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THE CONCEPT OF TIME AND FIELD THEORY

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1 Introduction

In this paper, we formulate main principles of the theory of time that is capable to answer the following questions of fundamental importance: Is the time only a coordinate? Does the connection exist between the Euclidean metric on a manifold and Lorentz metric, i.e. between a positive definite metric and the one with signature (+ - - -)? How can an invariant definition of the rate of change of the field be introduced into the theory, in analogy with classical mechanics? Why is one of the coordinates parametrizing a smooth real manifold called the time? The latter question requires explanation. According to the modern standpoint, space-time theory is any theory that possesses a mathematical representation whose elements are a smooth four-dimensional manifold and geometrical objects defined on that manifold. However, the definition of a manifold given in [1] contains nothing that could impartially, i.e. not going beyond the scope of this notion, point to one of the coordinates on a manifold being distinguished. All the coordinates appear on equal status.

The paper is organized as follows. In the next section, we present the definition of time, consider the problem on the rate of change of the field, formulate the general concept of the evolution form of field equations. In sec.3, we give the general-covariant definition of the strength of electric and magnetic fields on a four-dimensional manifold. The Maxwell equations are written in the evolution form, which allows us to establish the connection of the Euclidean metric with the Lorentz one. Geometrical interpretation of the Lorentz metric is given in terms of the Euclidean metric. In sec.4, we analyze the connection between the gravitational field and temporal field, the field of time. Equations of the temporal field are derived. In sec.5, the obtained results are summarized.

2 Time

Geometrically, Time is defined as a congruence of lines (lines of time) on a smooth real four-dimensional manifold. We recall that the congruence of lines is a set of lines characterized by that the only element of the set crosses every point of the manifold or its part. According to the definition, lines belonging to the congruence do not intersect and fill either the whole manifold or its part. If the congruence of lines fills just a part of the manifold, Time will be local, otherwise, Time will be global. The lines of local time being continued outside the region of definition can either begin to branch or converge at one point, 'the source'. The simplest example is the congruence of rays coming from one point of the Euclidean space. If Time is global, the Euler–Poincare characteristic of the manifold should be zero. For instance, for a 4-dimensional sphere it equals 2 and therefore a manifold like that is not globally stratified into lines of time. When a manifold is equipped with the structure of Time it will be called the space-time manifold.

It is intuitively clear from the above geometrical definition of time that Time can be oriented. To give a strict definition of the time direction, note that Time can be given by indicating the field of nonzero vectors t^i, tangent to the time lines on the manifold or on its part. This field will be called the field of time or temporal field. The temporal field defines not only the time direction but the coordinate of time and special systems of coordinates on a manifold too. Analytically, Time is defined by the following system of differential equations:

$$\frac{dx^i}{dt} = c\, t^i(x^1(t), x^2(t), x^3(t), x^4(t)), \quad i = 1,2,3,4, \qquad (1)$$

where c is a scale constant. According to the Whitney theorem, any smooth manifold admits a positive definite (Euclidean) metric g_{ij} and hence it can be considered as a structure element of the

manifold. For this reason, the unit vector field

$$g(t,t) = g_{ij}t^it^j = 1. \qquad (2)$$

will be called the temporal field in what follows. The independent variable t is the time coordinate. Equations (1) and (2) uniquely define the time coordinate up to the translation $t \rightarrow t+a$, where a is a constant. It can be shown that if the functions $\varphi^i(t)$ are solutions to eqs. (1), the functions $\psi^i(t) = \varphi^i(t+a)$ will also be solutions to the same equations. Thus, the interval of change of the time coordinate can be considered symmetric with respect to the change $t \rightarrow -t$. By definition, the time coordinate is a measurable quantity and therefore the scale constant in eq. (1) is of the dimension of velocity since coordinates are of the dimension of length and components of the temporal field are dimensionless.

Introduction of the temporal field allows us to build one more bridge between geometry and physics, the equations of the temporal field. (Derivation of these equations will follow.) If $ds^2 = g_{ij}dx^idx^j$, it follows from eqs. (1) that $ds/dt = c$. Thus, the length of the time line is a linear function of the time coordinate. It is natural to assume that the scale factor does not change when passing from one time to another and is a universal constant.

We will assume that the temporal field is directed along the increase of the time coordinate. If we invert the direction of temporal field $\bar{t}^i = -t^i$ we obtain a new time coordinate $\bar{t}$, defined by the new temporal field. From eq. (1) it is not difficult to deduce that between the old and new time coordinates there is a one-to-one continuous correspondence given by the equation $\bar{t} = -t$. Then we may conclude that the existence of the so-called 'time arrow' gives evidence that not all fundamental equations of physics are invariant under the reversal of the temporal field.

Let us now show how the temporal field enters into equations of other physical fields. The magnitude of a physical field and the rate of its change in time are analogous to the position and velocity of a particle. From classical mechanics it is known that the velocity of a particle is a well-defined quantity, which is also true from a geometrical point of view. It is to be supposed that the rate of the field change is also a well-defined quantity provided that Time is defined correctly. We will show that the above definition of Time allows an invariant coordinate-independent definition of the rate of change of the field. To do this , we will make some preliminary mathematical comments. All points of the manifold are indistinguishable of each other. There is no objective property that could distinguish between points. Hence it follows that if coordinates of one point take given numerical values, there should exist the system of coordinates in which the coordinates of any other point assume the same numerical values. If a manifold admits the transitive group of transformations the above situation is always realizable. Let P and Q be two arbitrary points, then there exists a transformation σ,such that $\sigma(P) = Q$. Let $x^i(P)$, be the initial coordinate system and let us introduce a new coordinate system as a function of the initial one $x^i(P)$, setting $\bar{x}^i(P) = x^i(\sigma^{-1}(P))$. In the system of coordinates $\bar{x}^i$, point Q has the coordinates

$$\bar{x}^i(Q) = x^i(\sigma^{-1}(Q)) = x^i(\sigma^{-1}(\sigma(P))) = x^i(P)$$

Q.E.D. A similar reasoning can be applied to construct new fields from the given ones. Let a field be given in the coordinate system x^i. A new field is introduced so that its components in a new coordinate system take the same values as the old-field components in the initial coordinate system. We present the corresponding analytic expressions to be required below. In the coordinate system x^i the transformation σ is given by smooth functions

$$\sigma : x^i \Rightarrow \varphi^i(x), \quad \sigma^{-1} : x^i \Rightarrow f^i(x); \quad \varphi^i(f(x)) = x^i.$$

To be specific, we take the covector field $A_i(x)$, and derive the following transformation formula for it:

$$\tilde{A}_i(x) = A_j(f(x))\frac{\partial f^j(x)}{\partial x^i}. \tag{3}$$

Then we write the solution to eq.(1) in the form $\varphi^i(t,x)$, with $\varphi^i(0,x) = x^i$. From the theory of differential equations it is known that $\varphi^i(t,\varphi(s,x)) = \varphi^i(t+s,x)$. It is clear that Time defines a one-parameter set of transformations of a manifold. By using a formula of the type (3) one can construct a'line' in the space of field under consideration similar to the time line on the basis of the given field. According to (3), for the covector field we obtain the following analytic expression of the 'line':

$$A_i(t,x) = A_j(\varphi(-t,x))\frac{\partial \varphi^j(-t,x)}{\partial x^i}. \tag{4}$$

Hence it follows that the rate of change of the field at the initial moment of time can naturally be defined as follows:

$$\dot{A}_i(x) = \frac{d}{dt}A_i(t,x)|_{t=0}.$$

The rate at any subsequent moment of time can be found by the previous formula but, as will be shown below, the determination of the field evolution with time requires the knowledge of the rate of change of the field only at the initial moment. From (1) and (4) we derive the following formula for the rate of the covector-field change:

$$\dot{A}_i = t^k\frac{\partial A_i}{\partial x^k} + A_k\frac{\partial t^k}{\partial x^i}. \tag{5}$$

and for the vector field we have

$$\dot{A}^i = t^k\frac{\partial A^i}{\partial x^k} - A^k\frac{\partial t^i}{\partial x^k}. \tag{6}$$

The above formulae are easily generalized to any geometrical objects. In mathematics, they are known since 1931 and are called the Lie derivatives [2],[3]. In what follows, the Lie derivatives along the temporal field will be denoted by D_t.

The evolution equation of a field Ψ can be written in the form

$$D_t\Psi = H(\Psi), \tag{7}$$

where the operator H is given not only by the temporal field buy also some other structures on a manifold. Equation (7) allows us to calculate the formal exponential

$$\Psi(t) = exp(tD_t)\Psi.$$

So, from eq.(7) it follows that to determine the dynamics of a field, it is sufficient to know its rate at the initial moment of time if the operators D_t and H commute. In the next section, the above definitions will be specified for the Maxwell equations, the basis of the whole field theory.

3 Time in electromagnetic field theory

Before to proceed to consider the problem of the role of time in electromagnetic field theory, we clarify some details connected with the positive definite metric g_{ij} on a manifold. Indices will, as usual, be raised and lowered through the metric fields g_{ij} and g^{ij}. The symbol ∇_i denoted the covariant derivative with respect to the connection

$$\Gamma^i_{jk} = \frac{1}{2}g^{il}(\partial_j g_{kl} + \partial_k g_{jl} - \partial_l g_{jk})$$

of the metric g_{ij}; ε_{ijkl} is a skew-symmetric Levi-Civita tensor with the main component $\sqrt{g}$, where g is the metric-tensor determinant. Since the metric is positive definite, $g > 0$.

First, we consider the second group of equations since they written on an arbitrary manifold do not require any structures apart from the structure of a smooth manifold. Let F_{ij} be a tensor of the electromagnetic field, then the equations are written in the form

$$\partial_i F_{jk} + \partial_j F_{ki} + \partial_k F_{ij} = 0. \tag{8}$$

To write these equations in the evolution form given by eq.(7), we introduce the strengths of electric and magnetic fields on a manifold

in an invariant coordinate-independent way. If the temporal field is known, we can make this as follows. Introduce two covector fields setting

$$E_i = t^k F_{ki}, \quad H_i = t^k \tilde{F}_{ki}, \tag{9}$$

where

$$\tilde{F}_{ki} = \frac{1}{2}\varepsilon_{kijl}F^{jl}$$

is the electromagnetic-field tensor dual to the tensor F_{ij}, $\tilde{\tilde{F}}_{ij} = F_{ij}$. Equations (9) can be inverted, and as a result, we arrive at the relation

$$F_{ij} = t_i E_j - t_j E_i + \frac{1}{2}\varepsilon_{ijkl}(t^k H^l - t^l H^k). \tag{10}$$

Equations (9) and (10) are compatible since the latter is invariant under the transformations $E_i \to E_i + \lambda t_i$, $H_i \to H_i + \mu t_i$, and, consequently, with an appropriate choice of scalars λ and μ we obtain the following relations

$$t^i E_i = 0, \quad t^i H_i = 0. \tag{11}$$

Inserting (10) into the equations $\nabla_i \tilde{F}^{ij} = 0$, adequate to eqs.(8) we arrive at the equations

$$D_t H^l - \frac{1}{2}(\nabla_i E_j - \nabla_j E_i)\varepsilon^{ijkl}t_k + H^l \nabla_i t^i - t^l \nabla_i H^i = 0, \tag{12}$$

under the condition that the temporal field obeys the equations

$$\nabla_i t_j - \nabla_j t_i = 0. \tag{13}$$

From eqs. (1) and (13) it follows that the congruence of time lines is a geodesic. From eqs. (2) and (13) we get $D_t t_i = t^k\partial_k t_i + t_k \partial_i t^k = t^k \nabla_k t_i + t_k \nabla_i t^k = 0$. So, from eqs. (12) it follows that

$$\nabla_i H^i = 0. \tag{14}$$

In analogy with eqs.(12) we will write the first group of the Maxwell equations

$$\partial_t E^l + \frac{1}{2}(\nabla_i H_j - \nabla_j H_i)\varepsilon^{ijkl} t_k + E^l \nabla_i t^i - t^l \nabla_i E^i = J^l, \quad (15)$$

where J^l is the vector of current. The validity of the first group of Maxwell equations in the form (15) will be proved below. Decomposing the current vector J^l over the temporal field and its orthogonal field

$$J^l = t^l(t_i J^i) + J^l - t^l(t_i J^i),$$

from eqs.(15) we derive the following equations

$$D_t E^l + \frac{1}{2}(\nabla_i H_j - \nabla_j H_i)\varepsilon^{ijkl} t_k + E^l \nabla_i t^i = J^l - t^l(t_i J^i), \quad (16)$$

$$\nabla_i E^i + t_i J^i = 0. \quad (17)$$

Using the notation without indices, we will write basic operations of the vector algebra and vector analysis on a four-dimensional manifold in the form of the relations

$$(A \circ B) = A_i B^i,$$

$$[A \times B]^l = \frac{1}{2}(A_i B_j - A_j B_i)\varepsilon^{ijkl} t_k,$$

$$(Rot\, A)^l = \frac{1}{2}(\nabla_i H_j - \nabla_j H_i)\varepsilon^{ijkl} t_k,$$

$$Div\, A = \nabla_i A^i, \quad (Grad\, \phi)^i = \nabla^i \phi = g^{ij}\nabla_j \phi = g^{ij}\partial_j \phi.$$

Setting that

$$E \to E^i, \quad H \to H^i, \quad T \to t^i,$$

$$4\pi J \to J^l - t^l(t_i J^i), \quad 4\pi\rho = -t_i J^i,$$

we write eqs. (12), (14), (16), and (17) in the canonical form

$$D_t\, E + Rot\, H = 4\pi J,$$

$$Div\, E = 4\pi\rho,$$

$$-D_t\, H + Rot\, E = 0,$$

$$Div\, H = 0.$$

These equations should be supplemented with the orthogonality conditions of E and H to T and the temporal field equations obtained so that the electromagnetic field equations look as simple as possible

$$(E \circ T) = (H \circ T) = 0,$$

$$(T \circ T) = 1, \quad T = Grad\, \phi, \quad Div\, T = 0.$$

Thus, it has been shown that the strength of electric and magnetic fields can be introduced on an arbitrary 4-dimensional manifold and a canonical 4-dimensional system of equations can be written for them if the temporal field is given. The problem remains open concerning the equivalence of the first group of Maxwell equations as equations for the electromagnetic field tensor. The answer is of interest and, which is important, admits the geometrical interpretation on the basis of a simple connection between the Euclidean and Lorentz metrics. Before proceeding to that problem, we dwell upon two very important points. As is known [4], the vector field t^i can be connected with a special coordinate system where the field components acquire the form (0,0,0,1) simultaneously at all points. In this coordinate system it follows from (1) that $x^4 = ct$ and, in accordance with (6), we obtain

$$D_t E^i = \frac{1}{c}\frac{\partial E^i}{\partial t}.$$

So, the definition of the rate of field change given above is the invariant coordinate-independent form of the field change rate as a partial derivative with respect to time. Let a manifold be realized as a 4- dimensional Euclidean space related with the system of coordinates $x^1 = x$, $x^2 = y$, $x^3 = z$, $x^4 = ct$, where

$t^i = (0,0,0,1)$ and $g_{ij} = (1,1,1,1)$. Then provided the orthogonality conditions $t^i E_i = t^i H_i = 0$, hold from the Maxwell equations (12), (14),(16),(17) we obtain the so-called canonical three-dimensional form of these equations which contains the temporal field **implicitly.**

By $\bar{g}_{ij}$ we denote the metric Lorentz tensor and by $\bar{g}^{ij}$ its reciprocal tensor. It may be shown that equations (15) are derived by the variational method from the standard Lagrangian

$$L = -\frac{1}{4} F_{ik} F_{jl} \bar{g}^{ij} \bar{g}^{kl} + A_i J^i,$$

if we set

$$\bar{g}_{ij} = 2t_i t_j - g_{ij}, \quad \bar{g}^{ij} = 2t^i t^j - g^{ij}. \tag{18}$$

It is seen that the Lorentz metric usually used for the description of the gravitational field and for writing equations of other fields is simply expressed in terms of the Euclidean metric and temporal field. However, the Lorentz metric, as was shown above for the example of equations of the electromagnetic field, does not allows us to represent equations of physical fields in the evolution dynamic form required for their physical interpretation. Nevertheless, the Lorentz metric is highly suitable for writing the equations in the geometric form since the transition from a geometric form of equations to their evolution form can be realized if the connection given by eqs.(15) is known. From the above consideration it follows that the Lorentz metric contains the temporal field implicitly.

Let us show that equations (18) admit an intriguing and unexpected geometrical interpretation. Let $(A \circ B) = g_{ij} A^i B^j = |A||B| \cos\varphi$ be a scalar product of vectors given by the positive definite (Euclidean) metric. We expand the vector A into the temporal field and its orthogonal field

$$A^i = A^i_- + A^i_\perp = t^i(t_j A^j) + A^i - t^i(t_j A^j).$$

The vector

$$A^i_s = A^i_- - A^i_\perp$$

is a mirror image of the vector A relative to the temporal field T. Consider the Euclidean scalar product of vector A and its mirror image A_s:

$$(A \circ A_s) = |A|^2 \cos 2\varphi = (2t_i t_j - g_{ij}) A^i A^j = \bar{g}_{ij} A^i A^j,$$

where φ is the angle between the vector A and temporal field. Hence it follows that the Lorentz scalar product is defined as the Euclidean scalar product of a vector and its mirror image. The role of mirror plays just the temporal field. According to the terminology accepted for the Lorentz metrics, vectors are divided into three classes: space-like, time-like, and isotropic vectors. In accordance with the given interpretation of the Lorentz metric, vectors belong to a given class depending on their angle with the temporal field. For instance, a vector orthogonal to the temporal field will be space-like, and a vector orthogonal to its mirror image with be isotropic. As we see, the division of vectors into three classes has a simple geometrical meaning from the viewpoint of the Euclidean metric. We have derived the temporal field equations by requiring the Maxwell equations being as simple as possible. In the next section, the equations of temporal field will be derived on the basis of relations (18).

4 Equations of temporal field

The Lagrangian of the temporal field will be obtained as follows. From relation (18) between the Lorentz and Euclidean metric we derive the connection between the Christoffel symbols belonging to these metrics. Then we find the connection between the Riemannian tensors, Ricci tensors, and scalar curvatures. Let

$$\Gamma^i_{jk} = \frac{1}{2} g^{il}(\partial_j g_{kl} + \partial_k g_{jl} - \partial_l g_{jk})$$

be Christoffel symbols of the Euclidean metric, and, respectively,

$$\bar{\Gamma}^i_{jk} = \frac{1}{2} \bar{g}^{il}(\partial_j \bar{g}_{kl} + \partial_k \bar{g}_{jl} - \partial_l \bar{g}_{jk})$$

be Christoffel symbols of the Lorentz metric. Let us find explicit expression for the tensor

$$Q^i_{jk} = \bar{\Gamma}^i_{jk} - \Gamma^i_{jk}$$

in terms of t^i and g_{ij}. We have $\nabla_i g_{jk} = 0$,

$$\nabla_k \bar{g}_{ij} = \partial_k \bar{g}_{ij} - \bar{g}_{lj}\Gamma^l_{ki} - \bar{g}_{il}\Gamma^l_{kj}, \quad 0 = \partial_k \bar{g}_{ij} - \bar{g}_{lj}\bar{\Gamma}^l_{ki} - \bar{g}_{il}\bar{\Gamma}^l_{kj}.$$

From these equations it follows that

$$\nabla_k \bar{g}_{ij} = \bar{g}_{lj} Q^l_{ki} - \bar{g}_{il} Q^l_{kj}. \tag{19}$$

Since $\nabla_i g_{jk} = 0$, from (19) we obtain

$$Q^i_{jk} = (2t^i t^l - g^{il})(\nabla_j(t_k t_l) + \nabla_k(t_j t_l) - \nabla_l(t_j t_k)).$$

For calculations, it is convenient to represent this relation in another form. We set

$$u_{ij} = \nabla_i t_j - \nabla_j t_i, \quad u_i = t^k u_{ik}, \quad h_{ij} = \nabla_i t_j + \nabla_j t_i.$$

As a result, we have

$$Q^i_{jk} = 2t^i(t_j u_k + t_k u_j) + (t_j u_{lk} + t_k u_{lj})g^{il} + t^i h_{jk}. \tag{20}$$

Note that eqs. (15) can be derived from the equations $\bar{\nabla}_i \bar{F}^{ik} = J^k$ with the help of (10) and (20). From (2) and (20) it follows that the covector Q^i_{ij} equals zero,

$$Q^i_{ik} = 0. \tag{21}$$

With the equality (21) and the transformation law of the Riemann tensor under the connection transformation

$$\bar{R}_{ijk}{}^l = R_{ijk}{}^l + \nabla_i Q^l_{jk} - \nabla_i Q^l_{jk} + Q^l_{im} Q^m_{jk} - Q^l_{jm} Q^m_{ik}$$

we obtain the connection between the Ricci tensors $\bar{R}_{jk} = \bar{R}_{ljk}{}^l$, $R_{jk} = R_{ljk}{}^l$,

$$\bar{R}_{jk} = R_{jk} + \nabla_l Q^l_{jk} - Q^l_{jm} Q^m_{kl}.$$

From this relation and (2), (20), upon some algebra, we obtain the connection between scalar curvatures

$$\bar{R} = -R + 2u_i u^i - u_{ij}u^{ij} + 2t^i t^j R_{ij} + \nabla_i(2u^i - 2t^i \nabla_k t^k) \tag{22}$$

The Ricci identity $\nabla_i \nabla_j t^k - \nabla_j \nabla_i t^k = R_{ijl}{}^k t^l$ allows us to establish the relationship

$$-u_{ij}u^{ij} + 2t^i t^j R_{ij} = -2\nabla_i t_j \nabla^i t^j + 2(\nabla_i t^i)^2 - \nabla_i(2u^i + 2t^i \nabla_k t^k).$$

Inserting this relation into eq. (22) we get the following equality

$$\bar{R} = -R - 2\nabla_i t_j \nabla^i t^j + 2u_i u^i + 2(\nabla_i t^i)^2 + \nabla_i(-4t^i \nabla_k t^k). \tag{23}$$

From eqs. (18) one can derive also a simple relation between determinants of the Euclidean and Lorentz metrics $g = -\bar{g}$. If we neglect an unessential divergent term in eq. (23), we find that the Einstein - Hilbert action splits into the action for the gravitational field and the action for the temporal field. In accordance with (23) we write the total Lagrangian of the gravitational and temporal fields in the form

$$L = L_g + L_T = -\frac{1}{4}R + \frac{1}{2}(-\nabla_i t_j \nabla^i t^j + u_i u^i + (\nabla_i t^i)^2). \tag{24}$$

from which by varying we obtain the temporal field equations

$$(\delta^i_k - t^i t_k)(\nabla_j \nabla^j t_i - (\nabla_i + u_i)\nabla_j t^j - t^j(\nabla_j u_i + \nabla_i u_j)) = 0. \tag{25}$$

Note that any solution to the equations $\nabla_i t_j - \nabla_j t_i = 0$, $\nabla_i t^i = 0$, that have been obtained in writing the Maxwell equations in the evolution form will be a solution to eqs. (25) if the gravitational field obeys the equations $R_{ij} = \lambda g_{ij}$.

5 Conclusion

We will summarize the results obtained and outline some future problems. The basis of any space-time theory is a smooth real 4-dimensional manifold. It has been shown that Time is a geometrical structure on a manifold and is a specific field tightly connected with the gravitational field. Invariant definition has been given for the rate of change of the field and the evolution form of field equations. Basic relations of the vector algebra and vector analysis are written on a 4- dimensional manifold. Evolution form of the Maxwell equations on an arbitrary space-time manifold are derived. It has been found that the gravitational field is described by the positive definite (Euclidean) metric whereas the temporal field connected with the gravitational field is described by the unit vector field for which the field equations are deduced. Generally speaking, every physical process should be described by its own time. To find this time is apparently a principal task of the theory of time whose fundamentals have been formulated in this paper. The conclusion on the metric corresponding to the gravitational field being positive definite makes substantial many fields of studies recently discovered. We mention the instanton theory [5] and quantization of the gravitational field by the path integral method [6], where the Euclidean metric for a description of the gravitational field has been introduced on the other basis. It goes without saying that difficulties are removed which are caused by the complex structure introduced on the manifold [7]. We note also the significance of the Eguchi–Hanson space [8] acquired in the context of this paper. The question suggests itself concerning the connection of Eguchi– Hanson metric with the Schwarzschild metric. Topological aspects of the introduced notion of time are related to the problem of existence of nonzero unit vector fields on 4-dimensional manifold. Maybe, this is tightly connected with the problem of singularities in General Relativity [9]. Of great interest is the problem of the evolution form of the Dirac equation on an arbitrary space-time manifold.

But this is a separate topic.

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