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V.S.Barashenkov[1], W.A.Rodrigues, Jr.[2]

LAUNCHING OF NON-DISPERSIVE SUB- AND SUPERLUMINAL BEAMS

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[1]E-mail: barashenkov@lcta30.jinr.dubna.su
[2]Institute of Mathematics, Statistics and Scientific Computation
IMECC — UNICAMP; CP 6065 13081-970, Campinas, SP, Brasil
E-mail: walrod@ime.unicamp.br

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1. — Introduction

As it is known (see, e. g., [1] for a review and references) the hypothesis of faster-than-light bodies (tachyons) contradicts drastically the causality principle if we suppose the strict validity of the principle relativity. One of the logical contradictions which exists in the standard formulation of the tachyon theory is that accepting the validity of the principle of relativity for faster-than-light bodies permits us in some situations to send informations to the past. E. Recami argued [2, 3] that situations like that never occurs in reality, however, his arguments are not very convincing as discussed by many authors (see the review [1]). The second serious difficulty of tachyon theories is the impossibility of a non-contradictory superluminal generalization of the Lorentz transformations what is necessary for any consistent description of the tachyon kinematics [4].

Meanwhile tachyon-like objects appear in various string models, in theories with high-order lagrangians, by supersymmetric generalizations. Many authors are of the opinion that this fact is not only a disappointing theoretical failure and think that an improvement of our view of the universe that produces a space-time model compatible with superluminal phenomena and free of logical contradictions is necessary. Researches in this direction can be found, e.g., in papers [2, 6].

There are now many strict mathematical investigations proving the existence of families of non-dispersive wave packets propagating in homogeneous media even in vacuum, with arbitrary superluminal speed $v > c = 3.10^{10} cm/s$ (see, e.g., papers [7, 8, 9, 10, 11, 12, 13] and, especially, a review [14]) [1]. Such packets correspond to solutions of the homogeneous wave equations, Klein-Gordon, Dirac, Weyl and Maxwell equations which are, dispersion-free, i. e. in contrast to the usual wave packets made of superpositions of plane waves they don't spread even in media and, therefore, can be considered as completely independent space-time localized material objects ("wave torpedos", purely electromagnetic particles etc.). We call these kind of solutions

[1] Now we are not interested in priority questions and cite only the papers where one can find the more detailed bibliography.

"undistorted progressive waves" (UPWs), a name suggested in [13, 14] and which seems to the authors to correctly express the essence of their nature.

It is important to emphasize that like the plane-wave solutions of the relativistic wave equations UPWs have infinite energy. However, it is possible to exhibit arbitrary ($0 \leq v < \infty$) speed solutions of such equations that have finite energy. Making special superposition of UPWs, in particular, in some cases using the Rayleigh-Sommerfeld theory of diffraction, it is possible to obtain finite aperture approximations (FAA) to a given solution of the relativistic wave equations that have finite energy [13, 14]. One can verify also that such finite energy solutions (subluminal, luminal or superluminal) are quasi-undistorted progressive waves (QUPWs). For the QUPWs solutions of the Maxwell equations it can be proved that they decay into solutions travelling with the usual light speed c. In this sense QUPWs looks like instable particles.

The existence of QUPWs-type solutions for the case of sound has been proved by J.-Y. Lu and J. E. Greenleaf [12]. The existence of QUPWs satisfying eq. (1.1) and traveling with speeds either $v < c_*$ or $v > c_*$ where c_*, called speed of sound, is a characteristic of the properties of the medium, e. g., the temperature,Young modulus etc is proved also experimentally [14, 15] [2].

It is very important for all considerations that follows to take into account the following. Usually the velocity of propagation of energy of a wave satisfying the equation

$$\frac{1}{c_*^2}\frac{\partial^2\Phi}{\partial t^2} - \nabla^2\Phi = 0, \tag{1.1}$$

where $\Phi(t,x)$ is the pressure at the point x at the time t is defined as

$$v_e = S/u. \tag{1.2}$$

[2]In the experiments the so-called X-wave packet travels only with speed 0.2441(8)% greater than c_* and skeptics can have doubts. A new experiments with QUPWs having speeds $V \gg c_*$ should be done. Such "superfast waves", especially in water, are of great practical importance.

Here S is the momentum flux and u is the energy density given by

$$S = \nabla\Phi\partial\Phi/\partial t, \quad u = (1/2)[(\nabla\Phi)^2 + (1/c_*^2)(\partial\Phi/\partial t)^2]. \tag{1.3}$$

From these expressions it follows that

$$v_e \leq c_* \tag{1.4}$$

One can easily prove that for plane waves $v_e = c_*$ indeed. However, the acoustic experiments described in the papers [14, 15] show that the FAA to the limited band sound X-wave travels with a speed $v > c_*$. This speed is *also* the speed of the propagation of the energy carried by the wave, since the hydrophone located at a distance d from the transducer is activated by the energy carried by the wave at a time $t = d/v$ after the launching of the wave at a moment $t = 0$. It means that the definition (1.2) is devoid of sense. For the case of the FAA to the limited band sound X-wave the speed v_e is a complicated function of (x,t), so we must be careful when discussing the velocity of a propagation of energy of UPWs and QUPWs solutions, particularly, of Maxwell equations.

From the viewpoint of current quantum theory a sound wave is composed of phonons and excitations that travel in the medium with speed c_*. The problem of understanding the acoustic waves travelling with speeds $v < c_*$ or $v > c_*$ using quantum field theory will be analyzed in another paper where we show that they correspond to a new kind of boson condensate.

Another important remark is necessary to be emphasized here. The superluminal UPWs or QUPWs solutions of the free Maxwell equations share with the above mentioned classical tachyons only the property that both travel with speed $v > c$. The analogy ends here because any classical tachyon is a material object (an elementary particle, a macroscopic body) whose Lorentz change of the longitudinal dimension in the laboratory frame is defined by the cut of the tachyon world tube by a plane $t =$ constant. Since tachyons are always moving we can write

$$L(v)/L(v') = (v^2/c^2 - 1)^{1/2}/(v'^2/c^2 - 1)^{1/2}. \tag{1.5}$$

This means that the tachyon mass and energy distributions in the laboratory frame must be ellipsoidal also, but in contrast to the case of subluminal bodies instead of a contraction the mass and energy distributions will show a dilation which increases with the tachyon speed v [16]. It differs essentially from the X-like form of some UPWs or QUPWs solutions of the wave equation. The conclusion about X-like shape of the moving tachyons obtained by E. Recami [2] is a consequence of the formal Lorentz transformation to the laboratory frame from a superluminal reference frame. However, as mentioned above, such a transformation is contradictory. So, the coincidence of the tachyon shape predicted by E. Recami with the X-wave solutions of the Maxwell equations should be considered as accidental and doesn't prove the breakdown of the well known relativistic shape change law for the tachyons.

After these remarks we complete the introduction by saying that the main purpose of our paper is to analyze the physical meaning of the various wave velocities that appear (i) in some extraordinary solutions of the Schrödinger equation (Section 2), (ii) in the case of UPWs $v > c_*$ solutions of the wave equation for sound waves (Section 3) and (iii) in UPWs $v > c$ solutions of the free Maxwell equations (Section 4) where, in particular, we analyze the energy velocity paradox quoted by W. Band [9].

Obviously, with the analysis of the superluminal electromagnetic UPWs in Section 4 we are not proving that it is sure that such waves can be launched in physical space, and, of course, if we believe in the strict validity of the principle of relativity which is one of the main dogmas of current physics, to launch a "wave torpedo" with $v > c$ is impossible. Nevertheless the detailed computer simulation to the finite aperture approximation to the superluminal electromagnetic waves presented in the paper [14] suggests that it is worth to try the experiment. Sure, if the FAA to a given superluminal electromagnetic wave can be launched, we will see a breakdown of the principle relativity as it is known today, thus determining the limits of validity of the special theory of relativity.

2. — Non-dispersive solutions of Schrödinger equation

In dealing with waves we have several velocities involved, e. g., phase and group velocities, velocity of transport of energy, signal speed, front velocity etc... Of course, the meaning of all these quantities depends on the particular theory to which the wave motion is associated. In order to reveal their relations more clear let us take the case of non-relativistic quantum mechanics where a free particle has associated to a wave function Ψ satisfying the equation

$$i\hbar\frac{\partial\Psi}{\partial t}+\frac{\hbar^2}{2m}\nabla^2\Psi=0. \tag{2.1}$$

Now, if the particle is moving with the kinetic energy $E = mv^2/2$ and the momentum $p = mvz$, then the associated wave function in the solution of eq. (2.1) given by the plane wave

$$\Psi = Ae^{i(\omega t-kz)} \tag{2.2}$$

with A is a constant and

$$E=\hbar\omega=mv^2/2, \quad p=\hbar k=mv \tag{2.3}$$

The function Ψ is simultaneously eigenfunction of energy and momentum operators:

$$\hat{E}\Psi\equiv(i\hbar\partial/\partial t)\Psi=\hbar\omega\Psi, \quad \hat{p}\Psi\equiv(-i\hbar\partial/\partial z)\Psi=\hbar k\Psi. \tag{2.4}$$

The connection between ω and k in order for eq. (2.2) to be a solution of eq. (2.1) gives the dispersion relation

$$\omega=\hbar k^2/2m. \tag{2.5}$$

The phase and group velocities associated with Ψ are

$$v_{ph}=v/2, \quad v_g=d\omega/dk=v \tag{2.6}$$

and we see that for plane waves the speed of the particle is equal to v_g.

Now, as it well known, the function Ψ given by eq. (2.2) is not an element of the Hilbert space (H) of wave functions. A probability wave

$\Psi(z,t) \in H$ describing a particle moving with speed $v = vz$ is given by the wave packet

(2.7) $$\Psi(z,t) = A \int dk B(k) e^{i(kz-\omega t)},$$

where $B(k)$ is a weight function centered in k_o and decaying rapidly outside the interval $k_o - \Delta k < k < k_o + \Delta k$. For Ψ given by eq.(2.7) the group velocity is defined by

(2.8) $$v_g = (d\omega/dk)_{k_o}.$$

The function Ψ is than interpreted as associated with a particle moving with expectation kinetic energy and momentum

(2.9) $$E_o = mv^2/2 = \langle\Psi|\hat{E}|\Psi\rangle, \quad p_o = mv_o = \langle\Psi|\hat{p}_z|\Psi\rangle.$$

The important point to be emphasized here is that Ψ is a *spreading* wave packet, and the question naturally arises here: are there any non-spreading wave packets which are solutions of Schrödinger equation? If the answer is positive, which is the meaning of such solutions?

If we solve eq. (2.1) in cylindrical co-ordinates for a wave moving along the z-axis, we immediately get a family of solutions

(2.10) $$\Psi_{J_n} = A_n e^{in\theta} J_n(\alpha\rho) e^{i(kz-\omega t)},$$

where A_n is a constant, $\rho = (x^2+y^2)^{1/2}$ and α is the so-called separation constant what means that α is not a function of (x,t) but, of course, it may be a general function of k, ω and other parameters.

In what follows let us consider for simplicity the solution with $n = 0$.

The dispersion relation

(2.11) $$\omega = \frac{\hbar}{2m}(k^2+\alpha^2)$$

must hold in order for the function Ψ_{J_o} to be a solution of eq. (2.1). To interpret the physical meaning of this relation we recall that Ψ_{J_o} is a simultaneous eigenfunction of $\hat{E}$ and $\hat{p}$. Indeed, we have

(2.12) $$\hat{E}\Psi_{J_o} = \hbar\omega\Psi_{J_o} = E\Psi_{J_o}, \quad \hat{p}\Psi_{J_o} = \hbar k\Psi_{J_o} = p\Psi_{J_o}.$$

Then, we may write

(2.13) $$E = \frac{\hbar^2}{2m}(k^2+\alpha^2) = \frac{\hbar^2}{2m} + mc^2, \quad p = \hbar k$$

where we put

(2.14) $$m = \hbar\alpha/\sqrt{2}c$$

interpreted as the mass of the particle. We have then, as usual, the particle, phase and group velocities

(2.15) $$v = p/m = \hbar k/m = \sqrt{2}ck/\alpha, \quad v_{ph} = \omega/k, \quad v_g = d\omega/dk = v$$

Let us now introduce; following J.-Y. Lu and J. F. Greenleaf [15], the axicon angle η by

(2.16) $$k = \omega\xi\cos\eta, \quad \alpha = \omega\xi\sin\eta$$

where ξ is determined by the dispersion relation (2.11) as

(2.17) $$\xi = (2m/\hbar\omega)^{1/2} = \sqrt{2}c^{-1}\sin\eta.$$

Taking into account now the relation between α and m, given by the eq. (2.14) we have

(2.18) $$\omega = mc^2/\hbar\sin^2\eta$$

Then we have for the velocities in the eq. (2.15:

(2.19) $$v = \sqrt{2}c\cot\eta$$

(2.20) $$v_{ph} = v_g = \sqrt{2}c/\sin 2\eta = v/2\cos^2\eta$$

In contrast to the plane wave solution here $v_{ph} = v_g$, but in general $v_g \neq v$. For all this, as the Schrödinger equation is a non-relativistic,

(2.21) $$v/c = \sqrt{2}\cot\eta \ll 1,$$

i. e. the axicon angle has to be close to $\pi/2$, and, respectively,

(2.22) $$v \simeq \sqrt{2}c\cos\eta \ll c$$

(2.23) $$v_{ph} = v_g \simeq \sqrt{2}c/\cos\eta.$$

The interpretation of the velocities are now clear. If we want that Ψ_{J_o} describes a free particle moving with the speed v, then we cannot attach a physical meaning to v_{ph} or v_g, i. e. in this case the transport of energy will not be given by v_g.

We can now construct a wave packet

(2.24) $$\Psi_n = A_n e^{in\theta}\int d\omega B(\omega)J_n(\omega\sqrt{2}c_{-1}\rho\sin^2\eta)e^{i\omega(2zv^{-1}\cos^2\eta - t)}.$$

which is, obviously, dispersive solution of eq. (2.1) if we consider the mass (2.14) as a constant, because in this case the axicon angle η and, respectively, the velocities (2.19), (2.20) are frequency dependent. In this connection it should be noted that the results presented in the paper [17] claiming the existence of X-wave solutions for the Schrödinger equation are seen unfortunately to be incorrect for analogous reasons.

In a paper [18] A. Barut tried to construct non-dispersive solutions of the Schrödinger equation using spherical symmetry. If we suppose that there is a stationary solution of eq.(2.1) of the form

(2.25) $$\Psi(x,t) = e^{-imc^2t/2\hbar}f(x)$$

then putting Ψ into eq. (2.1) we see that the function $f(x)$ satisfies the Helmholtz differential equation

(2.26) $$\nabla^2 f(x) + \alpha^2 f(x) = 0$$

with $\alpha = mc/\hbar$.

A simple solution of this equation in spherical coordinates is $\sin\alpha r/r$ where $r = (x^2+y^2+z^2)^{1/2}$. Then, really a non-dispersive packet exist which is stationary in the laboratory frame and is given by

(2.27) $$\Psi(x,t) = \frac{\sin\alpha r}{r}e^{-imc^2t/2\hbar}.$$

It is clear that $\Psi(x,t)$ given by eq. (2.27) is not an eigenfunction of the momentum operator, but, nevertheless, it is an eigenfunction of the energy operator.

A. Barut claims to have constructed a non-dispersive wave packet solution of the Schrödinger equation by first writing

(2.28) $$\Psi(x,t) = \psi(\dot{x},t)e^{-imc^2t/2\hbar}$$

where the function $\psi(x,t)$ satisfies the equation

(2.29) $$i\hbar\frac{\partial\psi}{\partial t} + \frac{\hbar^2}{2m}\nabla^2\psi + \frac{mc^2}{2}\psi = 0.$$

Then the author of the paper [18] used a ansatz

(2.30) $$\psi(x,t) = f(\xi)\exp\left[-i\hbar^{-1}mv(z - vt/2)\right]$$

with $\xi = [x^2+y^2+(z-vt)^2]^{1/2}$. Substituting this expression into eq. (2.29) gives

(2.31) $$i\hbar\frac{\partial}{\partial t}f(\xi) + \left[\frac{\hbar^2}{2m}\nabla^2 f(\xi) + \frac{mc^2}{2}f(\xi)\right] = 0.$$

As the next step A. Barut proposed to put

(2.32) $$\nabla^2 f(\xi) + (mc/\hbar)^2 f(\xi) = 0.$$

However,to be satisfied it is necessary that

(2.33) $$\partial f(\xi)\partial t = 0.$$

This equation has a solution only when $v = 0$ which, particularly, produces as a possible solution our stationary wave packet given by eq. (2.27).

We can think that an X-wave solution of Schrödinger equation can be constructed by relaxing the condition that α in eq. (2.13) is a constant. Unfortunately even in this case a simple calculation shows that there is no such a solution. Nevertheless, it is worth to try to construct ion beams with the wave function like that given in eq. (2.10) in an experiment analogous to a light experiment by Durnin [8] but using instead of an optical lens a magnetic one.

3. — UPWs solutions of the wave equation

The solution of the homogenous wave equation in cylindrical coordinates has the same form (2.10) but with the different dispersion relation

$$\omega^2 = c_*^2(k^2 + \alpha^2). \tag{3.1}$$

Consider a quantum mechanical meaning for these expressions. If we write $E = \hbar\omega$ and $p_z = c_* c^{-1}\hbar k$ then

$$E^2/c^2 - p_z^2 = m^2, \quad m = \hbar\alpha c_*/c. \tag{3.2}$$

Putting now

$$k = c_*\omega\cos\eta, \quad \alpha = c_*\omega\sin\eta \tag{3.3}$$

we can define the velocities

$$v = p_z c^2/E = c\cos\eta, \tag{3.4}$$

$$v_{ph} = v_g = \omega/k = c_*/cos\eta. \tag{3.5}$$

We see that the massless wave equation has solutions propagating with phase and group velocities (3.5), and v would be the velocity of a excitation-like particle with the mass depending on the frequency:

$$m = \hbar c_* c^{-1}\omega\sin\eta. \tag{3.6}$$

Using relations (2.10) and (3.5), one can build the packet

$$\Phi_{Xn} = A_n e^{in\theta}\int d\omega B(\omega) J_n(c_*^{-1}\omega\rho\sin\eta) e^{i\omega(c_*^{-1}\cos\eta - t)} \tag{3.7}$$

which moves rigidly, without any distortion:

$$\Phi_{X_n}(x,y,z,t) = \Phi_{X_n}(x,y,z-(c_*/\cos\eta)t, 0). \tag{3.8}$$

We remark that the packet Φ_{Xn}, if interpreted, for example, as a classical sound wave, has infinite energy. However, a finite approximation to Φ_{Xn} with an appropriate function $B(\omega)$ has been seen to travel with the speed $c_*/\cos\eta$ [14]. Thus, for this case, as said in Section 1, this is the velocity of propagation of the wave energy — a non-trivial fact showing again that the interpretation of the velocities associated to a wave depends on the theory, that the wave is supposed to describe.

From the quantum-mechanical point of view sound is composed of phonons. Ψ_{Xn} is a kind of field configuration defining a new kind of boson condensate. This will be studied later.

4. — Superluminal wave packets

If $c_* = c$ the homogenous wave equations have superluminal solutions with $v_{ph} = v_g > c$ (see eq.(3.5)). As shown in [13, 14] such solutions exist also for massive Dirac and Klein-Gordon particles. If we strictly believe in the presently known relativistic physics these superluminal packets, of course, cannot be generated. What, however, is the physical meaning of such faster-than-light solutions in this case?

Solutions of this kind describe inertia-free processes like, for example, a neon advertisement string where each letter flashes independently of the preceding one. Neither information nor energy is transferred in these processes and the problem concerning the velocity of energy transport simply doesn't exist here.

A superluminal electromagnetic beam can be launched in physical space with a boundary, like in W. Band's gedanken experiment [9] where a charged cylinder with an appropriate charge density is used. Band's solution with $v_g > c$ describes an inertia-free process if we *change* the charge of each tiny cylinder segment ("switch" it up) quite independently according to Band's solution. One should say that the situation is here quite clear, again there is no transfer of information and energy, and Band's problem concerning the ratio $|S|/u < 1$ (of Poynting vector over energy density) is trivially no problem at all.

As we said already in the Introduction, the "mathematical experiments" done in [14] for the electromagnetic X-waves solutions of free Maxwell equations seem, however, to indicate that eventually these waves can be generated with appropriate antennas. Of course, if this really can be done we will find a violation of the principle of relativity.

5. — Conclusions

We see that both the sub- and superluminal non dispersive soliton-like solutions of the homogeneous linear relativistic equations describe physical situations that can be realized in practice. For superluminal solutions there is surely a real transfer of energy and information, superluminal wave packets describe inertia-free processes without any transfer. Nevertheless it remains to verify if *real*, i. e. energy transferring, faster-than-light electromagnetic pulses suggested by mathematical simulation can be launched. The properties of UPWs are so extraordinary that only experiment can decide, and if such beams exist then we have a first case of a breakdown of the current form of the theory of relativity.

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