

ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

Дубна

96-53

E2-96-53

V.M.Dubovik, V.E.Kuznetsov

THE TOROID MOMENT
OF MAJORANA NEUTRINO

Submitted to «Canadian Journal of Physics»

1996

I. INTRODUCTION

The properties of neutrinos are of great interest in the physical literature motivated by different puzzles in neutrino physics [1]. A direction of investigations of neutrinos is the study of their electromagnetic properties. The usual electromagnetic characteristics: magnetic, electric dipole moments and neutrino charge radius have been widely discussed everywhere [2]. Here we will investigate a more exotic kind of electromagnetic characteristic of Majorana neutrinos, the toroid moment.

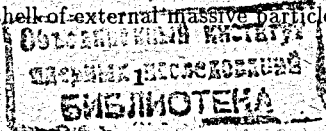
As early as in 1939, Pauli remarked that Majorana neutrinos have neither magnetic dipole moment nor electric dipole moment [3]. However as was pointed out a long time ago in Ref. [4], for the T-invariant interaction with non-conservation of P and C symmetries, there is a diverse dipole moment of a particle with 1/2-spin: the anapole moment. Subsequently there was pointed out a more convenient characteristic, the toroid dipole moment, which is the first term of the third multipole family, toroid moments [5].¹ This type of static multipole moments does not produce any external fields in vacuum and only reduces a free-field (gauge-invariant) transverse-longitudinal potential [6] responsible for topological effects like Aharonov - Bohm ones. In brief, the experts of neutrino physics show that the toroid dipole moment [7], is a single electromagnetic characteristic of Majorana neutrinos valid for any values of the spin [8]. The Majorana neutrinos possessing the toroid moment provide very interesting results in different media [9,10]. Indeed, as was pointed out in Ref. [10], the toroid dipole moment moving in a medium with a sufficiently high dielectric constant and magnetic permeability, $(v/c)\sqrt{\epsilon\mu} > 1$, can generate the Vavilov-Cherenkov radiation. This radiation may be detected in experiment and gives a new possibility for investigating neutrino properties.

A calculation of toroid dipole moment (anapole) has been started in [11] and by diploma student of JINR A. A. Chepkasov in 1976. Then a number of articles about the problems of renormalizability, gauge non-invariance and consequently observability of anapole moment and neutrino charge radius was published [11-13]. However, as was pointed out in [14], these quantities are finite and well-defined in the Standard Model as being its axial-vector and vector contact interactions with an external electromagnetic field, respectively. Since a great interest to neutrino properties at present and development of experimental setups for detection of neutrinos, we re-analysed the previous results and present here the calculation of the toroid moment of Majorana neutrino for a general class of modern gauge theories of electroweak interactions.

II. ONE-LOOP RESULT

In the case of Majorana neutrinos, a toroid moment can be defined in the one-loop approximation of the Standard Model of electroweak interactions from the Feynman

¹The toroid dipole is well-defined in the classic limit that is not a case of anapole which coincides with it only on mass-shell of external massive particles.



graphs shown in Fig. 1. The electromagnetic vertex of a neutrino, $\Gamma_\mu(q)$, has anapole or toroid parametrization:

$$\Gamma_\mu(q) = \left\{ G(q^2) [q^2 \gamma_\mu - \hat{q} q_\mu] \gamma_5 \right\}_{\text{anapole}} = \left\{ G(q^2) [i \varepsilon_{\mu\nu\lambda\sigma} P_\nu q_\lambda \gamma_\sigma \gamma_5] \gamma_5 \right\}_{\text{toroid}}. \quad (1)$$

They coincide identically only on mass-shell and different off mass-shell [5], which follows from the identity:

$$\bar{u}_f(\mathbf{p}') \left\{ (m_f - m_i) \sigma_{\mu\nu} q^\nu + [q^2 \gamma_\mu - \hat{q} q_\mu] + i \varepsilon_{\mu\nu\lambda\sigma} P_\nu q_\lambda \gamma_\sigma \gamma_5 \right\} \gamma_5 u_i(\mathbf{p}) = 0, \quad (2)$$

where $\varepsilon_{\mu\nu\lambda\sigma}$ is an antisymmetric tensor, $P_\nu = p_\nu + p'_\nu$ and $q_\nu = p_\nu - p'_\nu$. As can see from (2) the transition toroid moment is equal to diagonal one plus part, which is proportional to the neutrino mass difference, therefore for calculation of diagonal toroid moment and estimation of transition one it is enough only anapole parametrization.

For illustration, we write down some details of the calculation of two graphs with $\ell\ell W$ states, see Fig. 2. It is easy to verify that contributions of particle and antiparticle currents are equal to each other and we will consider one of them multiplying the amplitude by factor 2. Using the Feynman rules, summarized in Appendix A, we can write the amplitude in the following form

$$\mathcal{M} = 2 \int \frac{d^4 k_1}{(2\pi)^4} \frac{d^4 k_2}{(2\pi)^4} (2\pi)^4 \delta^4(p_1 - p_2 - (k_1 - k_2)) \bar{u}(p_1) [i \Gamma_\lambda^{(\ell N)}] i \Delta_F(k_1) \times (-i e \gamma_\mu) i \Delta_F(k_2) [i \Gamma_\nu^{(\ell N)}] u(p_2) [i \Delta_W^\nu(p_1 - k_1)] \mathcal{A}^\mu(k_1 - k_2). \quad (3)$$

Here $\mathcal{A}^\mu(k_1 - k_2)$ is an external electromagnetic field, a source of virtual photons.² For convenience of our calculations, we pass to the t -channel where the momenta of particles transform as

$$p_1 \rightarrow p_-, \quad p_2 \rightarrow -p_+, \quad k_1 \rightarrow k_- \equiv k_1, \quad k_2 \rightarrow -k_+ \equiv -k_2,$$

$$q = (k_1 + k_2) = (p_- + p_+),$$

and using the transformation

$$\frac{1}{k^2 - m^2} \rightarrow (-2\pi i) \delta(k^2 - m^2) \Theta(k_0), \quad (4)$$

valid when we take into account the unitary condition for the S-matrix [15,16], we can write the imaginary part of the amplitude as

²In the case of real photons, $q^2 = 0$ and $\varepsilon \cdot q = 0$, the toroid (anapole) interaction disappears.

$$\text{Im} \mathcal{M} = e \int d\tau \frac{\bar{u}(p_-) \Gamma_\lambda^{(\ell N)}(\hat{k}_1 + m_\ell) \gamma_\mu (\hat{k}_2 - m_\ell) \Gamma_\nu^{(\ell N)} g^{\lambda\nu} v(p_+)}{(p_- - k_1)^2 - M_W^2} \mathcal{A}^\mu(q). \quad (5)$$

Here we have denoted the two-body phase-space factor as

$$d\tau = \frac{1}{(2\pi)^2} d^4 k_1 d^4 k_2 \delta^4(p_- + p_+ - k_1 - k_2) \delta(k_1^2 - m_\ell^2) \Theta(k_{10}) \delta(k_2^2 - m_\ell^2) \Theta(k_{20}).$$

Now, keeping only terms with $\gamma_\mu \gamma_5$ and performing the two-particle phase space integration, following Refs. [11], we obtain

$$\begin{aligned} \text{Im} G(t) &= \frac{1}{16\pi} \left[(A_L^{(\ell N)})_{1t} (A_L^{(\ell N)*})_{t2} - (A_R^{(\ell N)})_{1t} (A_R^{(\ell N)*})_{t2} \right] \frac{1}{\sqrt{t(t-4m_1^2)}} \\ &\times \left\{ t \ln c - \frac{1}{2} (t-4m_\ell^2) \left(\ln c + 2\frac{a}{b} - \frac{a^2}{b^2} \ln c \right) - t \sqrt{\frac{t-4m_\ell^2}{t-4m_1^2}} \left(2 - \frac{a}{b} \ln c \right) \right. \\ &- \frac{1}{2} (t-4m_\ell^2) \left(\frac{m_1^2 - m_2^2}{t-4m_1^2} \right) \left(\ln c + 6\frac{a}{b} - 3\frac{a^2}{b^2} \ln c \right) \\ &\left. + 2m_1(m_1 - m_2) \sqrt{\frac{t-4m_\ell^2}{t-4m_1^2}} \left(2 - \frac{a}{b} \ln c \right) \right\}, \quad (6) \end{aligned}$$

$$a = M_W^2 + \frac{1}{2} (t - 2m_\ell^2 - m_1^2 - m_2^2), \quad b = \frac{1}{2} \sqrt{t-4m_1^2} \sqrt{t-4m_2^2},$$

$$c = |(a+b)/(a-b)|, \quad t = q^2.$$

The real part of the toroid form factor can be derived by using the dispersion relation with one subtraction [15]. For $t < 0$, with the new variable $x = t/2M_W^2$ and in $m_\nu = m_1 = m_2 = 0$ approximation we find

$$G(t) = \frac{1}{32\pi^2 M_W^2} \left(|A_L^{(\ell N)}|^2 - |A_R^{(\ell N)}|^2 \right) \int_{2\gamma}^{\infty} \frac{F(x, \gamma) dx}{x(x+\alpha)}, \quad (7)$$

where $\alpha = -t/2M_W^2 > 0$ and the integrand reads

$$\begin{aligned} F &= \left(\frac{\gamma-1}{x} - 3 \right) \sqrt{1 - \frac{2\gamma}{x}} + \left[2 \left(1 + \frac{1}{2x} \right)^2 - \frac{\gamma}{x} - \frac{\gamma}{x^2} + \frac{\gamma^2}{2x^2} \right] \\ &\times \ln \left(\frac{1+x-\gamma+\sqrt{x(x-\gamma)}}{1+x-\gamma-\sqrt{x(x-\gamma)}} \right), \end{aligned}$$

with $\gamma = m_\ell^2/M_W^2$. Finally, using the definition of matrices $A_{L,R}^{(\ell N)}$ and $B_{L,R}^{(\ell N)}$ in the Standard Model (A.3) and performing the elementary integrations for these two graphs

and other ones (making appropriate expansions in $m_f^2/m_{W,Z}^2$, $f = e, \mu, \tau, u, d, s, c, b, t$) we obtain for $|t| = 0$:

$$G(0) = \frac{\sqrt{2}G_F}{16\pi^2} \left\{ \sum_{\ell=e,\mu,\tau} (C_{\ell e W} + C_{\ell e \phi} + C_{W W \ell} + C_{W \phi \ell} + C_{\phi W \ell} + C_{\phi \phi \ell}) \right. \\ \left. + C_{W W Z} + C_{W \phi Z} + C_{\phi \phi Z} + \sum_f C_{f f Z} \right\} \\ C_{\ell e W} = |\mathcal{K}_{\ell i}|^2 \left\{ \frac{22}{9} - \frac{4}{3} \ln \frac{4m_f^2}{M_W^2} + \frac{m_f^2}{M_W^2} \left(\frac{7}{3} + \frac{2}{3} \ln 2 - \frac{3}{2} \ln 3 \right) + \mathcal{O}(\gamma^2) \right\}, \\ C_{\ell e \phi} = |\mathcal{K}_{\ell i}|^2 \frac{m_f^2}{M_W^2} \left(\frac{2}{9} - \frac{2}{3} \ln \frac{4m_f^2}{M_W^2} + \mathcal{O}(\gamma^2) \right), \\ C_{W W \ell} = |\mathcal{K}_{\ell i}|^2 \left(-\frac{10}{9} \right), \\ C_{W \phi \ell} = C_{\phi W \ell} = |\mathcal{K}_{\ell i}|^2 \left(-\frac{m_f^2}{6M_W^2} \right), \\ C_{\phi \phi \ell} = |\mathcal{K}_{\ell i}|^2 \left(-\frac{m_f^2}{9M_W^2} \right), \\ C_{f f Z} = \frac{\Omega_{ii}}{2} \sum_{f \neq i} (g_L^f + g_R^f) \left\{ \frac{5}{9} - \frac{4}{3} \ln \frac{4m_f^2}{M_Z^2} + \frac{m_f^2}{M_Z^2} \left(\frac{20}{3} \ln 2 - 3 \ln 3 - \frac{28}{3} \right) + \mathcal{O}(\gamma^2) \right\} \\ + \frac{\Omega_{ii}}{2} (g_L^i + g_R^i) \cdot (-0.139), \quad m_{\text{top}} = 180 \text{ GeV}, \\ C_{W W Z} = -\frac{\Omega_{ii}}{3} \left[\frac{77}{3} + 4d - \sqrt{d}(27 + 4d) \arctan(d^{-1/2}) \right], \\ C_{W \phi Z} = -\frac{\Omega_{ii}}{2} \sin^2 \theta_W \left[\frac{2}{3} + d - \sqrt{d}(1 + d) \arctan(d^{-1/2}) \right], \\ C_{\phi \phi Z} = -\frac{\Omega_{ii}}{3} (1 - 2 \sin^2 \theta_W) \left[\frac{1}{3} - d + d^{3/2} \arctan(d^{-1/2}) \right], \quad (8)$$

where $d = 4 \cos^2 \theta_W - 1$; $\mathcal{K}_{\ell i}$ and Ω_{ii} are elements of the mixing matrices K and Ω , see Appendix A.

III. REMARKS ON SOME PROPERTIES OF TOROID MOMENT

Let us discuss some general properties of toroid interaction of neutrinos. In the non-relativistic limit, the energy of interaction with external electromagnetic fields is defined, in general, by three characteristics: magnetic ($\boldsymbol{\mu}$), electric (\mathbf{d}) and toroid (\mathbf{T}) dipole moments:

$$\mathcal{H}_{\text{int}} \sim -(\boldsymbol{\mu} \cdot \mathbf{H}) - (\mathbf{d} \cdot \mathbf{E}) - [\mathbf{T} \cdot (\text{rot} \mathbf{H} \text{ or } \dot{\mathbf{E}})]. \quad (9)$$

In the case of Majorano neutrinos, imposing the restriction of CPT-invariance and using C-, P-, T-properties of \mathcal{H}_{int} , which we have combined in Table I, we see that

magnetic and electric dipole moments are absent in the static limit ($m_i = m_f = m_\nu$). This means that Majorano neutrino can possess only one electromagnetic characteristic, the toroid (anapole) moment if masses of initial and final neutrino eigenstates are equal to each other. The electromagnetic matrix element connecting two different mass eigenstates of the Majorano neutrino can be described, in principle, by all three moments [7]. Adding all contributions from (8) and using the standard definitions of dipole moments [5,9,14] we define the toroid moment of Majorano neutrino as being its axial-vector contact interaction with an external electromagnetic field:³

$$T_\mu = \frac{eG(0)}{2m_\nu} \bar{u}(0) \gamma_\mu \gamma_5 u(0), \quad \mathbf{T} = eG(0) \boldsymbol{\varphi}^\dagger \boldsymbol{\sigma} \boldsymbol{\varphi},$$

$$G(0) = 1.28 \times 10^{-33} (|\mathcal{K}_{ei}|^2 + 0.55|\mathcal{K}_{\mu i}|^2 + 0.31|\mathcal{K}_{\tau i}|^2 - 0.21\Omega_{ii}) \text{ (cm}^2\text{)},$$

where $\boldsymbol{\varphi}$ is the Pauli spinor. It has the density $g(\mathbf{r}) = [\mathbf{r}(\mathbf{J}\mathbf{r}) - 2r^2\mathbf{J}]/10$ in the coordinate space and the following interaction with an external electromagnetic field [17]:

$$\mathcal{H}_{\text{int}} = J_\mu^{\text{EM}}(x) \mathcal{A}^\mu(x) = eG(q^2) \bar{N}(x) \left[q^2 \gamma_\mu \mathcal{A}^\mu(x) - \gamma_\mu q^\nu q_\nu \mathcal{A}^\nu(x) \right] \gamma_5 N(x) \\ = eG(q^2) \bar{N}(x) \gamma_\mu \gamma_5 N(x) \left[\frac{\partial^2 \mathcal{A}^\mu(x)}{\partial x^\nu \partial x_\nu} - \frac{\partial^2 \mathcal{A}^\nu(x)}{\partial x^\nu \partial x_\mu} \right] \\ = eG(q^2) \bar{N}(x) \gamma_\mu \gamma_5 N(x) \frac{\partial F^{\mu\nu}(x)}{\partial x^\nu}.$$

Here $F^{\mu\nu}(x)$ is the tensor of the electromagnetic field that produces the external current

$$j^\mu(x) = -\partial_\nu F^{\mu\nu}(x).$$

In the non relativistic limit we obtain

$$\mathcal{H}_{\text{int}} = -eG(0) \boldsymbol{\varphi}^\dagger \boldsymbol{\sigma} \boldsymbol{\varphi} \mathbf{j} \equiv -eG(0) \boldsymbol{\varphi}^\dagger \boldsymbol{\sigma} \boldsymbol{\varphi} (\text{rot} \mathbf{H} - \dot{\mathbf{E}}).$$

How can we think of the toroid moment of the Majorana neutrino? The answer to this question was given in the original idea of Zel'dovich [4]: a conventional solenoid folded into a torus having poloidal currents is a classical example of a toroid dipole. For such a solenoid there is neither azimuthal component of the poloidal currents nor electric fields around torus that are responsible for the absence of electromagnetic fields outside the torus and the presence of a non zero magnetic field inside the torus. That is why the fields outside the toroid dipole are zero in vacuum. However when the toroid dipole moves in a medium, the medium should be regarded as permitting

³Here we have used the normalization $\bar{u}(\mathbf{p})u(\mathbf{p}) = 2m_\nu$ and chiral representation of gamma matrices.

the dipole itself and the fields outside the dipole to appear producing, for instance [10], the Vavilov-Cherenkov radiation.

In conclusion, we have calculated the diagonal toroid dipole moment of Majorana neutrino without specifying the gauge group under consideration. In the Standard Model, it has a finite value and does not depend on the neutrino mass, i.e. it is different from zero in the case of massless neutrinos. The toroid moment of Dirac or Majorana neutrinos should be taken into account in various situations: it gives an extra contribution to the total cross section of scattering of neutrinos by a spinless nucleus and it has a finite value in different media [9]; it is responsible for the Vavilov-Cherenkov and transition radiations when neutrinos move through an uniform medium [10] and it may play a sensible role in neutrino oscillations, since the toroid interaction with an external source of electromagnetic fields (in media it can be the electron current) should be added to the Hamiltonian of evolution of neutrinos. For instance, the evolution equation for three neutrino flavors (in the presence of non zero $\text{rot}\mathbf{H}$ or $\mathbf{\dot{E}}$) can be written as

$$i\frac{d\vec{\nu}_\ell}{dt} = K \left[\frac{1}{2E} \text{diag}(m_1^2, m_2^2, m_3^2) + T \right] K^\dagger \vec{\nu}_\ell,$$

where the matrix T is, in general, a 3×3 matrix whose elements are different from zero (they are defined by the diagonal and transition toroid moments). This problem is an analog of the well-known Wolfenstein equation for propagation of neutrinos through the medium [18].

Note added. After this manuscript was completed an article by Boyarkin and Rein [19] appeared. These authors studied the transition in a system of Majorana neutrinos with anapole and transition magnetic moments propagating in matter and twisting non potential magnetic field investigated within the asymmetric left-right model. It was shown that resonance conversion of neutrinos appears not only in response to influence of matter but also by the availability of electromagnetic moments. Since our calculations showed that toroid (anapole) moment is non-zero quantity in the Standard Model, consequently the result of Ref. [19] should be appreciated more carefully in relation to neutrino oscillations and explanation of solar neutrino deficit.

ACKNOWLEDGMENTS

We are grateful to Prof. V. A. Naumov for useful comments and discussions.

TABLE I. C-, P-, T-properties of spin, electromagnetic fields and their interactions.

	σ	\mathbf{H}	\mathbf{E}	$\text{rot}\mathbf{H}, \mathbf{\dot{E}}$	$\sigma \cdot \mathbf{H}$	$\mathbf{j} \sigma \cdot \mathbf{E}$	$\sigma \cdot \text{rot}\mathbf{H}, \sigma \cdot \mathbf{\dot{E}}$
C	+	-	-	-	-	-	-
P	+	+	-	-	+	-	-
T	-	-	+	-	+	-	+

APPENDIX A. FEYNMAN RULES

Here we give a short list of the Feynman rules used in our calculations. The weak interactions of Majorana neutrinos N and charged leptons ℓ with gauge bosons W^\pm, Z^0 , non physical scalars ϕ^\pm, ϕ and Higgs particles ϕ^0 may be described by five Lagrangians [20]:

$$\begin{aligned} \mathcal{L}_{\text{int}}^{NW^\pm} &= \bar{N} \Gamma_\mu^{(\ell N)} \ell W^{+\mu} + \bar{\ell} \bar{\Gamma}_\mu^{(\ell N)} N W^{-\mu}, \\ \mathcal{L}_{\text{int}}^{NZ^0} &= (\bar{N} \Gamma_\mu^{(N)} N + \bar{\ell} \bar{\Gamma}_\mu^{(\ell)} \ell) Z^\mu, \\ \mathcal{L}_{\text{int}}^{N\phi^\pm} &= \bar{N} \Gamma_\mu^{(\ell N)} \ell \phi^\pm + \bar{\ell} \bar{\Gamma}_\mu^{(\ell N)} N \phi^\mp, \\ \mathcal{L}_{\text{int}}^{N\phi} &= (\bar{N} \Gamma_\mu^{(N\phi)} N + \bar{\ell} \bar{\Gamma}_\mu^{(\ell\phi)} \ell) \phi, \\ \mathcal{L}_{\text{int}}^{N\phi^0} &= (\bar{N} \Gamma^{(N)} N + \bar{\ell} \bar{\Gamma}^{(\ell)} \ell) \phi^0. \end{aligned}$$

The respective Feynman rules are:

- $i\Gamma_\mu^{(\ell N)}$ for outgoing W^- or incoming W^+ ,
- $i\bar{\Gamma}_\mu^{(\ell N)}$ for outgoing W^+ or incoming W^- ,
- $i\Gamma^{(\ell N)}$ for outgoing ϕ^- or incoming ϕ^+ ,
- $i\bar{\Gamma}^{(\ell N)}$ for outgoing ϕ^+ or incoming ϕ^- ,
- $i(\Gamma_\mu^{(N)} + \Gamma_\mu^{(NC)})$ for Z^0 ,
- $i(\Gamma^{(N)} + \Gamma^{(NC)})$ for ϕ^0 ,
- $i(\Gamma^{(N\phi)} + \Gamma^{(N\phi C)})$ for ϕ .

Here $\Gamma_\mu^{(NC)} \equiv C [\Gamma_\mu^{(N)}]^T C^{-1}$ and $\Gamma^{(NC)} \equiv C [\Gamma^{(N)}]^T C^{-1}$ (similarly, for $\Gamma^{(N\phi C)}$).

As was pointed out in Ref. [20], in real calculations we should take the following rule for the Dirac-Majorana transition in a Feynman graph: for an incoming (outgoing) Dirac particle the outgoing (incoming) Majorana neutrino must be treated as a particle, and vice versa, for antiparticles.

Introducing the notation $P_{L,R} \equiv \frac{1}{2}(1 \mp \gamma_5)$, we can write the general forms for all vertices

$$\Gamma_\mu^{(x)} = \gamma_\mu (P_L A_L^{(x)} + P_R A_R^{(x)}), \quad x = \ell, N, \ell N, \quad (\text{A.1})$$

$$\bar{\Gamma}_\mu^{(\ell N)} \equiv \gamma_0 [\Gamma_\mu^{(\ell N)}]^\dagger \gamma_0 = \gamma_\mu (P_L A_L^{(\ell N)*} + P_R A_R^{(\ell N)*}),$$

and

$$\Gamma^{(x)} = P_L B_L^{(x)} + P_R B_R^{(x)}, \quad x = N, N\phi, \ell N, \quad (\text{A.2})$$

$$\bar{\Gamma}^{(\ell N)} \equiv \gamma_0 [\Gamma^{(\ell N)}]^\dagger \gamma_0 = P_R B_L^{(\ell N)*} + P_L B_R^{(\ell N)*}.$$

FIGURES

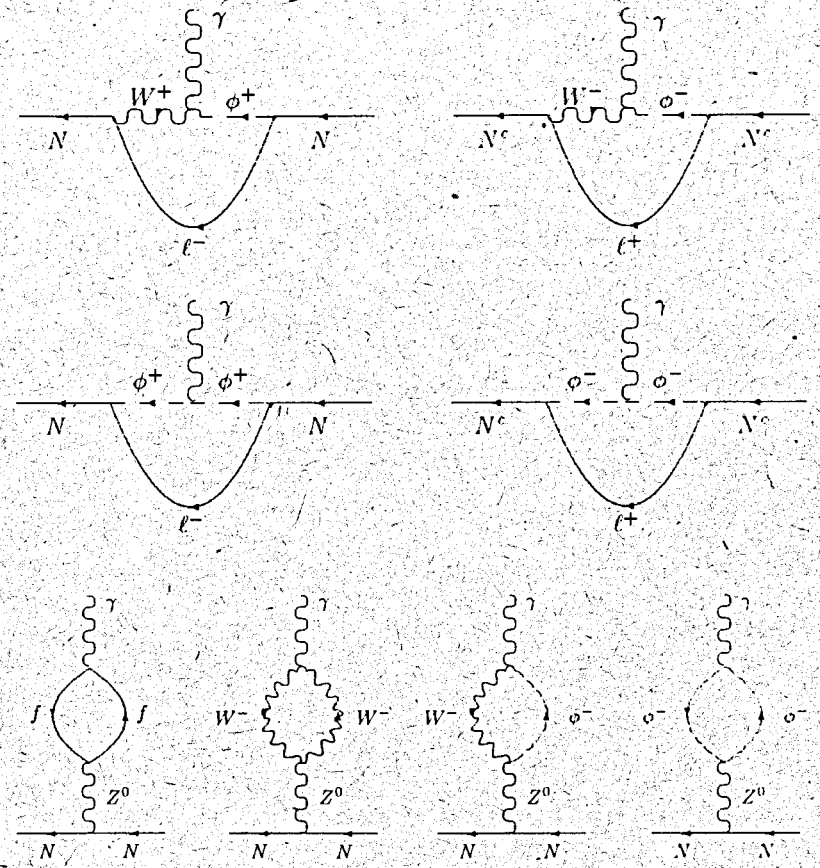
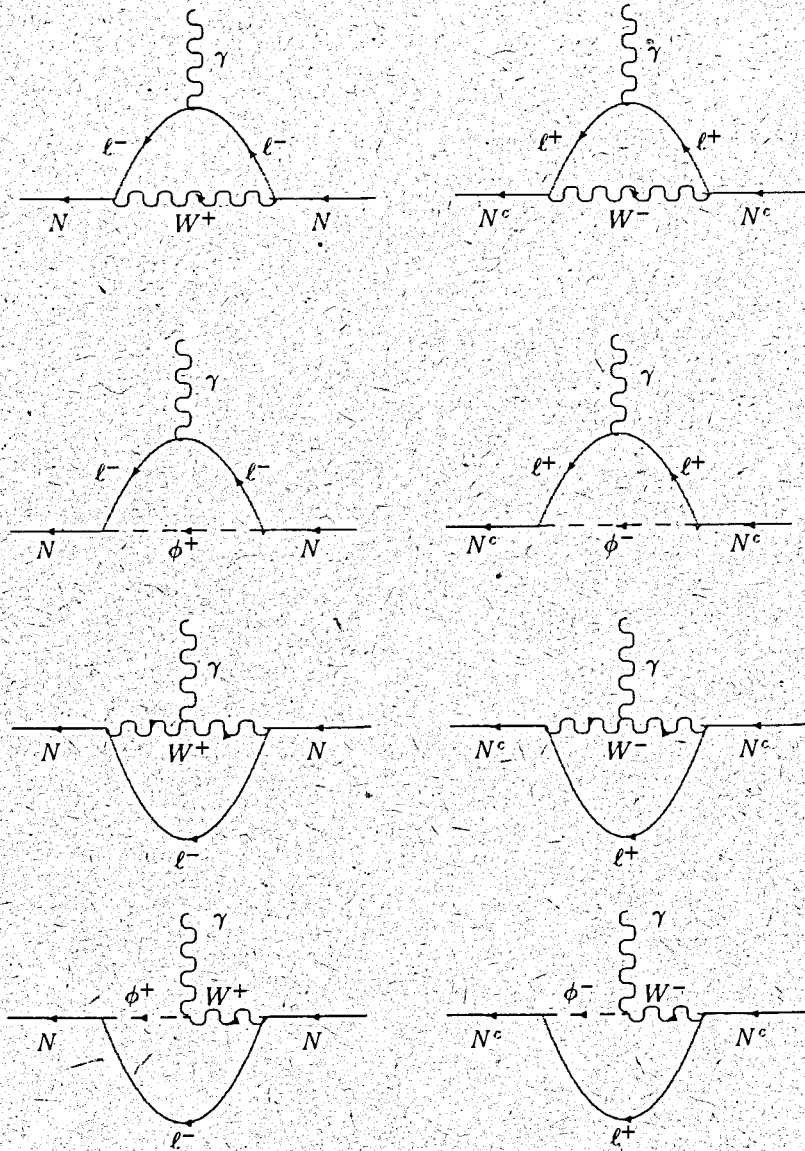


FIG. 1. Feynman diagrams which are responsible for toroid moment of Majorana neutrino.

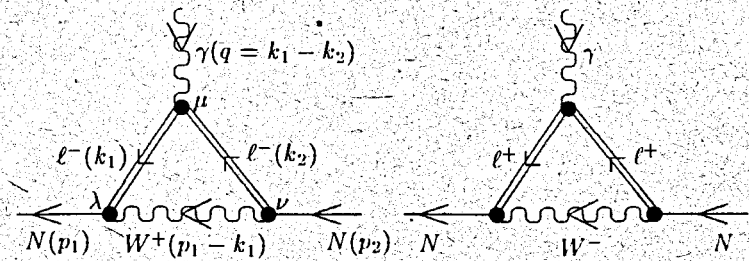


FIG. 2. Feynman graphs with llW intermediate states.

REFERENCES

- [1] For references see, e.g., S. M. Bilenky, S. T. Petcov, *Rev. Mod. Phys.*, **59**, 671 (1987); P. B. Pal, *Int. J. Mod. Phys. A*, **7**, 5387 (1992).
- [2] B. W. Lee and R. E. Shrock, *Phys. Rev. D*, **16**, 1444 (1977); J. L. Lucio, A. Rosado, A. Zepeda, *Phys. Rev. D*, **31**, 1091 (1985); G. Degrassi, A. Sirlin, W. J. Marciano, *Phys. Rev. D*, **39**, 287 (1989); P. Vogel, J. Engel, *Phys. Rev. D*, **39**, 3378 (1989).
- [3] W. Pauli, *Rev. Mod. Phys.*, **13**, 203 (1939).
- [4] Ya. B. Zel'dovich, *Sov. Phys. JETP*, **33**, 1531 (1957).
- [5] V. M. Dubovik, A. A. Cheshkov, *Phys. Part. Nucl.*, **5**, 791 (1974).
- [6] V. M. Dubovik, *JINR Rapid Communication*, **3** [36], 39 (1989); V. M. Dubovik, S. V. Shabanov, *Phys. Lett. A*, **142**, 203 (1989); *J. Math. A: Math. Gen.*, **23**, 3245 (1990); *In special issue "Essays on the formal aspects of electromagnetic theory." Edited by A. Lakhakia. World Scientific. Singapore. 1993. pp. 399-474.*
- [7] I. Yu. Kobzarev, L. B. Okun, *In Problems of Theoretical Physics. Publishing House "Nauka". Moscow. 1972. pp. 219-224*; B. Kayser, *Phys. Rev. D*, **26**, 1662 (1982); *Phys. Rev. D*, **30**, 1023 (1984); J. F. Nieves, *Phys. Rev. D*, **26**, 3152 (1982);
- [8] E. Radescu, *Phys. Rev. D*, **26**, 1266 (1985).
- [9] V. B. Semikoz, Ya. A. Smorodinsky, *Sov. Phys. JETP*, **95**, 35 (1989).
- [10] V. L. Ginzburg, V. N. Tsytovich, *Sov. Phys. JETP*, **88**, 84 (1985).
- [11] Z. Z. Aydin, S. A. Baran, A. O. Barut, *Nucl. Phys. B*, **55**, 601 (1973); S. A. Baran, *Nucl. Phys. B*, **62**, 333 (1973); N. Dombey, A. D. Kennedy, *Phys. Lett. B*, **91**, 428 (1980); M. Abak, C. Aydin, *Europhysics Letters*, **4** (8), 881 (1987).
- [12] H. Czyz, K. Kolodziej, M. Zralek, *Can. J. Phys.*, **66**, 132 (1988); M. J. Musolf, B. R. Holstein, *Phys. Rev. D*, **43**, 2956 (1991).
- [13] V. M. Dubovik, L. A. Tosnyan, *Sov. J. Part. Nucl.*, **14**, 504 (1983); V. M. Dubovik, V. V. Tugushev, *Phys. Rep.*, **187**, 146 (1990).
- [14] A. Gongora-T., R. G. Stuart, *Z. Phys. C*, **55**, 101 (1992).
- [15] E. M. Lifshitz, L. P. Pitaevsky. *Relativistic quantum theory. Moscow. 1971.*
- [16] A. I. Achiezer, V. B. Berestetsky. *Quantum electrodynamics. Moscow. 1971.*
- [17] S. Weinberg, G. Feinberg, *Phys. Rev. Lett.*, **3**, 111 (1959); A. M. Perelomov, Ya. B. Zel'dovich, *Sov. Phys.-JETP*, **39**, 1115 (1960).
- [18] L. Wolfenstein, *Phys. Rev. D*, **17** 2369 (1978); *Phys. Rev. D*, **20** 2634 (1979); S. P. Mikheyev and A. Yu. Smirnov, *Yad. Eiz.*, **42** 1441 (1985) [*Sov. J. Nucl. Phys.* **42** 913 (1985)]; *Nuovo Cimento C*, **9** 17 (1986).
- [19] O. Boyarkin, D. Rein, *Z. Phys. C*, **67**, 607 (1995).
- [20] J. Gluza, M. Zralek, *Phys. Rev. D*, **45**, 1693 (1992); *Phys. Rev. D*, **48**, 5093 (1993).
- [21] L.-F. Li, T.-P. Cheng. *Gauge theory of elementary particle physics. Clarendon Press. Oxford. 1984*; K. Aoki et al., *Suppl. Prog. Theor. Phys.*, **73** (1982).
- [22] J. Schechter, J. W. F. Valle, *Phys. Rev. D*, **22**, 2227 (1980).

Received by Publishing Department
on February 16, 1996.

Дубовик В.М., Кузнецов В.Е.

E2-96-53

Тороидный момент майорановского нейтрино

Рассмотрен полный набор электромагнитных характеристик майорановских нейтрино. Показано, что в статическом пределе ($m_i = m_f = m_\nu$) майорановские нейтрино обладают только одной электромагнитной характеристикой — тороидным дипольным моментом (анополем). С помощью дисперсионного метода вычислен диагональный тороидный момент (формфактор) майорановского нейтрино в однопетлевом приближении стандартной модели. Все внешние частицы находятся на массовых поверхностях, и не возникает проблем с физической интерпретацией конечного результата. Также обсуждаются различные приложения тороидного момента майорановского нейтрино.

Работа выполнена в Лаборатории теоретической физики им. Н.Н.Боголюбова ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна, 1996

Dubovik V.M., Kuznetsov V.E.

E2-96-53

The Toroid Moment of Majorana Neutrino

The total set of electromagnetic characteristics of Majorana neutrinos is considered. It is shown that in the static limit ($m_i = m_f = m_\nu$) the Majorana neutrinos possess only one electromagnetic characteristic, the toroid dipole moment (anapole). We have calculated the diagonal toroid moment (form factor) of the Majorana neutrino in the one-loop approximation of the Standard Model by the dispersion method. All external particles are on the mass shells and there are no problems with the physical interpretation of the final result. Different applications of the toroid moment of Majorana neutrino are also discussed.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna, 1996