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ON SPINOR REPRESENTATIONS IN THE WEYL GAUGE THEORY

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1 Introduction

In ref.[1] it has been shown that the congruent transference introduced by Weyl [2] in 1921 defines a non-Abelian gauge field. The Weyl gauge theory is a realization of abstract theory of gauge fields in the framework of classical differential geometry which does not assume separation between space - time and a gauge space. At the same time, contemporary gauge models assume an exact local separation between space - time and a gauge field. It is just this point that the Weyl theory opens a new possibility.

It is shown that the space of all covariant antisymmetric tensor fields is a spinor representation of the Weyl gauge group and allows the construction of a spinor current - source in a gauge theory of that type. Status of the Cartan torsion field within the Weyl gauge theory is considered and it is shown that the torsion is not a gauge field, however, in a certain gauge, the theory admits geometric interpretation in terms of the Riemann - Cartan geometry.

2 Gauge potential

The Weyl connection which defines the congruent transference of a vector is of the form

$$\Gamma^i_{jk} = \{^{\,i}_{jk}\} + F^i_{jk}, \tag{1}$$

where $\{^{\,i}_{jk}\}$ are Christoffel symbols of the Riemann connection of the metric g_{ij} :

$$\{^{\,i}_{jk}\} = \frac{1}{2} g^{il}(\partial_j g_{kl} + \partial_k g_{jl} - \partial_l g_{jk}), \tag{2}$$

and $F^i_{jk} = F_{jkl} g^{il}$ are components of the Weyl gauge potential that is a covariant third-rank tensor, skew-symmetric in the last two indices

$$F_{jkl} + F_{jlk} = 0. \tag{3}$$

According to (1), vector components under the congruent transference change by the law

$$dv^i = -\{^{\,i}_{jk}\}\, dx^j\, v^k - F^i_{jk}\, dx^j\, v^k, \qquad (4)$$

which includes the displacement belonging to the Riemann geometry (the first term) and the rotation determined by the metric g_{ij} and the bivector $F_{jkl}\, dx^j$. Denote by $\overset{w}{\nabla}_i$ the covariant derivative with respect to the connection Γ^i_{jk}. Then with allowance for (3) we obtain

$$\overset{w}{\nabla}_i\, g_{jk} = 0. \qquad (5)$$

Thus, the Weyl connection is metric. Under congruent transference, the length of a vector does not change since in accordance with (4) $d(g_{ij}\, v^i\, v^j) = 0$.

The Weyl geometric construction presented above has a simple group- theoretical meaning. Let S^i_j be components of a tensor field S of type (1,1) obeying the condition $det(S^i_j) \neq 0$. In this case there exists a tensor field S^{-1} with components T^i_j such that $S^i_k T^k_j = \delta^i_j$. It is obvious that the tensor field S can be regarded as a linear transformation

$$\bar{v}^i = S^i_j\, v^j \qquad (6)$$

in the space of vector fields; S^{-1} is the inverse transformation. Since under congruent transference the length of a vector remains constant, among the transformations (6) we distinguish those that do not change the length of a vector; they are given by the equations

$$g_{ik} S^k_j = g_{jk} T^k_i. \qquad (7)$$

Transformations of the form (6) and (7) form a group that is a gauge group, as will be shown below; we denote it by G_W. The gauge group establishes an equivalence relation in the space of vector fields. It can be shown that if a vector v^i in an equivalence class undergoes congruent transference, then any vector $\bar{v}^i$ equivalent to it in the sense of the group G_W, also undergoes congruent transference. Then, the gauge potential should be transformed by the following law

$$\bar{F}_{lkm} = F_{lij}\, T^i_k\, T^j_m + g_{ij}\, T^i_{m;l}\, T^j_k. \qquad (8)$$

From (7) it follows that the tensor $\bar{F}_{lkm}$ obeys equation (3), and hence, the Weyl connection

$$\bar{\Gamma}^i_{jk} = \{^{\,i}_{jk}\} + \bar{F}^i_{jk}$$

also determines congruent transference. Consider an infinitesimal gauge transformation $S^i_j = \delta^i_j + u^i_j$, $T^i_j = \delta^i_j - u^i_j$, which upon substitution into (7) gives $g_{ik}u^k_j + g_{jk}u^k_i = 0$. Hence it follows that any antisymmetric covariant tensor field of second rank (2-form) $u_{ij} = -u_{ji}$ determines an infinitesimal gauge transformation since $u^i_j = u_{jk}g^{ik}$. Consider infinitesimal gauge transformations of the potential. From (8) we obtain

$$\bar{F}_{ijk} = F_{ijk} + \nabla_i u_{jk} + F_{ijl}u^l_k - F_{ikl}u^l_j, \qquad (9)$$

where ∇_i is covariant derivative with respect to the Riemann connection of the metric g_{ij}, whose Christoffel symbols are given by (2). Let us now construct the strength tensor of the gauge field

$$B_{ijkl} = \nabla_i F_{jkl} - \nabla_j F_{ikl} + F_{ikm} F^m_{jl} - F_{jkm} F^m_{il} + R_{ijkl}, \qquad (10)$$

where R_{ijkl} is the Riemann curvature tensor of the metric g_{ij}. From (9) it follows that the strength tensor is gauge-transformed by the law

$$\bar{B}_{ijkl} = B_{ijkl} + B_{ijkm}u^m_l - B_{ijlm}u^m_k.$$

Let us interpret the Riemann curvature tensor in expression (10) from a group-theoretical and geometric point of view. We set $B_{ijkl} = H_{ijkl} + R_{ijkl}$ and from (9) and (10) we get

$$\bar{H}_{ijkl} = H_{ijkl} + H_{ijkm}u^m_l - H_{ijlm}u^m_k + (\nabla_i\nabla_j - \nabla_j\nabla_i)u_{kl}.$$

According to the Bianchi identities,

$$(\nabla_i \nabla_j - \nabla_j \nabla_i) u_{kl} = R_{ijkm} u_l^m - R_{ijlm} u_k^m,$$

which clearly shows the role of the Riemann curvature tensor under gauge transformations. The tensor (10) has a simple geometric meaning. It can be shown that the curvature tensor of the Weyl connection coincides with the strength tensor of the Weyl gauge field whereas the gauge potential is considered as a deformation tensor of the Riemann connection.

Thus the tensor field F_{ijk}, entering into the Weyl connection is a gauge field, and the tensor B_{ijkl} is the strength tensor of that field. We stress that the gauge group in the case under consideration is defined by the metric, while the gauge field has a direct geometrical meaning (congruent transference) and no extra internal or gauge space is to be introduced. Here gauge symmetry reflects the fact that there does not exist any objective property that could distinguish the geometry defined by their connection Γ from the one defined by the connection $\bar{\Gamma}$.

3 Gauge-field equations

We write the gauge-invariant Lagrangian in the form

$$L = -\frac{1}{16} B_{ijkl} B^{ijkl} + \frac{1}{4} F_{ijk} S^{ijk}, \tag{11}$$

where S^{ijk} is an unknown current-source of the gauge field that should be a quadratic function of components of the quantity defining a spinor representation of the gauge group G_W. Variational procedure results in the following equations of the gauge field:

$$\nabla_i B^{ijkl} + F_{im}^k B^{ijml} - F_{im}^l B^{ijmk} + S^{jkl} = 0. \tag{12}$$

From these equations we derive the equations for the gauge-field current- source

$$\nabla_i S^{ikl} + F_{im}^k S^{iml} - F_{im}^l S^{imk} = 0. \tag{13}$$

Next, consider the current vector

$$Q^j = \frac{1}{2} v_{kl} (F_{im}^k B^{ijml} - F_{im}^l B^{ijmk} + S^{jkl}).$$

From the field equations it follows that the current is conserved if the bivector v_{ij} obeys the equation $\nabla_i v_{jk} = 0$. However, the corresponding conserved quantity is not gauge-invariant. The same holds true also in the abstract theory of gauge fields. In all the previous formulas it was assumed that the gauge potential is of dimension of the inverse length. To introduce the constant of interaction with the gauge field, we should make the substitution $F_{ikl} \Rightarrow \frac{\varepsilon}{\hbar c} F_{ikl}$. In the limit $\varepsilon \Rightarrow 0$ the Lagrangian (11) transforms into the pure gravitational one

$$L = -\frac{1}{16} R_{ijkl} R^{ijkl},$$

which is known [3] to be renormalizable.

Let us now compare the Weyl gauge theory with the abstract theory of gauge fields. The latter is based on an arbitrary semisimple Lie group with structure constants f_{bc}^a and a set of vector fields. Space-time indices are raised and lowered with the metric tensor g_{ij}; whereas parametric indices, with the group metric [4]: $g_{ab} = f_{am}^n f_{bn}^m$. In the Weyl theory, the metric tensor is also a group tensor and structure constants are absent. The reason is that for some Lie groups the coordinates on a group can be regarded as tensor fields in space - time, which just leads to the situation when space-time and gauge space are not separated like in the abstract theory.

4 Spinor representation

Let us consider the field that is a source of the Weyl gauge field and defines a spinor representation of the group G_W. The spinor representation of the Weyl gauge group is a 16-component object which can be defined as space of all covariant antisymmetric tensor fields $f_{i_1\cdots i_p}(p = 0,1,2,3,4)$ on a space - time manifold with the metric g_{ij}. Mathematically, a shorten notation 'differential form' is adopted. So, the form is the following quantity

$$F = (f, \quad f_i, \quad f_{ij}, \quad f_{ijk}, \quad f_{ijkl}). \qquad (14)$$

Objects of that sort were first considered in ref. [5] (see also [6, 7, 8]). The history of the problem and further references can be found in ref. [9].

To prove the above statement, we determine the natural Lagrangian for the field (14) and show that it is invariant under gauge transformations which define the symmetry aspect of the Weyl gauge field. We define the scalar bracket of two fields of the type (14) as follows

$$(F,H) = \bar{f}h + \bar{f}_i h^i + \frac{1}{2!}\bar{f}_{ij}h^{ij} + \frac{1}{3!}\bar{f}_{ijk}h^{ijk} + \frac{1}{4!}\bar{f}_{ijkl}h^{ijkl},$$

where the bar means complex conjugation. If F is a form, the generalized curl operator d is given as follows

$$dF = (0,\ \partial_i f,\ 2\partial_{[i}f_{j]},\ 3\partial_{[i}f_{jk]},\ 4\partial_{[i}f_{jkl]}). \qquad (15)$$

Here square brackets denote alternation; $\partial_i = \partial/\partial x^i$. The simplest Lagrangian for the field F that can be constructed in terms of the operator d is of the form

$$L_d(F) = (F, dF) + (dF, F) + m(F,F). \qquad (16)$$

Note that the operator of external differentiation (15) is the only linear operator of first order that commutes with transformations of the group of diffeomorphisms, the group of symmetry of gravitational interactions. Therefore, the Lagrangian (16) is defined uniquely. If ∇_i is a covariant derivative with respect to the Riemann connection of the metric $\bar{g}_{ij}$, defined by relations (2), then partial derivatives in (15) can be replaced by covariant derivatives.

The Lagrangian (16) is not suitable for the investigation since the operator d is not self-conjugate with respect to the scalar product

$$< F|H >= \int (F,H)\sqrt{-g}d^4 .$$

Using the identity

$$\sum_{p=0}^{4}\frac{1}{p!}f_{ki_1\cdots i_p}h^{ki_1\cdots i_p} = \sum_{p=0}^{4}\frac{1}{p!}(pf_{i_1\cdots i_p})h^{i_1\cdots i_p}$$

we can easily verify that the operator $\nabla = \delta + d$, possesses the required property, where δ is the operator of generalized divergence

$$\delta F = (-\nabla^m f_m,\ -\nabla^m f_{mi},\ -\nabla^m f_{mij},\ -\nabla^m f_{mijk},\ 0).$$

The Lagrangian (16) in terms of the operator $\nabla = \delta + d$ reads

$$L_d(F) = \frac{1}{2}(F,\nabla F) + \frac{1}{2}(\nabla F, F) + m(F,F) + \nabla_i T^i,$$

where

$$T^i = \sum_{p=0}^{4}\frac{1}{p!}\bar{f}_{i_1\cdots i_p}f^{ki_1\cdots i_p} + \bar{f}^{ki_1\cdots i_p}f_{i_1\cdots i_p}.$$

So, the Lagrangian (16) is equivalent to the Lagrangian

$$L(F) = \frac{1}{2}(F,\nabla F) + \frac{1}{2}(\nabla F, F) + m(F,F), \qquad (17)$$

which will be now analyzed. We define a numerical operator Λ, by setting

$$\Lambda F = (f,\ -f_i,\ f_{ij},\ -f_{ijk},\ f_{ijkl}).$$

It is not difficult to verify the validity of the following relations

$$\Lambda^2 = 1, \quad \Lambda d + d\Lambda = 0, \quad \nabla\Lambda + \Lambda\nabla = 0. \qquad (18)$$

Since $\nabla d + d\nabla = \nabla^2$, then

$$\nabla(\frac{1}{2}\nabla - d) + (\frac{1}{2}\nabla - d)\nabla = 0. \quad (19)$$

From (18) and (19) it follows that the operator

$$\overset{*}{\nabla} = (\nabla - 2d)\Lambda \quad (20)$$

commutes with the operator ∇, whereas their squares are equal to

$$\nabla \overset{*}{\nabla} = \overset{*}{\nabla} \nabla, \quad \nabla^2 = \overset{*}{\nabla}{}^2.$$

We will call the operator $\overset{*}{\nabla}$ dual to the operator ∇. In accordance with the principle of 'minimal electromagnetic interaction', we make the substitution $\nabla_i \Rightarrow \nabla_i - \frac{ie}{\hbar c}A_i$ in the operators ∇ and $\overset{*}{\nabla}$, denote the new operators by D and $\overset{*}{D}$, respectively, and determine their squares. We have

$$\overset{*}{D}{}^2 = \nabla^2 - \frac{ie}{\hbar c}Q(F_{ij}) + \frac{2ie}{\hbar c}A^i\nabla_i + \frac{e^2}{\hbar^2 c^2}A_iA^i + \frac{ie}{\hbar c}\nabla_i A^i,$$

where F_{ij}, is a bivector of the electromagnetic field. A similar formula follows for the dual operator D with the change of the operator $Q(F_{ij})$ by the dual operator $\overset{*}{Q}(F_{ij})$. The operators Q and $\overset{*}{Q}$ are defined by antisymmetric tensor fields of second rank (2-forms). Let us write the operators $Q(u_{ij})$, $\overset{*}{Q}(u_{ij})$ in an explicit form

$$\begin{aligned} Q(u_{ij})F = & (\frac{1}{2}u^{mn}f_{mn}, \frac{1}{2}u^{mn}f_{mni} + u_{mi}f^m, \\ & \frac{1}{2}u^{mn}f_{mnij} + 2u_{m[i}f^m_{.j]} - u_{ij}f, \\ & 3u_{m[i}f^m_{.jk]} - 3u_{[ij}f_{k]}, 4u_{m[i}f^m_{.jkl]} - 6u_{[ij}f_{kl]}), \quad (21) \end{aligned}$$

$$\begin{aligned} \overset{*}{Q}(u_{ij})F = & (-\frac{1}{2}u^{mn}f_{mn}, -\frac{1}{2}u^{mn}f_{mni} + u_{mi}f^m, \\ & -\frac{1}{2}u^{mn}f_{mnij} + 2u_{m[i}f^m_{.j]} + u_{ij}f, \\ & 3u_{m[i}f^m_{.jk]} + 3u_{[ij}f_{k]}, 4u_{m[i}f^m_{.jkl]} + 6u_{[ij}f_{kl]}). \end{aligned}$$

It can be shown that the operators $Q(u_{ij})$ and $\overset{*}{Q}(u_{ij})$ commute. Algebra of the operators $J(u_{ij}) = \frac{1}{2}Q(u_{ij})$ is closed with respect to the Lie bracket operation, i.e.

$$[J(u_{ij}), J(v_{ij})] = J(w_{ij}), \quad (22)$$

where

$$w_{ij} = u_{im}v^m_{j.} - u_{jm}v^m_{i.}. \quad (23)$$

From (23) it follows that the operators $J(u_{ij})$ define realization of the Lie algebra of the considered Weyl group. Since

$$(F, J(u_{ij})H) = -(J(u_{ij})F, H), \quad (24)$$

then the Lagrangian (17) will be invariant under the gauge transformations

$$F \Rightarrow \bar{F} = exp(J(u_{ij}))F, \quad (25)$$

provided that

$$[J(u_{ij}), \quad \nabla] = 0. \quad (26)$$

The relation (26) holds valid if the bivector u_{ij} satisfies the equations

$$\nabla_i u_{jk} = 0. \quad (27)$$

The conditions of integrability of equations (27) follow from the Bianchi identities and are of the form $R_{ijk.}{}^m u_{ml} + R_{ijl.}{}^m u_{km} = 0$. When $R_{ijl.}{}^m = K(g_{il}\delta^m_j - g_{jl}\delta^m_i)$, equations (27) will not have solutions at all. Thus, the Lagrangian (17) in the space of constant curvature will be invariant under the transformations (25) only upon introducing a gauge field of a definite type. The latter can be determined as follows. Consider variations of the type $\delta F = J(u_{ij})F$.

This class of variations, up to the Lagrange derivative, yields for the Lagrangian (17)

$$\delta L(F) = \frac{1}{4}\nabla_i u_{jk} S^{ijk},$$

where S^{ijk} is a tensor field of third rank antisymmetric in the last two indices

$$\begin{aligned} S^{jkl} &= \sum_{p=0}^{4} \frac{1}{p!}(\bar{f}^{j}_{.i_1\cdots i_p} f^{kli_1\cdots i_p} + 2g^{j[k}\bar{f}^{l]}_{.i_1\cdots i_p} f^{i_1\cdots i_p} - \\ &\quad -2\bar{f}^{i_1\cdots i_p j[k} f^{l]}_{.i_1\cdots i_p} - \bar{f}_{i_1\cdots i_p} f^{jkli_1\cdots i_p}] + c.c.\,. \end{aligned} \tag{28}$$

So, the Lagrangian (17) is to be supplemented with a term of the form

$$L_I = \frac{1}{2} F_{jkl} S^{jkl}$$

to ensure gauge invariance. We added the same term to the Lagrangian (11) of the gauge field. Thus, the explicit form of the current source of the gauge field is determined uniquely. From (28) it follows that under transformations $\delta F = J(u_{ij})F$, the tensor S^{jkl} is transformed by the law

$$\delta S^{jkl} = u^k_{m.} S^{ijm} - u^j_{m.} S^{ikm}.$$

Hence we obtain that the gauge field F_{ijk} is transformed as follows:

$$\bar{F}_{ijk} = F_{ijk} + \nabla_i u_{jk} + F_{ijm} u^m_{k.} - F_{ikm} u^m_{j.}.$$

According to (7), the field F_{ikl} is the Weyl gauge field, whereas the field F is shown to be its spinor source. That the transformations (25) define the spinor representation of the group G_W can easily be verified by comparing them with the transformations (5). Theory of the field F_{ijk} has been already formulated above, and in the next section we dwell upon the relation between the Weyl gauge potential and the Cartan torsion. That this relation does exist follows from both the fields being tensor fields of the same type.

5 Torsion and gauge symmetry

At present, the torsion discovered by Cartan is the subject of numerous studies aimed at establishing its physical meaning and the connection of general relativity with the physics of microworld. We will consider this question in the framework of the Weyl gauge theory. Let a linear connection be given, and Γ^i_{jk} be its components, the Christoffel symbols. Then, as it was first shown by Cartan [10], the linear connection uniquely defines a tensor field $K^i_{jk} = 1/2(\Gamma^i_{jk} - \Gamma^i_{kj})$, that is called the torsion tensor. The Riemann-Cartan geometry is given by the metric and torsion tensor of the linear connection compatible with the metric. Thus, the connection called the metric connection is defined unambiguously because

$$\Gamma^i_{jk} = \{^{\,i}_{jk}\} + K^i_{jk} + g^{il} K^m_{lj} g_{mk} + g^{il} K^m_{lk} g_{mj}. \tag{29}$$

The problem is formulated as follows. We take the gauge-invariant Lagrangian

$$L = L(F) + \frac{1}{4} F_{ijk} S^{ijk} \tag{30}$$

and determine its variation with respect to F. Then we replace the covariant derivative ∇_i with respect to the Riemann connection in the Lagrangian $L(F)$ by the covariant derivative $\overset{*}{\nabla}_i$ with respect to the connection (29) of the Riemann-Cartan space. This peculiar substitution introduces the torsion field into the Lagrangian (17). A new Lagrangian will be denoted by $L_K(F)$. According to (29), this Lagrangian for the field F in the Riemann-Cartan space can be represented as a sum of the Lagrangian (17) and an extra term to be denoted as $L_A(F)$. Next we vary both the Lagrangians with respect to F. As a result, we have

$$\delta L = \delta L(F) + \frac{1}{2}\sum_{p=0}^{4}\frac{1}{p!}\delta \bar{f}^{i_1\cdots i_p}(F^m f_{mi_1\cdots i_p} + pF_{[i_1} f_{i_2\cdots i_p]} +$$

$$+\frac{p}{2}D_{mn[i_1}f^{mn}_{..i_2\cdots i_p]}+\frac{p(p-1)}{2}D_{[i_1i_2|m|}f^{m}_{i_3\cdots i_p]}-$$
$$-\frac{1}{3!}C^{jkl}f_{jkli_1\cdots i_p}-\frac{1}{3!}p(p-1)(p-2)C_{[i_1i_2i_3}f_{i_4\cdots i_p]})+$$
$$c.c., \tag{31}$$

where $F_m = g^{jk}F_{jkm}$,

$$C_{ijk}=3F_{[ijk]},\quad D_{ijk}=-C_{ijk}+2F_{ijk}.$$

Indices sandwiched between vertical lines are not subject to the operation of alternation. For the new Lagrangian we get

$$\delta L_K(F) = \delta L(F)+\frac{1}{2}\sum_{p=0}^{4}\frac{1}{p!}\delta f^{i_1\cdots i_p}(-K^m f_{mi_1\cdots i_p}-pK_{[i_1}f_{i_2\cdots i_p]}-$$
$$-pK_{[i_1|mn|}f^{mn}_{..i_2\cdots i_p]}-p(p-1)K_{m[i_1i_2}f^{m}_{i_3\cdots i_p]})+c.c., \tag{32}$$

where $K_i = K^m_{\ .mi}$ is the covector of torsion. When varying the Lagrangian L_K, we should take into account that

$$\overset{*}{\nabla}_i A^i=\frac{1}{\sqrt{-g}}\partial_i(\sqrt{-g}A^i)+2K_iA^i,$$

where g is the determinant of the metric tensor. From comparison of (31) and (32) it can be seen that these expressions will coincide if the gauge condition $C_{ijk}=0$ is imposed on the field F_{ijk}, i.e. if we set its completely antisymmetric part to zero,

$$F_{ijk}+F_{jki}+F_{kij}=0$$

and then set $F^i_{jk}=-2K^i_{\ .jk}$.

Thus, we have shown that the field- F equations derived by varying the Lagrangian (30) can, in a certain gauge, be represented as equations in the Riemann-Cartan space. We will also show that the Cartan torsion is not a gauge field. Since, as follows from (5), the Weyl connection is also a metric connection, then from comparison of (1) and (29) we obtain the relation between components of the torsion tensor and gauge potential

$$F_{ijk}=-K_{ijk}+K_{ikj}+K_{jki}, \tag{33}$$

where $K_{ijk}=K^l_{ij}g_{lk}$. From (33) it follows that if F_{ijk} is known, we can determine the torsion tensor components

$$K_{ijk}=-1/2(F_{ijk}-F_{jik}). \tag{34}$$

Considering the relation (34), we can construct the tensor

$$\bar{K}_{ijk}=-1/2(\bar{F}_{ijk}-\bar{F}_{jik}) \tag{35}$$

and pose the entirely natural question of the relationship between the tensors $\bar{K}_{ijk}$ and K_{ijk}. However, such a relationship that contains only the tensors $\bar{K}_{ijk}$, K_{ijk} and the elements of the gauge group does not exist. Indeed, since the tensor F_{ijk} is a skew - symmetric with respect to the second and third indices, whereas the torsion tensor is skew - symmetric with respect to the first two indices, in the relation (34) the index that participates in the gauge transformation (8) and an index that is not affected by it are confused.

The conclusion is that the torsion tensor is not a geometrical quantity from the point of view of gauge symmetry. Specifying the torsion tensor, we fix the gauge. Thus, the fundamental geometrical object is the tensor F_{ijk} that determines the congruent transport. It is for this tensor that the gauge - invariant equations (12), which are in fact determined uniquely by the gauge symmetry, are written down. It is now easy to understand why for the torsion tensor all possible Lagrangians are encountered and investigated in literature with equal success. If one does pose the question of equations for the torsion, then it is most natural to do this end to fix the gauge in accordance with what was said earlier.

We note an interesting connection between gauge transformations and Riemannian geometry. The second term on the right -

hand side of relation (8) vanishes if $T^i_{m;l} = 0$. In the standard theory of gauge fields, this corresponds to transition from local to global transformations. In the considered case, the equations $T^i_{m;l} = 0$ may not have any nontrivial solutions at all, for example, in the case when g_{ij} is the metric of a space of constant curvature. Thus, a Riemannian geometry in general requires a local (gauge) symmetry. We note also that geometrical relationships, like physical laws, depend neither on the choice of the coordinate system nor on the choice of the basis in the studied vector spaces, so that all the relations that have been established above can be expressed in any coordinate system and in any basis, including an orthogonal one.

6 Conclusions

We summarize the obtained results and present some problems. The interpretation of congruent transport given here makes it possible to establish a deep connection between classical differential geometry and the theory of gauge fields. It is important to emphasize once more the fundamental significance of this relationship, which is that in the considered case it is not necessary to introduce an abstract gauge space. The equations for interacting fields can in fact be uniquely derived. The relations established for the Weyl gauge field and the Cartan torsion make it possible to consider, from a new point of view, the problem of physical interpretation of the torsion in the framework of the gauge principle. The existence of the spinor source of the Weyl gauge field is an interesting feature of this field that dictates the question about possible physical manifestations of this kind of interactions. In the Minkowski space - time equations (27) are quite integrable. Thus, the gauge symmetry can be considered in this case as a global one. With respect to this global symmetry a space of forms (14) is reducible. Associated reduction of the space of forms (14) gives the Dirac theory in which we find only well known interactions. In contrast with this case, there is a more interesting possibility, when equations (27) have no

solutions at all. As it was mentioned above, this situation occurs in the space of constant curvature, where the appearance of the Weyl gauge field in a definite sense becomes simple a necessity because of the absent of global symmetry. A very interesting space - time of this kind is the de Sitter one, which is usually considered as a cosmological model. So, the Weyl forces could be manifested on the cosmological scale. The general remark is that all questions and problems discussed in literature in relation to the physical interpretation of torsion can be investigated in a more suitable framework of the Weyl gauge theory.

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