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SQUEEZED CONDENSATE AND CONFINEMENT IN A SCALAR MODEL

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1 Introduction

Phenomenological models have been developed for the low energy sector of QCD which are rather successful in describing hadrons and their properties. Here the aspect of chiral symmetry is understood much better than that of quark and gluon confinement. The mechanism for dynamical chiral symmetry breaking is identified fairly well since the work by Nambu and Jona-Lasinio [1]. For recent reviews on the application of the NJL-model to low energy QCD, see [2]. For the explanation of confinement many different models have been discussed. We will refer in the present work to approaches based on nonlocal interactions that address quark and gluon confinement via the criterion of absence of poles for the propagators [3], for a review see [4]. These confining quark models use effective gluon propagators, which have infrared singularities like $1/k^4$ or a delta function $\delta(k)$ at low energy [3, 5, 6]. Despite the success of these phenomenological approaches in explaining low energy hadronic observables [6], the question for the mechanism which leads to the infrared singularities of the gluon propagator remains open. There is evidence from detailed studies [7, 8] of the Schwinger-Dyson equations of QCD for a strong infrared enhancement of the gluon propagator due to the non Abelian character of the theory and in particular due to the gluon-gluon self coupling. On the other hand there are several approaches which relate the confinement problem to the question of the true physical vacuum of QCD.

In more rigorous treatments of QCD, the problem of the vacuum is well known. For instance, the simple perturbative vacuum is unstable [9], and there is no stable (gauge invariant) coherent vacuum in Minkowski space [10]. Therefore, in the context of the construction of a gauge invariant, stable QCD vacuum in Minkowski space, the squeezed condensate of gluons has become a topic of great interest [11] - [15]. From the physical point of view the squeezed state differs from the coherent one by the condensation of colour singlet gluon pairs rather than condensation of single gluons. It has been discussed before that a squeezed vacuum in form of Gaussian fields [12, 14] leads to confinement of quarks via the criterion of the linearly rising static potential [12] and via the area law behaviour of the Wilson loop [14]. These confinement criteria suit only for heavy (static) external quarks.

The present paper is devoted to the study of confinement as a possible consequence of the squeezed vacuum. Our approach differs from the previous ones addressing the squeezed vacuum [12, 13, 14] in the following way. We will not try to generate the squeezed vacuum by treating selfinteractions of gluons but we will use the squeezed condensate as a phenomenological input. Furthermore, in contrast to previous work we discuss confinement by using the criterion of the absence of poles for the propagators [3]. We will prepare the squeezed vacuum by macroscopically populating it with zero momentum gluon pairs and study the changes in the analytical properties of the propagators caused by the change of the vacuum structure. This procedure parallels the original idea by Bogoliubov [16] for the explanation of the Landau sound in a superfluid liquid according to which the change in the excitation spectrum of the theory at low energies is due to the condensation of a macroscopic number of particles in the zero momentum state.

As a first step towards discussing the squeezed gluon vacuum as a possible scenario for quark confinement in full QCD we examine here a simplified model field theory where quarks and gluons are considered as scalar fields interacting via Yukawa coupling. The Lagrangian of this model is given by

$$\mathcal{L}(x) \equiv \mathcal{L}_0^{\Psi}(x) + \mathcal{L}_0^{\varphi}(x) + g\Psi^*(x)\Psi(x)\varphi(x) . \tag{1}$$

Here $\mathcal{L}_0^{\Psi}$ is the free Lagrangian of a complex scalar quark field and $\mathcal{L}_0^{\varphi}$ denotes the free Lagrangian of a scalar gluon field. We show in this paper that the interaction of scalar quarks with the squeezed condensate of scalar gluon pairs leads to the occurence of a permanent, nonlocal coupling of quarks (δ- function in momentum space) and that this removes the poles of the effective quark propagator constructed by using the Schwinger-Dyson equation.

A type of delta function interaction has been considered for the computation of the meson spectra on the level of Schwinger-Dyson and Bethe-Salpeter equations in both Minkowski space [3, 5] and Euclidean space [3, 4, 5, 6]. In this paper we prefer to work directly in Minkowski space.

The content of the paper is the following: In section 2 we compute the generating functional for the squeezed vacuum in the gluon sector and derive an effective gluon propagator mediating the quark-quark interaction. In section 3 we discuss how this can lead to confinement of quarks using the Schwinger-Dyson equation and show that a scalar meson-like solution of the corresponding Bethe–Salpeter equation exists.

2 Squeezed vacuum in the gluon sector

In order to study quark confinement in the presence of a squeezed condensate of the scalar gluons in our model (1), we shall first derive an effective quark-quark interaction by eliminating the gluon degrees of freedom. For this purpose we first consider only the sector of scalar gluons and treat the quark fields as an external classical source $J(x) = g\Psi^*(x)\Psi(x)$. The effective Lagrangian of the gluon sector reads

$$\mathcal{L}_J^{\varphi}(x) \equiv \mathcal{L}_0^{\varphi}(x) + \varphi(x)J(x) = \frac{1}{2}\left(\partial_\mu\varphi(x)\right)^2 - \frac{m^2}{2}\varphi^2(x) + \varphi(x)J(x) . \tag{2}$$

Equivalent to the solution of the quantum field theory is to derive a corresponding generating functional $Z[J]$ from which all Green functions can be obtained via functional differentiation with respect to the sources $J(x)$. For the free theory (2) the result is well known [17, 18] and one obtains the action functional $W[J] = -i \ln Z[J]$ which describes the interaction of the external (scalar) quark sources via the standard (scalar) gluon propagator. The generating functional can be obtained either by

functional integration over the (classical) fields $\varphi(x)$ or by calculating field operator expectation values in the perturbative vacuum $|0>$. In order to fix our notations both ways of calculating the generating functional are briefly reviewed in Appendix A.

We now generalize the generating functional to the case of a squeezed gluon vacuum by using the operator approach and simply replacing the perturbative vacuum by the squeezed one. The squeezed vacuum is obtained from the perturbative vacuum $|0>$ by a unitary transformation

$$|0_{\mathrm{sq}}> \equiv U_{\mathrm{sq}}^{-1}|0> . \tag{3}$$

The squeezing operator for homogeneous fields is

$$U_{\mathrm{sq}} = \exp\left[i\frac{f_0}{2}\left(\varphi_0\pi_0 + \pi_0\varphi_0\right)\right] , \tag{4}$$

where φ_0 and π_0 are the zero-momentum components of the field operators φ_I and π_I (see Appendix A). Under the squeezing transformation (4), the fields transform as

$$\varphi_{\mathrm{sq}} = U_{\mathrm{sq}}\varphi_0 U_{\mathrm{sq}}^{-1} = \mathrm{e}^{f_0}\ \varphi_0 , \tag{5}$$

$$\pi_{\mathrm{sq}} = U_{\mathrm{sq}}\pi_0 U_{\mathrm{sq}}^{-1} = \mathrm{e}^{-f_0}\ \pi_0 , \tag{6}$$

and the commutation relations remain unchanged $i[\varphi_0, \pi_0] \equiv i[\varphi_{\mathrm{sq}}, \pi_{\mathrm{sq}}] = 1$. The open parameter f_0 is considered as a phenomenological parameter characterizing the modification of the perturbative gluon vacuum due to pairing correlations. The consequences of a nonvanishing squeezing parameter f_0 will be discussed in detail in Section 3.

We will consider the above model (2) in the massless limit $m \to 0$ where the squeezing transformation (4) becomes Lorentz invariant, see [19]. Consider for example the zero momentum mode of the field operator $\varphi(\mathbf{x})$ in this limit. It can be seen to be Lorentz invariant from the fact that the zero momentum component of the corresponding Heisenberg operator $\varphi(t,\mathbf{x}) \equiv \exp(iH_0 t)\ \varphi(\mathbf{x})\ \exp(-iH_0 t)$ becomes space and time independent [1] and therefore commutes with the infinitesimal generator $M^{\mu\nu}$ of Lorentz transformations.

Generalizing the operator approach given in Appendix A to the case of a squeezed vacuum the generating functional is defined according to

$$Z_{\mathrm{sq}}[J] = <0_{\mathrm{sq}}|T\exp\left\{i\int_{-\infty}^{+\infty} dt \int_V d\mathbf{x}\varphi_I(\mathbf{x},t)J(\mathbf{x},t)\right\}|0_{\mathrm{sq}}> . \tag{7}$$

Using the Wick theorem (37) in analogy to the ordinary perturbative vacuum which is explained in Appendix B we can simplify (7) to the form

$$Z_{\mathrm{sq}}[J] = \exp\left\{-\frac{1}{2}\int d^4x \int d^4y\ J(x) <0_{\mathrm{sq}}|T\varphi_I(x)\varphi_I(y)|0_{\mathrm{sq}}> J(y)\right\} . \tag{8}$$

[1] Since for $\mathbf{p} = 0$ also the energy $\omega(p) = \sqrt{p^2 + m^2}$ vanishes for $m \to 0$.

The calculation of the squeezed generating functional has thus been reduced to the calculation of the squeezed two-point Green function

$$D_{sq}(x,y) \equiv <0_{sq}|T\varphi_I(x)\varphi_I(y)|0_{sq}> \ . \tag{9}$$

In order to evaluate (9) we use again the Wick formula (37) and obtain

$$\begin{aligned} D_{sq}(x,y) &= <0|T\varphi_I(x)\varphi_I(y)|0> + <0_{sq}| :\varphi_I(x)\varphi_I(y): |0_{sq}> \\ &= D(x-y) + D_f(x,y) \ . \end{aligned} \tag{10}$$

The first term is the ordinary causal Green function $D(x-y)$ given in (39). The second term

$$D_f(x,y) \equiv <0|U_{sq} :\varphi_I(x)\varphi_I(y): U_{sq}^{-1}|0> \tag{11}$$

appears due to the modification of the vacuum. The evaluation of this term which is performed in Appendix B makes use of a finite mass $m \neq 0$ in order to avoid infrared singularities, the Wick reordering and the explicit form of the squeezing transformation (4) in terms of creation and annihilation operators in the perturbative vacuum. At the end of the calculation, the massless limit and the infinite volume limit are performed, the latter one being in analogy to the thermodynamic limit in the Bogoliubov model [16]. As discussed in Appendix B, we can separate the homogeneous (zero momentum) part of the squeezed propagator (10) according to

$$D_{sq}(\mathbf{x}-\mathbf{y};x_0,y_0) \equiv \tilde{D}(\mathbf{x}-\mathbf{y};x_0-y_0) + C_0 \ , \tag{12}$$

where the spatial average of the fluctuating part $\tilde{D}$ vanishes. The constant part C_0 is determined by the parameter f_0 of the squeezing transformation (4), see Appendix B. In the following, it will be used instead of f_0 as an empirical parameter to characterize the squeezed vacuum state.

With this result for the squeezed two-point Green function, the squeezed generating functional becomes

$$Z_{sq}[J] = \exp\left\{-\frac{1}{2}\int d^4x \int d^4y J(x)\tilde{D}(x-y)J(y) - \frac{C_0}{2}\left(\int d^4x J(x)\right)^2\right\} \ , \tag{13}$$

which is our central result. It is to be understood in the massless limit and has all the symmetries (including Lorentz invariance) of the generating functional (25) in this limit. The first term in the exponent of (13) corresponds to the fluctuating part of the propagator in the unsqueezed vacuum (25). The second term represents the zero momentum sector of the theory which is modified compared to (25) due to the squeezing operation (3), (4). The coefficient C_0 which replaces the squeezing factor e^{2f_0} as described by Eq. (50) of Appendix B is a measure of the squeezing strength and is an open parameter in our treatment. The exponent of the generating functional (13) represents a modified current-current interaction where in addition

to the ordinary bilocal coupling a permanent and nonlocal coupling of currents with the strength C_0 occurs due to the presence of the squeezed condensate. This second term corresponds to a δ -function propagator in momentum space. It has been shown by Munczek and Nemirovsky [5] for quark (vector-) currents that such an interaction leads to the absence of poles for the propagator thus providing quark confinement. For our simple scalar field theory confinement of scalar quarks will be discussed in the next section.

3 Confinement in the quark sector

The form of the generating functional in the squeezed vacuum (13) is related to confinement of scalar quarks. As a reasonable criterion for confinement of quarks in the vacuum we will use the absence of poles on the real-k^2 axis in the quark propagator [3, 5].

In order to investigate quark confinement in our scalar quark-gluon model we consider now the full Lagrangian (1) in the homogeneous squeezed gluon vacuum

$$\mathcal{L}^{\Psi}_{eff}(x) \equiv \partial^\mu\Psi^*(x)\partial_\mu\Psi(x) - M^2\Psi^*(x)\Psi(x) + \mathcal{L}_I(x) \ , \tag{14}$$

where the effective quark-quark interaction $\mathcal{L}_I(x)$ is found from the effective action in the squeezed gluon sector

$$W_{sq}[J] = -i\ln Z_{sq}[J] = \int d^4x \mathcal{L}_I(x) \ . \tag{15}$$

The treatment of the gluon sector in the previous section leads to the result (13) which after identifying the external source $J(x)$ with the quark current $J(x) \equiv g\Psi^*(x)\Psi(x)$ corresponds to the four-quark interaction term

$$W_{sq}[J] = i\frac{g^2}{2}\int d^4x d^4y \Psi^*(x)\Psi(x)\left[\tilde{D}(x-y) + C_0\right]\Psi^*(y)\Psi(y) \ . \tag{16}$$

Fourier transformation leads to

$$W_{sq}[J] = i\frac{g^2}{2}\int \frac{d^4k_1}{(2\pi)^4}\cdots\frac{d^4k_4}{(2\pi)^4}\Psi^*(k_1)\Psi(k_2)D_{sq}(k_1-k_2)(2\pi)^4\delta^4(k_1-k_2-k_3+k_4)\Psi^*(k_3)\Psi(k_4) \ , \tag{17}$$

with

$$D_{sq}(q) = \left[\frac{i}{q^2+i\epsilon} + (2\pi)^4\delta^4(q)C_0\right] \ . \tag{18}$$

The first term corresponds to the usual propagator of a massless boson, whereas the second is a delta function interaction which leads to confinement of scalar quarks as will be shown in the following. Using just this δ- function interaction and the rainbow approximation for the vertex function in the Schwinger–Dyson equation for the quark self energy [2]

$$i\Sigma(q) = g^2 \int \frac{d^4k}{(2\pi)^4} D_{sq}(k-q)G(k) , \tag{19}$$

the inverse dressed quark propagator is given by an algebraic equation

$$iG^{-1}(q) \equiv q^2 - M^2 - \Sigma(q) = q^2 - M^2 + g^2 C_0 \frac{1}{q^2 - M^2 - \Sigma(q)} , \tag{20}$$

which can be solved analytically. One obtains the solution

$$iG^{-1}(q) = \frac{q^2 - M^2}{2} + \sqrt{\left(\frac{q^2 - M^2}{2}\right)^2 + g^2 C_0} , \tag{21}$$

which has no zeros on the real q^2 axis[3]. According to Refs. [3, 5] this property of the propagator is signalling confinement.

The Bethe-Salpeter equation for the vertex function $\Gamma(p, P)$ of scalar quark-antiquark correlations reads

$$\Gamma(p, P) = -g^2 \int \frac{d^4q}{(2\pi)^4} D_{sq}(q-p)\ G(q + \frac{P}{2})\ \Gamma(q, P)\ G(q - \frac{P}{2}) , \tag{22}$$

where $P = p_1 - p_2$ and $p = (p_1 + p_2)/2$ are total and relative momentum of the particle-antiparticle state, respectively. For the δ- function interaction, the Bethe-Salpeter equation reduces to an algebraic equation and for the special case of vanishing relative momentum it reads

$$G^{-1}(P/2)G^{-1}(P/2)\ \Gamma(0, P) = g^2 C_0\ \Gamma(0, P) , \tag{23}$$

and has the solution $P^2 = (2M)^2$ which corresponds to a pole in the two-particle propagator. In contrast to the one-particle sector where due to the confinement property no pole on the real q^2 axis exists and consequently no asymptotic free (scalar) quarks, the two-particle sector contains the physical excitations which are the bound states of the (confined) constituents.

The remaining interaction $D(q)$ in $D_{sq}(q)$ which is not included so far will be responsible for perturbative interactions at large momentum transfer. In the absence of a condensate ($C_0 = 0$) it can lead to the formation of ordinary bound states as e.g. discussed in [20] for the above model.

[2]For charged scalar quarks there is only the Fock contribution to the self energy; the Hartree term vanishes for zero net charge of the system.

[3]Note that this special solution coincides in the limit $C_0 = 0$ with the free inverse propagator $iG_0^{-1} = q^2 - M^2$.

4 Conclusions

We have prepared a Lorentz invariant vacuum as squeezed condensate of zero momentum particles with a macroscopic occupation number in the infinite volume limit and investigated the corresponding generating functional. This squeezed condensate has been characterized by a phenomenological parameter C_0.

The squeezed vacuum is constructed by applying a unitary operator to the perturbative vacuum. This operator connects two representations of the commutation relations which become unitary nonequivalent to each other in the large volume limit.

We have shown that such a squeezed vacuum can lead to a confinement type interaction of currents. In the present approach the confinement type interaction has been enforced by introducing the squeezed vacuum per construction. We have not yet shown how it might be generated by the self-interactions of the fields.

We expect that the squeezed condensate may provide a good frame for developing effective approaches to low energy QCD which include confinement. We considered here a very simple model in order to present a possible mechanism for confinement in a transparent way. An improved model should contain the color and flavor degrees of freedom of QCD and should treat quarks and gluons as Dirac spinor and vector boson fields, respectively. Work in this direction is in progress [15].

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A Generating functional for the free scalar field

In this Appendix, we remind the reader on the calculation of the generating functional for the model Lagrangian (2) in two equivalent approaches, the functional integral as well as the operator one.

The standard representation of the generating functional as a path integral over all field configurations is [17]

$$Z[J] = \mathcal{N} \int D\varphi \exp\left\{ i \int d^4x \mathcal{L}_J^{\varphi}(x) \right\} . \tag{24}$$

Here $\mathcal{N}$ is the normalization which is chosen such that $Z[0] = 1$. Performing the

Gaussian integration over φ one obtains

$$Z[J] = \exp\left\{-\frac{1}{2}\int d^4x \int d^4y J(x)D(x-y)J(y)\right\} , \qquad (25)$$

with the causal Green function

$$D(x-y) = \int \frac{d^4k}{(2\pi)^4} i \frac{e^{-ik(x-y)}}{k^2 - m^2 - i\epsilon} = \int \frac{d^3\mathbf{k}}{(2\pi)^3} e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})} \frac{e^{-i\omega(\mathbf{k})|x_0-y_0|}}{2\omega(\mathbf{k})} , \qquad (26)$$

where

$$\omega(\mathbf{k}) = \sqrt{\mathbf{k}^2 + m^2} . \qquad (27)$$

There exists an alternative approach to obtain (25) which is formulated on the operator level. This will turn out to be much more appropriate for generalization to a vacuum with a squeezed condensate. Quantum field theory in the operator approach is consistently formulated in a finite volume $V = \int d^3\mathbf{x}$. Finally we shall take the limit $V \to \infty$ and compare with the functional integral approach. For free massive scalar field theory the Hamilton operator corresponding to $\mathcal{L}_0^\varphi$ in (2) is

$$H_0[\varphi, \pi] = \int d\mathbf{x} \left\{\frac{1}{2}\pi^2(\mathbf{x}) + \frac{1}{2}(\partial_i\varphi(\mathbf{x}))^2 + \frac{1}{2}m^2\varphi^2(\mathbf{x})\right\} , \qquad (28)$$

with the Schrödinger field operators $\varphi(\mathbf{x}), \pi(\mathbf{x})$ satisfying the canonical commutation relations

$$[\pi(\mathbf{x}), \varphi(\mathbf{x}')] = -i\delta(\mathbf{x}-\mathbf{x}') . \qquad (29)$$

In the momentum representation

$$\varphi(\mathbf{x}) = \frac{1}{\sqrt{V}}\sum_k e^{i\mathbf{k}\cdot\mathbf{x}}\varphi_k = \frac{1}{\sqrt{V}}\sum_k e^{i\mathbf{k}\cdot\mathbf{x}}\sqrt{\frac{1}{2\omega(\mathbf{k})}}(a_k + a_{-k}^+) , \qquad (30)$$

$$\pi(\mathbf{x}) = \frac{1}{\sqrt{V}}\sum_k e^{i\mathbf{k}\cdot\mathbf{x}}\pi_k = \frac{i}{\sqrt{V}}\sum_k e^{i\mathbf{k}\cdot\mathbf{x}}\sqrt{\frac{\omega(\mathbf{k})}{2}}(-a_k + a_{-k}^+) , \qquad (31)$$

the field operators φ_k and π_k satisfy the commutation relations $[\pi_k, \varphi_{k'}] = -i\delta_{k,-k'}$. The creation and annihilation operators a_k, a_k^+ satisfy the commutation relations $[a_k, a_{k'}^+] = \delta_{k,k'}$ and diagonalize H_0 for $\omega(k)$ given by (27). The vacuum $|0>$ of the Hamiltonian H_0 satisfies $a_k|0> = 0$, and the Fock space is defined as

$$\{|\Phi>\} = \{|0> \ , \ a_k^+|0> = |k>, \ a_{k_1}^+ a_{k_2}^+ |0> = |k_1k_2>, \cdots\} . \qquad (32)$$

In the interaction representation the field operators become time dependent

$$\varphi_I(\mathbf{x}, t) \equiv e^{iH_0t}\varphi(\mathbf{x})e^{-iH_0t} , \qquad (33)$$

$$\pi_I(\mathbf{x}, t) \equiv e^{iH_0t}\pi(\mathbf{x})e^{-iH_0t} , \qquad (34)$$

while the states are $|\Psi_I(t)> \equiv e^{iH_0t}|\Psi(t)>$. In particular for the vacuum $|0>$ and the other Fock space states $|n>$ we have

$$|n_I(t)> \equiv e^{iH_0t}e^{-iE_nt}|n> = |n> . \qquad (35)$$

In the interaction picture (33), generating functional (24) can then be written as the vacuum expectation value of the evolution operator

$$Z[J] = <0|T\exp\left\{i\int_{-\infty}^{+\infty} dt \int_V d\mathbf{x}\varphi_I(\mathbf{x}, t)J(\mathbf{x}, t)\right\}|0> , \qquad (36)$$

where T is the time ordering operator. In order to prove the equivalence of (36) to (25) one uses the Wick theorem [17, 18]

$$TF[\varphi_I] = \exp\left\{\frac{1}{2}\int d^4x \int d^4y <0|T\varphi_I(x)\varphi_I(y)|0> \frac{\delta}{\delta\varphi(x)}\frac{\delta}{\delta\varphi(y)}\right\} : F[\varphi] : , \qquad (37)$$

where $F(\varphi)$ is an arbitrary polynomial in the operator φ and $: F(\varphi) :$ denotes normal ordering. Then (36) becomes

$$Z[J] = \exp\left\{-\frac{1}{2}\int d^4x \int d^4y J(x) <0|T\varphi_I(x)\varphi_I(y)|0> J(y)\right\} , \qquad (38)$$

which agrees with (25) for $V \to \infty$ since

$$D(x-y) = <0|T\varphi_I(x)\varphi_I(y)|0> . \qquad (39)$$

In this way the functional integral approach is equivalent to the operator approach. In both cases, the evaluation of the generating functional, i.e. the elimination of the gluonic degrees of freedom, is reduced to the determination of the matrix elements of the gluon two-point function (propagator).

B Two-point function in the squeezed vacuum with homogeneous condensate

In this Appendix we calculate the squeezed two-point Green function

$$D_{\text{sq}}(x, y) \equiv <0_{\text{sq}}|T\varphi_I(x)\varphi_I(y)|0_{\text{sq}}> = <0|U_{\text{sq}}T\varphi_I(x)\varphi_I(y)U_{\text{sq}}^{-1}|0> \qquad (40)$$

for the finite mass model (2) and consider the massless limit $m \to 0, \ V \to \infty, \ mV =$ const.

Lorentz invariance dictates us to populate the squeezed vacuum with pairs of massless particles. Note that for $m = 0$ the representation (30) of the field operators, which diagonalizes the free Hamiltonian is singular in the zero momentum component which expresses the well-known infrared singularity of massless theories. In order to avoid the well-known infrared singularity of massless theories we shall do the calculations in the following for finite mass m. Our final result, however, taken in the limit $m \to 0$ at the end of our calculations will again be invariant with respect to all original symmetries.

In terms of creation and annihilation operators the squeezing operator (4) reads

$$U_{sq} = \exp\left[\frac{f_0}{2}\left(a_0^2 - (a_0^+)^2\right)\right] . \tag{41}$$

which shows that (4) can be interpreted as a Bogoliubov transformation to the new creation and annihilation operators b_0 and b_0^+ according to

$$b_0 \equiv U_{sq} a_0 U_{sq}^{-1} = \cosh(f_0) a_0 + \sinh(f_0) a_0^+ . \tag{42}$$

In order to calculate the squeezed two-point Green function (40) we use the Wick formula (37)

$$T\varphi_I(x)\varphi_I(y) =: \varphi_I(x)\varphi_I(y) : + < 0|T\varphi_I(x)\varphi_I(y)|0 > , \tag{43}$$

and thus have (taking the squeezed condensate expectation value of (43))

$$\begin{aligned} D_{sq}(x,y) &= < 0|T\varphi_I(x)\varphi_I(y)|0 > + < 0_{sq}| : \varphi_I(x)\varphi_I(y) : |0_{sq} > \\ &= D(x-y) + D_f(x,y) , \end{aligned} \tag{44}$$

with the causal Green function $D(x-y)$ given in (39) and

$$D_f(x,y) \equiv < 0|U_{sq} : \varphi_I(x)\varphi_I(y) : U_{sq}^{-1}|0 > . \tag{45}$$

The calculation of this expectation value is performed in the Fock space representation with respect to the creation and annihilation oprators a^+, a defined in Appendix A. The result of the calculation can be written in the form

$$\begin{aligned} D_f(x,y) &= \frac{1}{2mV}\Big\{e^{-im(x_0-y_0)}\left[\cosh^2(f_0) - 1\right] + e^{im(x_0-y_0)}\sinh^2(f_0) \\ &\quad +2\cos[m(x_0+y_0)]\sinh(f_0)\cosh(f_0)\Big\} \\ &= D_f(x_0,y_0) . \end{aligned} \tag{46}$$

For the further discussion it is useful to single out the zero momentum part of the squeezed propagator

$$D_{sq}(\mathbf{x}-\mathbf{y}; x_0, y_0) \equiv \tilde{D}(\mathbf{x}-\mathbf{y}; x_0 - y_0) + D_{(0)}(x_0,y_0) , \tag{47}$$

where

$$\tilde{D}(\mathbf{x}-\mathbf{y}; x_0 - y_0) = \frac{1}{V}\sum_{k\neq 0} e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}\frac{e^{-i\omega(\mathbf{k})|x_0-y_0|}}{2\omega(\mathbf{k})} \tag{48}$$

is the causal Green function (39) with the zero-momentum sector removed and treated separately together with D_f in

$$D_{(0)}(x_0,y_0) = \frac{1}{2mV}e^{-im|x_0-y_0|} + D_f(x_0,y_0) . \tag{49}$$

We are interested here in the zero mass and infinite volume limit $m \sim 1/V \to 0$ such that we can introduce instead of the squeezing parameter f_0 a new constant C_0 of dimension $(\text{mass})^{-2}$,

$$\lim_{V\to\infty, mV=\text{const}} e^{2f_0} = 2mVC_0 , \tag{50}$$

so that in this limit

$$\lim_{V\to\infty, mV=\text{const}} D_{(0)}(x_0,y_0) = C_0 . \tag{51}$$

This is the result which is exploited in Section 2 for the discussion of the generating functional in the squeezed vacuum.

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Бляшке Д. и др. E2-96-413

Сжатый конденсат и конфайнмент в скалярной модели

Обобщается производящий функционал в скалярной теории поля на случай сжатого вакуума. Сжатый конденсат получается макроскопическим заселением вакуума парами частиц с нулевой энергией. Показано, что соответствующий «кварковый» пропагатор не имеет полюсов на реальной оси k^2, что может быть интерпретировано как «кварковый» конфайнмент. В отличие от этого скалярный «мезон», связанное состояние «кварков», возникает как решение соответствующего уравнения Бете — Салпитера.

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Squeezed Condensate and Confinement in a Scalar Model

The generating functional of a free scalar field theory is generalized to the case of a squeezed vacuum. The squeezed vacuum is prepared by macroscopically populating the original vacuum with pairs of zero energy particles. It is shown that the corresponding quark propagator has no poles on the real-k^2 axis which can be interpreted as quark confinement. In contrast, a scalar meson-like bound state exists as solution of the corresponding Bethe—Salpeter equation.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

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