at $E_{pn} = 20$ MeV (Fig. 8) and in the same region $T_p = 0.5$–2 GeV the BFW relation underestimates the cross section of a triplet pn-pair formation by $\sim 15\%$, whereas the ADM one — in about two times.

In paper [9] on the basis of the relations (25), (27) (the BFW recipe) an attempt was undertaken to separate contributions of the singlet ($^1S_0$) and triplet ($^3S_1$-$^3D_1$) pn-pair states to the cross section of the deuteron break-up reaction discussed. But, for experimental resolution of the relative energy $\Delta E_{pn} \sim 15$–20 MeV, the accuracy of the relation (25) with the function $f_{\mathrm{BFW}}(k)$ is about 15%. The contribution of the rest partial waves (besides $^1S_0$ and $^3S_1$-$^3D_1$ ones) to the cross section gives around 10% of the latter. Hence, the total error of separation of the pure $^1S_0$-state contribution by such a method can be estimated in 25% of the full cross section. Therefore, the attempt of the authors of paper [9] to realize this separation, when the $^1S_0$-state contribution is expected to be less than 30% of the full cross section, and at poor relative energy resolution is, most probably, an exceeding of the accuracy level of the relation (25).

Nevertheless, it is worth to mention that at low energy E_{pn} the cross section of a $^3S_1$-$^3D_1$-state pair formation accounts for a great deal of the full cross section given by Eq. (34) including all the partial waves in the pn-pair state (cf. the solid and dashed lines in Figs. 7, 8). Thus, at $E_{pn} = 3$ MeV, the relative weight of the $(pn)_t$-pair contribution in the full cross section defined by Eq. (34) exceeds 90%. At $E_{pn} = 20$ MeV due to the growing influence of the highest partial waves this ratio becomes somewhat smaller, but remains rather high, 75–85%. So, the ADM and BFW relations can be used for some estimation of total contribution of the triplet and singlet pairs to the cross section of deuteron break-up reaction in quasi-two-body kinematics [3].

---

[3] This statement violates, if E_{pn} is very close to zero, since at such conditions the singlet $^1S_0$-state dominates.

Conclusion

We investigated the influence of the off-energy-shell effects in the NN-scattering amplitude on the cross section of the reaction $p + d \rightarrow N(180°) + (NN)(0°)$ in the framework of the ONE mechanism. For the case of the forward going pp-pair a deep minimum in the corresponding cross section at the initial proton kinetic energy $T_p \sim 0.77$ GeV is expected, caused by the node of the $^1S_0 \rightarrow {}^1S_0$ transition half-off-shell NN-amplitude. Appearance of such a node is tightly connected with the properties of the NN-interaction in the region of nucleon overlap. As the relative energy E_{pp} of the protons in the final pp-pair increases, the minimum rapidly fills up because of amplification of contributions from the highest partial waves. The position of this minimum was found to be sensitive to the value of E_{pp}. It was shown that the necessary (but, probably, still not sufficient) condition of experimental observation of the minimum in question was high enough relative energy resolution $\Delta E_{pp} \leq 5$ MeV. Another important feature of the reaction $p + d \rightarrow n(180°) + (pp)(0°)$ considered in the framework of the ONE mechanism is an approximate constancy of the differential cross section in the region of the initial energies $T_p \sim 1.4$–2 GeV.

For the reaction of pn-pair formation in the same kinematics of the elastic backward pd-scattering the node of the $^1S_0 \rightarrow {}^1S_0$ transition amplitude (as well as of the $^3S_1 \rightarrow {}^3S_1$ one) is not distinctly seen in the corresponding cross section at any excitation energy E_{pn}, that is caused by significant contribution of the "nodeless" transition $^3D_1 \rightarrow {}^3S_1$. However, the nodes discussed may reveal themselves in polarization phenomena which have been studied so far only in the impulse approximation (i. e. without taking into account the off-shell effects) [21].

It was shown also that for the relative energy $E_{pn} \leq 3$ MeV the cross section of the reaction $p + d \rightarrow p(180^0) + (pn)_t$ of the triplet pn-pair formation can be estimated with an accuracy $\sim 5\%$ by means of the relations (25), (27) based on an analogy between the processes of deuteron break-up and elastic backward pd-scattering. Since the latter has been already well studied both theoretically and experimentally, separation of the channel of the reaction $p + d \rightarrow N + (NN)$ with the singlet NN-pair seems mostly interesting.

Appendix

Below we outline the main steps of proving that Eq. (25) remains approximately valid for deuteron considered as a two-channel (i. e. admitting the mixture of $^3S_1$- and $^3D_1$-states) system. The detailed proof can be found in Ref. [18].

It is more convenient for us to work in the coordinate representation. The two radial components of the deuteron w.f. in the coordinate space, $u_0(r)$ and $u_2(r)$, satisfy the system of two coupled differential equations (obtained from the Schrödinger equation for the initial w.f.):

$$u_L''(r) - \frac{L(L+1)}{r^2} u_L(r) + m \sum_{L''} v_{LL''}(r) u_{L''}(r) - \alpha_t^2 u_L(r) = 0 \qquad (A.1)$$

and the boundary conditions

$$u_L(0) = 0; \quad u_L(r \rightarrow \infty) = N_L e^{-\alpha_t r}. \qquad (A.2)$$

Here $v_{LL''}(r)$ is the radial component of the interaction potential, $\alpha_t = \sqrt{m|\varepsilon|}$, N_L is some constant.

Let us introduce four radial scattering w.f's of pn-system, $g_{LL'}(r, k)$, describing transitions between the states with the "deuteron" quantum numbers, i. e. $J = 1$, $S = 1$, $L = 0, 2$, $L' = 0, 2$. The corresponding system of equations and boundary conditions for $g_{LL'}(r, k)$ are

$$g_{LL'}''(r, k) - \frac{L(L+1)}{r^2} g_{LL'}(r, k) + m \sum_{L''} v_{LL''}(r) g_{L''L'}(r, k) + k^2 g_{LL'}(r, k) = 0, \quad (A.3)$$

$$g_{LL'}(0, k) = 0; \quad g_{LL'}(r \rightarrow \infty, k) = \frac{1}{2ik} \left[e^{ikr} \delta_{LL'} - (-1)^{L'} S_{LL'}^*(k) e^{-ikr} \right], \quad (A.4)$$

where $S_{LL'}^*(k)$ is the complex conjugated scattering matrix (for definiteness we consider $g_{LL'}(r, k)$ as the components of the scattering function $\phi^{(-)}(\mathbf{r}, \mathbf{k})$). The normalization is chosen to be

$$\sum_L \int_0^\infty dr \, [u_L(r)]^2 = 1, \qquad (A.5)$$

$$\sum_L \int_0^\infty dr \, g_{LL'}^*(r, k') g_{LL''}(r, k) = \frac{\pi}{2k^2} \delta_{L'L''} \delta(k - k'). \qquad (A.6)$$

Eqs. (A.1) and (A.3) differ from each other only by the last term on the left-hand-sides. If we substitute k by $-i\alpha_t$, then the difference disappears, and the equations become the same. The second boundary condition in Eq. (A.4) goes over into

$$g_{LL'}(r \rightarrow \infty, k \rightarrow -i\alpha_t) = \frac{1}{2\alpha_t} \left[e^{\alpha_t r} \delta_{LL'} - (-1)^{L'} S_{LL'}^*(k \rightarrow -i\alpha_t) e^{-\alpha_t r} \right]. \quad (A.7)$$

Since the S^*-matrix has a pole at $k = -i\alpha_t$, the term with $e^{\alpha_t r}$ is "suppressed", and we get

$$g_{LL'}(r \rightarrow \infty, k \rightarrow -i\alpha_t) = \frac{(-1)^{L'+1}}{2\alpha_t} S_{LL'}^*(k \rightarrow -i\alpha_t) e^{-\alpha_t r}, \qquad (A.8)$$

that up to a constant factor coincides with the second condition in Eq. (A.2). So far as the boundary condition at $r=0$ does not depend on k at all, we conclude that

$$u_L(r) = \lim_{k \to -i\alpha} \left[A_{L'}(k) g_{LL'}(r, k) \right], \qquad (A.9)$$

the coefficient A being independent of r and L. In an expanded form Eq. (A.9) is equivalent to the two systems of equalities

$$\begin{pmatrix} u_0(r) \\ u_2(r) \end{pmatrix} = \lim_{k \to -i\alpha_t} \left[A_0(k) \begin{pmatrix} g_{00}(r, k) \\ g_{20}(r, k) \end{pmatrix} \right] \qquad (A.10)$$

and

$$\begin{pmatrix} u_0(r) \\ u_2(r) \end{pmatrix} = \lim_{k \to -i\alpha_t} \left[A_2(k) \begin{pmatrix} g_{02}(r, k) \\ g_{22}(r, k) \end{pmatrix} \right]. \qquad (A.11)$$

Note that $A_0(k)$ and $A_2(k)$ are not unique, because they always can be multiplied by an arbitrary function $\varphi(k)$ satisfying the condition $\varphi(-i\alpha_t) = 1$. One possible choice is [18]

$$A_{0,2}(k) = -\sqrt{\frac{2\alpha_t(k^2 + \alpha_t^2)}{\cos 2\epsilon_1}} \, e^{-i\delta_{0,2}}, \qquad (A.12)$$

where $\delta_{0,2}$ are the scattering phase shifts of the $^3S_1$- and $^3D_1$-states, respectively, ϵ_1 is the mixing parameter. All the quantities $\delta_{0,2}$, ϵ_1 relate to the so-called "nuclear bar" parametrization [22].

In Ref. [18] it was shown that Eq. (A.10) with the coefficient $A_0(k)$ from Eq. (A.12) holds approximately true, even if we remove the sign of limit and take the right-hand-side of Eq. (A.10) at small real values of k. Since under these conditions $|e^{i\delta_0}| = 1$, $\cos 2\epsilon_1 \approx 1$, we get

$$\begin{pmatrix} |u_0(r)| \\ |u_2(r)| \end{pmatrix} \approx \sqrt{2\alpha_t(k^2 + \alpha_t^2)} \begin{pmatrix} |g_{00}(r, k)| \\ |g_{20}(r, k)| \end{pmatrix}. \qquad (A.13)$$

Of course, Eq. (A.13) is valid with good accuracy, if k is close to the threshold, and $r < a$, where a is the characteristic radius of the interaction potential. Going over to the momentum representation by the formulas

$$u_L(q') = 4\pi(-i)^L \int_0^\infty dr \, r \, u_L(r) j_L(q'r), \qquad (A.14)$$

$$g_{LL'}(q', k) = 4\pi(-i)^L \int_0^\infty dr \, r \, g_{LL'}(r, k) j_L(q'r) \qquad (A.15)$$

($j_L(q'r)$ is a spherical Bessel function) and taking into account the connection between the functions $g_{LL'}(q', k)$ and the previously introduced scattering amplitudes $t_{LL'}^{JS}(q', k)$

$$g_{LL'}(q', k) = \frac{t_{LL'}^{11}(q', k)}{4m(q'^2 - k^2 - i0)}, \qquad (A.16)$$

22

we obtain

$$(q'^2 - k^2)^2 \left[u_0^2(q') + u_2^2(q') \right] \approx \frac{\alpha_t(k^2 + \alpha_t^2)}{8m^2} \left[|t_{00}^{11}(q', k)|^2 + |t_{20}^{11}(q', k)|^2 \right]. \qquad (A.17)$$

Although, strictly speaking, Eq. (A.11) can not be treated in the same way, as Eq. (A.10), and the relation between $u_0(q')$, $u_2(q')$ and the amplitudes $t_{02}^{11}(q', k)$, $t_{22}^{11}(q', k)$, analogous to Eq. (A.17), does not exist, at $ka \ll 1$ the sum $|t_{02}^{11}(q', k)|^2 + |t_{22}^{11}(q', k)|^2$ is small compared to $|t_{00}^{11}(q', k)|^2 + |t_{20}^{11}(q', k)|^2$. Therefore, without significant change of accuracy we may extend Eq. (A.17) to include into its right-hand-side all the four amplitudes:

$$(q'^2 - k^2)^2 \left[u_0^2(q') + u_2^2(q') \right] \approx \frac{\alpha_t(k^2 + \alpha_t^2)}{8m^2} \left[|t_{00}^{11}(q', k)|^2 + |t_{20}^{11}(q', k)|^2 \right.$$
$$\left. + |t_{02}^{11}(q', k)|^2 + |t_{22}^{11}(q', k)|^2 \right]. \qquad (A.18)$$

Numerical estimations show that even at $k^2/m = 20$ MeV the relative difference between the right-hand-sides of Eqs. (A.17) and (A.18) is less than 6%, when q' varies from 0 up to 1 GeV.

Now, substituting Eq. (A.18) into Eq. (37) and comparing the result with Eq. (36) (keeping in mind that $q'^2 \gg k^2 \sim m|\varepsilon|$), we reproduce in the nonrelativistic limit Eq. (25) with the function $f_{BFW}(k)$ defined by Eq. (27).

On the basis of the same arguments we conclude that Eq. (25) with the function $f_{ADM}(k)$ (Eq. (26)), being true for the single $^3S_1$-state, holds also for the case of coupled $^3S_1$–$^3D_1$ channels.

References

[1] P. Bosted et al. Phys. Rev. Lett. **49**(1982) 1380.

[2] V. G. Ableev et al. Pis'ma ZhETF **47** (1988)558; B. Kuehn et al., Phys. Lett. **B334** (1994) 298.

[3] V. Punjabi, C. F. Perdrisat, P. Ulmer et al. Phys.Rev. **C39** (1989) 608.

[4] G. I. Lykasov. EPAN **24** (1993) 140.

[5] A. P. Kobushkin, A. Syamtonov, L. Ya. Glozman. Yad. Fiz. **59** (1996) 833.

[6] O. Imambekov, Yu. N. Uzikov. Yad. Fiz.**52** (1990) 1361.

[7] COSY proposal N° 20, KFA Juelich (1992), V. I. Komarov spokes-person.

[8] V. V. Anisovich, L. G. Dahno, M. M. Makarov. Yad.Fiz. **32** (1980) 1521.

[9] A. Boudard, G. Fäldt, C. Wilkin. Preprint TSL/ISV-96-0142, Uppsala, 1996.

23

[10] V. A. Karmanov. Fiz. El. Chast. At. Yadra, **19** (1988) 525; Sov. J. Part. Nucl. 19 (1988) 228.

[11] V. A. Karmanov and A. V. Smirnov. Nucl. Phys. **A546** (1992) 691; **A575** (1994) 520.

[12] L. P. Kaptari *et al.* Phys. Lett. **B351** (1995) 400.

[13] B. L. G. Bakker, L. A. Kondratyuk, M. V. Terentjev. Nucl. Phys. **B158** (1979) 497.

[14] P. L. Chung, F. C. Coester, B. D. Keister, W. P. Polyzou. Phys.Rev. **C37** (1988) 2000.

[15] Yu. N. Uzikov. Yad. Fiz. **55** (1992) 2374.

[16] M. Lacombe *et al.* Phys. Rev. **C21** (1980) 861.

[17] J. Haidenbauer and W. Plessas. Phys. Rev. **C30** (1984) 1822.

[18] A. V. Smirnov. P. N. Lebedev Physical Institute Preprint **No.34, Moscow, 1997**.

[19] COSY proposal $N°38$, KFA (1996) V. P. Koptev spokes-person

[20] L. A. Kondratyuk, F. M. Lev, L. V. Schevchenko. Yad. Fiz. **23** (1981) 1208.

[21] V. P. Ladygin. Preprint JINR, E2-96-333, Dubna, 1996.

[22] H. Stapp *et al.*, Phys. Rev. **105** (1957) 302

Смирнов А.В., Узиков Ю.Н.   E2-97-140

Расщепление дейтрона протонами
в кинематике квазиупругого *pd*-рассеяния

Рассматривается эксклюзивная реакция расщепления дейтрона $p + d \rightarrow N\,(180°) + (NN)\,(0°)$ в рамках механизма однонуклонного обмена при кинетической энергии налетающих протонов $0{,}2 - 2$ ГэВ и относительной энергии вылетающей вперед конечной *NN*-пары $E_{NN} = 0 - 20$ МэВ, что соответствует кинематическим условиям эксперимента, планируемого на COSY. Исследованы внеэнергетические эффекты, проявляющиеся, в частности, при наличии узла парциальной амплитуды *NN*-рассеяния наполовину внеэнергетической поверхности в $^{1}S_{0}$-состоянии, а также роль высших парциальных волн в подпроцессе $pN \rightarrow pN$. Указанные эффекты оказались очень существенными, особенно в реакции $p + d \rightarrow n\,(180°) + (pp)\,(0°)$. В рамках рассматриваемого механизма проверена точность некоторых приближенных соотношений между сечениями упругого *pd*-рассеяния назад и расщепления дейтрона.

Smirnov A.V., Uzikov Yu.N.   E2-97-140

Deuteron Disintegration by Protons
in Kinematics of the Quasi-Elastic Backward *pd*-Scattering

The deuteron break-up exclusive reaction $p + d \rightarrow N\,(180°) + (NN)\,(0°)$ is considered in the framework of the one-nucleon-exchange mechanism at the incident proton kinetic energy $0.2 - 2$ GeV and the relative energy of nucleons in the forward going final *NN*-pair $E_{NN} = 0 - 20$ MeV, that is close to the kinematical conditions of the planned experiment at COSY. The off-energy-shell effects resulting, in particular, in the node of the half-of-shell $^{1}S_{0}$-state partial *NN*-scattering amplitude and the role of higher partial waves in the subprocess $pN \rightarrow pN$ are analysed and found to be very important, especially for the reaction $p + d \rightarrow n\,(180°) + (pp)\,(0°)$. The accuracy of some approximate relations between the cross sections of elastic *pd*-scattering and deuteron break-up is examined in the framework of the mechanism in question.