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PHASE OF THE COULOMB AMPLITUDE IN THE SECOND BORN APPROXIMATION

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Experimental determination of the paramcters of elastic scattering is very important for the development of the modern strong interaction theory [1]. The interaction potential of charged hadrons is a sum of Coulomb and nuclear interactions. So, after the eikonal summation, terms with the Coulomb and nuclear interactions appear. The differential cross sections measured in experiment are described by the square of the scattering amplitude

$$d\sigma/dt = \pi\ (F_C^2(t) + (1+\rho^2(s,t))\ ImF_N^2(s,t)$$
$$\mp 2(\rho(s,t)+\alpha\varphi))\ F_C(t)\ ImF_N(s,t)), \tag{1}$$

where $F_C = \mp 2\alpha G^2/|t|$ is the Coulomb amplitude; α is the fine-structure constant and $G^2(t)$ is the proton electromagnetic form factor squared; $Re\ F_N(s,t)$ and $Im\ F_N(s,t)$ are the real and imaginary parts of the nuclear amplitude; $\rho(s,t) = Re\ F(s,t)/Im\ F(s,t)$. Just this formula is used for the fit of experimental data determining the Coulomb and hadron amplitudes and the Coulomb-hadron phase to evaluate $\rho(s,t)$. The phase of the Coulomb-hadron interaction has been calculated and discussed by many authors [2] and has the form [3]

$$\varphi(s,t) = \mp[\gamma + \ln(B(s,t)|t|/2) + \ln(1+8/(B(s,t)\Lambda^2)) +$$
$$(4|t|/\Lambda^2)\ \ln(4|t|/\Lambda^2) + 2|t|/\Lambda^2], \tag{2}$$

where $B(s,t)$ is the slope of the nuclear amplitude; Λ is a constant entering into the dipole form factor.

Two experiments UA4 [4] and UA4/2 [5] gave very different values of ρ. In work [6] it was shown that this value strongly depends on the used procedure and parameters in (2). That is why one should more accurately know all quantities in (2). Recently, M.

Block has noted the importance of the magnetic interaction of hadrons for the definition of ρ [7]. Now there are the large spin programs at RHIC and LHC. These programs include the measure of the spin correlation parameters in the diffraction range of elastic scattering. The phenomena of the interference of the hadronic and the coulombic amplitudes may give an important contribution not only at very small transfer momenta but also in the range of the diffraction minimum [8]. So one should know the phase of the interference of the coulombic and hadronic amplitude at sufficiently large transfer momenta too.

The last two terms in (2), symbolized as ν, appear in the phase of the Coulomb amplitude in the second Born approximation when the hadron form factor is taken into account. They were calculated by R. Cahn [3] with the form factor of hadrons in the form:

$$G(t) = [\Lambda^2/(\Lambda^2 + q^2)]^{1/2}. \tag{3}$$

Then, taking the new value of $\Lambda^2 = 0.71/4\ GeV^2$ to coincide with the dipole form factor at very small transfer momenta and using some approximation he calculated an additional term and showed that it is to be taken into account. However, really we don't know the accuracy and the working range of additional terms in this case.

A number of models [9,10] predicts the existence of the nondisappearing spin-flip amplitude at high energies in the diffraction peak region. A possibility for appearing this amplitude at $\sqrt{s} = 540\ GeV$ in the proton-antiproton scattering has been shown in [6]. At present, new experiments are proposed on high precision investigations of polarization phenomena in the diffraction region of the proton-proton scattering at RHIC, see for example [8,11]. There also we need exactly the contribution of the coulombic amplitude.

In this work, we exactly calculate the phase of the Coulomb amplitude in the second Born approximation with the form factor in the monopole form. Our expansion of the monopole form factor allows us to make a good approximation of the dipole form factor up to few GeV^2 of transfer momenta and then calculate ν which can work up to the range of the diffraction minimum.

For the hadron interaction, in the absence of nuclear forces we have only the Coulomb amplitude that in the eikonal representation has the form

$$F_c(q^2) = \frac{1}{2i}\int_0^\infty bdbJ_0(bq)(1 - exp(\chi_c(b))), \tag{4}$$

$$\chi_c(b) = 2i\int_0^\infty bdbJ_0(bq)\, F_c^B(q^2). \tag{5}$$

Let us take the hadron form factor in the form:

$$G^2(q) = \Lambda^4/(\Lambda^2 + q^2)^2, \tag{6}$$

where Λ^2 is some constant. Let us expand the form factor into three parts

$$G^2(q) \equiv 1 - \frac{q^2}{\Lambda^2 + q^2} - \frac{\Lambda^2 q^2}{(\Lambda^2 + q^2)^2}. \tag{7}$$

In this case, we obtain the part of the scattering amplitude which does not depend on the form factor and the part with a new form factor. Note that the first two terms give the square of the form factor as in the work by R. Cahn.

For the second Born term with our form factor we have

$$F_c^{2b} = \frac{1}{2i}\int_0^\infty [\{2\otimes 2\} + \{2\otimes 3\}]J_0(bq)bdb \tag{8}$$

where

$$\{2\otimes 2\} = \chi_0^{'}\chi_0^{''} - \chi_0^{'}\chi_1^{''} - \chi_1^{'}\chi_0^{''} + \chi_1^{'}\chi_1^{''},$$

$$\{2\otimes 3\} = -\chi_0^{'}\chi_2^{''} - \chi_2^{'}\chi_0^{''} + \chi_1^{'}\chi_2^{''} + \chi_2^{'}\chi_1^{''} + \chi_2^{'}\chi_2^{''}$$

. The quantity χ_0 is the eikonal with the first term, χ_1 and χ_2 are the eikonals with the second and third terms of our form factor. Note that the terms $\{2\otimes 2\}$ represent the second Born approximation with the Cahn form factor.

Let us develop our procedure of calculating the Coulomb phase with the form factor for the simplest examples. So, the first term of (8) is the second Born term of the scattering amplitude without the form factor. For it we have

$$F_{c0}^{2B}(q^2) = \frac{1}{2i}\,\phi(\chi_0^{'}\chi_0^{''}) = \frac{1}{2i}\int_0^\infty \chi_0^{'}\chi_0^{''}\, bdbJ_0(bq); \tag{9}$$

and we obtain, introducing λ as a fictitious photon mass and representing a power of q^2 as an eponential integral,

$$\chi_0'(b) = -2i\alpha \int_0^\infty \int_0^\infty exp(-(\lambda^2+q^2)x)\ J_0(qb)qdqdx, \tag{10}$$
$$\chi_0''(b) = -2i\alpha \int_0^\infty \int_0^\infty exp(-(\lambda^2+q'^2)y)\ J_0(q'b)q'dq'dy.$$

Integrating over q we obtain for the scattering amplitude

$$F_{c0}^{2B} = \frac{i\alpha^2}{2} \int_0^\infty [\int_0^\infty \int_0^\infty \frac{dxdy}{xy} exp(-x\lambda^2 - \frac{b^2}{4x})\ exp(-y\lambda^2 - \frac{b^2}{4y})] J_0(bq)bdb \tag{11}$$

Upon integrating over b and introducing new variables w and z

$$2x = w(1+z); 2y = w(1-z)$$

we obtain

$$F_{c0}^{2B} = i\alpha^2 \int_{-1}^1 dz \int_0^\infty dw\ exp(-w\lambda^2 - q^2(1-z^2)w/4). \tag{12}$$

These integrals are calculated exactly

$$F_{c0}^{2B} = \frac{i\alpha^2}{q^2} \ln|\frac{q/2 + \sqrt{(\lambda^2+q^2/4)}}{q/2 - \sqrt{(\lambda^2+q^2/4)}}|. \tag{13}$$

and for $\lambda \to 0$ we obtain

$$F_{c0}^{2B} = \frac{-i\alpha^2}{q^2} \ln(\frac{\lambda^2}{q^2}). \tag{14}$$

So, for the sum of the first and second Born terms without the form factor we have the ordinary representation

$$F_{c0} = -\frac{\alpha}{q^2} - \frac{i\alpha^2}{q^2}\ln(\frac{\lambda^2}{q^2}) = -\frac{\alpha}{q^2}\, exp(i\alpha\,\varphi_c(q^2)), \tag{15}$$

where

$$\varphi_c(q^2) = \ln(\frac{\lambda^2}{q^2}).$$

For the second and third terms (8) we have

$$F_{(c2),(c3)}^{2B} = \frac{i\alpha^2}{2} \int_0^\infty [\int_0^\infty \int_0^\infty \frac{dxdy}{xy} exp(-x\lambda^2 - \frac{b^2}{4x})\ exp(-y\Lambda^2 - \frac{b^2}{4y})] J_0(bq)bdb. \tag{16}$$

Again, upon introducing the new variables w and z and integrating, we find

$$F_{(c2),(c3)}^{2B} = \frac{i\alpha^2}{2} \int_{-1}^1 \int_0^\infty dw\, dz\ exp(-w[(1-z)\lambda^2/2 + (1+z)\Lambda^2/2 + q^2(1-z^2)/4])$$
$$= -i\alpha^2\{\frac{1}{2(\Lambda^2+q^2)} \ln|\frac{\lambda^2}{q^2}| + \frac{1}{2(\Lambda^2+q^2)} \ln|\frac{\Lambda^2\cdot q^2}{(\Lambda^2+q^2)^2}|\}. \tag{17}$$

Now we must calculate the fourth term of (8). In the above calculations we had the same form as for the first term but with the change $\lambda \to \Lambda$. However, at the end, we cannot neglect Λ^2 in any terms, and we obtain

$$F_{c4}^{2B} = -i\alpha^2\ \phi(\chi_1'\chi_1'') = -i\alpha^2\, [\frac{-1}{q\sqrt{(4\Lambda^2+q^2)}} \ln|\frac{4\Lambda^2}{(\sqrt{(4\Lambda^2+q^2)}+q)^2}|]. \tag{18}$$

Using this method of calculation we can obtain the result for other terms of (8) which are sufficiently complicated but eventually exactly calculated, and thus we finally have

$$\phi(\chi_0'\chi_0'') = \frac{-1}{q^2}\ln[\frac{\lambda^2}{q^2}], \tag{19a}$$
$$\phi(-\chi_0'\chi_1'' - \chi_1'\chi_0'') = \frac{2}{\Lambda^2+q^2}\ln[\frac{\lambda\Lambda}{\lambda^2+q^2}], \tag{19b}$$
$$\phi(\chi_1'\chi_1'') = \frac{-2}{q\sqrt{(4\Lambda^2+q^2)}}\ln(\frac{2\Lambda}{\sqrt{(4\Lambda^2+q^2)}+q}), \tag{19c}$$
$$\phi(-\chi_0'\chi_2'' - \chi_2'\chi_0'') = \frac{2\Lambda^2}{(\Lambda^2+q^2)^2}[\ln[\frac{\lambda\Lambda}{\lambda^2+q^2}] + \frac{\Lambda^2-q^2}{2\Lambda^2}], \tag{19d}$$

$$\phi(\chi_1'\chi_2'' + \chi_2'\chi_1'') = \frac{1}{4\Lambda^2+q^2} - \frac{2\Lambda^2}{q(4\Lambda^2+q^2)^{3/2}}\ln[\frac{2\Lambda}{\sqrt{(4\Lambda^2+q^2)}+q}], \tag{19e}$$

$$\phi(\chi_2'\chi_2'') = \frac{2\Lambda^2(\Lambda^2+q^2)}{q^2(4\Lambda+q^2)^2}\ \{1 - \frac{\Lambda^2+q^2}{2(\Lambda^2+q^2)} - \frac{4\Lambda^2}{q\sqrt{(4\Lambda^2+q^2)}}\ln[\frac{2\Lambda}{\sqrt{(4\Lambda^2+q^2)}}]\}. \tag{19f}$$

Summing all the terms and collecting the leading terms with $\ln(\lambda^2/q^2)$ we have

$$F_c^{2b}(q) = -\frac{i\alpha^2}{q^2}\ln(\frac{\lambda^2}{q^2}) + i\alpha^2\frac{1}{(\Lambda^2+q^2)}\ln(\frac{\lambda^2}{q^2})$$

$$+i\alpha^2\frac{\Lambda^2}{(\Lambda^2+q^2)^2}\ln(\frac{\lambda^2}{q^2}) + \frac{\Lambda^4}{q^2(\Lambda^2+q^2)^2}\ \nu_s. \tag{20}$$

Hence, for the total Coulombic scattering amplitude we have the eikonal approximation of the second order in α

$$F_c(q) = F_c^{1B} + F_c^{2B} = -\frac{\alpha}{q^2}[\frac{\Lambda^4}{(\Lambda^2+q^2)^2}][1+i\alpha(\{\ln(\frac{\lambda^2}{q^2}) + \nu_s\}], \tag{21}$$

where

$$\nu_s = A\ln(\frac{(\Lambda^2+q^2)^2}{\Lambda^2 q^2}) + B\ln(\frac{4\Lambda^2}{(\sqrt{(4\Lambda^2+q^2}+q)^2}) + C, \tag{22}$$

with

$$A = \frac{q^2(2\Lambda^2+q^2)}{\Lambda^4}; \quad C = \frac{2\Lambda^4 - 17\Lambda^2 q^2 - q^4}{(4\Lambda^2+q^2)^2},$$

$$B = \frac{(\Lambda^2+q^2)^2[4\Lambda^4(\Lambda^2+7q^2)+q^4(10\Lambda^2+q^2)]}{\Lambda^4 q(4\Lambda^2+q^2)^{5/2}}.$$

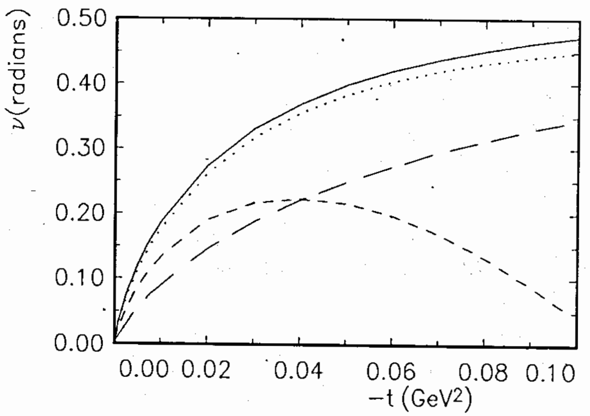

Fig. 1. The behavior of ν with q^2. (the long dushed line ν_s- the equations (22); the short dushed line - ν_c of the work [3]. the dotted line - $\nu_{\Lambda/3}$; the solid line - ν_{fit} .

Fig.1 shows the result of paper [3] - ν_c and our calculation of ν_s with $\Lambda^2 = 0.71/3\ GeV^2$. Note that our form factor is sufficiently close to the usual dipole form factor with $\Lambda^2 = 0.71/3\ GeV^2$ up to $q^2 \approx 1\ GeV^2$. It is easily seen that as $q \to 0$, our $\nu_s \to \nu_c$ and vanishes in the limit of a point charge. The numerical calculation shows (see fig.1) that at small q^2 the difference between ν_s and ν_c is small, but after $q^2 = 3.10^{-2}\ GeV^2$ it is rapidly growing. So, there is an essential difference of their behavior with growing q^2 as ν_s continues slowly growing with q^2 and ν_c quickly decreases and become negative. It is clear that the solution of ν_c must be bounded at $-t = 3.10^{-2}\ GeV^2$.

Our method allows us to make exact calculations of ν with the dipole form factor of hadrons. However, the corresponding expressions will be very complicated and it is better to make the calculation with fitting the dipole form factor. If we multiply the last two terms and Λ in (7) by free parameters, we can make a good fit of the dipole form factor up to $q^2 = 5\ GeV^2$ and then calculate $\nu_{\rm fit}$ using (22). This calculation of $\nu_{\rm fit}$ for this form factor are shown in Fig.1. It is clear that the complete fit gives the upper bound of ν and for the practical calculation we can use our formulas with $\Lambda^2 = 0.71/3\ GeV^2$. It means that we can calculate the contribution of the coulombic interaction, taking into account the hadron form factor that will be valid in the whole range of the diffraction peak and it is especially exactly calculated for small transfer momenta. The method developed here gives the possibility of calculating the hadron-coulomb phase with the hadron eikonal phase which can describe the experimental data up to diffraction minimum. We hope make it in a subsequent work. The author expresses his gratitude to V.Meshcheryakov and D.Shirkov for support in this work and to P.Gauron, S.Goloskokov, S.Kuleshov, B.Nicolescu and M.Smondyrev for fruitful discussions.

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Селюгин О.В. E2-96-58

Фаза кулоновской амплитуды во втором борновском приближении

Предложен метод вычисления кулоновской фазы во втором борновском приближении. Фаза модифицированной кулоновской амплитуды с учетом формфактора адронов вычислена точно. Сделана оценка фазы с учетом дипольного формфактора. В результате изменяется зависимость полной фазы кулон-адронной интерференции от переданного момента.

Работа выполнена в Лаборатории теоретической физики им. Н.Н.Боголюбова ОИЯИ.

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Selyugin O.V. E2-96-58

Phase of the Coulomb Amplitude in the Second Born Approximation

The method of calculating the Coulomb phase in the second Born order with allowance for the hadron form factor is presented. The phase of the modified Coulomb amplitude is exactly calculated with taking account of the form factor of hadrons. The phase with the dipole form factor is estimated, as a result the behaviour of the total phase of the Coulomb-hadron interference changes as a function of the transfer momentum.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

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