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RESULTS ON $K \to 2\pi$ DECAYS AT $O(p^6)$ AND ε'/ε FROM AN EFFECTIVE CHIRAL LAGRANGIAN APPROACH

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1 Introduction

The starting point for most advanced calculations of nonleptonic kaon decays is effective weak lagrangians of the form [2, 3]

$$\mathcal{L}_w^q(|\Delta S| = 1) = \sqrt{2}\, G_F\, V_{ud} V_{us}^* \sum_i C_i\, \mathcal{O}_i \tag{1}$$

which can be derived with the help of the Wilson operator product expansion from elementary quark processes, with additional gluon exchanges. In the framework of perturbative QCD the coefficients C_i are to be understood as scale- and normalization scheme dependent functions. There exist extensive next-to-leading order (NLO) calculations [4, 5] in the context of kaon decays, among others. These calculations are based on the possibility of factorization of short- and long-range contributions, i.e. into Wilson coefficient functions C_i and mesonic matrix elements of four-quark operators $\mathcal{O}_i$, respectively. The latter, however, can presently be obtained only by using nonperturbative, i.e. model-dependent, methods. But still it seems not possible so far, to fulfil the obvious requirement of scale- and renormalization scheme invariance of the resulting amplitudes in a satisfactory way.

Usually, the results of calculations are displayed with the help of B-factors in the form

$$T_{K\to 2\pi} = \sqrt{2}\, G_F\, V_{ud} V_{us}^* \sum_i \Big[C_i(\mu) B_i(\mu)\Big] < \pi\pi|\mathcal{O}_i|K >_{vac.sat.}\,,$$

where the mesonic matrix elements of four-quark operators are approximated by their vacuum saturation values which are, of course, μ-independent. In principle, $B_i(\mu)$ should be estimated by some higher-order calculations in the long-range regime, for instance, in $1/N_c$-expansion [6] in the form $1 + O(1/N_c)$, or from the lattice approach. The preliminary stage of these calculations is best characterized by the problem to explain the well-known $\Delta I = 1/2$ rule quantitatively. Of course, the lack of such calculations for long-range effects severely restricts the predictive power of (1), leaving only the possibility of some semi-phenomenological treatment [4, 1, 7], with correspondingly large theoretical uncertainties. As a matter of fact, some of the combinations $\big[C_i(\mu)B_i(\mu)\big]$ have to be fixed by experimental data, while others are restricted by additional theoretical arguments.

A logical improvement of this approach, pursued in [8, 9], is to take the lagrangian (1) as defining the structure of the $|\Delta S| = 1$ weak interaction and to submit it to a global confrontation with data by sandwiching it between appropriate initial and

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final states (for instance, charged and neutral kaons, respectively 2π, 3π, $n\pi\gamma$,... states). Of course, there would be hardly any gain in predictive power, if we stuck to the vacuum saturation approximation because there may be large unknown factors involved, when switching between different initial/final states of the same operator. It is evident that everything could be absorbed into the B_i-factors which thereby would not only be scheme- and scale-dependent, but would also change with the initial/final states (becoming even unknown (complex) functions of dynamic variables for final states). On the other hand, these latter changes should be avoided or, at least, diminished when using a consistent higher-order calculation of the mesonic matrix elements in the framework of chiral theory which today is, certainly, the most developed approach to long-range hadronic phenomena in general. These higher-order calculations after the inclusion of meson loops have their own scale dependent renormalization ambiguities. In the ideal case these meson loops should cancel the scale dependence of the Wilson coefficients from the perturbative QCD calculation (resulting in B-factors equal to 1). As a complete match is lacking so far, the best one can do is to fix the long-range ambiguities in a definite way at an explicitly or implicitly given scale while considering the Wilson coefficients times (unknown) B-factors as phenomenological constants.

As the above-mentioned perturbative QCD calculations in NLO are believed to be reliable down to a scale of $O(1\text{ GeV})$ (in [1, 7] the charm quark mass $m_c = 1.3$ GeV is used as a matching scale), it is very desirable, however, that the effective scale for the long-range calculation reaches this value too. Therefore, our strategy includes a consistent calculation of the matrix elements up to $O(p^6)$, and furthermore some consideration of the effects from vector, axial-vector and scalar resonances which can be included by the procedure of reduction [10, 11], i.e. a certain recalculation of structure coefficients of the effective chiral lagrangians involved.

In this note we reconsider in the above manner especially the $K \to 2\pi$ channels with matrix elements of four-quark operators calculated at $O(p^6)$ in momentum expansions within the chiral lagrangian approach. This approach is based on the bosonized version of the weak lagrangian (1) and chiral effective meson lagrangians of higher orders with the structure coefficients fixed by bosonization of the four-quark NJL-type interaction. According to Weinberg's power-counting scheme [12], the calculation involves tree-level, one- and two-loop diagrams. The method of superpropagator regularization [13] was used to fix UV divergences arising at the loop level. The aim of this note is to give the first numerical results of this calculation

concerning $K \to 2\pi$ amplitudes as they lead to new estimates of ε'/ε (see section 3). In section 2 and appendices A and B we repeat all relevant definitions taken from earlier work.

2 Lagrangians and currents

In the present paper we use the operators $\mathcal{O}_i$ in the representation given in [2, 14]:

$$\begin{aligned}
\mathcal{O}_1 &= \bar{u}_L\gamma_\mu u_L\, \bar{d}_L\gamma^\mu s_L - \bar{d}_L\gamma_\mu u_L\, \bar{u}_L\gamma^\mu s_L\,, \\
\mathcal{O}_2 &= \bar{u}_L\gamma_\mu u_L\, \bar{d}_L\gamma^\mu s_L + \bar{d}_L\gamma_\mu u_L\, \bar{u}_L\gamma^\mu s_L + 2\bar{d}_L\gamma_\mu d_L\, \bar{d}_L\gamma^\mu s_L + 2\bar{s}_L\gamma_\mu s_L\, \bar{d}_L\gamma^\mu s_L\,, \\
\mathcal{O}_3 &= \bar{u}_L\gamma_\mu u_L\, \bar{d}_L\gamma^\mu s_L + \bar{d}_L\gamma_\mu u_L\, \bar{u}_L\gamma^\mu s_L + 2\bar{d}_L\gamma_\mu d_L\, \bar{d}_L\gamma^\mu s_L - 3\bar{s}_L\gamma_\mu s_L\, \bar{d}_L\gamma^\mu s_L\,, \\
\mathcal{O}_4 &= \bar{u}_L\gamma_\mu u_L\, \bar{d}_L\gamma^\mu s_L + \bar{d}_L\gamma_\mu u_L\, \bar{u}_L\gamma^\mu s_L - \bar{d}_L\gamma_\mu d_L\, \bar{d}_L\gamma^\mu s_L\,, \\
\mathcal{O}_5 &= \bar{d}_L\gamma_\mu\lambda_c^a s_L \left(\sum_{q=u,d,s} \bar{q}_R\,\gamma^\mu\,\lambda_c^a\, q_R\right), \quad \mathcal{O}_6 = \bar{d}_L\gamma_\mu s_L \left(\sum_{q=u,d,s} \bar{q}_R\,\gamma^\mu\, q_R\right), \\
\mathcal{O}_7 &= 6\bar{d}_L\gamma_\mu s_L \left(\sum_{q=u,d,s} \bar{q}_R\,\gamma^\mu\, Q\, q_R\right), \quad \mathcal{O}_8 = 6\bar{d}_L\gamma_\mu\lambda_c^a s_L \left(\sum_{q=u,d,s} \bar{q}_R\,\gamma^\mu\,\lambda_c^a\, Q\, q_R\right),
\end{aligned}$$

where $q_{L,R} = \frac{1}{2}(1 \mp \gamma_5)q$; λ_c^a are the generators of the $SU(N_c)$ color group; Q is the matrix of electric quark charges. The operators $\mathcal{O}_{5,6}$ containing right-handed currents are generated by gluonic penguin diagrams and the analogous operators $\mathcal{O}_{7,8}$ arise from electromagnetic penguin diagrams. The operators $\mathcal{O}_{1,2,3,5,6}$ and $\mathcal{O}_4$ describe the transitions with $\Delta I = 1/2$ and $\Delta I = 3/2$, respectively, while the operators $\mathcal{O}_{7,8}$ contribute to the transition with both $\Delta I = 1/2$ and $\Delta I = 3/2$.

The Wilson coefficients C_i of the effective weak lagrangian (1) with four-quark operators $\mathcal{O}_i$ are connected with the Wilson coefficients c_i corresponding to the basis of four-quark operators Q_i given in Refs. [1, 4], by the following linear relations:

$$\begin{aligned}
&C_1 = c_1 - c_2 + c_3 - c_4 + c_9 - c_{10}\,, \quad C_2 = \frac{1}{5}(c_1 + c_2 - c_9 - c_{10}) + c_3 + c_4\,, \\
&C_3 = \frac{1}{5}C_4 = \frac{1}{5}\left(\frac{2}{3}(c_1 + c_2) + c_9 + c_{10}\right), \\
&C_5 = c_6\,, \quad C_6 = 2\left(c_5 + \frac{1}{3}c_6\right), \quad C_7 = \frac{1}{2}\left(c_7 + 2c_8\right), \quad C_8 = \frac{1}{4}c_8\,. \qquad (2)
\end{aligned}$$

The bosonized version of the effective Lagrangian (1) can be expressed in the form [8, 9]:

$$\mathcal{L}_w^{mes} = \widetilde{G}_F\Big\{(-\xi_1 + \xi_2 + \xi_3)\Big[(J^1_{L\mu} - iJ^2_{L\mu})(J^4_{L\mu} + iJ^5_{L\mu}) - (J^3_{L\mu} + \frac{1}{\sqrt{3}}J^8_{L\mu})(J^6_{L\mu} + iJ^7_{L\mu})\Big]$$

$$+(\xi_1+5\,\xi_2)\sqrt{\frac{2}{3}}\,J^0_{L\mu}(J^6_{L\mu}+iJ^7_{L\mu})+\frac{10}{\sqrt{3}}\,\xi_3\,J^8_{L\mu}(J^6_{L\mu}+iJ^7_{L\mu})$$

$$+\xi_4\left[(J^1_{L\mu}-iJ^2_{L\mu})(J^4_{L\mu}+iJ^5_{L\mu})+2\,J^3_{L\mu}(J^6_{L\mu}+iJ^7_{L\mu})\right]$$

$$-4\,\xi_5\left[(J^1_R-iJ^2_R)(J^4_L+iJ^5_L)-(J^3_R-\frac{1}{\sqrt{3}}J^8_R-\sqrt{\frac{2}{3}}\,J^0_R)(J^6_L+iJ^7_L)\right.$$
$$\left.-\sqrt{\frac{2}{3}}\,(J^6_R+iJ^7_R)(\sqrt{2}J^8_L-J^0_L)\right]$$

$$+\xi_6\sqrt{\frac{3}{2}}\,(J^4_{L\mu}+iJ^5_{L\mu})J^0_R+6\,\xi_7\,(J^6_{L\mu}+iJ^7_{L\mu})(J^3_{R\mu}+\frac{1}{\sqrt{3}}\,J^8_{R\mu})$$

$$-16\,\xi_8\left[(J^1_R-iJ^2_R)(J^4_L+iJ^5_L)+\frac{1}{2}\,(J^3_R-\frac{1}{\sqrt{3}}J^8_R-\sqrt{\frac{2}{3}}\,J^0_R)(J^6_L+iJ^7_L)\right.$$
$$\left.\left.+\frac{1}{\sqrt{6}}\,(J^6_R+iJ^7_R)(\sqrt{2}\,J^8_L-J^0_L)\right]\right\}+\text{h.c.} \qquad (3)$$

Here $\widetilde{G}_F = \sqrt{2}\,G_F\,V_{ud}V^*_{us}$, $J^a_{L/R\mu}$ and $J^a_{L/R}$ are bosonized $(V \mp A)$ and $(S \mp P)$ meson currents corresponding to the quark currents $\bar{q}\gamma_\mu\frac{1}{4}(1\mp\gamma^5)\lambda^a q$ and densities $\bar{q}\frac{1}{4}(1\mp\gamma^5)\lambda^a q$, respectively ($\lambda^a$ are the generators of the $U(3)_F$ flavor group);

$$\xi_1 = C_1\left(1-\frac{1}{N_c}\right), \qquad \xi_{2,3,4} = C_{2,3,4}\left(1+\frac{1}{N_c}\right),$$
$$\xi_{5,8} = C_{5,8}\left(1-\frac{1}{N_c^2}\right)+\frac{1}{2N_c}C_{6,7}, \qquad \xi_{6,7} = C_{6,7}, \qquad (4)$$

where the color factor $1/N_c$ originates from the Fierz-transformed contribution to the nonleptonic weak effective chiral Lagrangian [8, 9].

Only the even-intrinsic-parity sector of the chiral lagrangian is required to describe nonleptonic kaon decays up to and including $O(p^6)$. In the bosonization approach this sector is obtained from the modulus of the logarithm of the quark determinant of the NJL-type models [15] using the path-integral technique (see [16] and references therein). The meson currents $J^a_{L/R\mu}$ and $J^a_{L/R}$ are obtained from this quark determinant by variation over additional external sources associated with corresponding quark currents and densities [8, 9]. From the momentum expansion of the quark determinant to $O(p^{2n})$ one can derive the strong lagrangian for mesons $\mathcal{L}_{eff}$ of the same order and the corresponding currents $J^a_{L/R\mu}$ and $J^a_{L/R}$ to the order $O(p^{2n-1})$ and $O(p^{2n-2})$, respectively.

For example, from the terms of quark determinant of $O(p^2)$ one gets

$$\mathcal{L}^{(p^2)}_{eff} = -\frac{F_0^2}{4}\,\mathrm{tr}\,(L_\mu^2)+\frac{F_0^2}{4}\,\mathrm{tr}\,(\chi U^\dagger + U\chi^\dagger)\,,$$

$$J^{(p^1)a}_{L\mu} = \frac{iF_0^2}{4}\,\mathrm{tr}\,(\lambda^a L_\mu)\,, \quad J^{(p^0)a}_{L} = \frac{F_0^2}{4}\overline{m}R\,\mathrm{tr}\,(\lambda^a U)\,, \qquad (5)$$

where $U = \exp\left(\frac{i\sqrt{2}}{F_0}\varphi\right)$, with φ being the pseudoscalar meson matrix, and $L_\mu = D_\mu U\,U^\dagger$, $D_\mu U = \partial_\mu U + (A^L_\mu U - UA^R_\mu)$ and $A^{R/L}_\mu = V_\mu \pm A_\mu$ are right/left-handed combinations of vector and axial-vector fields. Furthermore, $F_0 \approx 90$ MeV is the bare coupling constant of pion decay, $\chi = \mathrm{diag}(\chi^2_u, \chi^2_d, \chi^2_s) = -2m_0{<}\bar{q}q{>}F_0^{-2}$ is the meson mass matrix, m_0 is the current quark mass-matrix, ${<}\bar{q}q{>}$ is the quark condensate, $\overline{m} \approx 265$ MeV is an average constituent quark mass, and $R = {<}\bar{q}q{>}/(\overline{m}F_0^2)$.

At $O(p^4)$ one gets

$$\mathcal{L}^{(p^4)}_{eff} \;\Rightarrow\; \left(L_1-\frac{1}{2}L_2\right)(\mathrm{tr}\,L_\mu^2)^2 + L_2\,\mathrm{tr}\left(\frac{1}{2}[L_\mu,L_\nu]^2+3(L_\mu^2)^2\right)+L_3\,\mathrm{tr}\left[(L_\mu^2)^2\right]$$
$$-L_4\,\mathrm{tr}\,(L_\mu^2)\;\mathrm{tr}\,(\chi U^\dagger + U\chi^\dagger) - L_5\,\mathrm{tr}\left(L_\mu^2(\chi U^\dagger + U\chi^\dagger)\right)$$
$$+L_8\,\mathrm{tr}\left((\chi^\dagger U)^2+(\chi U^\dagger)^2\right)+H_2\,\mathrm{tr}\,\chi\chi^\dagger\,,$$
$$J^{(p^3)a}_{L\mu} \;\Rightarrow\; i\,\mathrm{tr}\left\{\lambda^a\left[L_4 L_\mu\,\mathrm{tr}(\chi U^\dagger + U\chi^\dagger)+\frac{1}{2}L_5\{L_\mu,(\chi U^\dagger + U\chi^\dagger)\}\right]\right\},$$
$$J^{(p^2)a}_{L} \;\Rightarrow\; -\overline{m}R\,\mathrm{tr}\left\{\lambda^a\left[L_4 U\,\mathrm{tr}(L_\mu^2)+L_5(L_\mu^2 U) - 2L_8 U\chi^\dagger U - H_2\chi\right]\right\}, \qquad (6)$$

where L_i and H_2 are structure constants introduced by Gasser and Leutwyler [17]. For the sake of brevity, here and in following expressions for the lagrangian and currents generated at $O(p^6)$, given in appendix A, we restrict ourselves to the terms which are necessary to calculate the decay $K \to 2\pi$ at $O(p^6)$. We do not show explicitly the terms of the effective action at $O(p^8)$ generating the scalar current $J^{(p^6)}_L$ which is necessary for the full calculation of the tree-level matrix elements at $O(p^6)$ for the penguin operators, since the corresponding contributions turn out to be negligibly small.

The structure constants L_i, H_i and Q_i should be obtained from the modulus of the logarithm of quark determinant of the NJL-type model which explicitly contains, apart from the pseudoscalar Goldstone bosons, also scalar, vector and axial-vector resonances as dynamic degrees of freedom. However, in order to avoid double counting in calculating pseudoscalar meson amplitudes when taking into account resonance degrees of freedom, one has to integrate out (reduce) these resonances in the generating functional of the bosonization approach. As a consequence of this procedure, the structure coefficients of pseudoscalar low-energy interactions will be strongly modified. In this way one effectively takes into account resonance-exchange contributions

[10, 11]. The explicit expressions for the structure constants of non-reduced and reduced effective meson lagrangians are given in appendix B.

In the context of the power counting rules we have to give some comments concerning the additional symmetry breaking term $\sim \operatorname{tr}(m_0 D^2 U)$ which appears in [6, 18, 19]. This term leads to nonzero meson matrix element of the gluonic penguin operator due to the appearance of an additional contribution to the scalar density $\sim \operatorname{tr}(D^2 U)$. In the bosonization approach this term of $O(p^4)$ arises from that term of the divergent part of the quark determinant which also generates the kinetic term of $O(p^2)$:

$$
\begin{aligned}
\mathcal{L}_{div} \;\Rightarrow\; & \frac{N_c}{16\pi^2}\, y \operatorname{tr}\left[D^\mu(\overline{m}U + m_0)\,\overline{D}_\mu(\overline{m}U + m_0)^\dagger\right] \\
= & \frac{F_0^2}{4}\operatorname{tr}(D^\mu U\,\overline{D}_\mu U^\dagger) - \frac{F_0^2}{4}\frac{1}{\Lambda_\chi^2}\operatorname{tr}(\chi^\dagger D^2 U + \chi \overline{D}^2 U^\dagger)\,,
\end{aligned}
\tag{7}
$$

where $\overline{D}_\mu U^\dagger = \partial_\mu U^\dagger + (A^R_\mu U^\dagger - U^\dagger A^R_\mu)$, $y = 4\pi^2 F_0^2/(N_c \overline{m}^2)$, $\Lambda_\chi^2 = \overline{m}^2/x$ with $x = -\overline{m}F_0^2/(2<\overline{q}q>)$. For the phenomenological value of the quark condensate $<\overline{q}q>^{1/3} \approx -220$ MeV one obtains $\Lambda_\chi \approx 839$ MeV (to be compared with $\Lambda_\chi \approx 1020$ MeV in [1, 4]). The term $\sim \operatorname{tr}(m_0 D^2 U)$ does not appear explicitly in the effective lagrangian (6) because after transformations of double derivatives according to the equations of motion

$$
\begin{aligned}
D^2 U &= -D_\mu U\,\overline{D}^\mu U^\dagger \cdot U - \frac{1}{2}(U\chi^\dagger U - \chi) - \frac{1}{6}U \operatorname{tr}(\chi U^\dagger - U\chi^\dagger)\,, \\
\overline{D}^2 U^\dagger &= -U^\dagger D_\mu U\,\overline{D}^\mu U^\dagger - \frac{1}{2}(U^\dagger \chi U^\dagger - \chi^\dagger) + \frac{1}{6}U^\dagger \operatorname{tr}(\chi U^\dagger - U\chi^\dagger)\,,
\end{aligned}
$$

it is transformed into a combination of terms contributing to the structure coefficients L_5, L_8 and H_2:

$$
\begin{aligned}
-\frac{F_0^2}{4}\frac{1}{\Lambda_\chi^2}\operatorname{tr}(\chi^\dagger D^2 U + \chi\overline{D}^2 U^\dagger) \;=\; & \frac{N_c}{16\pi^2}\Big[-xy \operatorname{tr}\left(L_\mu^2(\chi U^\dagger + U\chi^\dagger)\right) \\
& + \frac{1}{2}xy \operatorname{tr}\left((\chi^\dagger U)^2 + \chi U^\dagger)^2\right) - xy \operatorname{tr}\chi\chi^\dagger \\
& - \frac{1}{6}xy\left(\operatorname{tr}(\chi U^\dagger - U\chi^\dagger)\right)^2\Big]\,.
\end{aligned}
\tag{8}
$$

It is easy to see that the contributions of Eq. (8) to the nonreduced structure constants (see Eq. (15) in appendix B) contain the product xy. The corresponding numerical contributions to the structure coefficients are $L_5 = 2.9\cdot 10^{-3}$, $L_8 = 1.4\cdot 10^{-3}$ and $H_2 = -2.9\cdot 10^{-3}$. Other contributions to L_5, L_8 and H_2 in Eq. (15) arise from the other terms of the divergent and finite parts of the quark determinant at $O(p^4)$ and lead to numerical values $L_5 = 0.98\cdot 10^{-3}$, $L_8 = 0.36\cdot 10^{-3}$ and $H_2 = 1.01\cdot 10^{-3}$.

3 Amplitudes and phenomenological results

Using isospin relations, the $K \to 2\pi$ decay amplitudes can be parameterized as

$$
\begin{aligned}
T_{K^+\to\pi^+\pi^0} &= \frac{\sqrt{3}}{2}A_2\,, \\
T_{K^0_S\to\pi^+\pi^-} &= \sqrt{\frac{2}{3}}A_0 + \frac{1}{\sqrt{3}}A_2\,, \qquad T_{K^0_S\to\pi^0\pi^0} = \sqrt{\frac{2}{3}}A_0 - \frac{2}{\sqrt{3}}A_2\,.
\end{aligned}
$$

The isotopic amplitudes $A_{2,0}$ determine the $K \to 2\pi$ transitions into states with isospin $I = 2, 0$, respectively:

$$
A_2 = a_2 e^{i\delta_2}\,, \qquad A_0 = a_0 e^{i\delta_0}\,,
$$

where $\delta_{2,0}$ are the phases of $\pi\pi$-scattering. It is well known that direct CP violation results in an additional (small) relative phase between a_2 and a_0. Let us next introduce the contributions of the four-quark operators $\mathcal{O}_i$ to the isotopic amplitudes $\mathcal{A}_I^{(i)}$ by the relations

$$
A_I = \mathcal{F}_I \mathcal{A}_I\,, \quad \mathcal{A}_I = -i\sum_{i=1}^{8}\xi_i \mathcal{A}_I^{(i)}\,,
\tag{9}
$$

where $\mathcal{F}_2 = \sqrt{2}\mathcal{F}_0 = \frac{\sqrt{3}}{2}\widetilde{G}_F F_0(m_K^2 - m_\pi^2)$.

At $O(p^2)$ we obtain for the nonzero tree-level amplitudes $\mathcal{A}_I^{(i)}$ the following expressions:

$$
\begin{aligned}
\mathcal{A}_0^{(1)} &= -\mathcal{A}_0^{(2,3)} = -1 = -\mathcal{A}_0^{(4)}\,, \quad \mathcal{A}_0^{(7)} = -\mathcal{A}_2^{(7)} = 2\,, \quad \mathcal{A}_0^{(5)} = -32\left(\frac{R\overline{m}}{F_0}\right)^2 L_5\,. \\
\mathcal{A}_0^{(8)} &= \frac{16(R\overline{m})^2}{m_K^2 - m_\pi^2}\Big\{1 - \frac{2}{F_0^2}\Big[6L_4(\chi_s^2 + \chi_d^2 + \chi_u^2) \\
&\qquad + (L_5 - 4L_8)(\chi_s^2 + 3\chi_d^2 + 2\chi_u^2) + 2L_5 m_\pi^2\Big]\Big\}\,, \\
\mathcal{A}_2^{(8)} &= \frac{8(R\overline{m})^2}{m_K^2 - m_\pi^2}\Big\{1 - \frac{2}{F_0^2}\Big[6L_4(\chi_s^2 + \chi_d^2 + \chi_u^2) \\
&\qquad + (L_5 - 4L_8)(\chi_s^2 + 3\chi_d^2 + 2\chi_u^2) + 2L_5 m_K^2\Big]\Big\}\,.
\end{aligned}
\tag{10}
$$

The L_8 and H_2 contributions in the penguin operators $\mathcal{O}_{5,8}$ also have a tadpole contribution from $K \to$ (vacuum) included through strong rescattering. $K \to \pi\pi K$ with $K \to$ (vacuum). At $O(p^2)$, in case of the penguin operator $\mathcal{O}_5$, the L_8 and H_2 contributions in the direct matrix element from $K \to 2\pi$ vertices, are fully cancelled by the tadpole diagrams [1]. However, such cancellation does not take place at $O(p^4)$ and $O(p^6)$.

Some interesting observations on the difference of the momentum behavior of penguin and nonpenguin operators can be drawn from power-counting arguments. According to Eq. (5) the leading contributions to the vector and scalar currents are of $O(p^1)$ and $O(p^0)$, respectively. Since in our approach the nonpenguin operators are constructed out of the products of $(V-A)$-currents $J^a_{L\mu}$, while the penguin operators are products of $(S-P)$-currents J^a_L, the lowest-order contributions of nonpenguin and penguin operators are of $O(p^2)$ and $O(p^0)$, respectively. However, due to the well-known cancellation of the contribution of gluonic penguin operator $\mathcal{O}_5$ at the lowest order [20], the leading gluonic penguin as well as nonpenguin contributions start from $O(p^2)$ [2]. Consequently, in order to derive the $(V-A)$-currents which contribute to the nonpenguin transition operators at leading order, it is sufficient to use the terms of the quark determinant to $O(p^2)$ only. At the same time the terms of the quark determinant to $O(p^4)$ have to be kept for calculating the penguin contribution at $O(p^2)$ since it arises from the combination of $(S-P)$-currents from Eqs. (5) and (6), which are of $O(p^0)$ and $O(p^2)$, respectively. In this subtle way a difference in momentum behaviour is revealed between matrix elements for these two types of weak transition operators; it manifests itself more drastically in higher-order lagrangians and currents.

Using the truncated lagrangian (7) only, the gluonic penguin operator matrix element at $O(p^2)$ is

$$\mathcal{A}_0^{(5)} = 4R \approx -20.0\,. \qquad (11)$$

However, taking into account the additional contribution from other part of the quark determinant at $O(p^4)$, we get a suppression of this matrix element which after substitution of the full expression for non-reduced structure constant L_5 from Eq. (15) becomes

$$\mathcal{A}_0^{(5)} = 4R\left(1-\frac{1}{y}\right) \approx -6.8\,.$$

This suppression of the gluonic penguin contribution in $\Delta I = 1/2$ transition leads to dramatic consequences for the phenomenological estimates of ε'/ε shown below.

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[1] We thank W.A. Bardeen and A.J. Buras for drawing our attention to this point.

[2] There is no cancellation of the contribution of the electromagnetic penguin operator $\mathcal{O}_8$ at the lowest order and the first terms in the expressions (10) for $\mathcal{A}^{(8)}_{0,2}$ correspond to the contributions at $O(p^0)$.

The modification of the structure constant after reduction of resonances ($L_5^{red} = 1.64 \cdot 10^{-3}$) leads to an increase in absolute value of the gluonic penguin matrix element to $\mathcal{A}_0^{(5)} \approx -11.2$, which, however, is still about factor 2 smaller than (11).

Table 1 presents the modification of the amplitudes $\mathcal{A}_I^{(i)}$ when including successively the higher order corrections $O(p^4)$ and $O(p^6)$ and the reduction of meson resonances. Our calculations involve Born and one- and two-loop meson diagrams and take into account isotopic symmetry breaking ($\pi^0-\eta-\eta'$ mixing). In our approach, the UV divergences resulting from meson loops at $O(p^4)$ and $O(p^6)$ were separated using the superpropagator regularization method [13] which is particularly well suited to the treatment of loops in nonlinear chiral theories. The result is equivalent to the dimensional regularization technique, the difference being that the scale parameter μ is no longer arbitrary but fixed by the inherent scale of the chiral theory $\hat{\mu} = 4\pi F_0 \approx 1$ GeV, and the UV divergences have to be replaced by a finite term using the substitution

$$(\mathcal{C} - 1/\varepsilon) \rightarrow C_{SP} = -1 + 4\mathcal{C} + \beta\pi\,,$$

where $\mathcal{C} = 0.577$ is Euler's constant, $\varepsilon = (4-D)/2$ and β is an arbitrary constant introduced by the Sommerfeld-Watson integral representation of the superpropagator. The splitting of the decay constants F_π and F_K is used at $O(p^4)$ to fix $C_{SP} \approx 3.0$.

The strong interaction phases $\delta_{2,0}$ arise first at $O(p^4)$, but for the quantitative description of the phases it is necessary to go beyond $O(p^4)$. At $O(p^4)$, for the $\pi\pi$-scattering phase shifts and their difference $\Delta = \delta_0 - \delta_2$, we have obtained the values of $\delta_0 \approx 22^\circ$, $\delta_2 \approx -13^\circ$, $\Delta \approx 35^\circ$ which are in agreement with [21]. At $O(p^6)$, we have obtained $\delta_0 \approx 35^\circ$, $\delta_2 \approx -9^\circ$, $\Delta \approx 44^\circ$, in better agreement with the experimental value $\Delta^{exp} = (48 \pm 4)^\circ$ [22].

In our approach the parameters ξ_i in Eq. (9) are treated as phenomenological (μ-independent) parameters to be fixed from the experimental data. They can be related to the μ-dependent QCD predicted $\xi_i(\mu)$ with the help of some μ-dependent B_i-factor defined as

$$\xi_i^{ph} = \xi_i(\mu) B_i(\mu).$$

Table 2 shows the QCD predictions for the coefficients $\xi_i(\mu) = \xi_i^{(z)}(\mu) + \tau\xi_i^{(y)}(\mu)$ which correspond to the Wilson coefficients

$$c_i(\mu) = z_i(\mu) + \tau y_i(\mu), \quad \tau = -\frac{V_{td}V_{ts}^*}{V_{ud}V_{us}^*}\,,$$

from the table XVIII of Ref. [1] calculated numerically from perturbative QCD at $\mu = 1$ GeV for $m_t = 170$ GeV in leading (LO) and next-to-leading order in different renormalization schemes (NDR and HV). $\xi_i^{(z)}$ and $\xi_i^{(y)}$ were obtained from z_i and y_i, respectively, using the Eqs. (2) and (4).

As we cannot calculate the factors $B_i(\mu)$ theoretically, they can only be fixed from data in the spirit of the semi-phenomenological approach [1, 4, 7]. Table 1 shows that the amplitudes of $K \to 2\pi$ decays are dominated by the contribution of the operators $\mathcal{O}_i$ with $i = 1,2,3,4,5,8$. Moreover, in case of the operators $\mathcal{O}_{1,2,3}$, the first term in the combination $(-\xi_1 + \xi_2 + \xi_3)$ dominates in the effective weak meson lagrangian (3). Thus, the isotopic amplitudes can be given in the approximation of the dominating contributions of four-quark operators as

$$\begin{aligned} \mathcal{A}_I &= \mathcal{A}_I^{(z)} + \tau \mathcal{A}_I^{(y)}, \\ \mathcal{A}_I^{(z,y)} &= \Big[-\xi_1^{(z,y)}(\mu) + \xi_2^{(z,y)}(\mu) + \xi_3^{(z,y)}(\mu) \Big] B_1(\mu)\, \mathcal{A}_I^{(1)} + \xi_4^{(z,y)}(\mu) B_4(\mu)\, \mathcal{A}_I^{(4)} \\ &\quad + \xi_5^{(z,y)}(\mu) B_5(\mu)\, \mathcal{A}_I^{(5)} + \xi_8^{(z,y)}(\mu) B_8(\mu)\, \mathcal{A}_I^{(8)} \Big], \end{aligned} \tag{12}$$

and

$$A_I = \left(a_I^{(z)} + \tau a_I^{(y)}\right) e^{i\delta_I}.$$

At least two factors B_1 and B_4 can be estimated from the experimental values $\mathcal{A}_0^{exp} \approx 10.9$ and $\mathcal{A}_2^{exp} \approx 0.347$ while the other two (penguin) factors B_5 and B_8 should be fixed from other data or restricted by theoretical arguments.

The parameter ε' of direct CP-violation in $K \to 2\pi$ decays can be expressed by the formulae

$$\varepsilon' = -\frac{\omega}{\sqrt{2}} \frac{\mathrm{Im}\, a_0}{\mathrm{Re}\, a_0} (1 - \Omega)\, e^{i(\pi/2 + \delta_2 - \delta_0)}, \quad \omega = \frac{\mathrm{Re}\, a_2}{\mathrm{Re}\, a_0}, \quad \Omega = \frac{1}{\omega} \frac{\mathrm{Im}\, a_2}{\mathrm{Im}\, a_0},$$

and the ratio ε'/ε can be estimated as

$$\varepsilon'/\varepsilon = \mathrm{Im}\,\lambda_t\, (P_0 - P_2), \quad P_I = 65.06\, a_I^{(y)}/a_I^{(z)}, \tag{13}$$

with $\mathrm{Im}\,\lambda_t = \mathrm{Im}\, V_{ts}^* V_{td} = |V_{ts}||V_{td}| \sin\delta$ in the standard parameterization of the CKM matrix. In our estimates for ε'/ε we will use the restriction [1, 7]

$$0.86 \cdot 10^{-4} \leq \mathrm{Im}\,\lambda_t \leq 1.71 \cdot 10^{-4} \tag{14}$$

obtained from the phenomenological analysis of indirect CP violation in $K \to 2\pi$ decay and $B^0 - \overline{B}^0$ mixing.

Table 3 gives the estimates of ε'/ε from a semi-phenomenological approach obtained after fixing the correction factors B_1 and B_4 for isotopic amplitudes in the representation (12) by experimental (CP-conserving) data on $\mathrm{Re}\,\mathcal{A}_{0,2}$, and setting $B_5 = B_8 = 1$. We have used the matrix elements of the operators $\mathcal{O}_i$ displayed in table 1, and the theoretical values $\xi_i(\mu)$ from table 2. Taking into account the dependence of the results on the renormalization scheme, we have obtained without reducing resonances (see table 1a) the following upper and lower bounds for ε'/ε, corresponding to the interval (14) for $\mathrm{Im}\,\lambda_t$:

$$-4.7 \cdot 10^{-4} \leq \varepsilon'/\varepsilon \leq -0.4 \cdot 10^{-4}.$$

The peculiarity of these results lies in the observation that all estimates of ε'/ε lead to negative values. This is related to the fact that in the case corresponding to table 1a the contribution of gluonic penguins to $\Delta I = 1/2$ transitions appears to be suppressed, leading, after the interplay between gluonic and electromagnetic penguins, to the relation $P_0 < P_2$ for the two competing terms in (13). Generally speaking, $\Delta I = 1/2$ transitions loose importance compared to $\Delta I = 3/2$ when estimating ε'/ε. The situation changes after the reduction of resonances, due to a relative enhancement of the matrix element for the operator $\mathcal{O}_5$ (see table 1b). The bounds obtained in this case are

$$-3.2 \cdot 10^{-4} \leq \varepsilon'/\varepsilon \leq 2.7 \cdot 10^{-4},$$

that substantially agrees with the bounds given by [1, 7].

So, our calculations have shown that especially the penguin matrix elements are most sensitive to various refinements of the theory; here they are determined in leading order $O(p^2)$ by the structure constants L_4, L_5 and L_8. The current experimental status of the effective chiral lagrangian at $O(p^4)$ has been discussed in some details in [23] where the following phenomenological estimates for L_i are given:

$$L_4^{exp} = (-0.3 \pm 0.5) \cdot 10^{-3}, \quad L_5^{exp} = (1.4 \pm 0.5) \cdot 10^{-3}, \quad L_8^{exp} = (0.9 \pm 0.3) \cdot 10^{-3}.$$

It should be added that the modification of penguin matrix elements, discussed in this note, is much more important for gluonic than for electromagnetic penguin transitions. This is obvious from the observation that the latter at lowest order contain terms of $O(p^0)$ which are left unchanged when taking into account the additional terms derived from the quark determinant at $O(p^4)$.

Finally, we give some results concerning the dependence of the above semi-phenomenological estimates for ε'/ε on the choice of the penguin correction factors B_5 and B_8. In table 4 we present the B_5-dependence of ε'/ε estimated with reducing meson resonances for the central value $\mathrm{Im}\,\lambda_t = 1.29 \cdot 10^{-4}$ and $B_8 = 1$. The corresponding B_8-dependence of ε'/ε ($B_5 = 1$) is given in table 5. The sensitivity of these dependences to a small ($\sim 5\%$) uncertainty from the phenomenological fixation of $<\bar{q}q>^{1/3}$ is also shown in figures 1 and 2.

Since our results are very sensitive to the relative contribution of the gluonic penguin operator, the question of its phenomenological separation in $K \to 2\pi$ decays becomes critical, in the context of the $\Delta I = 1/2$ rule as well as for the very important problem of direct CP-violation, where at least one experimental result [24] would lead to some revision of the present picture. CP-conserving $K \to 2\pi$ data alone are clearly not sufficient for such a separation. It could be accomplished, on the other hand, when taking into account Dalitz-plot data for $K \to 3\pi$ as well as differential distributions for radiative decays $K \to 2\pi\gamma$, $K \to \pi 2\gamma$, which are described by the same lagrangian (1). The reason for this possibility is found in the difference in momentum power counting behaviour between penguin and non-penguin matrix elements, as emphasized above, which appears in higher orders of chiral theory, when calculating various parameters of differential distributions, for instance, slope parameters of Dalitz-plot for $K \to 3\pi$. A substantial improvement in the accuracy of such experimental data (mostly being of older dates) would be very helpful for such a phenomenological improvement of the theoretical situation for ε'/ε (see [9] for a discussion of this point and [25, 26] for some recent measurements). Of course, for all these model developments the new experiments at CERN, FNAL and planned at Frascati have to be considered as crucial if the accuracy level of 10^{-4} for ε'/ε is obtained.

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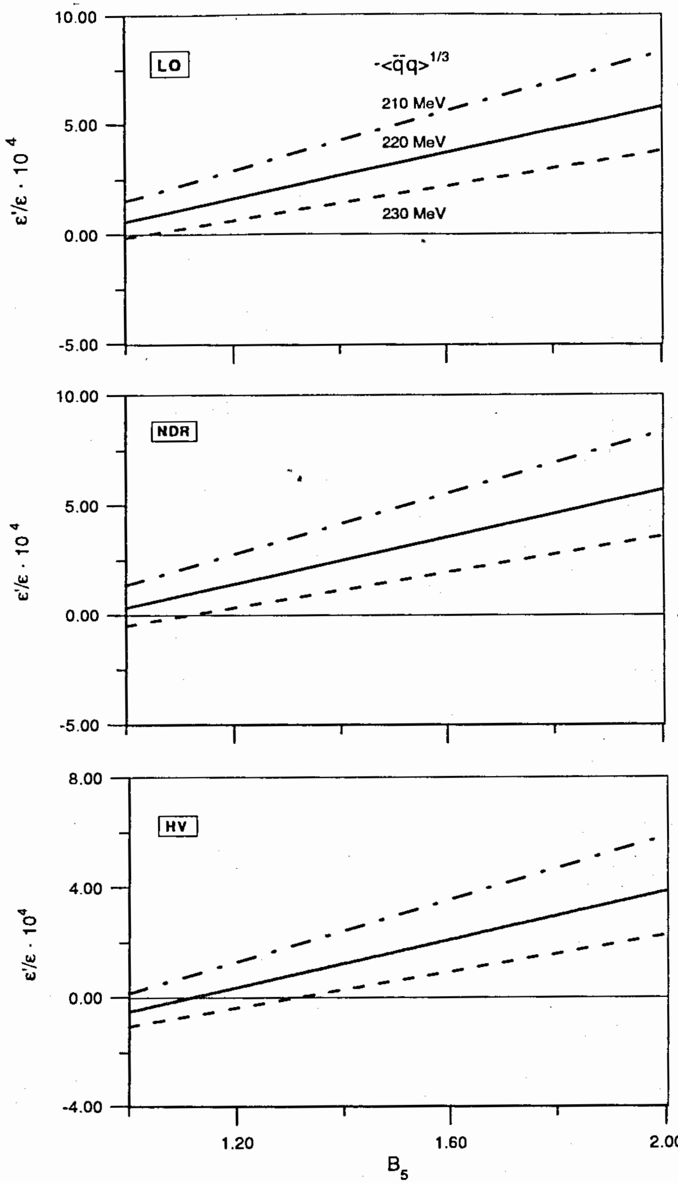

Figure 1. B_5-dependence of ε'/ε for different values of the quark condensate $\Lambda^{(4)}_{\overline{MS}} = 325$ MeV.

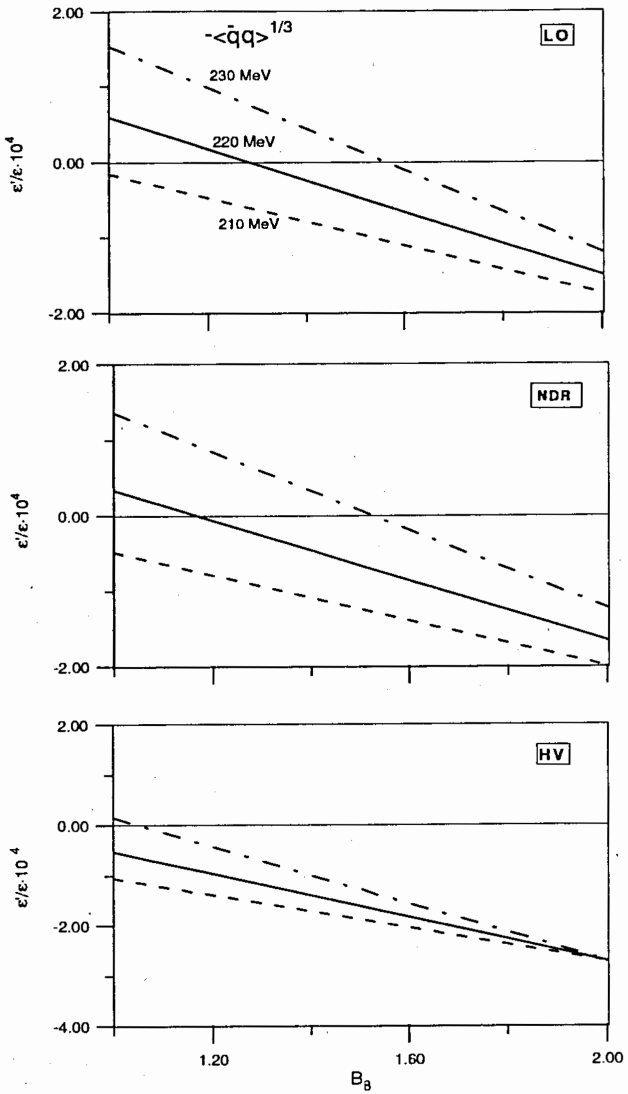

Figure 2. B_8-dependence of ε'/ε for different values of the quark condensate $\Lambda^{(4)}_{\overline{MS}} = 325$ MeV.

Table 1. Isotopic amplitudes of the $K \to 2\pi$ decays with successive inclusion of higher-order corrections.

a) Without reduction of meson resonances:

		$\mathcal{O}_1$	$\mathcal{O}_2$	$\mathcal{O}_3$	$\mathcal{O}_4$	$\mathcal{O}_5$	$\mathcal{O}_6$	$\alpha\mathcal{O}_7$	$\alpha\mathcal{O}_8$
$O(p^2)$	$\mathrm{Re}\mathcal{A}_0^{(i)}$	–1.000	1.000	1.000	0.000	–6.833	0.000	0.016	1.015
	$\mathrm{Re}\mathcal{A}_2^{(i)}$	0.000	0.000	0.000	1.000	0.000	0.000	–0.016	0.454
$O(p^4)$	$\mathrm{Re}\mathcal{A}_0^{(i)}$	–1.184	1.173	1.090	0.021	–7.450	0.003	0.016	1.141
	$\mathrm{Im}\mathcal{A}_0^{(i)}$	–0.482	0.482	0.482	0.000	–3.265	0.000	0.008	0.439
	$\mathrm{Re}\mathcal{A}_2^{(i)}$	–0.005	0.016	0.035	0.867	–0.088	–0.003	–0.015	0.433
	$\mathrm{Im}\mathcal{A}_2^{(i)}$	0.000	0.000	0.000	–0.213	0.000	0.000	0.003	–0.043
$O(p^6)$	$\mathrm{Re}\mathcal{A}_0^{(i)}$	–1.014	1.003	0.888	0.022	–6.811	0.003	0.012	0.991
	$\mathrm{Im}\mathcal{A}_0^{(i)}$	–0.707	0.709	0.682	0.000	–3.662	–0.001	0.011	0.599
	$\mathrm{Re}\mathcal{A}_2^{(i)}$	–0.004	0.016	0.035	0.812	–0.085	–0.003	–0.015	0.408
	$\mathrm{Im}\mathcal{A}_2^{(i)}$	0.000	0.001	0.001	–0.114	0.003	0.000	0.002	–0.043

b) After reduction of resonances:

		$\mathcal{O}_1$	$\mathcal{O}_2$	$\mathcal{O}_3$	$\mathcal{O}_4$	$\mathcal{O}_5$	$\mathcal{O}_6$	$\alpha\mathcal{O}_7$	$\alpha\mathcal{O}_8$
$O(p^2)$	$\mathrm{Re}\mathcal{A}_0^{(i)}$	–1.000	1.000	1.000	0.000	–11.248	0.000	0.016	1.325
	$\mathrm{Re}\mathcal{A}_2^{(i)}$	0.000	0.000	0.000	1.000	0.000	0.000	–0.016	0.574
$O(p^4)$	$\mathrm{Re}\mathcal{A}_0^{(i)}$	–1.195	1.187	1.101	0.021	–13.176	0.002	0.016	1.369
	$\mathrm{Im}\mathcal{A}_0^{(i)}$	–0.482	0.482	0.482	0.000	–5.398	0.000	0.008	0.328
	$\mathrm{Re}\mathcal{A}_2^{(i)}$	–0.006	0.014	0.035	0.879	–0.148	-0.002	–0.015	0.531
	$\mathrm{Im}\mathcal{A}_2^{(i)}$	0.000	0.000	0.000	–0.213	0.000	0.000	0.003	–0.030
$O(p^6)$	$\mathrm{Re}\mathcal{A}_0^{(i)}$	–0.974	0.967	0.798	0.024	–10.665	0.002	0.010	1.165
	$\mathrm{Im}\mathcal{A}_0^{(i)}$	–0.643	0.646	0.619	0.000	–5.232	–0.001	0.010	0.509
	$\mathrm{Re}\mathcal{A}_2^{(i)}$	-0.005	0.014	0.036	0.779	–0.139	–0.002	0.015	0.491
	$\mathrm{Im}\mathcal{A}_2^{(i)}$	0.000	0.001	0.001	–0.126	0.008	0.000	0.002	–0.044

Table 2. QCD predictions for the parameters $\xi_i(\mu) = \xi_i^{(z)}(\mu) + \tau \xi_i^{(y)}(\mu)$ calculated with Wilson coefficients $c_i(\mu) = z_i(\mu) + \tau y_i(\mu)$ at $\mu = 1$ GeV for $m_t = 170$ GeV [1].

	$\Lambda_{\overline{MS}}^{(4)} = 215$ MeV			$\Lambda_{\overline{MS}}^{(4)} = 325$ MeV			$\Lambda_{\overline{MS}}^{(4)} = 435$ MeV		
	LO	**NDR**	**HV**	**LO**	**NDR**	**HV**	**LO**	**NDR**	**HV**
$\xi_1^{(z)}$	-1.286	-1.061	-1.165	-1.443	-1.159	-1.325	-1.624	-1.270	-1.562
$\xi_2^{(z)}$	0.187	0.195	0.198	0.172	0.176	0.182	0.157	0.150	0.165
$\xi_3^{(z)}$	0.129	0.143	0.137	0.122	0.137	0.130	0.115	0.131	0.121
$\xi_4^{(z)}$	0.645	0.714	0.687	0.609	0.684	0.650	0.573	0.654	0.599
$\xi_5^{(z)}$	-0.008	-0.020	-0.008	-0.012	-0.032	-0.013	-0.016	-0.056	-0.023
$\xi_6^{(z)}$	0.000	-0.003	0.000	-0.001	-0.007	-0.001	-0.002	-0.021	-0.007
$\xi_7^{(z)}/\alpha$	0.002	0.003	-0.001	0.004	0.008	0.001	0.006	0.015	0.032
$\xi_8^{(z)}/\alpha$	0.000	0.002	0.001	0.001	0.004	0.002	0.001	0.009	0.067
$\xi_1^{(y)}$	0.044	0.038	0.048	0.054	0.048	0.053	0.065	0.060	0.069
$\xi_2^{(y)}$	-0.028	-0.029	-0.030	-0.029	-0.033	-0.030	-0.030	-0.033	-0.030
$\xi_3^{(y)}$	-0.002	-0.002	0.001	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002
$\xi_4^{(y)}$	-0.009	-0.010	0.004	-0.008	-0.009	-0.009	-0.008	-0.009	-0.008
$\xi_5^{(y)}$	-0.081	-0.076	-0.067	-0.109	-0.111	-0.092	-0.143	-0.173	-0.132
$\xi_6^{(y)}$	-0.033	-0.042	-0.021	-0.049	-0.076	-0.033	-0.071	-0.139	-0.051
$\xi_7^{(y)}/\alpha$	0.033	0.004	0.006	0.044	0.013	0.016	0.057	0.027	0.032
$\xi_8^{(y)}/\alpha$	0.031	0.028	0.031	0.043	0.041	0.045	0.058	0.061	0.067

Table 3. Predictions for the parameters of $K \to 2\pi$ decays in the semi-phenomenological approach ($B_5 = B_8 = 1$). The ratio ε'/ε is given in units 10^{-4}.

a) Without reduction of meson resonances:

	$\Lambda_{\overline{MS}}^{(4)} = 215$ MeV			$\Lambda_{\overline{MS}}^{(4)} = 325$ MeV			$\Lambda_{\overline{MS}}^{(4)} = 435$ MeV		
	LO	**NDR**	**HV**	**LO**	**NDR**	**HV**	**LO**	**NDR**	**HV**
B_1	5.52	6.29	5.91	5.08	5.92	5.39	4.63	5.53	4.75
B_4	0.65	0.59	0.62	0.69	0.61	0.65	0.74	0.64	0.70
P_0	0.66	0.22	-0.36	1.72	1.35	0.77	3.13	3.87	2.47
P_2	1.50	1.29	2.40	2.48	2.42	2.33	3.74	4.35	3.83
$(\varepsilon'/\varepsilon)_{min}$	-1.4	-1.8	-4.7	-1.3	-1.8	-2.7	-1.0	-0.8	-2.3
$(\varepsilon'/\varepsilon)_{max}$	-0.7	-0.9	-2.4	-0.7	-0.9	-1.4	-0.5	-0.4	-1.2

b) After reduction of resonances:

	$\Lambda_{\overline{MS}}^{(4)} = 215$ MeV			$\Lambda_{\overline{MS}}^{(4)} = 325$ MeV			$\Lambda_{\overline{MS}}^{(4)} = 435$ MeV		
	LO	**NDR**	**HV**	**LO**	**NDR**	**HV**	**LO**	**NDR**	**HV**
B_1	5.86	6.65	6.26	5.38	6.25	5.71	4.90	5.79	5.00
B_4	0.68	0.61	0.64	0.72	0.64	0.67	0.76	0.66	0.73
P_0	1.95	1.44	0.69	3.45	3.14	2.22	5.42	6.67	4.56
P_2	1.82	1.61	2.57	2.90	2.88	2.59	4.29	5.12	4.22
$(\varepsilon'/\varepsilon)_{min}$	0.1	-0.3	-3.2	0.5	0.2	-0.6	1.0	1.3	0.3
$(\varepsilon'/\varepsilon)_{max}$	0.2	-0.1	-1.6	1.0	0.4	-0.3	1.9	2.7	0.6

Table 4. Dependence of the predictions for the ratio ε'/ε (in units 10^{-4}) on the factor B_5 ($B_8 = 1$).

	$\Lambda^{(4)}_{\overline{MS}} = 215$ MeV			$\Lambda^{(4)}_{\overline{MS}} = 325$ MeV			$\Lambda^{(4)}_{\overline{MS}} = 435$ MeV		
B_5	LO	NDR	HV	LO	NDR	HV	LO	NDR	HV
1.0	0.08	-0.20	-2.43	0.59	0.34	-0.52	1.32	2.00	0.38
1.1	0.46	0.16	-2.12	1.11	0.87	-0.08	2.00	2.84	1.02
1.2	0.85	0.53	-1.80	1.63	1.41	0.35	2.68	3.67	1.66
1.3	1.23	0.89	-1.48	2.15	1.94	0.79	3.36	4.51	2.30
1.4	1.62	1.26	-1.16	2.66	2.48	1.23	4.05	5.34	2.94
1.5	2.01	1.63	-0.84	3.18	3.01	1.67	4.73	6.17	3.58
1.6	2.39	1.99	-0.52	3.70	3.55	2.10	5.41	7.00	4.22
1.7	2.78	2.36	-0.21	4.22	4.08	2.54	6.09	7.83	4.86
1.8	3.16	2.72	0.11	4.73	4.61	2.98	6.77	8.66	5.50
1.9	3.55	3.09	0.43	5.25	5.15	3.41	7.45	9.49	6.14
2.0	3.93	3.46	0.75	5.77	5.68	3.85	8.13	10.31	6.78

Table 5. Dependence of the predictions for the ratio ε'/ε (in units 10^{-4}) on the factor B_8 ($B_5 = 1$).

	$\Lambda^{(4)}_{\overline{MS}} = 215$ MeV			$\Lambda^{(4)}_{\overline{MS}} = 325$ MeV			$\Lambda^{(4)}_{\overline{MS}} = 435$ MeV		
B_8	LO	NDR	HV	LO	NDR	HV	LO	NDR	HV
1.0	0.08	-0.21	-2.43	0.59	0.34	-0.52	1.32	2.00	0.38
1.1	-0.08	-0.34	-2.58	0.39	0.14	-0.74	1.03	1.71	0.09
1.2	-0.23	-0.48	-2.73	0.18	-0.06	-0.96	0.75	1.41	-0.20
1.3	-0.38	-0.61	-2.88	-0.03	-0.26	-1.18	0.47	1.11	-0.49
1.4	-0.53	-0.75	-3.04	-0.24	-0.46	-1.40	0.18	0.82	-0.77
1.5	-0.68	-0.88	-3.19	-0.45	-0.66	-1.62	-0.10	0.52	-1.05
1.6	-0.83	-1.02	-3.34	-0.66	-0.86	-1.83	-0.39	0.23	-1.33
1.7	-0.98	-1.15	-3.49	-0.87	-1.06	-2.05	-0.67	-0.07	-1.61
1.8	-1.14	-1.29	-3.64	-1.08	-1.26	-2.27	-0.95	-0.37	-1.88
1.9	-1.29	-1.42	-3.79	-1.29	-1.45	-2.49	-1.24	-0.66	-2.15
2.0	-1.44	-1.56	-3.94	-1.50	-1.65	-2.71	-1.52	-0.95	-2.42

Appendix A

At $O(p^6)$ one gets [3]

$$\begin{aligned}
\mathcal{L}^{(p^6)}_{eff} \Rightarrow \mathrm{tr}\Big\{ & Q_{12}\Big(\chi R^\mu U^\dagger(D_\mu D_\nu U + D_\nu D_\mu U)U^\dagger L^\nu \\
& +\chi^\dagger L^\mu U(\overline{D}_\mu\overline{D}_\nu U^\dagger + \overline{D}_\nu\overline{D}_\mu U^\dagger)UR^\nu\Big) \\
& +Q_{13}\Big[\chi(\overline{D}_\mu\overline{D}_\nu U^\dagger L^\mu L^\nu + R^\nu R^\mu U\overline{D}_\mu\overline{D}_\nu U^\dagger) \\
& +\chi^\dagger(D_\mu D_\nu U R^\mu R^\nu + L^\nu L^\mu D_\mu D_\nu U)\Big] \\
& +Q_{14}\Big[\chi\Big(U^\dagger D_\mu D_\nu U\overline{D}^\mu\overline{D}^\nu U^\dagger + \overline{D}_\mu\overline{D}_\nu U^\dagger D^\mu D^\nu U U^\dagger\Big) \\
& +\chi^\dagger\Big(U\overline{D}_\mu\overline{D}_\nu U^\dagger D^\mu D^\nu U + D_\mu D_\nu U\overline{D}^\mu\overline{D}^\nu U^\dagger U\Big)\Big] \\
& +Q_{15}\chi^\dagger L_\mu\chi R^\mu + Q_{16}\Big(\chi^\dagger\chi R_\mu R^\mu + \chi\chi^\dagger L_\mu L^\mu\Big) \\
& +Q_{17}\Big(U\chi^\dagger U\chi^\dagger L_\mu L^\mu + U^\dagger\chi U^\dagger\chi R_\mu R^\mu\Big) + Q_{18}\Big[(\chi U^\dagger L_\mu)^2 + (\chi^\dagger U R_\mu)^2\Big] \\
& +Q_{19}\Big[(\chi U^\dagger)^3 + (\chi^\dagger U)^3\Big] + Q_{20}\Big(U^\dagger\chi\chi^\dagger\chi + U\chi^\dagger\chi\chi^\dagger\Big)\Big\},
\end{aligned}$$

where Q_i are structure constants introduced in [28], whereas $R_\mu = U^\dagger D_\mu U$. The corresponding terms of $(V \mp A)$ and $(S \mp P)$ bosonized meson currents are given by

$$\begin{aligned}
J^{(p^5)a}_{L\mu} \Rightarrow i\frac{1}{4}\mathrm{tr}\Big\{\lambda^a\Big[& -2Q_{14}\Big[(U\chi^\dagger + \chi U^\dagger)D_\mu D_\nu U\,U^\dagger L^\nu + D_\mu D_\nu U(U^\dagger\chi U^\dagger + \chi^\dagger)L^\nu \\
& -U\overline{D}^\nu\Big((U^\dagger\chi + \chi^\dagger U)\overline{D}_\nu\overline{D}_\mu U^\dagger + \overline{D}_\nu\overline{D}_\mu U^\dagger(U\chi^\dagger + \chi U^\dagger)\Big) \\
& +L^\nu U\Big((U^\dagger\chi + \chi^\dagger U)\overline{D}_\mu\overline{D}_\nu U^\dagger + \overline{D}_\mu\overline{D}_\nu U^\dagger(U\chi^\dagger + \chi U^\dagger)\Big) \\
& +D^\nu\Big((U\chi^\dagger + \chi U^\dagger)D_\nu D_\mu U + D_\nu D_\mu U(U^\dagger\chi + \chi^\dagger U)\Big)U^\dagger\Big] \\
& +2Q_{15}(U\chi^\dagger L_\mu\chi U^\dagger + \chi U^\dagger L_\mu U\chi^\dagger) + 2Q_{16}(\{U\chi^\dagger\chi U^\dagger, L_\mu\} + \{\chi\chi^\dagger, L_\mu\}) \\
& +2Q_{17}(\{(U\chi^\dagger)^2, L_\mu\} + \{(\chi U^\dagger)^2, L_\mu\}) \\
& -4Q_{18}(U\chi^\dagger L_\mu U\chi^\dagger + \chi U^\dagger L_\mu\chi U^\dagger)\Big]\Big\},
\end{aligned}$$

and

$$\begin{aligned}
J^{(p^4)a}_{L} \Rightarrow \overline{m}R\,\mathrm{tr}\Big\{\lambda^a\Big[& Q_{12}\,L^\mu U\{\overline{D}_\mu, \overline{D}_\nu\}U^\dagger U R^\nu + Q_{13}(L^\nu L^\mu D_\mu D_\nu U + D_\mu D_\nu U\cdot R^\mu R^\nu) \\
& +Q_{14}(U\overline{D}^\nu\overline{D}^\mu U^\dagger D_\nu D_\mu U + D_\nu D_\mu U\overline{D}^\nu\overline{D}^\mu U^\dagger\cdot U)
\end{aligned}$$

[3] The rather lengthy full expression for bosonized effective lagrangian at $O(p^6)$ was presented in refs.[27, 28].

$$+Q_{15}\,L^\mu\chi R_\mu + Q_{16}\left(\chi R_\mu^2 + L_\mu^2\chi\right) + Q_{17}(U\chi^\dagger U R_\mu^2 + L_\mu^2 U\chi^\dagger U)$$
$$+2Q_{18}\,L^\mu U\chi^\dagger L_\mu U + Q_{19}(U\chi^\dagger)^2 U + Q_{20}(\chi U^\dagger\chi + \chi\chi^\dagger U + U\chi^\dagger\chi)\Big]\Big\}.$$

Appendix B

Without reduction of resonance degrees of freedom the structure constants $L_i = N_c/(16\pi^2)\cdot l_i$, $H_2 = N_c/(16\pi^2)\cdot h_i$ and $Q_i = N_c/(32\pi^2\overline{m}^2)\cdot q_i$ are fixed from the bosonization of an NJL-type model as

$$l_1 = \frac{1}{2}l_2 = \frac{1}{24}, \quad l_3 = -\frac{1}{6}, \quad l_4 = 0, \quad l_5 = xy - x,$$
$$l_8 = \frac{1}{2}xy - x^2y - \frac{1}{24}, \quad h_2 = -xy - 2x^2y + \frac{1}{12} + \frac{16\pi^2}{N_c}\frac{xF_0^2}{4\overline{m}^2}, \qquad (15)$$

and

$$q_{12} = \frac{1}{60}, \quad q_{13} = -\frac{1}{3}\left(\frac{1}{20} - x + c\right), \quad q_{14} = \frac{x}{6},$$
$$q_{15} = \frac{2}{3}x(1-x) - \left(\frac{1}{3} - 2x\right)c, \quad q_{16} = -\frac{1}{120} + \frac{4}{3}x^2 + \frac{x}{6}(1-4x) - 2\left(x - \frac{1}{6}\right)c,$$
$$q_{17} = \frac{1}{120} + \frac{x}{6}(1-4x) - \left(x + \frac{1}{6}\right)c, \quad q_{18} = \frac{4}{3}x^2 + \left(\frac{1}{6} - x\right)c,$$
$$q_{19} = -\frac{1}{240} - x^2 + \frac{2}{3}x^3 + x(1+2xy)c,$$
$$q_{20} = \frac{1}{240} + x^2 + 2(1-2y)x^3 - x(1+2xy)c,$$

where $x = -\overline{m}F_0^2/(2<\overline{q}q>)$, $y = 4\pi^2F_0^2/(N_c\overline{m}^2) = 1.5$ and $c = 1 - 1/(6y)$. After reduction of the resonances, the structure coefficients get the form

$$l_1^{red} = \frac{1}{2}l_2^{red} = \frac{1}{12}\left[Z_A^8 + 2(Z_A^4 - 1)\left(\frac{1}{4}\tilde{y}(Z_A^4 - 1) - Z_A^4\right)\right],$$
$$l_3^{red} = -\frac{1}{6}\left[Z_A^8 + 3(Z_A^4 - 1)\left(\frac{1}{4}\tilde{y}(Z_A^4 - 1) - Z_A^4\right)\right],$$
$$l_4^{red} = 0, \quad l_5^{red} = (\tilde{y} - 1)\frac{1}{4}Z_A^6, \quad l_8^{red} = \frac{\tilde{y}}{16}Z_A^4, \quad h_2^{red} = \tilde{y}Z_A^2\left(\frac{Z_A^2}{8} - x\right).$$

and

$$q_{12}^{red} = q_{13}^{red} = 0, \quad q_{14}^{red} = \frac{1}{24}Z_A^6,$$
$$q_{16}^{red} = q_{17}^{red} = -\frac{Z_A^6}{64}\left\{\tilde{y} - Z_A^2\left[4 - 6\left(1 + 4(1 - Z_A^2)\right)(1 - \tilde{y}) + 4\left(1 + 16(1 - Z_A^2)\right)\frac{1 - \tilde{y}}{\tilde{y}}\right]\right\},$$

$$q_{15}^{red} = -2q_{18}^{red} = \frac{1}{48}Z_A^6\left[3\tilde{y} - 2Z_A^2\left(5 - 12(1 - Z_A^2)\frac{(1-\tilde{y})^2}{\tilde{y}}\right)\right],$$
$$q_{19}^{red} = \frac{1}{3}q_{18}^{red} = -\frac{1}{192}Z_A^6(3\tilde{y} - 2),$$

where $\tilde{y} = 4\pi^2F_0^2/(Z_A^2N_c\overline{m}^2) = 2.4$, and $Z_A^2 = 0.62$ is the $\pi - A_1$ mixing factor.

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Результаты по распадам $K \to 2\pi$ в p^6-порядке и ε'/ε в подходе эффективных киральных лагранжианов

В рамках полуфеноменологического подхода анализируются результаты объединения новых систематических вычислений мезонных матричных элементов в p^6-порядке киральной теории и вильсоновских коэффициентов из работы [1], полученных в рамках пертурбативной КХД. Полные выражения для амплитуд распадов $K \to 2\pi$ приводят к новым оценкам для ε'/ε.

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Bel'kov A.A. et al. E2-97-139

Results on $K \to 2\pi$ Decays at $O(p^6)$ and ε'/ε from an Effective Chiral Lagrangian Approach

We have combined a new systematic calculation of mesonic matrix elements at $O(p^6)$ from an effective chiral lagrangian approach using Wilson coefficients taken from [1], derived in the framework of perturbative QCD, and restricted partly by experimental data. We derive complete expressions for $K \to 2\pi$ amplitudes and give new estimates for ε'/ε.

The investigation has been performed at the Laboratory of Particle Physics, JINR.

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