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THE PSEUDOSCALAR AND VECTOR EXCITED MESONS IN THE $U(3) * U(3)$ CHIRAL MODEL

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1. Introduction

Investigation of the radial excitations of the light mesons is of great interest in hadronic physics. So far there are the questions connected with the experimental and theoretical descriptions of the radial excitations of pseudoscalar mesons. For instance, the π' meson with the mass $(1300 \pm 100) MeV$ is usually identified as the first radial excitation of the pion [1]. However, indications of a light resonance in diffractive production of 3π–states have lead to speculations that the mass of the π' may be considerably lower, at $\sim 750\,\mathrm{MeV}$ [2]. So far there are no experimental data concerning the excited states of the kaons [1]. In this paper we attempt to give the theoretical predictions for the masses and the weak decay constants of these excited mesons.

A theoretical description of radially excited pions poses some interesting challenges. The physics of normal pions is completely governed by the spontaneous breaking of chiral symmetry. A convenient way to derive the properties of soft pions is by way of an effective Lagrangian based on a non-linear realization of chiral symmetry [3]. When attempting to introduce higher resonances to extend the effective Lagrangian description to higher energies, one must ensure that the introduction of new degrees of freedom does not spoil the low–energy theorems for pions, which are universal consequences of chiral symmetry.

A useful guideline in the construction of effective meson Lagrangians is the Nambu–Jona-Lasinio (NJL) model, which describes the spontaneous breaking of chiral symmetry at quark level using a four–fermion interaction [4, 5, 6]. The bosonization of this model and the derivative expansion of the resulting fermion determinant reproduce the Lagrangian of the linear sigma model, which embodies the physics of soft pions as well as higher–derivative terms. With appropriate couplings the model allows to derive also a Lagrangian for vector and axial–vector mesons. This not only gives the correct structure of the terms of the Lagrangian as required by chiral symmetry, but also quantitative predictions for the coefficients, such as f_π, f_K, g_π, g_ρ, *etc.*, which are in good agreement with phenomenology. One may therefore hope that a suitable generalization of the NJL–model may provide a means for deriving an effective Lagrangian including also the excited mesons.

When extending the NJL model to describe radial excitations of mesons, one has to introduce non-local (finite–range) four–fermion interactions. Many non-local generalizations of the NJL model have been proposed, using either covariant–euclidean [7] or instantaneous (potential–type) [8, 9] effective quark interactions. These models generally require bilocal meson fields for bosonization, which makes it difficult to perform a consistent derivative expansion leading to an effective Lagrangian. A simple alternative is the use of separable quark interactions. There are a number of advantages of working with such a scheme. First, separable interactions can be bosonized by introducing local meson fields, just as the usual NJL–model. One can thus derive an effective meson Lagrangian directly in terms of local fields and their derivatives. Second, separable interactions allow one to introduce a limited number of excited states and only in a given channel. An interesting method for describing excited meson states in this approximation was proposed in [10]. Furthermore, the separable interaction can be defined in Minkowski space in a 3-dimensional (yet covariant) way, with form factors depending only on the part of the quark–antiquark relative momentum transverse to the meson momentum [9, 11, 12]. This is essential for a correct description of excited states, since it ensures the absence of spurious relative–time excitations [13]. Finally, as we have shown [12], the form factors defining the separable interaction can be chosen so that the gap equation of the generalized NJL–model coincides with the one of the usual NJL–model, whose solution is a constant (momentum–independent) dynamical quark mass. Thus, in this approach it is possible to describe radially excited mesons above the usual NJL vacuum. Aside from the technical simplification the latter means that the separable generalization contains all the successful quantitative results of the usual NJL model.

In our previous paper [12] the theoretical foundations for the choice of the pion-quark form factors in a simple extension of the NJL model to the nonlocal quark interaction were discussed. It was shown that we can choose these form factors such that the gap equation conserves the usual form and gives the solution with a constant constituent quark mass. The quark condensate also does not change after including the excited states in the model, because the tadpole connected with the excited scalar field is equal to zero (the quark loop with the one excited scalar vertex - vertex with form factor).

Now we shall use these form factors for describing the first excited states of the pseudoscalar and vector meson nonets in the framework of the more realistic $U(3) * U(3)$ chiral model [4, 5, 6]. We shall take into account the connections of the scalar and vector coupling constants which have appeared in this model and the additional renormalization of the pseudoscalar fields connected with the pseudoscalar – axial–vector transitions. For simplicity, we shall suppose that the masses of the *up* and *down* quarks are equal to each other and shall take into account only the mass difference between (*up*, *down*) and *strange* quarks (m_u and m_s). Then we have in this model the five basic parameters: m_u, m_s, Λ_3 (3-dimensional cut-off parameter), G_1 and

G_2 (the four–quark coupling constants for the scalar–pseudoscalar coupling (G_1) and for the vector – axial–vector coupling (G_2)). For the definition of these parameters we shall use the experimental values: the pion decay constant $f_\pi = 93 MeV$, the ρ–meson decay constant $g_\rho \approx 6.14$ ($\frac{g_\rho^2}{4\pi} \approx 3$), the pion mass $M_\pi \approx 140 MeV$, ρ–meson mass $M_\rho = 770 MeV$ and the kaon mass $M_K \approx 495 MeV$. Using these five parameters we can describe the masses of the four meson nonets (pseudoscalar, vector, scalar and axial–vector) [1] and all the meson coupling constants describing the strong interactions of the meson with each other and with the quarks.

For the investigation of the excited states of the mesons it is necessary to consider the nonlocal four–quark interactions. We have shown that for description of the excited states of the pseudoscalar and vector meson nonets it is enough to use only three different form factors in the effective four–quark interactions. These form factors contain three arbitrary parameters and describe the excited states of the mesons consisting : 1) of the u and d quarks (π', ρ', ω'); 2) of the (u, d) and the strange quarks (K', $K^{*'}$); 3) of the strange quarks (ϕ'). For the determination of these parameters we shall use the experimental values of the masses of the excited vector mesons ρ', $K^{*'}$ and ϕ'. Then we can calculate the masses of the excited state π' and K' mesons and their weak decay constants $f_{\pi'}$ and $f_{K'}$. In our next work we are going to calculate the excited states of the η and η' mesons without any additional parameters.[2]

In section 2, we introduce the effective quark interaction in the separable approximation and describe its bosonization. We discuss the choice of the three different form factors necessary to describe the excited states of the pseudoscalar and vector meson nonets. In section 3, we derive the effective Lagrangian for the pseudoscalar mesons, and perform the diagonalization leading to the physical meson ground and excited states. In section 4, we perform it for the vector mesons. In section 5, we fix the parameters of the model and evaluate the masses of the excited states π' and K' and their weak decay constants $f_{\pi'}$ and $f_{K'}$. In section 6, we discuss the obtained results.

[1] The mass formulae for the axial–vector mesons and , especially, for the scalar mesons give only qualitative results ($20 - 30\%$ accuracy).

[2] Remind, one more additional parameter , connected with the gluon anomaly, was used in the usual NJL model, when we described the ground states of the η and η' mesons ($U(1)$ problem) [5].
The problem of the radial excitations of the light mesons, including the η and η', in the framework of the potential model was discussed in works [14].

2. $U(3)*U(3)$ chiral Lagrangian with the excited meson states

In the usual $U(3)*U(3)$ NJL model a local (current–current) effective quark interaction is used

$$L[\bar{q},q] = \int d^4x\, \bar{q}(x)\left(i\partial\!\!\!/ - m^0\right) q(x) \; + \; L_{\rm int}, \tag{1}$$

$$\begin{aligned} L_{\rm int} &= \int d^4x [\frac{G_1}{2}(j_S^a(x)j_S^a(x) + j_P^a(x)j_P^a(x)) \\ &- \frac{G_2}{2}(j_V^a(x)j_V^a(x) + j_A^a(x)j_A^a(x))], \end{aligned} \tag{2}$$

where m^0 is the current quark mass matrix. We suppose that $m_u^0 \approx m_d^0$. $j^a_{S,P,V,A}(x)$ denote, respectively, the scalar, pseudoscalar, vector and axial–vector currents of the quark field ($U(3)$–flavor),

$$\begin{aligned} j_S^a(x) &= \bar{q}(x)\lambda^a q(x), \qquad & j_P^a(x) = \bar{q}(x)i\gamma_5\lambda^a q(x), \\ j_V^{a,\mu}(x) &= \bar{q}(x)\gamma^\mu\lambda^a q(x), \qquad & j_A^{a,\mu}(x) = \bar{q}(x)\gamma_5\gamma^\mu\lambda^a q(x). \end{aligned} \tag{3}$$

Here λ^a are the Gell-Mann matrices, $0 \le a \le 8$. The model can be bosonized in the standard way by representing the 4–fermion interaction as a Gaussian functional integral over scalar, pseudoscalar,vector and axial–vector meson fields [4, 5, 6]. The effective meson Lagrangian, which is obtained by integration over the quark fields, is expressed in terms of local meson fields. By expanding the quark determinant in derivatives of the local meson fields one then derives the chiral meson Lagrangian.

The Lagrangian (2) describes only ground–state mesons. To include excited states, one has to introduce effective quark interactions with a finite range. In general, such interactions require bilocal meson fields for bosonization [7, 9]. A possibility to avoid this complication is the use of a separable interaction, which is still of current–current form, eq.(2), but allows for non-local vertices (form factors) in the definition of the quark currents, eqs.(3),

$$\begin{aligned} \tilde{L}_{\rm int} = \int d^4x \sum_{i=1}^N \Bigg[&\frac{G_1}{2}\left[j^a_{S,i}(x)j^a_{S,i}(x) + j^a_{P,i}(x)j^a_{P,i}(x)\right] \\ &-\frac{G_2}{2}\left[j^a_{V,i}(x)j^a_{V,i}(x) + j^a_{A,i}(x)j^a_{A,i}(x)\right]\Bigg], \end{aligned} \tag{4}$$

$$j^a_{S,i}(x) = \int d^4x_1 \int d^4x_2\, \bar{q}(x_1)F^a_{S,i}(x;x_1,x_2)q(x_2), \tag{5}$$

$$j^a_{P,i}(x) = \int d^4x_1 \int d^4x_2\, \bar{q}(x_1)F^a_{P,i}(x;x_1,x_2)q(x_2), \tag{6}$$

$$j^{a,\mu}_{V,i}(x) = \int d^4x_1 \int d^4x_2\, \bar{q}(x_1)F^{a,\mu}_{V,i}(x;x_1,x_2)q(x_2), \tag{7}$$

$$j^{a,\mu}_{A,i}(x) = \int d^4x_1 \int d^4x_2\, \bar{q}(x_1)F^{a,\mu}_{A,i}(x;x_1,x_2)q(x_2). \tag{8}$$

Here, $F^{a,\mu}_{U,i}(x;x_1,x_2)$, $\quad i = 1,\dots N$, denote a set of non-local scalar, pseudoscalar, vector and axial–vector quark vertices (in general momentum– and spin–dependent), which will be specified below. Upon bosonization we obtain

$$\begin{aligned} L_{\rm bos}(\bar{q},q;\sigma,\phi,P,A) = &\int d^4x_1 \int d^4x_2\, \bar{q}(x_1)[\left(i\partial\!\!\!/_{x_2} - m^0\right)\delta(x_1-x_2) \\ &+ \int d^4x \sum_{i=1}^N \left(\sigma_i^a(x)F^a_{\sigma,i}(x;x_1,x_2) + \phi_i^a(x)F^a_{\phi,i}(x;x_1,x_2)\right. \\ &\left. + V_i^{a,\mu}(x)F^{a,\mu}_{V,i}(x;x_1,x_2) + A_i^{a,\mu}(x)F^{a,\mu}_{A,i}(x;x_1,x_2)\right)]q(x_2) \\ - \int d^4x \sum_{i=1}^N &\left[\frac{1}{2G_1}\left(\sigma_i^{a\,2}(x) + \phi_i^{a\,2}(x)\right) - \frac{1}{2G_2}\left(V_i^{a,\mu\,2}(x) + A_i^{a,\mu\,2}(x)\right)\right]. \end{aligned} \tag{9}$$

This Lagrangian describes a system of local meson fields, $\sigma_i^a(x)$, $\phi_i^a(x)$, $V_i^{a,\mu}(x)$, $A_i^{a,\mu}(x)$, $i = 1,\dots N$, which interact with the quarks through non-local vertices. These fields are not yet to be associated with physical particles , which will be obtained after determining the vacuum and diagonalizing the effective meson Lagrangian.

In order to describe the first radial excitations of mesons (N = 2), we take the form factors in the form (see [12])

$$\begin{aligned} F^a_{\sigma,2}(\mathbf{k}) &= \lambda^a f_a(\mathbf{k}),; \quad F^a_{\phi,2}(\mathbf{k}) = i\gamma_5\lambda^a f_a(\mathbf{k}), \\ F^{a,\mu}_{V,2}(\mathbf{k}) &= \gamma^\mu\lambda^a f_a(\mathbf{k}),; \quad F^{a,\mu}_{A,2}(\mathbf{k}) = \gamma_5\gamma^\mu\lambda^a f_a(\mathbf{k}), \end{aligned} \tag{10}$$

$$f_a(\mathbf{k}) = c_a(1 + d_a\mathbf{k}^2). \tag{11}$$

We consider here the form factors in the momentum space and in the rest frame of the mesons ($\mathbf{P}_{meson} = 0$. k and P are the relative and total momentum of the quark-antiquark pair.). For the ground states of the mesons the functions $f_a^0(\mathbf{k}) = 1$.

After integrating over the quark fields in eq.(9), one obtains the effective Lagrangian of the $\sigma_1^a, \sigma_2^a, \phi_1^a, \phi_2^a, V_1^{a,\mu}, V_2^{a,\mu}, A_1^{a,\mu}$ and $A_2^{a,\mu}$ fields. ($u_1 = u, u_2 = \bar{u}$)

$$L(\sigma', \phi, V, A, \bar{\sigma}, \bar{\phi}, \bar{V}, \bar{A}) =$$

$$-\frac{1}{2G_1}(\sigma_a'^2+\phi_a^2+\bar\sigma_a^2+\bar\phi_a^2)+\frac{1}{2G_2}(V_a^2+A_a^2+\bar V_a^2+\bar A_a^2)$$
$$-iN_c \,\mathrm{Tr}\,\log[i\hat\partial - m^0 + (\sigma_a' + i\gamma_5\phi_a + \gamma_\mu V_a^\mu + \gamma_5\gamma_\mu A_a^\mu)\lambda^a$$
$$+(\bar\sigma_a + i\gamma_5\bar\phi_a + \gamma_\mu \bar V_a^\mu + \gamma_5\gamma_\mu \bar A_a^\mu)\lambda^a f_a]\,. \qquad (12)$$

Now let us remind how we fix the basic parameters in the usual NJL model without the excited state of mesons [5].

Firstly, define the vacuum expectation of the σ_a' fields

$$<\frac{\delta L}{\delta\sigma_a'}>_0 \;=\; -iN_c\,\mathrm{tr}\int_{\Lambda_3}\frac{d^4k}{(2\pi)^4}\frac{1}{(\hat k - m^0 + <\sigma_a'>_0)} - \frac{<\sigma_a'>_0}{G_1} \;=\; 0. \quad (13)$$

Introduce the new sigma fields whose vacuum expectations are equal to zero

$$\sigma_a = \sigma_a' - <\sigma_a'>_0 \qquad (14)$$

and redefine the quark masses

$$m_a = m_a^0 - <\sigma_a'>\,. \qquad (15)$$

Then eq. (13) can be rewritten in the form of the usual gap equation

$$m_i = m_i^0 + 8G_1 m_i I_1(m_i), \qquad (i=u,d,s) \qquad (16)$$

where

$$I_n(m_i) = -iN_c\int_{\Lambda_3}\frac{d^4k}{(2\pi)^4}\frac{1}{(m_i^2-k^2)^n} \qquad (17)$$

and m_i are the constituent quark masses.

In order to obtain the correct coefficients of kinetic terms of the mesons in the one-quark-loop approximation, we have to make the renormalization of the meson fields in eq. (12)

$$\sigma_a = g_\sigma^a\sigma_a^r, \quad \phi_a = g_\sigma^a\phi_a^r, \quad V_a^\mu = g_V^a V_a^{\mu,r}, \quad A_a^\mu = g_V^a A_a^{\mu,r}, \qquad (18)$$

where

$$g_\sigma^{a_{i,j}} = [4I_2(m_i,m_j)]^{-\frac{1}{2}}, \quad I_2(m_i,m_j) = -iN_c\int_{\Lambda_3}\frac{d^4k}{(2\pi)^4}\frac{1}{(m_i^2-k^2)(m_j^2-k^2)}, \qquad (19)$$

$$g_V^a = \sqrt{6}g_\sigma^a. \qquad (20)$$

After taking into account the pseudoscalar – axial-vector transitions ($\phi_a \to A_a$), the additional renormalization of the pseudoscalar fields appears

$$g_\phi^a = Z_a^{-\frac{1}{2}} g_\sigma^a, \qquad (21)$$

where $Z_\pi = 1 - \frac{6m_u^2}{M_{a_1}^2} \approx 0.7$ for pions. ($M_{a_1} = 1.23GeV$ is the mass of the axial-vector a_1 meson, [1], $m_u = 280MeV$ [5]). We shall assume that all $Z_a \approx Z_\pi \approx 0.7$.

After these renormalizations the part of the Lagrangian (12) describing the ground states of mesons takes the form

$$L(\sigma,\phi,V,A) = -\frac{1}{2G_1}(g_\sigma^{a2}\sigma_a^2 + g_\phi^{a2}\phi_a^2) + \frac{g_V^{a2}}{2G_2}(V_a^2+A_a^2)$$
$$-iN_c\,\mathrm{Tr}\,\log\left[i\hat\partial - m^0 + \left(g_\sigma^a\sigma_a + i\gamma_5 g_\phi^a\phi_a + \frac{g_V^a}{2}(\gamma_\mu V_a^\mu + \gamma_5\gamma_\mu A_a^\mu)\right)\lambda^a\right]. \quad (22)$$

For simplicity we omitted here the index r of the meson fields.

From the Lagrangian (22) in the one-loop approximation the following expressions for the meson masses are obtained [5]

$$M_\pi^2 \;=\; g_\pi^2\left[\frac{1}{G_1} - 8I_1(m_u)\right] = \frac{g_\pi^2}{G_1}\frac{m_u^0}{m_u}, \quad g_\pi^2 = \frac{1}{4ZI_2(m_u,m_u)}, \qquad (23)$$

$$M_K^2 = g_K^2\left[\frac{1}{G_1} - 4(I_1(m_u)+I_1(m_s))\right] + Z^{-1}(m_s-m_u)^2,$$
$$g_K^2 = \frac{1}{4ZI_2(m_u,m_s)}, \qquad (24)$$

$$M_\rho^2 = \frac{g_\rho^2}{4G_2} = \frac{3}{8g_2 I_2(m_u,m_u)}, \qquad (25)$$

$$M_\phi^2 = M_\rho^2\frac{I_2(m_u,m_u)}{I_2(m_s,m_s)}, \qquad (26)$$

$$M_{K^*}^2 = M_\rho^2\frac{I_2(m_u,m_u)}{I_2(m_u,m_s)} + \frac{3}{2}(m_s-m_u)^2. \qquad (27)$$

Now let us fix our basic parameters. For that we shall use the five experimental values [4, 5]:

1) The pion decay constant $f_\pi = 93MeV$.

2) The ρ-meson decay constant $g_\rho \approx 6.14$.

Then from the Goldberger-Treimann identity we obtain

$$m_u = f_\pi g_\pi \qquad (28)$$

and from eqs. (20) and (21) we get

$$g_\pi = \frac{g_\rho}{\sqrt{6Z}}, \quad m_u = \frac{f_\pi g_\rho}{\sqrt{6Z}} = 280MeV. \qquad (29)$$

From eqs. (19) and (20) we can obtain (see [15])

$$I_2(m_u, m_u) = \frac{3}{2g_\rho^2}, \qquad \Lambda_3 = 1.03GeV. \tag{30}$$

3) $M_\pi \approx 140MeV$. The eq. (23) gives $G_1 = 3.48GeV^{-2}$ (see [15]).
4) $M_\rho = 770MeV$. The eq. (25) gives $G_2 = 16GeV^{-2}$.
5) $M_K \approx 495MeV$. The eq. (24) gives $m_s = 460MeV$.
After that the masses of η, η' and K^*, ϕ mesons can be calculated with a satisfactory accuracy. [3] It is possible also to give the qualitative estimations for the masses of the scalar and axial–vector mesons, using the formulae

$$M^2_{A_{i,j}} = M^2_{V_{i,j}} + 6m_i m_j, \tag{31}$$
$$M^2_{\sigma_{i,j}} = M^2_{\phi_{i,j}} + 4m_i m_j. \tag{32}$$

We can calculate the values of all the coupling constants, describing the strong interactions of the scalar, pseudoscalar, vector and axial–vector mesons with each other and with the quarks, and describe all the main decays of these mesons (see [5]).

3. The effective Lagrangian for the ground and excited states of the pions and kaons

To describe the first excited states of the all meson nonets, it is enough to use only three different form factors $f_a(\mathbf{k})$ (see eq. (11))

$$\begin{aligned} f_{uu}(\mathbf{k}) &= c_{uu}(1 + d_{uu}\mathbf{k}^2), \\ f_{us}(\mathbf{k}) &= c_{us}(1 + d_{us}\mathbf{k}^2), \\ f_{ss}(\mathbf{k}) &= c_{ss}(1 + d_{ss}\mathbf{k}^2). \end{aligned} \tag{33}$$

Following our work [12] we can fix the parameters d_{uu}, d_{us} and d_{ss} by using the conditions

$$I_1^{f_{uu}}(m_u) = 0, \qquad I_1^{f_{us}}(m_u) + I_1^{f_{us}}(m_s) = 0, \qquad I_1^{f_{ss}}(m_s) = 0, \tag{34}$$

where

$$I_1^{f_a..f_a}(m_u) = -iN_c \int_{\Lambda_3} \frac{d^4k}{(2\pi)^4} \frac{f_a..f_a}{(m_i^2 - k^2)}. \tag{35}$$

The eqs. (34) allows us to conserve the gap equations in the form usual for the NJL model (see eqs. (16)), because the tadpoles with the excited scalar external fields do not contribute to the quark condensates and to the constituent quark masses.

[3] To calculate the masses of the η and η' mesons, it is necessary to take into account the gluon anomaly [5].

Using eqs. (34) we obtain for all d_a close values

$$d_{uu} = -1.78\ GeV^{-2}, \quad d_{us} = -1.75\ GeV^{-2}, \quad d_{ss} = -1.72\ GeV^{-2}. \tag{36}$$

Now let us consider the free part of the Lagrangian (12). For the pseudoscalar meson we obtain

$$L^{(2)}(\phi) = \frac{1}{2} \sum_{i,j=1}^{2} \sum_{a=0}^{8} \phi_i^a(P) K_{ij}^{ab}(P) \phi_j^b(P). \tag{37}$$

Here

$$\sum_{a=1}^{3} (\phi_i^a)^2 = (\pi_i^0)^2 + 2\pi_i^+\pi_i^-, \quad (\phi_i^4)^2 + (\phi_i^5)^2 = 2K_i^+K_i^-,$$
$$(\phi_i^6)^2 + (\phi_i^7)^2 = 2K_i^0\bar{K}_i^0, \quad (\phi_i^0)^2 = (\phi_i^u)^2, \quad (\phi_i^8)^2 = (\phi_i^s)^2. \tag{38}$$

ϕ_i^u and ϕ_i^s are the components of the η and η' mesons. The quadratic form $K_{ij}^{ab}(P)$, eq.(37), is obtained as

$$K_{ij}^{ab}(P) \equiv \delta^{ab} K_{ij}^a(P), \tag{39}$$

$$K_{ij}^a(P) = -\delta_{ij}\frac{1}{G_1}$$
$$- i N_c \,\mathrm{tr} \int_{\Lambda_3} \frac{d^4k}{(2\pi)^4} \left[\frac{1}{\not k + \frac{1}{2}\not P - m_q^a} i\gamma_5 f_i^a \frac{1}{\not k - \frac{1}{2}\not P - m_{q'}^a} i\gamma_5 f_j^a \right],$$
$$f_1^a \equiv 1, \qquad f_2^a \equiv f_a(\mathbf{k}). \tag{40}$$

$$\begin{aligned} m_q^a &= m_u \ (a = 0, ..., 7); \ m_q^8 = m_s; \\ m_{q'}^a &= m_u \ (a = 0, ..., 3); \ m_{q'}^a = m_s \ (a = 4, ..., 8). \end{aligned} \tag{41}$$

m_u and m_s are the constituent quark masses ($m_u \approx m_d$). The integral (40) is evaluated by expanding in the meson field momentum, P. To order P^2, one obtains

$$\begin{aligned} K_{11}^a(P) &= Z_1^a(P^2 - M_1^{a2}), \qquad K_{22}^a(P) = Z_2^a(P^2 - M_2^{a2}), \\ K_{12}^a(P) &= K_{21}^a(P) = \gamma^a(P^2 - \Delta^2\delta_{ab}|_{b=4,...,7}), \quad (\Delta = m_s - m_u) \end{aligned} \tag{42}$$

where

$$Z_1^a = 4I_2^a Z, \qquad Z_2^a = 4I_2^{ffa}, \qquad \gamma^a = 4I_2^{fa}, \tag{43}$$
$$M_1^{a2} = (Z_1^a)^{-1}[\frac{1}{G_1} - 4(I_1^a(m_q^a) + I_1^a(m_{q'}^a)] + Z^{-1}\Delta^2\delta_{ab}|_{b=4,...,7}, \tag{44}$$
$$M_2^{a2} = (Z_2^a)^{-1}[\frac{1}{G_1} - 4(I_1^{ffa}(m_q^a) + I_1^{ffa}(m_{q'}^a)] + \Delta^2\delta_{ab}|_{b=4,...,7}. \tag{45}$$

Here, I_n^a, I_n^{fa} and I_n^{ffa} denote the usual loop integrals arising in the momentum expansion of the NJL quark determinant, but now with zero, one or two factors $f_a(\mathbf{k})$, eqs.(33), in the numerator (see (35) and below)

$$I_2^{f..fa}(m_q, m_{q'}) = -iN_c \int_{\Lambda_3} \frac{d^4k}{(2\pi)^4} \frac{f_a(\mathbf{k})..f_a(\mathbf{k})}{(m_q^{a2} - k^2)(m_{q'}^{a2} - k^2)}. \tag{46}$$

The evaluation of these integrals with a 3–momentum cutoff is described *e.g.* in ref.[15]. The integral over k_0 is taken by contour integration, and the remaining 3–dimensional integral is regularized by the cutoff. Only the divergent parts are kept; all finite parts are dropped. We point out that the momentum expansion of the quark loop integrals, eq.(40), is an essential part of this approach. The NJL–model is understood here as a model only for the lowest coefficients of the momentum expansion of the quark loop, not its full momentum dependence (singularities *etc.*). Z is the additional renormalization of the ground pseudoscalar meson states taking into account the $\phi^a \to A^a$ transitions (see eq.(21)).

After the renormalization of the meson fields

$$\phi_i^{ar} = \sqrt{Z_i^a}\phi_i^a \tag{47}$$

the part of the Lagrangian (37), describing the pions and kaons, takes the form

$$L_\pi^{(2)} = \frac{1}{2}\left[(P^2 - M_{\pi_1}^2)\ \pi_1^2 + 2\Gamma_\pi P^2\ \pi_1\pi_2 + (P^2 - M_{\pi_2}^2)\ \pi_2^2\right], \tag{48}$$

$$\begin{aligned} L_K^{(2)} &= \frac{1}{2}[(P^2 - M_{K_1}^2 - \Delta^2)\ K_1^2 + (P^2 - M_{K_2}^2 - \Delta^2)\ K_2^2 \\ &+ 2\Gamma_K(P^2 - \Delta^2)\ K_1K_2]. \end{aligned} \tag{49}$$

Here

$$\Gamma_a = \frac{\gamma_a}{\sqrt{Z_1^a Z_2^a}} = \frac{I_2^{fa}}{\sqrt{I_2^a I_2^{ffa}}}. \tag{50}$$

$$\begin{aligned} M_{\pi_1}^2 &= (4ZI_2(m_u, m_u))^{-1}[\frac{1}{G_1} - 8I_1(m_u)] = \frac{m_u^0}{4Zm_uI_2(m_u, m_u)}, \\ M_{\pi_2}^2 &= (4I_2^{ff}(m_u, m_u))^{-1}[\frac{1}{G_1} - 8I_1^{ff}(m_u)], \end{aligned} \tag{51}$$

$$\begin{aligned} M_{K_1}^2 &= (4ZI_2(m_u, m_s))^{-1}[\frac{1}{G_1} - 4(I_1(m_u) + I_1(m_s))] + (Z^{-1} - 1)\Delta^2 \\ &= \frac{\frac{m_u^0}{m_u} + \frac{m_s^0}{m_s}}{4ZI_2(m_u, m_s)} + (Z^{-1} - 1)\Delta^2, \\ M_{K_2}^2 &= (4I_2^{ff}(m_u, m_s))^{-1}[\frac{1}{G_1} - 4(I_1^{ff}(m_u) + I_1^{ff}(m_s))]. \end{aligned} \tag{52}$$

After the transformations of the meson fields

$$\begin{aligned} \phi^a &= \frac{1}{\sqrt{2}}(\sqrt{1+\Gamma_a}\ sin\theta_a + \sqrt{1-\Gamma_a}\ cos\theta_a)\phi_1^{ar} \\ &+ \frac{1}{\sqrt{2}}(\sqrt{1+\Gamma_a} sin\ \theta_a - \sqrt{1-\Gamma_a}\ cos\theta_a)\phi_2^{ar}, \\ \phi'^a &= \frac{1}{\sqrt{2}}(\sqrt{1-\Gamma_a}\ sin\theta_a - \sqrt{1+\Gamma_a}\ cos\theta_a)\phi_1^{ar} \\ &- \frac{1}{\sqrt{2}}(\sqrt{1-\Gamma_a}\ sin\theta_a + \sqrt{1+\Gamma_a}\ cos\theta_a)\phi_2^{ar} \end{aligned} \tag{53}$$

the Lagrangians (48) and (49) take the diagonal forms

$$L_\pi^{(2)} = \tfrac{1}{2}(P^2 - M_\pi^2)\ \pi^2 + \tfrac{1}{2}(P^2 - M_{\pi'}^2)\ \pi'^2, \tag{54}$$

$$L_K^{(2)} = \tfrac{1}{2}(P^2 - M_K^2)\ K^2 + \tfrac{1}{2}(P^2 - M_{K'}^2)\ K'^2. \tag{55}$$

Here

$$\begin{aligned} M_{(\pi,\pi')}^2 &= \frac{1}{2(1-\Gamma_\pi^2)}[M_{\pi_1}^2 + M_{\pi_2}^2 \\ &(-,+)\ \sqrt{(M_{\pi_1}^2 - M_{\pi_2}^2)^2 + (2M_{\pi_1}M_{\pi_2}\Gamma_\pi)^2}], \end{aligned} \tag{56}$$

$$\begin{aligned} M_{(K,K')}^2 &= \frac{1}{2(1-\Gamma_K^2)}[M_{K_1}^2 + M_{K_2}^2 + 2\Delta^2(1-\Gamma_K^2) \\ &(-,+)\ \sqrt{(M_{K_1}^2 - M_{K_2}^2)^2 + (2M_{K_1}M_{K_2}\Gamma_K)^2}]. \end{aligned} \tag{57}$$

and

$$\tan 2\theta_a = \sqrt{\frac{1}{\Gamma_a^2} - 1}\ \left[\frac{M_{\phi_1^a}^2 - M_{\phi_2^a}^2}{M_{\phi_1^a}^2 + M_{\phi_2^a}^2}\right]. \tag{58}$$

In the chiral limit we obtain: $M_{\pi_1} = 0$, $M_{\pi_2} \neq 0$ (see eqs. (51)) and

$$M_\pi^2 = M_{\pi_1}^2 + \mathcal{O}(M_{\pi_1}^4), \tag{59}$$

$$M_{\pi'}^2 = \frac{M_{\pi_2}^2 + M_{\pi_1}^2\Gamma_\pi}{1-\Gamma_\pi^2} + \mathcal{O}(M_{\pi_1}^4). \tag{60}$$

Thus, in the chiral limit the effective Lagrangian eq.(48) describes a massless Goldstone pion, π, and a massive particle, π'. We obtained similar results for the kaons.

For the weak decay constants of the pions and kaons we obtain (see [12])

$$\begin{aligned} f_\pi &= m_u\sqrt{2ZI_2(m_u, m_u)}\ (sin\theta_\pi\sqrt{1+\Gamma_\pi} + cos\theta_\pi\sqrt{1-\Gamma_\pi}), \\ f_{\pi'} &= m_u\sqrt{2ZI_2(m_u, m_u)}\ (sin\theta_\pi\sqrt{1-\Gamma_\pi} - cos\theta_\pi\sqrt{1+\Gamma_\pi}), \end{aligned} \tag{61}$$

$$f_K = \frac{m_u+m_s}{\sqrt{2}}\sqrt{ZI_2(m_u,m_s)}\;(sin\theta_K\sqrt{1+\Gamma_K}+cos\theta_K\sqrt{1-\Gamma_K}),$$
$$f_{K'} = \frac{m_u+m_s}{\sqrt{2}}\sqrt{ZI_2(m_u,m_s)}\;(sin\theta_K\sqrt{1-\Gamma_K}-cos\theta_K\sqrt{1+\Gamma_K}). \quad (62)$$

In the chiral limit we have

$$sin\theta_a = \sqrt{\frac{1+\Gamma_a}{2}}, \qquad cos\theta_a = \sqrt{\frac{1-\Gamma_a}{2}} \quad (63)$$

and

$$f_\pi = \frac{m_u}{g_\pi}, \quad f_K = \frac{(m_u+m_s)}{2g_K}, \quad f_{\pi'} = 0, \quad f_{K'} = 0. \quad (64)$$

Here we used eqs.(19) and (21). Therefore, in the chiral limit we obtain the Goldberger-Treimann identities for the coupling constants g_π and g_K. The matrix elements of the divergences of the axial currents between meson states and the vacuum equal (PCAC relations)

$$\langle 0|\partial^\mu A^a_\mu|\phi^b\rangle = m^2_\phi f_\phi \delta^{ab}, \quad (65)$$
$$\langle 0|\partial^\mu A^a_\mu|\phi'^b\rangle = m^2_{\phi'} f_{\phi'} \delta^{ab}. \quad (66)$$

Then from eqs. (59) and (64) we can see that these axial currents are conserved in the chiral limit, because their divergences equal zero, according to the low-energy theorems.

4. The effective Lagrangian for the ground and excited states of the vector mesons

The free part of the effective Lagrangian (12) describing the ground and excited states of the vector mesons has the form

$$L^{(2)}(V) = -\tfrac{1}{2}\sum_{i,j=1}^{2}\sum_{a=0}^{8} V^{\mu a}_i(P)R^{\mu\nu a}_{ij}(P)V^{\nu a}_j(P), \quad (67)$$

where

$$\sum_{a=0}^{3} V^{\mu a}_i = (\omega^\mu_i)^2 + (\rho^{0\mu}_i)^2 + 2\rho^{+\mu}_i\rho^{-\mu}_i, \quad (V^{4\mu}_i)^2+(V^{5\mu}_i)^2 = 2K^{*+\mu}_iK^{*-\mu}_i,$$
$$(V^{6\mu}_i)^2+(V^{7\mu}_i)^2 = 2K^{*0\mu}_iK^{*0\mu}_i, \quad (V^{8\mu}_i)^2 = (\phi^\mu_i)^2 \quad (68)$$

and

$$R^{\mu\nu a}_{ij}(P) = -\frac{\delta_{ij}}{G_2}g^{\mu\nu}$$

$$-\,i\,N_c\,\mathrm{tr}\int_{\Lambda_3}\frac{d^4k}{(2\pi)^4}\left[\frac{1}{\not k+\frac{1}{2}\not P - m^a_q}\gamma^\mu f^a_i \frac{1}{\not k-\frac{1}{2}\not P - m^a_{q'}}\gamma^\nu f^a_j\right].$$
$$f^a_1 \equiv 1, \qquad f^a_2 \equiv f_a(\mathbf{k}). \quad (69)$$

To order P^2, one obtains

$$R^{\mu\nu a}_{11} = W^a_1[P^2g^{\mu\nu} - P^\mu P^\nu - g^{\mu\nu}(\bar{M}^a_1)^2],$$
$$R^{\mu\nu a}_{22} = W^a_2[P^2g^{\mu\nu} - P^\mu P^\nu - g^{\mu\nu}(\bar{M}^a_2)^2],$$
$$R^{\mu\nu a}_{12} = R^{\mu\nu a}_{21} = \bar{\gamma}^a[P^2g^{\mu\nu} - P^\mu P^\nu - \frac{3}{2}\Delta^2 g^{\mu\nu}\delta^{ab}|_{b=4..7}]. \quad (70)$$

Here

$$W^a_1 = \frac{8}{3}I^a_2, \quad W^a_2 = \frac{8}{3}I^{ffa}_2, \quad \bar{\gamma}^a = \frac{8}{3}I^{fa}_2, \quad (71)$$
$$(\bar{M}^a_1)^2 = (W^a_1G_2)^{-1} + \frac{3}{2}\Delta^2\delta^{ab}|_{b=4..7}, \quad (72)$$
$$(\bar{M}^a_2)^2 = (W^a_2G_2)^{-1} + \frac{3}{2}\Delta^2\delta^{ab}|_{b=4..7}. \quad (73)$$

After renormalization of the meson fields

$$V^{\mu ar}_i = \sqrt{W^a_i}\;V^{\mu a}_i \quad (74)$$

we obtain the Lagrangians

$$L^{(2)}_\rho = -\tfrac{1}{2}[(g^{\mu\nu}P^2 - P^\mu P^\nu - g^{\mu\nu}M^2_{\rho_1})\rho^\mu_1\rho^\nu_1$$
$$+\ 2\Gamma_\rho(g^{\mu\nu}P^2 - P^\mu P^\nu)\rho^\mu_1\rho^\nu_2 + (g^{\mu\nu}P^2 - P^\mu P^\nu - g^{\mu\nu}M^2_{\rho_2})\rho^\mu_2\rho^\nu_2], \quad (75)$$

$$L^{(2)}_\phi = -\tfrac{1}{2}[(g^{\mu\nu}P^2 - P^\mu P^\nu - g^{\mu\nu}M^2_{\phi_1})\phi^\mu_1\phi^\nu_1$$
$$+\ 2\Gamma_\phi(g^{\mu\nu}P^2 - P^\mu P^\nu)\phi^\mu_1\phi^\nu_2 + (g^{\mu\nu}P^2 - P^\mu P^\nu - g^{\mu\nu}M^2_{\phi_2})\phi^\mu_2\phi^\nu_2]. \quad (76)$$

$$L^{(2)}_{K^*} = -\tfrac{1}{2}[(g^{\mu\nu}P^2 - P^\mu P^\nu - g^{\mu\nu}(\frac{3}{2}\Delta^2 + M^2_{K^*_1}))K^{*\mu}_1K^{*\nu}_1$$
$$+\ 2\Gamma_{K^*}(g^{\mu\nu}P^2 - P^\mu P^\nu - g^{\mu\nu}\frac{3}{2}\Delta^2)K^{*\mu}_1K^{*\nu}_2$$
$$+\ (g^{\mu\nu}P^2 - P^\mu P^\nu - g^{\mu\nu}(\frac{3}{2}\Delta^2 + M^2_{K^*_2}))K^{*\mu}_2K^{*\nu}_2]. \quad (77)$$

Here

$$M^2_{\rho_1} = \frac{3}{8G_2I_2(m_u,m_u)}, \quad M^2_{K^*{}_1} = \frac{3}{8G_2I_2(m_u,m_s)},$$
$$M^2_{\phi_1} = \frac{3}{8G_2I_2(m_s,m_s)}, \quad M^2_{\rho_2} = \frac{3}{8G_2I^{ff}_2(m_u,m_u)},$$
$$M^2_{K^*{}_2} = \frac{3}{8G_2I^{ff}_2(m_u,m_s)}, \quad M^2_{\phi_2} = \frac{3}{8G_2I^{ff}_2(m_s,m_s)}, \quad (78)$$

$$\Gamma_{a_{i,j}} = \frac{I_2^{fa}(m_i, m_j)}{\sqrt{I_2^a(m_i, m_j) I_2^{ffa}(m_i, m_j)}}. \quad (79)$$

After trasformations of the vector meson fields, similar to eqs. (53) for the pseudoscalar mesons, the Lagrangians (75,76,77) take the diagonal form

$$\begin{aligned} L^{(2)}_{V^a,\bar{V}^a} = -\tfrac{1}{2}\left[(g^{\mu\nu}P^2 - P^\mu P^\nu - M^2_{V^a})V^{a\mu}V^{a\nu}\right. \\ \left. +(g^{\mu\nu}P^2 - P^\mu P^\nu - M^2_{\bar{V}^a})\bar{V}^{a\mu}\bar{V}^{a\nu}\right], \end{aligned} \quad (80)$$

where V^a and $\bar{V}^a$ are the physical ground and excited states vector mesons

$$\begin{aligned} M^2_{\rho,\bar{\rho}} &= \frac{1}{1-\Gamma^2_\rho}\left[M^2_{\rho_1} + M^2_{\rho_2}\ (-,+)\ \sqrt{(M^2_{\rho_1} - M^2_{\rho_2})^2 + (2M_{\rho_1}M_{\rho_2}\Gamma_\rho)^2}\right] \\ &= M^2_{\omega,\bar{\omega}}, \end{aligned} \quad (81)$$

$$M^2_{\phi,\bar{\phi}} = \frac{1}{1-\Gamma^2_\phi}\left[M^2_{\phi_1} + M^2_{\phi_2}\ (-,+)\ \sqrt{(M^2_{\phi_1} - M^2_{\phi_2})^2 + (2M_{\phi_1}M_{\phi_2}\Gamma_\phi)^2}\right], \quad (82)$$

$$\begin{aligned} M^2_{K^*,\bar{K}^*} = \frac{1}{1-\Gamma^2_{K^*}}\left[M^2_{K_1^*} + M^2_{K_2^*} + 3\Delta^2(1-\Gamma^2_{K^*})\right. \\ \left.(-,+)\ \sqrt{(M^2_{K_1^*} - M^2_{K_2^*})^2 + (2M_{K_1^*}M_{K_2^*}\Gamma_{K^*})^2}\right]. \end{aligned} \quad (83)$$

To describe the excited states of the vector mesons, we shall use the same form factors f_a like in the case of the pseudoscalar mesons. Therefore, we can fix the parameters c_{uu}, c_{us} and c_{ss} using the experimental values of the masses of the vector meson excited states and then make the predictions for the masses of the excited states of the pseudoscalar mesons and vice versa. We shall here the first version.

5. Numerical estimations

We can now estimate numerically the masses of the pseudoscalar and vector mesons and the weak decay constants f_π, $f_{\pi'}$, f_K and $f_{K'}$ in our model.

Because the masses formulae and others equations (for instance, Goldberger – Treimann identity and so on) have new forms in the NJL model with the excited states of mesons as compared with the usual NJL model, where the excited states of mesons were ignored, the values of basic parameters of this model (m_u, m_s, Λ_3, G_1, G_2) can change. However, we see that one can use the former values of the parameters $\Lambda_3 = 1.03\ GeV$ and $G_1 = 3.48\ GeV^{-2}$, because eqs. (23) and (30) change only slightly after including the excited

states of mesons. For the quark masses we shall use the values $m_u = 285\ MeV$ and $m_s = 470\ MeV$, which are also very close to the former values. For the coupling constant G_2 the new value $G_2 = 13.1\ GeV^{-2}$ will be used, which more noticeably differs from the former value $G_2 = 16\ GeV^{-2}$ (see section 2). It is a consequence of the fact that the mass M_{ρ_1} noticeably differs from the physical mass of the ground state $\rho - M_\rho$ (see eqs. (78) and (81)).

Using these basic parameters and the internal form factor parameter $d_{uu} = -1.78\ GeV^{-2}$ (see eq. (36)) and choosing the external form factor parameter $c_{uu} = 1.45$, one finds

$$\begin{aligned} M_\rho &= 770\ MeV, \quad M_{\rho'} = 1.5\ GeV, \\ M_\pi &= 137\ MeV, \quad M_{\pi'} = 1.3\ GeV. \end{aligned} \quad (84)$$

$$\Gamma_\pi = 0.647, \quad \Gamma_\rho = 0.54. \quad (85)$$

$\Gamma_\rho = \sqrt{Z}\Gamma_\pi$ (see eqs. (50) and (79)). The experimental values are equal to

$$\begin{aligned} M^{exp}_\rho &= 769.9 \pm 0.8\ MeV, \quad M^{exp}_{\rho'} = 1465 \pm 25\ MeV, \\ M^{exp}_{\pi^+} &= 139.57\ MeV, \quad M^{exp}_{\pi^0} = 134.98\ MeV, \\ M^{exp}_{\pi'} &= 1300 \pm 100\ MeV. \end{aligned} \quad (86)$$

From eq. (61), one obtains

$$\begin{aligned} f_\pi = 93\ MeV, \quad f_{\pi'} = 0.86\ MeV, \\ \frac{f_{\pi'}}{f_\pi} \approx \sqrt{\frac{1}{\Gamma^2_\pi - 1}}\left(\frac{M_\pi}{M_{\pi'}}\right)^2 \end{aligned} \quad (87)$$

Using the internal form factor parameter $d_{us} = -1.75\ GeV^{-2}$ (see eq. (36)) and choosing the external form factor parameter $c_{us} = 1.51$, one finds

$$\begin{aligned} M_{K^*} &= 880\ MeV, \quad M_{K^{*\prime}} = 1450\ MeV, \\ M_K &= 495\ MeV, \quad M_{K'} = 1455\ MeV, \end{aligned} \quad (88)$$

$$\Gamma_K = 0.56, \quad \Gamma_{K^*} = 0.466 = \sqrt{Z}\Gamma_K. \quad (89)$$

The experimental values are equal to

$$\begin{aligned} M^{exp}_{K^*} &= 891.59 \pm 0.24\ MeV, \quad M^{exp}_{K^{*\prime}} = 1412 \pm 12\ MeV, \\ M^{exp}_{K^+} &= 493.677 \pm 0.016\ MeV, \quad M^{exp}_{K^0} = 497.672 \pm 0.031\ MeV, \\ M^{exp}_{K'} &= 1460\ MeV(?). \end{aligned} \quad (90)$$

From the eq. (62), one gets

$$f_K = 1.16 f_\pi = 108\ MeV, \qquad f_{K'} = 11\ MeV. \quad (91)$$

And for the ϕ and ϕ' we obtain, using the form factor parameters $d_{ss} = -1.72\ GeV^{-2}$ (see eq. (36)) and $c_{ss} = 2$

$$M_\phi = 1019\ MeV, \quad M_{\phi'} = 1650\ MeV, \quad \Gamma_\phi = 0.4 \tag{92}$$

The experimental values are equal to

$$M_\phi^{exp} = 1019.413 \pm 0.008\ MeV, \quad M_{\phi'}^{exp} = 1680 \pm 50\ MeV. \tag{93}$$

We can use the parameters d_{ss} and c_{ss} for the calculations of the masses of the η and η' meson excited states. However, to calculate the masses of the ground and excited states of the eta–mesons it is necessary to take into account the gluon anomaly (see [5]) and the mixing of the η, η', $\bar\eta$ and $\bar\eta'$ states. Since it is a very complicated problem, we will consider this task in future in a separate paper.

6. Summary and conclusions

Let us discuss the obtained results. In the first case where we considered the mesons consisting of the *up* and *down* quarks (ρ, ω, π) we used the five experimental values: the masses M_ρ, $M_{\rho'}$, M_π and the decay constants f_π and g_ρ in order to fix the parameters m_u, Λ_3, G_1, G_2 and c_{uu}. It allows us to predict the mass $M_{\pi'} = 1.3\ GeV$ and the weak decay constant $f_{\pi'} = 0.86\ MeV$. [4] Our prediction for the mass of π' is consistent with the modern experimental data [1]. However, the mass of the first radial excited state of pion has an interesting history. Three years ago the new experimental information about excited states in the few-GeV region, e.g on the π' meson, was obtained at IHEP (Protvino). Indications of the light resonance in diffractive production of 3π–states have lead to speculations that the mass of the π' may be considerably lower at $\approx 750 MeV$ [2]. Our calculations showed that the first radial excited state of the ρ–meson corresponds to the first radial excited state of the pion with the mass 1.3 GeV. Therefore, the excited pion state with the mass 0.75 GeV is forbidden.

Using the experimental values of the masses M_ϕ and $M_{\phi'}$ we fix the parameters $m_s = 470\ MeV$ and $c_{ss} = 2$. We could make some predictions in this quark sector, if we consider the eta–meson states. However, this task will be solved in our subsequent paper.

A few predictions one can make for the strange mesons. Indeed, using only one experimental value of the mass $K^{*'}$ mason in order to fix parameter $c_{us} = 1.5$, we have calculated the masses of the K^*, K, K' mesons and the weak decay constants f_K and $f_{K'}$. We would like to emphasize that the excited

[4] We also can calculate all the strong meson coupling constants.

state of the pseudoscalar kaon K' is not well–determined experimentally at present (see [1]). Therefore, our prediction only points out the place where it is necessary to look for this state.[5]

In conclusion, we would like to note that the pseudoscalar and vector meson masses are unambiguously connected with each other in the NJL–model. Indeed, in the usual NJL–model we can use the masses of the pseudoscalar mesons for fixing the model parameters and after that predict the masses of the vector mesons and vice versa (see [5, 16]). The same situation occurs for their excited states. Using the masses of the excited states of the vector mesons we can predict the masses of the excited states of the pseudoscalar mesons and vice versa.

We have considered here the simplest extension of the NJL–model with polynomial meson–quark form factors and have shown that this model can be useful for describing the excited states of mesons. We have used assumption that the quark-current form factors for the scalar, pseudoscalar, vector and axial–vector currents with the same flavours are equal to each other and the obtained results have supported this assumption.

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