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RADIATIVE TAIL IN π_{e2} DECAY AND SOME COMMENTS TO μ—e UNIVERSALITY

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As a first step in calculation of the spectra of radiative pion decays we reproduce the results obtained by Berman and Kinoshita [1], considering the pion as a point–like particle. In paper of T. Kinoshita [1] was calculated the positron energy spectrum in radiative pion decay:

$$\frac{d\Gamma}{\Gamma_0 dy} = \frac{\alpha}{\pi}[\frac{1+y^2}{1-y}(L-1) - 1 + y - \frac{1}{2}(1-y)\ln(1-y) + \frac{1+y^2}{1-y}\ln y], \quad (1)$$

$$y_{min} \le y \le 1 + \frac{m_e^2}{m_\pi^2},$$

where $y = \frac{2\varepsilon_e}{m_\pi}$ is the positron's energy fraction,ε_e is its energy(we imply here and below the reference frame of resting pion), $L = \ln\frac{m_\pi}{m_e} = 5.6$ is *large logarithm, m_π*, m_e are the masses of pion and positron. The quantity

$$\Gamma_0 = \frac{G^2|V_{u\,d}|^2}{8\pi} f_\pi^2 m_e^2 m_\pi (1 - \frac{m_e^2}{m_\pi^2})^2 = 2.53 \cdot 10^{-14}\mathrm{MeV}, \quad (2)$$

is the total width of π_{e2} decay, calculated in the born approximation.

We will calculate now the photon spectrum. Consider first the emission of the soft real photon. Corresponding contribution to the total width may be obtained by standard integration of the differential ones:

$$\frac{d\Gamma^{soft}}{\Gamma_0} = -\frac{\alpha}{4\pi^2}\int \frac{d^3k}{\omega}\left(\frac{P}{Pk} - \frac{p_e}{p_e k}\right)^2_{\omega \le \Delta\varepsilon \ll \frac{m_\pi}{2}}, \quad (3)$$

where P, p_e, k are the four–momenta of pion, positron and photon respectively, $P^2 = m_\pi^2$, $p_e^2 = m_e^2$, $k^2 = \lambda^2$, and λ is the photon mass. Result has the form

$$\frac{\Gamma^{soft}}{\Gamma_0} = \frac{\alpha}{\pi}[-b(\sigma)\ln\frac{2\Delta\varepsilon}{\lambda} + 1 - \frac{1+\sigma}{2(1-\sigma)}\ln\sigma - \frac{1+\sigma}{4(1-\sigma)}\ln^2\sigma - \frac{1+\sigma}{1-\sigma}\mathrm{Li}_2(1-\sigma)], \quad (4)$$

where

$$b(\sigma) = \frac{1+\sigma}{1-\sigma}\ln\sigma + 2, \quad \sigma = \frac{m_e^2}{m_\pi^2}, \quad \mathrm{Li}_2(x) = -\int_0^x \frac{dt}{t}\ln(1-t). \quad (5)$$

Consider now the hard photon emission process

$$\pi^+(P) \to e^+(p_e) + \nu_e(p_\nu) + \gamma(k). \quad (6)$$

Standard procedure of final–states summing of the squared in modulo of its matrix element and integration on neutrino phase volume leads to spectral distribution on photon energy fraction $x = 2k^0/m_\pi$:

$$\frac{d\Gamma}{\Gamma_0 dx} = \frac{\alpha}{2\pi}\frac{x(1-x-\sigma)}{(1-\sigma)^2}\Big[-\frac{4(1-\sigma)}{x^2} - \frac{1}{1-x} + \frac{1}{x(1-x-\sigma)}[\frac{1}{x}(1+(1-x)^2) + 2\sigma - 2\frac{\sigma^2}{x}]\ln\frac{1-x}{\sigma}\Big]. \quad (7)$$

Further integration of this spectrum give the result:

$$\int\limits_{x_{min}}^{1-\sigma}\frac{d\Gamma}{\Gamma_0 dx}dx = \frac{\alpha}{2\pi}[-2b(\sigma)\ln\frac{1-\sigma}{x_{min}} - 2\frac{1+\sigma}{1-\sigma}\mathrm{Li}_2(1-\sigma) + \frac{3(1-2\sigma)}{2(1-\sigma)^2}\ln\sigma + \frac{19-25\sigma}{4(1-\sigma)}], \quad x_{min} = \frac{2k^0_{min}}{m_\pi}. \quad (8)$$

Putting in this formula $k^0_{min} = \Delta\varepsilon$ and adding the soft photon's contribution we obtain(in agreement with the T. Kinoshita 1959 year result) the contribution to the width from the inner bremsstrahlung of point–like pion[1]:

$$\frac{\Gamma_{IB}}{\Gamma_0} = \frac{\alpha}{\pi}[b(\sigma)[\ln\frac{\lambda}{m_\pi} - \ln(1-\sigma) - \frac{1}{4}\ln\sigma + \frac{3}{4}] - 2\frac{1+\sigma}{1-\sigma}\mathrm{Li}_2(1-\sigma) - \frac{\sigma(10-7\sigma)}{4(1-\sigma)^2}\ln\sigma + \frac{15-21\sigma}{8(1-\sigma)}]. \quad (9)$$

Return now to positron spectrum. Contributions to it, containing *large logarithm* L, may be associated with the known kernel of Altarelli–Parizi–Lipatov evolution equation (see[2]):

$$P^{(1)}(y) = \lim_{\Delta\to 0}[\frac{1+y^2}{1-y}\theta(1-y-\Delta) + (\frac{3}{2} + 2\ln\Delta)\delta(1-y) = (\frac{1+y^2}{1-y})_+. \quad (10)$$

Using the factorization theorem we may generalize this spectrum including the leading logarithmical terms in all orders of perturbation theory (PT). It may be done in terms of structure functions $D(y,\sigma)$[2]. In the case of photon spectrum $D(1-x;\sigma)$ appears. The function $D(y,\sigma)$ describes the probability to find a positron with energy fraction y inside initial positron. It may be present in form of sum of non–singlet and singlet contributions $D = D^\gamma + D^{e^+e^-}$. Iteration of evolution equations

[1]Result of my calculation of virtual corrections (excluding the ones related with mass operator) disagree with the T. Kinoshita result and have a form:

$$\Sigma|M|^2/\Sigma|M_0|^2 = 1 + \frac{\alpha}{\pi}[-b(\sigma)\ln\frac{\lambda}{m_\pi} + 3\ln\frac{\Lambda}{m_\pi} + \frac{1+\sigma}{4(1-\sigma)}\ln^2\sigma - 1 - \frac{1+3\sigma}{4(1-\sigma)}\ln\sigma],$$

where Λ is ultraviolet cut–off. The second term in the square bracket twice larger than the corresponding term in T. Kinoshita result.

give:

$$D(y,\sigma) = \delta(1-y) + P^{(1)}(y)\gamma + \frac{1}{2}(P^{(2)}(y) + P^{e^+e^-}(y))\gamma^2 + \ldots, \quad (11)$$

where

$$\gamma = -3\ln(1 - \frac{\alpha}{3\pi}(L-1)),$$

$$P^{(2)}(y) = \int\limits_y^1\frac{dt}{t}P^{(1)}(t)P^{(1)}(\frac{y}{t}) = \lim_{\Delta\to 0}[[(2\ln\Delta + \frac{3}{2})^2 - \frac{2\pi^2}{3}]\delta(1-y) + 2[\frac{1+y^2}{1-y}(2\ln(1-y) - \ln x + \frac{3}{2}) + \frac{1}{2}(1+y)\ln y - 1 + y]\theta(1-x-\Delta)], \quad (12)$$

$$P^{e^+e^-}(y) = \frac{2}{3}P^{(1)}(y) + \frac{(1-y)}{3y}(4 + 7y + 4y^2) + 2(1+y)\ln y.$$

It is convenient to use the smoothed form of them:

$$D^\gamma(y) = \frac{1}{2}b(1-y)^{\frac{1}{2}b-1}[1 + \frac{3}{2} - \frac{1}{48}b^2(\frac{2}{3}L + \pi^2 - \frac{47}{8})] - \frac{1}{4}b(1+y) + \frac{1}{32}b^2[4(1+y)\ln\frac{1}{1-y} + \frac{1+3y^2}{1-y}\ln\frac{1}{y} - 5 - y] + \mathcal{O}(b^3), \quad (13)$$

$$D^{e^+e^-}(y) = \frac{1}{3}(\frac{\alpha}{\pi}(L - \ln(1-y) - \frac{5}{6}))^2(1-y)^{\frac{1}{2}b-1}\,(1 + y^2 + \frac{1}{3}b(L - \ln(1-y) - \frac{5}{6})) + \frac{1}{96}b^2[\frac{1-y}{y}(4 + 7y + 4y^2) + 6(1+y)\ln y] + O(b^3),$$

$$b = \frac{4\alpha}{\pi}(L-1).$$

The expressions for spectra are as follows

$$\frac{d\Gamma}{\Gamma_0 dy} = D(y,\sigma)[1 + \frac{\alpha}{\pi}K_e(y)], \quad (14)$$

$$K_e(y) = 1 - y - \frac{1-y}{2}\ln(1-y) + \frac{1+y^2}{1-y}\ln y, \quad y = \frac{2\varepsilon_e}{m_\pi},$$

$$\frac{d\Gamma}{\Gamma_0 dx} = D(1-x,\sigma)[1 + \frac{\alpha}{\pi}K_\gamma(x)],$$

$$K_\gamma(x) = x + \frac{1 + (1-x)^2}{x}\ln(1-x), \quad x = \frac{2\omega}{m_\pi}.$$

Let us discuss the contribution of inelastic processes considered above to the ratio of widths of positron and muon modes of pion decay, $R_{\pi l2}$:

$$R_{\pi l2} = \frac{\Gamma(\pi\to e\nu) + \Gamma(\pi\to e\nu\gamma)}{\Gamma(\pi\to\mu\nu) + \Gamma(\pi\to\mu\nu\gamma)}. \quad (15)$$

A close attention was paid to this quantity some years ago [3,4], but the corrections of emission processes in higher orders of PT was not taken into account. Keeping in mind that quantity $P^{(1)}(y)$ have a property

$$\int_0^1 \mathrm{d}y\, P^{(1)}(y) = 0, \qquad (16)$$

we may do an important observation that as long as experiment is procced such that no cuts on energy of positron is imposed than no large logarithmic contributions appear. However if the cuts are such that the y-integration is restricted or convoluted with y-dependent function some proportional to large logarithm L terms will remain. Suggest now that there exists some minimal energy ε_{th} for detection of positron. An additional contribution(not considered in [4]) appears:

$$\frac{\Delta R}{R_0} = -\frac{\alpha}{\pi}\int_0^{x_{th}} \mathrm{d}x\, \frac{1+x^2}{1-x}(L-1) + \frac{\alpha}{\pi}\int_{x_{th}}^1 \mathrm{d}x\, D(x,\sigma)K_e(x), \quad x_{th} = \frac{2\varepsilon_{th}}{m_\pi}, \qquad (17)$$

where

$$R_0 = \frac{m_e^2}{m_\mu^2}\frac{(1-\frac{m_e^2}{m_\pi^2})^2}{(1-\frac{m_\mu^2}{m_\pi^2})^2} = 1.28347\cdot 10^{-4}. \qquad (18)$$

For typical values $x_{th} = 0.1$ this additional contribution will have a magnitude of order 10^{-3} and is to be taken into account on the accuracy level 0.1%.

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Радиационный хвост в π_{e2}-распаде и некоторые комментарии к μ—e универсальности

Используя аппарат структурных функций, результат вычисления электронного и фотонного спектров в радиационном распаде пиона обобщен на любые порядки теории возмущений в главном и следующем за ним приближении. Дополнительный к рассматриваемым в литературе источник радиационных поправок к отношению ширин электронного и мюонного каналов распада пиона, связанный с излучением виртуальных и реальных фотонов и пар, как показано, должен быть принят во внимание на уровне точности 0,1%.

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Radiative Tail in π_{e2} Decay and Some Comments to μ—e Universality

The results of lowest order of perturbation theory calculations of photon and positron spectra in radiative π_{e2} decay are generalized to all orders of perturbation theory using the structure functions method. An additional source of radiative corrections to the ratio of positron and muon channels of pion decays, connected with emission of virtual and real photons and pairs is searched. It depends on details of detection of final particles and large enough in magnitude to be taken into account in theoretical estimations on the level of accuracy 0.1%.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

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