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Search for stable solutions of nonlinear relativistic equations in three-dimensional space is, by now, of great interest, because such solutions can be interpreted as the classical models of extended particles. An attractive possibility of long-lived pulsed meson "bubbles" to exist has been demonstrated in ref. [1], where the bubble dynamics was described by the equation of Higgs field in spherically-symmetric (SS) geometry

$$u_{tt} - \Delta_{rr} u - m^2 u + g^2 u^3 = 0. \qquad (1)$$

This model becomes especially interesting if one takes into account that the quasi-plane solitons

$$u = \frac{m}{g} \mathrm{th}\,(m \frac{r - R_0}{\sqrt{2}}), \quad R_0 \gg \ell \sim \frac{1}{m}, \qquad (2)$$

which describe the initial states of bubbles, are apparently stable with respect to transverse (angular) perturbations unlike some other types of relativistic solitons [2]. Unfortunately, for the model under consideration one fails to estimate analytically the energy radiation rate and therefore the bubble life-time. Besides, it remained to be unclear whether the bubble comes back to initial soliton state (2) (or to close one) after reflection from the center*.

We investigate in present paper the dynamics of bubbles via computer solving of Eq. (1) using the grid method with finite-difference scheme conserving energy integral

$$E = \int_0^{r_m} \mathcal{H} dr, \quad \mathcal{H} = \frac{r^2}{2}\{(u_t)^2 + (u_r)^2 + \frac{1}{2}(u^2 - 1)^2\} \qquad (3)$$

with a very high accuracy when boundary conditions $u(r_m) = m/g$ "lock" wave energy inside the sphere of $r \le r_m = 2R_0$. Further wave radiation out of this sphere is allowed; the total energy flux $Q(t) = -r^2 u_t u_r$ at $r = r_m$ and functions $E(t)$ and $\mathcal{H}(r,t)$ are calculated during the computations. It is convenient to use variables r, t where $m = g = 1$**.

As it is well seen from Fig. 1 the bubble evolution depends essentially on R_0. The most regular picture of the bubble reflection occurs at $R_0 = 8$; here in two cycles of compression-expansion the bubble returns with a good accuracy to the initial soliton

*The computational technique used in [1] has led to a nonphysical energy dissipation and has not allowed the authors to answer these questions.

**These variables r, t relate to that of paper[1] r', t' as $r = 2r'$ and $t = 2t'$.

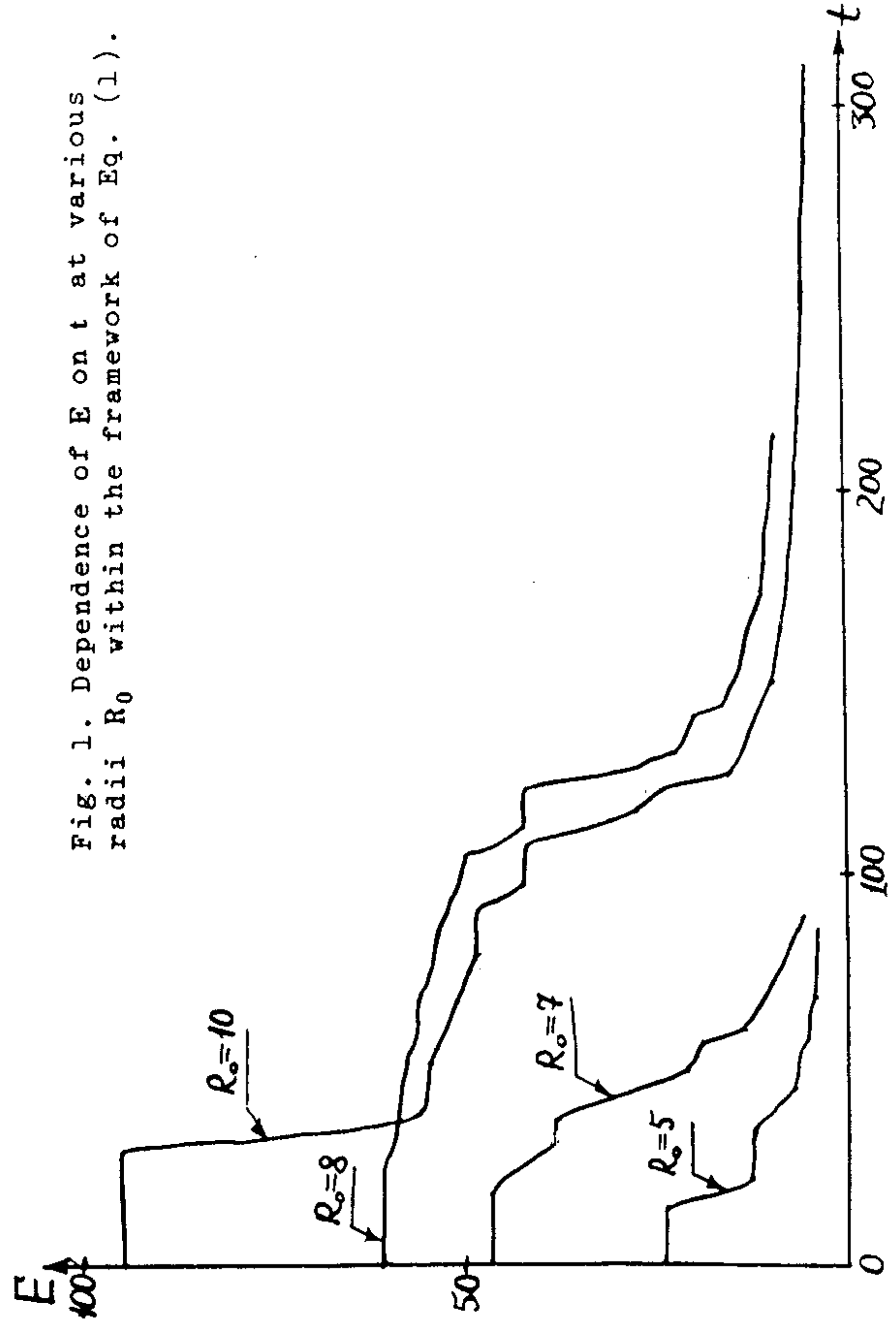

Fig. 1. Dependence of E on t at various radii R_0 within the framework of Eq. (1).

(2) state; in the following three pulsations return becomes not so full. Beginning from the third cycle separation of more and more energy portions from the main energy cluster takes place, and the flow $Q(t)$ out of the sphere increases. After the sixth reflection ($t \sim 110$) the cluster is divided into several spherical layers and some of them move to the boundary r_m and carry out of the region $r \le r_m$ about a half of energy $E(t)$ localized in it after five pulsations. The ratio $T/R_0 \sim 15$ which takes place at $R_0 = 8$ is apparently near maximum possible one which can be reached at varying R_0 (here T is a life-time of discussed pulsed solutions at initial data (2)).

At $R_0 = 5, 7, 10$ and 15 yet after the first reflection energy dividing occurs into separate spherical layers and one does not observe the return of the system to the initial soliton (2) state. A considerably less energy in comparison with initial one is drawn in the second compression-expansion cycle. The quantity $E(t)$ decreases quickly because of the strong radiation beginning from $t_{rad} \simeq 3R_0$. The most characteristic momenta of the bubble evolution at $R_0 = 10$ are shown in Fig. 2. At the time $t = 200$ we have $E(t) \approx 0.1 \times E(0)$ and the flux $Q(t)$ returns to be small. The "one-scaled" cluster of energy localized near the center (with characteristic half-width L being of order of its radius R_1) is formed which is described by nonstationary solution $u(r,t)$, oscillating near the vacuum solution $u_0 = 1$ (see Fig. 2, g,h). The life-time of this state ($T_1 \approx -E/(dE/dt)$) is of order of hundred values of its radius R_1. The question

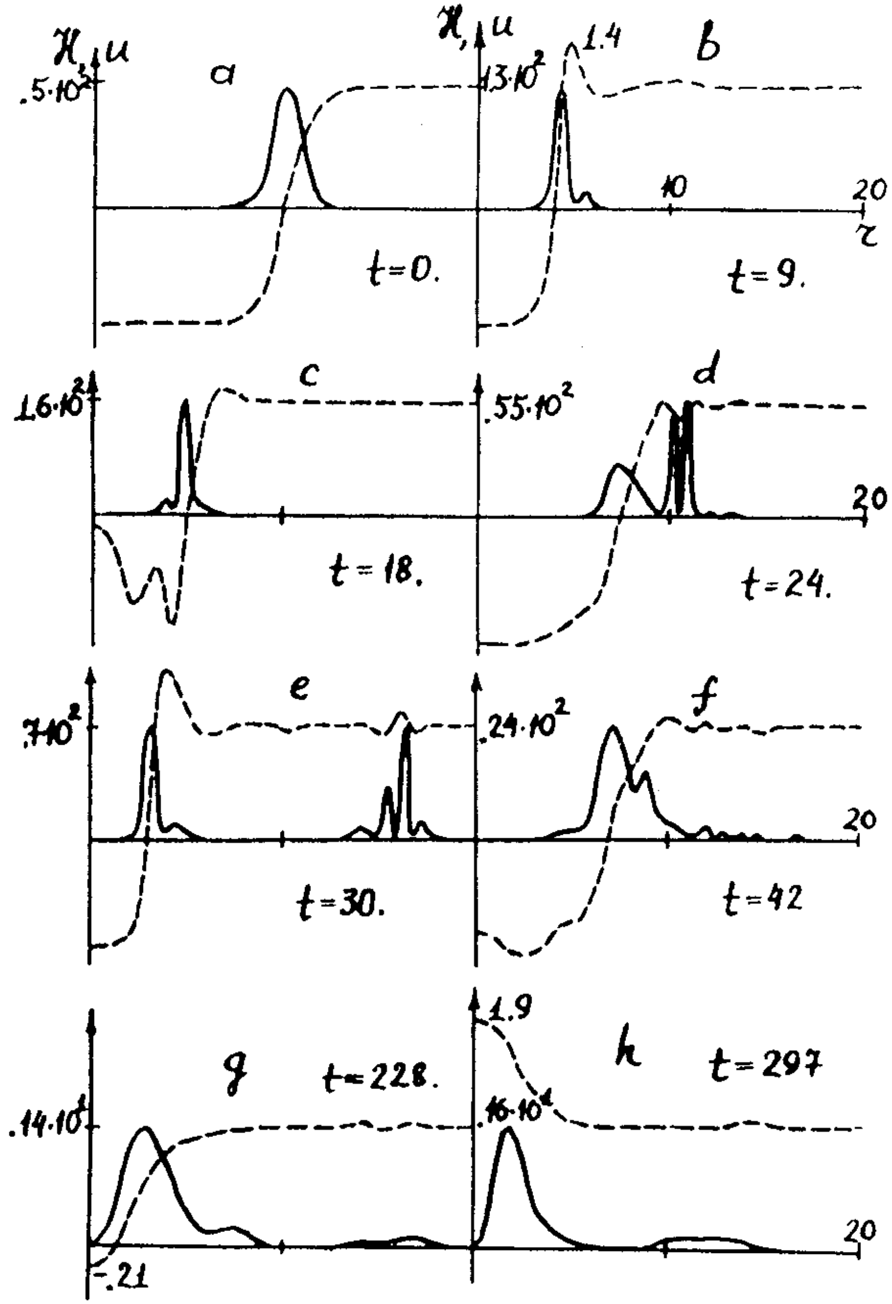

Fig. 2. The typical evolution picture of the bubbles ($R_0 = 10$).

if such oscillating solutions are of interest as models of mesons and "bags" for quark confinement [3] is now unclear.

Further we studied the evolution of soliton-like initial data, $u = 4\,\mathrm{arctg}\{\exp(r - R_0)\}$, $R_0 \gg 1$, within the framework of sine-Gordon equation in SS-geometry, trying to find some three-dimensional analogues of particular properties of this equation which it possesses in plane one-dimensional (x,t) case [4] (we mean the elastic interactions of solitons without radiation which are connected with complete integrability of this equation). In particular, if the radiation was forbidden in SS-geometry then it could lead to the infinite duration of strictly periodical bubble pulsations. Our computations at $R_0=12$ have found, however, the intense radiation just after the first reflection from the center (the evolution of energy clusters coincides qualitatively with that represented in Fig. 1). The energy of system has decreased more than twice by the moment $t = 88$. No principal difference has been observed between the bubble evolutions within the framework of Eq. (1) and sine-Gordon one. Thus we have not found in our computer experiments any extraordinary properties of sine-Gordon equation.

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