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SPIN-$\frac{3}{2}$ FIELDS IN THE HEAVY-BARYON EFFECTIVE THEORY

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The chiral symmetry and its breaking are the fundamental principles governing the dynamics of low-energy interactions of mesons and baryons [1]. The new formulation of the effective chiral lagrangians for the meson-baryon system given in Ref. [2] is based on the treatment of baryons as heavy static fields with the definite velocity. The main advantage of this approach is that the improved derivative expansion for the light velocity components makes it possible to include baryons in the power counting scheme of chiral perturbative theory and calculate the meson-baryon loops in the most systematical way. In such an approach some part of the higher order counterterms can be derived using the path integral techniques as $1/M$-corrections when the heavy velocity components of baryon fields are integrated out of the generating functional for the meson-baryon system.

It is well known that both free and interaction lagrangians for a spin-$\frac{3}{2}$ field coupled to a nucleon and a pseudoscalar meson field have to be constructed in such a way that the total lagrangian is invariant under the so called "point transformation" [3, 4]. This condition is a consequence of the invariance of the physical properties of spin-$\frac{3}{2}$ field with respect to rotations in the spin-$\frac{1}{2}$ space. The effective lagrangian introduced in the Ref. [2] is not invariant under point transformation and it has to be modified by introducing additional off-shell terms which restore its point transformation invariance. Such an extension also leads to the appearence of new terms when calculating $1/M$-corrections to the effective lagrangian for the light velocity components.

As an extension of the formalism from Ref. [2] the lagrangian for the system of spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ baryons coupled with pseudoscalar mesons, which is explicitly invariant under point transformation, is given in the form

$$\begin{aligned} \mathcal{L}_{tot} &= \overline{B}(i\not{D} - M - \gamma_5 \hat{\not{A}}_\Omega)B \\ &-\overline{T}^\mu\Big((i\not{D}-M)g_{\mu\nu} - \frac{1}{4}\gamma_\mu\gamma_\lambda(i\not{D}-M)\gamma^\lambda\gamma_\nu + \mathcal{H}\gamma_5\Omega_{\mu\nu}\Big)T^\nu \\ &+\mathcal{C}\Big(\overline{T}^\mu\Theta_{\mu\nu}A^\nu_\Omega B + \overline{B}A^\mu_\Omega\Theta_{\mu\nu}T^\nu\Big), \end{aligned} \tag{1}$$

where B is the spin-$\frac{1}{2}$ baryon field, while the field T_μ is related with the vector-spinor representation ψ_μ for spin-$\frac{3}{2}$ field, introduced by Rarita and Schwinger [5]

$$T_\mu = \mathcal{O}^A_{\mu\nu}\psi^\nu, \quad \mathcal{O}^A_{\mu\nu} = g_{\mu\nu} + \frac{1}{2}A\gamma_\mu\gamma_\nu, \tag{2}$$

with

$$\Theta_{\mu\nu} = g_{\mu\nu} + z\gamma_\mu\gamma_\nu, \tag{3}$$

where z and A are arbitrary parameters with $A \neq -\frac{1}{2}$. In Eq. (1) the covariant derivative $D^\mu * = \partial^\mu * + [V^\mu_\Omega, *]$ couples the baryon fields with the vector combination of the pseudoscalar meson matrix Ω and its derivative,

$$V^\mu_\Omega = \frac{1}{2}(\Omega\partial^\mu\Omega^\dagger + \Omega^\dagger\partial^\mu\Omega),$$

while the operators

$$\hat{A}^\mu_\Omega = D\{A^\mu_\Omega, B\} + F[A^\mu_\Omega, B], \tag{4}$$

$$\Omega_{\mu\nu} = \not{A}_\Omega g_{\mu\nu} + g_1(\gamma_\mu A_{\Omega\nu} + A_{\Omega\mu}\gamma_\nu) + g_2\gamma_\mu\not{A}_\Omega\gamma_\nu, \tag{5}$$

couple baryons with the axial-vector combination

$$A^\mu_\Omega = \frac{i}{2}(\Omega\partial^\mu\Omega^\dagger - \Omega^\dagger\partial^\mu\Omega).$$

In nonlinear realization of chiral symmetry in $SU(3)$ the matrix Ω for pseudoscalar mesons is given by

$$\Omega = \exp\Big(\frac{i}{\sqrt{2}F_0}\sum_{a=1}^{8}\varphi^a\frac{\lambda^a}{2}\Big),$$

where φ^a are the pseudoscalar octet fields, and F_0 is the bare π decay constant. For simplicity the flavour $SU(3)$ indices in the lagrangian (1) are omitted, and M being an avaraged $N\Delta$ mass.

As recently stressed by Pascalutsa [4], the representation (1) is convenient for the description of processes with spin-$\frac{3}{2}$ fields off-shell, as the full A-dependence is hidden now in the new fields (2) which are explicitly invariant under point transformation

$$\psi_\mu \to \psi'_\mu = \mathcal{O}^b_{\mu\nu}\psi^\nu, \quad A \to A' = \frac{A-b}{1+2b}, \tag{6}$$

where $b \neq -\frac{1}{2}$ is an arbitrary parameter. The invariance of the lagrangian (1) under the transformation (6) implies that the physical content of the theory does not depend on the choice of A. As the Rarita-Schwinger vector-spinor ψ_μ can be constructed from the direct product of the relativistic states ϵ_μ for spin-1 and u for spin-$\frac{1}{2}$, $\psi_{\mu,s}(p) = L^{(1)}_{\mu\nu}[\epsilon_\nu u(p)]_{3/2s}$, where the brackets denote the coupling of the Dirac spinor with the polarization vector ϵ_μ; $L^{(1)}(p)$ is the corresponding boost operator, it obeys in momentum space

$$p_\mu\psi^\mu = 0$$

corresponding to the Lorentz condition for ϵ_μ. In addition, on mass shell the supplementary condition

$$\gamma_\mu\psi^\mu = 0 \tag{7}$$

is satisfied, resulting from the elimination of the spin-$\frac{1}{2}$ components from the spinor-vector direct product. The on-shell coupling constants D, F, $\mathcal{H}$ and $\mathcal{C}$ in Eqs. (1) and (4) agree with Ref. [2], while g_1, g_2 and z in Eqs. (5) and (3) are additional off-shell parameters. Due to condition (7) the effective lagrangian (1) coincides with the form presented in Ref. [2] on the mass shell of spin-$\frac{3}{2}$ particles.

For meson-baryon interactions, at low energies when the momentum transfer from meson to baryon is small, if to compare with the baryon mass, the velocity of the baryon is conserved in the heavy mass limit. In this case the effective field theory can be written in terms of baryon fields with the definite four-velocity v. After indroducing the velocity components for the baryon fields and corresponding external sources in analogy with the heavy static quark approach [6], the generating functional corresponding to lagrangian (1) can be written as

$$\begin{aligned}\mathcal{Z}(R^B_v, \rho^b_v, R^T_v, \rho^t_v) = \int \mathcal{D}(B_v, T_v, \Omega) \exp\Big[i \int d^4x \mathcal{L}^{(v)}_{tot} \\ + i\int d^4x \big(\overline{R}^B_v B_v + \overline{B}_v R^B_v + \overline{\rho}^b_v b_v + \overline{b}_v \rho^b_v \\ + \overline{R}^T_{v\mu} T^\mu_v + \overline{T}^\mu_v R^T_{v\mu} + \overline{\rho}^t_{v\mu} t^\mu_v + \overline{t}^\mu_v \rho^t_{v\mu} \big)\Big] .\end{aligned} \tag{8}$$

Here $\mathcal{D}(B_v, T_v, \Omega)$ is the path integral measure including the velocity components

$$\begin{aligned} B &= \mathrm{e}^{-iM(v\cdot x)}(B_v + b_v), \quad B_v = \frac{1+\not{v}}{2}B, \quad b_v = \frac{1-\not{v}}{2}B, \\ T^\mu &= \mathrm{e}^{-iM(v\cdot x)}(T^\mu_v + t^\mu_v), \quad T^\mu_v = \frac{1+\not{v}}{2}T^\mu, \quad t^\mu_v = \frac{1-\not{v}}{2}T^\mu, \end{aligned} \tag{9}$$

R^B_v, ρ^b_v, $R^T_{v\mu}$ and $\rho^t_{v\mu}$ are the external sources coupling to the velocity components (9), and

$$\begin{aligned} \mathcal{L}^{(v)}_{tot} &= \overline{B}_v G B_v - \overline{b}_v \mathcal{G} b_v + \overline{B}_v H b_v + \overline{b}_v H B_v \\ &\quad - \overline{T}^\mu_v Q_{\mu\nu} T^\nu_v - \overline{t}^\mu_v \mathcal{Q}_{\mu\nu} t^\nu_v - \overline{T}^\mu_v R_{\mu\nu} t^\nu_v - \overline{t}^\mu_v R_{\nu\mu} T^\nu_v \\ &\quad + \mathcal{C}\Big[(\overline{T}^\mu_v + \overline{t}^\mu_v)\Theta_{\mu\nu} A^\nu_\Omega (B_v + b_v) + (\overline{B}_v + \overline{b}_v) A^\mu_\Omega \Theta_{\mu\nu} (T^\nu_v + t^\nu_v)\Big], \end{aligned} \tag{10}$$

with

$$\begin{aligned} G &= i(v\cdot D) - \gamma_5 \hat{A}_\Omega, \quad \mathcal{G} = i(v\cdot D) + 2M + \gamma_5 \hat{A}_\Omega, \quad H = i\not{D}^\perp - \gamma_5 \hat{A}_\Omega, \\ Q_{\mu\nu} &= i\not{D}^{\|} g_{\mu\nu} - \frac{1}{4}\gamma_\mu \gamma_\lambda i\not{D}^{\|}\gamma^\lambda \gamma_\nu + \mathcal{H}\gamma_5 \Omega_{\mu\nu}, \\ \mathcal{Q}_{\mu\nu} &= [i\not{D}^{\|} - 2M] g_{\mu\nu} - \frac{1}{4}\gamma_\mu\gamma_\lambda [i\not{D}^{\|} - 2M]\gamma^\lambda\gamma_\nu + \mathcal{H}\gamma_5\Omega_{\mu\nu}, \\ R_{\mu\nu} &= i\not{D}^\perp g_{\mu\nu} - \frac{1}{4}\gamma_\mu\gamma_\lambda i\not{D}^\perp \gamma^\lambda \gamma_\nu + \mathcal{H}\gamma_5\Omega_{\mu\nu}, \end{aligned}$$

where $D^{\|}_\mu = v_\mu (v\cdot D)$, $D^\perp_\mu = D_\mu - v_\mu (v\cdot D)$.

In order to derive Eq. (10) as the effective action for the velocity components (9), we have used the gauge condition $v_\mu T^\mu = 0$, which is equivalent to this $A = 0$ in (2). We are allowed to use this gauge condition as the physical properties of the spin-$\frac{3}{2}$ field are independent on the choice of parameter A due to its invariance with respect to point transformation.

In Eqs. (8) and (10) the light components B_v and T^μ_v correspond to massless effective baryon fields, while the heavy components b_v and t^μ_v have effective mass $2M$. As a consequence, they can be integrated from the generating functional (8) as the heavy degrees of freedom performing the Gaussian integration. According to the standard procedure, the heavy components $\overline{b}_v$, $\overline{t}^\mu_v$, b_v and t^μ_v have to be replaced in the exponent of the Eq. (8) by the solutions of the equations

$$\frac{\delta(\mathcal{L}^{(v)}_{tot} + \overline{\rho}^b_v b_v)}{\delta b_v} = 0, \quad \frac{\delta(\mathcal{L}^{(v)}_{tot} + \overline{\rho}^t_{v\mu} t^\mu_v)}{\delta t^\mu_v} = 0, \tag{11}$$

$$\frac{\delta(\mathcal{L}^{(v)}_{tot} + \overline{b}_v \rho^b_v)}{\delta \overline{b}_v} = 0, \quad \frac{\delta(\mathcal{L}^{(v)}_{tot} + \overline{t}^\mu_v \rho^t_{v\mu})}{\overline{t}^\mu_v} = 0, \tag{12}$$

which leads to the system of the equations for $\overline{b}_v$ and $\overline{t}^\mu_v$:

$$\begin{aligned} -\overline{b}_v \mathcal{G} + \mathcal{C}\,\overline{t}^\mu_v \Theta_{\mu\nu} A^\nu_\Omega + \overline{B}_v H + \mathcal{C}\,\overline{T}^\mu_v \Theta_{\mu\nu} A^\nu_\Omega + \overline{\rho}^b_v = 0, \\ \mathcal{C}\,\overline{b}_v A^\mu_\Omega \Theta_{\mu\nu} - \overline{t}^\mu_v \mathcal{Q}_{\mu\nu} + \mathcal{C}\,\overline{B}_v A^\mu_\Omega \Theta_{\mu\nu} - \overline{T}^\mu_v R_{\mu\nu} + \overline{\rho}^t_{v\nu} = 0, \end{aligned} \tag{13}$$

and similarly for b_v and t^μ_v.

We will calculate the effective meson-baryon action in terms of light velocity components of baryon fields up to and including corrections of $O(1/M)$. In such an approximation it is enough to keep only terms up to $O(1/M^3)$ in the solutions of the equations (13):

$$\begin{aligned} \overline{b}_v &= \big(\overline{B}_v H + \mathcal{C}\,\overline{T}^\mu_v \Theta_{\mu\nu} A^\nu_\Omega + \overline{\rho}^b_v\big)\mathcal{G}^{-1} \\ &\quad + \frac{\mathcal{C}}{4M^2}\big(\mathcal{C}\,\overline{B}_v A^\alpha_\Omega \Theta_{\alpha\beta} - \overline{T}^\alpha_v R_{\alpha\beta} + \overline{\rho}^t_{v\beta}\big)\tilde{\mathcal{Q}}^{\beta\mu}\Theta_{\mu\nu} A^\nu_\Omega + O\Big(\frac{1}{M^3}\Big), \\ \overline{t}^\mu_v &= \big(\mathcal{C}\,\overline{B}_v A_{\Omega\alpha}\Theta^{\alpha\nu} - \overline{T}_{v\alpha} R^{\alpha\nu} + \overline{\rho}^{t\nu}_v\big)\mathcal{Q}^{-1}_{\nu\mu} \\ &\quad + \frac{\mathcal{C}}{4M^2}\big(\overline{B}_v H + \mathcal{C}\,\overline{T}^\alpha_v \Theta_{\alpha\beta} A^\beta_\Omega + \overline{\rho}^b_v\big) A^\tau_\Omega \Theta_{\tau\nu} \tilde{\mathcal{Q}}^{\nu\mu} + O\Big(\frac{1}{M^3}\Big). \end{aligned}$$

In solving for the heavy velocity components in the preceeding equation, we stress the appearance of the inverse operators $\mathcal{G}^{-1}$ and $\mathcal{Q}^{-1}_{\mu\nu}$ given explicitly as

$$\mathcal{G}^{-1} = \frac{1}{2M}\Big[1 - \frac{1}{2M}\Big(i(v\cdot D) + \gamma_5 \hat{A}_\Omega\Big)\Big],$$

$$
\begin{aligned}
\mathcal{Q}^{-1}_{\mu\nu} &= \frac{1}{2M}\Big[-g_{\mu\nu}+\frac{1}{3}\gamma_\mu\gamma_\nu-\frac{1}{2M}\Big(\not{D}^{||}g_{\mu\nu}-\frac{1}{6}\gamma_\mu\not{D}^{||}\gamma_\nu-\frac{1}{3}\{\gamma_\mu\gamma_\nu,\not{D}^{||}\}\Big) \\
&\quad -\frac{\mathcal{H}}{2M}\gamma_5\Big(\not{A}_\Omega g_{\mu\nu}-\frac{2(1-g_1)-9g_2}{9}\gamma_\mu\not{A}_\Omega\gamma_\nu-\frac{1}{3}\{\gamma_\mu\gamma_\nu,\not{A}_\Omega\} \\
&\quad -\frac{g_1}{3}(\gamma_\mu A_{\Omega\nu}+A_{\Omega\mu}\gamma_\nu)\Big)\Big] \\
&\equiv \frac{1}{2M}\tilde{\mathcal{Q}}_{\mu\nu}+O(1/M^2),
\end{aligned}
$$

with

$$
\tilde{\mathcal{Q}}_{\mu\nu}=-g_{\mu\nu}+\frac{1}{3}\gamma_\mu\gamma_\nu\,,
$$

which obey the relations

$$
\mathcal{G}\mathcal{G}^{-1}=1+O(1/M^2)\,,\quad \mathcal{Q}_\mu{}^\alpha\mathcal{Q}^{-1}_{\alpha\nu}=g_{\mu\nu}+O(1/M^2)\,.
$$

Note that – consistent with the expansion of the effective lagrangian to $O(1/M)$ given below – there are *no* corrections of this order both in the inverse operators and the heavy velocity components. The solutions for b_v and t^μ_v can be obtained from the Eq. (12) in analogy.

After integrating out the heavy velocity components of spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ baryons, the generating functional contains only the light components B_v and T^μ_v:

$$
\begin{aligned}
\mathcal{Z} &= \mathcal{N}^{-1}\int\tilde{\mathcal{D}}(B_v,T_v,\Omega)\exp\Big\{i\int d^4x\Big[\overline{B}_v GB_v-\overline{T}^\mu_v Q_{\mu\nu}T^\nu_v \\
&\quad +\mathcal{C}\big(\overline{T}^\mu_v\Theta_{\mu\nu}A^\nu_\Omega B_v+\overline{B}_v A^\mu_\Omega\Theta_{\mu\nu}T^\nu_v\big) \\
&\quad +\frac{1}{2M}\Big[\overline{B}_v H^2 B_v+\overline{T}^\mu_v R_{\mu\alpha}\tilde{\mathcal{Q}}^{\alpha\beta}R_{\beta\nu}T^\nu_v \\
&\qquad -\mathcal{C}^2\big(\overline{B}_v A^\mu_\Omega\Theta_{\mu\alpha}\tilde{\mathcal{Q}}^{\alpha\beta}\Theta_{\beta\nu}A^\nu_\Omega B_v+\overline{T}^\mu_v\Theta_{\mu\alpha}A^\alpha_\Omega A^\beta_\Omega\Theta_{\beta\nu}T^\nu_v\big)\Big] \\
&\quad +O\Big(\frac{1}{M^2}\Big)\Big]\Big\}\,,
\end{aligned} \tag{14}
$$

where the determinants of the operators $\mathcal{G}$ and $\mathcal{Q}_{\mu\nu}$ are included in the normalization factor $\mathcal{N}$, and the terms containing external sources are omitted for the sake of simplicity.

Combining various relations, the leading pieces in the $1/M$ expansion of the effective lagrangian

$$
\mathcal{L}_v=\mathcal{L}^{(0)}_v(1/M^0)+\mathcal{L}^{(1)}_v(1/M^0)+O(1/M^2)
$$

in terms of the light components of baryon fields can be presented as

$$
\begin{aligned}
\mathcal{L}^{(0)}_v &= \overline{B}_v\big(i(v\cdot D)-\gamma_5\hat{\not{A}}_\Omega\big)B_v \\
&\quad -\overline{T}^\mu_v\Big[i\not{v}(v\cdot D)g_{\mu\nu}-\frac{1}{4}\gamma_\mu\gamma_\lambda i\not{v}(v\cdot D)\gamma^\lambda\gamma_\nu \\
&\quad +\mathcal{H}\gamma_5\big(\not{A}_\Omega g_{\mu\nu}+g_1(\gamma_\mu A_{\Omega\nu}+A_{\Omega\mu}\gamma_\nu)+g_2\gamma_\mu\not{A}_\Omega\gamma_\nu\big)\Big]T^\nu_v \\
&\quad +\mathcal{C}\big(\overline{T}^\mu_v(g_{\mu\nu}+z\gamma_\mu\gamma_\nu)A^\nu_\Omega B_v+\overline{B}A^\mu_\Omega(g_{\mu\nu}+z\gamma_\mu\gamma_\nu)T^\nu_v\big)\,.
\end{aligned} \tag{15}
$$

The corrections of $O(1/M)$ have the following form:

$$
\begin{aligned}
\mathcal{L}^{(1)}_v &= \frac{1}{2M}\overline{B}_v\Big\{-D^2+(v\cdot D)^2-\Big(1-\frac{2}{3}\mathcal{C}^2(1-z-2z^2)\Big)(\hat{A}^\mu_\Omega)^2 \\
&\quad +\frac{i}{2}\sigma_{\mu\nu}\Big[[D^\mu,D^\nu]+2v^\mu[D^\nu,(v\cdot D)] \\
&\qquad +\Big(1-\frac{1}{3}\mathcal{C}^2(1+2z+4z^2)\Big)[\hat{A}^\mu_\Omega,\hat{A}^\nu_\Omega]\Big] \\
&\quad +i\gamma_5\big([D^\perp_\mu,\hat{A}^\mu_\Omega]-i\sigma_{\mu\nu}\{D^{\perp\mu},\hat{A}^\nu_\Omega\}\big)\Big\}B_v \\
&- \frac{1}{2M}\overline{T}^\mu_v\Big\{\frac{1}{3}g_{\mu\nu}(-D^2+(v\cdot D)^2)-\frac{i}{3}\sigma_{\mu\nu}D^2 \\
&\quad +\frac{i}{6}(2\sigma_{\alpha\beta}g_{\mu\nu}+\gamma_\mu\sigma_{\alpha\beta}\gamma_\nu)\big([D^\alpha,D^\beta]+2v^\alpha[D^\beta,(v\cdot D)]\big) \\
&\quad -\frac{i}{3}\big(\not{D}\sigma_{\mu\nu}\not{D}-(v\cdot D)[\sigma_{\mu\nu},\not{D}]\big)+\frac{1}{6}\{\not{D},\gamma_\mu\not{D}\gamma_\nu\}+\frac{1}{3}(v\cdot D)^2\gamma_\mu\not{v}\gamma_\nu \\
&\quad +\frac{1}{6}\{(v\cdot D),\gamma_\mu\not{D}\gamma_\nu\}+\frac{1}{6}\big((v\cdot D)\gamma_\mu\not{v}\gamma_\nu\not{D}+\not{D}\gamma_\mu\not{v}\gamma_\nu(v\cdot D)\big) \\
&\quad -i\mathcal{H}\gamma_5\Big[\frac{1}{6}\big((4-g_1-4g_2)g_{\mu\nu}+i(g_1+4g_2)\sigma_{\mu\nu}\big)[D^\perp_\alpha,A^\alpha_\Omega] \\
&\quad -\frac{i}{6}\big(4\sigma^{\alpha\beta}g_{\mu\nu}-(g_1+4g_2)\gamma_\mu\sigma^{\alpha\beta}\gamma_\nu\big)\{D^\perp_\alpha,A_{\Omega\beta}\} \\
&\quad +\frac{1}{6}[\not{A}_\Omega,\gamma_\mu\not{D}^\perp\gamma_\nu]+\frac{1}{3}(g_1+g_2)[\gamma_\mu\not{A}_\Omega\gamma_\nu,\not{D}^\perp] \\
&\quad -\frac{1}{3}g_1(\not{D}^\perp\gamma_\mu A_{\Omega\nu}-A_{\Omega\mu}\gamma_\nu\not{D}^\perp)-\frac{5}{3}g_1(\gamma_\mu\not{D}^\perp A_{\Omega\nu}-A_{\Omega\mu}\not{D}^\perp\gamma_\nu)\Big] \\
&\quad -\Big(\frac{2}{3}\mathcal{H}^2\big(1+(g_1+g_2)(g_1-2g_2)\big)+\mathcal{C}^2z^2\Big)(A^\alpha_\Omega)^2g_{\mu\nu} \\
&\quad +\Big(\frac{2}{3}\mathcal{H}^2(g_1+g_2)(g_1-2g_2)-\mathcal{C}^2z^2\Big)i\sigma_{\mu\nu}(A^\alpha_\Omega)^2+\Big(\frac{4}{3}\mathcal{H}^2g_1^2+\mathcal{C}^2\Big)A_{\Omega\mu}A_{\Omega\nu} \\
&\quad -\Big(\frac{1}{3}\mathcal{H}^2g_1(3-g_1-4g_2)-\mathcal{C}^2z\Big)\big(A_{\Omega\mu}\not{A}_\Omega\gamma_\nu+\gamma_\mu\not{A}_\Omega A_{\Omega\nu}\big) \\
&\quad +\mathcal{H}^2\Big[\frac{1}{6}(3+g_1^2+4g_2^2+2g_1g_2)g_{\mu\nu}i\sigma_{\alpha\beta}[A^\alpha_\Omega,A^\beta_\Omega] \\
&\quad +\frac{1}{6}(g_1^2+4g_2^2+2g_1g_2))\sigma_{\mu\nu}\sigma_{\alpha\beta}[A^\alpha_\Omega,A^\beta_\Omega]-\frac{i}{3}\gamma_\alpha\sigma_{\mu\nu}\gamma_\beta A^\alpha_\Omega A^\beta_\Omega \\
&\quad +\frac{1}{3}(g_1+g_2)\{\not{A}_\Omega,\gamma_\mu\not{A}_\Omega\gamma_\nu\}+\frac{1}{3}g_1\big(\not{A}_\Omega\gamma_\mu A_{\Omega\nu}+A_{\Omega\mu}\gamma_\nu\not{A}_\Omega\big)\Big] \\
&\quad -\frac{1}{2}\mathcal{C}^2z^2\gamma_\mu i\sigma_{\alpha\beta}\gamma_\nu[A^\alpha_\Omega,A^\beta_\Omega]\Big\}T^\nu_v\,.
\end{aligned} \tag{16}
$$

The lagrangian (16) does not contain corrections for transitions between spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ baryons which appear at $O(1/M^2)$.

Now we discuss the comparison of the results given by Eqs. (15) and (16) with the effective lagrangians for light velocity components of baryon fields derived on bases of the effective meson-baryon lagrangian of Ref. [2] which corresponds to the Eq. (1) in absence of the off-shell terms contaning gamma-matreces contracted with spin-$\frac{3}{2}$:

$$\mathcal{L}_{tot} = \overline{B}(i\not{D} - M - \gamma_5\hat{\not{A}}_\Omega)B - \overline{T}^\mu(i\not{D} - M + \mathcal{H}\gamma_5\not{A}_\Omega)T_\nu + \mathcal{C}(\overline{T}^\mu A_{\Omega\mu}B + \overline{B}A_{\Omega\mu}T^\mu).$$

Following the path integral way described above we obtain in this case the lowest order lagrangian in the form which coincides with Eq. (15) if the off-shell terms are dropped out from it. The corresponding corrections at $O(1/M)$ get the form

$$\begin{aligned}\tilde{\mathcal{L}}_v^{(1)} \;=\; & \frac{1}{2M}\overline{B}_v\Big\{-D^2 + (v\cdot D)^2 - (1-\mathcal{C}^2)(\hat{A}^\mu_\Omega)^2 \\ & +\frac{i}{2}\sigma_{\mu\nu}\Big([D^\mu, D^\nu] + 2v^\mu[D^\nu,(v\cdot D)] + [\hat{A}^\mu_\Omega, \hat{A}^\nu_\Omega]\Big) \\ & +i\gamma_5\Big([D^\perp_\mu, \hat{A}^\mu_\Omega] - i\sigma_{\mu\nu}\{D^{\perp\mu}, \hat{A}^\nu_\Omega\}\Big)\Big\}B_v \\ & -\overline{T}^\mu_v\Big\{g_{\mu\nu}\Big[-D^2 + (v\cdot D)^2 - \mathcal{H}^2(A^\mu_\Omega)^2 \\ & +\frac{i}{2}\sigma_{\alpha\beta}\Big([D^\alpha, D^\beta] + 2v^\alpha[D^\beta,(v\cdot D)] + \mathcal{H}^2[A^\alpha_\Omega, A^\beta_\Omega]\Big) \\ & -i\mathcal{H}\gamma_5\Big([D^\perp_\alpha, A^\alpha_\Omega] - i\sigma_{\mu\nu}\{D^{\perp\alpha}, A^\beta_\Omega\}\Big)\Big] + \mathcal{C}^2A_{\Omega\mu}A_{\Omega\nu}\Big\}T^\nu_v . \qquad (17)\end{aligned}$$

The comparison of Eqs. (16) and (17) shows that the taking into account the off-shell terms in the effective lagrangian (1) leads not only to the additional off-shell terms in the expressions for $1/M$-correction, but also modifies the terms which does not disappear on mass shell of spin-$\frac{3}{2}$.

As the expressions derived above for $\mathcal{L}_v^{(0)}$ and $\mathcal{L}_v^{(1)}$ are faily cumbersome, it is tempting to look for a simpler representation of the heavy and light components of the spin-$\frac{3}{2}$ field. An interesting alternative was pursued in Ref. [7] based on the definitions

$$\begin{aligned} & T^\mu = e^{-iM(v\cdot x)}(T^\mu_v + t^\mu_v), \\ & T^\mu_v = \frac{1+\not{v}}{2}P_v^{(3/2)\mu\nu}T_\nu, \\ & t^\mu_v = \Big(g^{\mu\nu} - \frac{1+\not{v}}{2}P_v^{(3/2)\mu\nu}\Big)T_\nu = \Big(\frac{1-\not{v}}{2}P_v^{(3/2)\mu\nu} + P_v^{(1/2)\mu\nu}\Big)T_\nu , \qquad (18)\end{aligned}$$

using spin projection operators [8]

$$\begin{aligned} P_v^{(3/2)\mu\nu} &= g^{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{2}{3}v_\mu v_\nu - \frac{1}{3}\not{v}(\gamma_\mu v_\nu - v_\mu\gamma_\nu), \\ P_v^{(1/2)\mu\nu} &= g_{\mu\nu} - P_v^{(1/2)\mu\nu} .\end{aligned}$$

With this definition the light component T^μ_v corresponds to a massless pure spin-$\frac{3}{2}$ state, which satisfies the constraints

$$v_\mu T^\mu_v = 0, \quad \gamma^\nu T^\mu_v = 0, \qquad (19)$$

while the heavy component t^μ_v is the mixture of spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ states. In this case the operators $Q_{\mu\nu}$, $\mathcal{Q}_{\mu\nu}$ and $R_{\mu\nu}$ in Eq. (10) get the form

$$\begin{aligned} Q_{\mu\nu} &= [i(v\cdot D) + \mathcal{H}\gamma_5\not{A}_\Omega]g_{\mu\nu} \\ \mathcal{Q}_{\mu\nu} &= [i\not{D} - (1-\not{v})M]g_{\mu\nu} - \frac{1}{4}\gamma_\mu\gamma_\lambda[i\not{D} - (1-\not{v})M]\gamma^\lambda\gamma_\nu + \mathcal{H}\gamma_5\Omega_{\mu\nu}, \\ R_{\mu\nu} &= i\not{D}g_{\mu\nu} + \mathcal{H}\gamma_5(\not{A}_\Omega g_{\mu\nu} + g_1A_{\Omega\mu}\gamma_\nu).\end{aligned}$$

Due to the simple form of the operator $Q_{\mu\nu}$ and the constraints (19), the definitions (18) seem to be more preferable than we have used above. Unfortunately, it can be shown that the inverse operator $\mathcal{Q}^{-1}_{\mu\nu}$ does not exist in this case and the procedure of integrating out the heavy component t^μ_v can not be performed.

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Бельков А.А., Диллиг М. E2-96-432

Поля со спином $\frac{3}{2}$ в эффективной теории тяжелых барионов

В рамках подхода функциональных интегралов в пределе тяжелых барионов рассмотрен формализм зависящих от скоростей компонент барионных полей со спином $\frac{1}{2}$ и $\frac{3}{2}$, связанных с псевдоскалярными мезонами. Используя в качестве исходного наиболее общий вид кирального мезон-барионного лагранжиана, включающего инвариантность относительно точечного преобразования полей со спином $\frac{3}{2}$, мы рассматриваем различные проблемы, возникающие при интегрировании «тяжелых» компонент, зависящих от скорости барионных полей, и получении $1/M_{\text{baryon}}$-разложения эффективного лагранжиана для «легких» компонент.

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Spin-$\frac{3}{2}$ Fields in the Heavy-Baryon Effective Theory

In the framework of the path integral approach we develop the velocity component formalism for spin-$\frac{1}{2}$ and -$\frac{3}{2}$ baryons coupled with pseudoscalar mesons in the limit of heavy baryons. Starting from the most general chiral meson-baryon lagrangian including the invariance of spin-$\frac{3}{2}$ fields under point transformation we detail various problems in integrating out the heavy velocity components in the baryon fields and derive the effective lagrangian for the light components up to $O(1/M_{\text{baryon}})$.

The investigation has been performed at the Laboratory of Particle Physics, JINR.

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