

# ОБЪЕДИНЕННЫЙ ИНСтИТУт ЯДЕРНЫХ ИССЛЕДОВАНИЙ 

## Дубна

# 96-408 

E2-96-408
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## ON THE BRST APPROACH TO THE DESCRIPTION OF A REGGE TRAJECTORY

Submitted to «Modern Physics Letters A»

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## 1 Introduction

The main problem in the description of the higher spin particles is removing of unphysical degrees of freedom from the theory. As was shown in [1] the corresponding lagrangian must have some invariance which generalizes the gauge invariance of the electromagnetic field not only in the free case but for interacting fields as well.

On the free level such lagrangians was constructed both for massive [2]-[3] and for massless particles [4]-[5] of any spin as well as for massless supermultiplets [6]. In some sense the massless case is simpler and, hence, more investigated than the massive one. Some types of interacting lagrangians was constructed in the light-cone gauge [7] and some progress was made in the covariant description [8] of interactions of massless higher spin particles. The hope is that with the help of some Higgs-type effect some of the interacting particles acquire nonzero values of the mass, Up to now, the only existing example of higher spin interacting massive matter is the string theory.

Naturally higher spin particles arise after quantization of classical extended objects, such as string, relativistic oscillator [9] [10], discrete string [11] etc. Physically they correspond to exited levels of the system and belong to Regge trajectories, each including infinite sequence of states with a spin linearly depending on the square of the mass. There are infinite number of Regge trajectories in the string and relativistic oscillator models and only one such trajectory in the discrete string model due to existing of additional second-class constraints. Such subdivision of all higher spin particles on Regge trajectories leads to consideration of these Regge trajectories as independent objects for which it would be interesting to construct the lagrangian description.

It seems that one can not construct a consistent interaction of some finite set of higher spins until all higher spins up to infinity are included. This naturally leads to consideration of the whole families of the higher spin particles like Regge trajectory. In the infinite limit of the Regge slope all of the particles have the same value of the mass. In particular, the case when all of the particles have zero mass and this Regge trajectory is vertical massless tower of particles is very interesting.

One of the most economic and straightforward method of construction of lagrangians in gauge theories is the method using BRST charge of the corresponding firstly quantized theory. With the help of BRST-charge the lagrangian of the free field theory of infinite tower of massless higher spin particles was constructed [13]. This lagrangian describes the system which is infinitely degenerated on each spin level and only with the help of additional constraints we can delete all the extra states having precisely one particle of each spin. These additional constraints are of the second class and their inclusion in the BRST- construction is nontrivial.

The methods of such construction were discussed in [15]-[17]. With the help of additional variables one can modify the second class constraints in such a way that they become commuting, i.e the first class. At the same time the number of physical degrees of freedom for both systems of constraints is the same if the number
of additional variables coincides with the number of second class constraints.
In the second part of the paper we describe the most general system of constraints. We discuss also the possible truncations and simplifications of this general systems of constraints. In the third part of the paper we show that one of the truncated subsystem of constraints (first- and second-class) admits the construction of BRSTcharge along the line of [15]-[17]. As a result the correct free field lagrangian for Regge trajectory together with its daughter trajectories is constructed. The inclusion of additional second class constraints removing the daughter trajectories is discussed in the fourth part of the article. We describe the simple modification of the method of [15]-[17] which can be used for the BRST - construction of a single Regge trajectory.

## 2 The systems of constraints

To describe all higher spins simultaneously it is convenient to introduce auxiliary Fock space generated by creation and annihilation operators $a_{\mu}^{+}, a_{\mu}$ with vector Lorentz index $\mu=0,1,2, \ldots D-1$, satisfying the following commutation relations

$$
\begin{equation*}
\left[a_{\mu}, a_{\nu}^{+}\right]=-g_{\mu \nu}, g_{\mu \nu}=\operatorname{diag}(1,-1,-1, \ldots,-1) \tag{2.1}
\end{equation*}
$$

In addition the operators $a_{\mu}^{+}, a_{\mu}$ can have some internal indices leading to more complicated spectrum of physical states. For simplicity we consider in this paper $a_{\mu}^{+}, a_{\mu}$ without additional indices.

The general state of the Fock space

$$
\begin{equation*}
|\Phi\rangle=\sum \Phi_{\mu_{1} \mu_{2} \cdots \mu_{n}}^{(n)}(x) a_{\mu_{1}}^{+} a_{\mu_{2}}^{+} \cdots a_{\mu_{n}}^{+}|0\rangle \tag{2:2}
\end{equation*}
$$

depends on space-time coordinates $x_{\mu}$ and its components $\Phi_{\mu_{1} \mu_{2} \ldots \mu_{n}}^{(n)}(x)$ are tensor fields of rank $n$ in the space-time of arbitrary dimension $D$. The norm of states in this Fock space is not positively definite due to the minus sign in the commutation relation (2.1) for time components of creation and annihilation operators. It means that physical states must satisfy some constraints to have positive norm. These constraints arise naturally in the considerations of classical composite systems [10][11]. The corresponding quantum operators

$$
\begin{align*}
L_{0} & =-p^{2}-\alpha^{\prime} a_{\mu}^{+} a_{\mu}, \quad L_{1}=p a, \quad L_{-1}=p a^{+}=L_{1}^{+}  \tag{2.3}\\
L_{2} & \doteq \frac{1}{2} a a, \quad L_{-2}=\frac{1}{2} a^{+} a^{+}=L_{2}^{+} \tag{2.4}
\end{align*}
$$

form the algebra

$$
\begin{gather*}
{\left[L_{0}, L_{ \pm 1}\right]=\mp \alpha^{\prime} L_{ \pm 1} \quad\left[L_{0}, L_{ \pm 2}\right]=\mp 2 \alpha^{\prime} L_{ \pm 2}}  \tag{2.5}\\
{\left[L_{1}, L_{-2}\right]=-L_{-1} \quad\left[L_{-1}, L_{2}\right]=L_{1}} \tag{2.6}
\end{gather*}
$$

$$
\begin{equation*}
\left[L_{1}, L_{-1}\right]=-p^{2}\left[L_{2} \cdot L_{-2}\right]=-a_{\mu}^{+} a_{\mu}+\frac{D}{2} \equiv G_{0} \tag{2.9}
\end{equation*}
$$

The operators $L_{0}, L_{1}, L_{2}$ correspond to the mass shell, transversality and tracelessness conditions on the wavefunctions. The operators $L_{1}, L_{-1}$ and $L_{2}, L_{-2}$ are of second class and in general this system of constraints describes single Regge trajectory [10]-[12].

There exist some different possibilities in consideration of general system of constraints (2.3)-(2.4). The truncated systern $L_{0}, L_{ \pm 1}$ describes Regge trajectory together with its daughter trajectories. The operators $L_{ \pm 1}$ in this system are of second class as before: We describe the BRST - quantization of this system in the third part of the article.

The limit $\alpha^{\prime}=0$ of the system (2.3) corresponds to the massless infinite tower of spins with a single state at each value of the spin. In this case only the operators $L_{ \pm 2}$ are of second class. The BRST - quantization of this system as well as general system (2.3)-(2.4) will be given elsewhere.

The simplest system of first-class constraints

$$
\begin{equation*}
\tilde{L}_{0}=-p^{2}, L_{ \pm 1} \tag{2.10}
\end{equation*}
$$

corresponds to the massless tower of spins infinitely degenerated at each value of spin. The BRST - construction for this system of constraints was described in [13].

## 3 BRST-quantization of the Regge trajectory

In this section we consider the system with constraints

$$
\begin{equation*}
L_{0}=-p^{2}-\alpha^{\prime} a_{\mu}^{+} a_{\mu}+\alpha_{0}, \quad L_{1}=p a, \quad L_{1}^{+}=p a^{+} \tag{3.1}
\end{equation*}
$$

where parameter $\alpha_{0}$ plays the role of intercept for Regge trajectory. In some sense this system is intermediate between the systems in [13] and [12] because it describes the Regge trajectory together with all daughter trajectories (due to the absence of the constraints $L_{ \pm 2}$ ). The commutation relation $\left[L_{1}, L_{-1}\right]=-p^{2}$ means that $L_{ \pm 1}$ are the second class constraints. Following the prescription of [15]-[17] we introduce operators $b$ and $b^{+}$with the commutation relations $\left[b, b^{+}\right]=1$ and modify the constraints to the following expressions:

$$
\begin{align*}
& \tilde{L}_{0}=L_{0}+\alpha^{\prime} b^{+} b+\alpha^{\prime}  \tag{3.2}\\
& \tilde{L}_{-1}=L_{-1}+\sqrt{p^{2}} b^{+}  \tag{3.3}\\
& \tilde{L}_{1}=L_{1}+\sqrt{p^{2}} b  \tag{3.4}\\
& {\left[\tilde{L}_{1}, \tilde{L}_{-1}\right]=0, \quad\left[\tilde{L}_{0}, \tilde{L}_{1}\right]=-\alpha^{\prime} \tilde{L}_{1}, \quad\left[\tilde{L}_{0}, \tilde{L}_{-1}\right]=\alpha^{\prime} \tilde{L}_{-1} } \tag{3.5}
\end{align*}
$$

All of the modified constraints are of first class and BRST - charge construction is straightforward. Firstly we introduce additional set of anticommuting variables $\eta_{0}, \eta_{1}, \eta_{1}^{+}$having ghost number one and corresponding momenta $\mathcal{P}_{0}, \mathcal{P}_{1}^{+}, \mathcal{P}_{1}$ with commutation relations:

$$
\begin{equation*}
\left\{\eta_{0}, \mathcal{P}_{0}\right\}=\left\{\eta_{1}, \mathcal{P}_{1}^{+}\right\}=\left\{\eta_{1}^{+}, \mathcal{P}_{1}\right\}=1 \tag{3.6}
\end{equation*}
$$

The nilpotent BRST - charge has the following form:

$$
\begin{equation*}
Q=\eta_{1}^{+} \tilde{L}_{1}+\eta_{1} \tilde{L}_{-1}+\eta_{0} \tilde{L}_{0}+\alpha^{\prime} \eta_{0} \eta_{1}^{+} \mathcal{P}_{1}-\alpha^{\prime} \eta_{0} \eta_{1} \mathcal{P}_{1}^{+} \tag{3.7}
\end{equation*}
$$

Consider the total Fock space generated by creation operators $a_{\mu}^{+}, b^{+}, \eta_{1}^{+}, \mathcal{P}_{1}^{+}$. In addition each vector of the Fock space depends linearly on the real grassmann variable $\eta_{0}$ ( $\mathcal{P}_{0}$ considered as corresponding derivative $\mathcal{P}_{0}=\partial / \partial \eta_{0}$ )

$$
\begin{equation*}
|\chi\rangle=\left|\chi_{1}\right\rangle+\eta_{0}\left|\chi_{2}\right\rangle \tag{3.8}
\end{equation*}
$$

Ghost numbers of $\left|\chi_{1}\right\rangle$ and $\left|\chi_{2}\right\rangle$ are different if the state $|\chi\rangle$ has some definite one.
The BRST - invariant lagrangian in such Fock space can be written as

$$
\begin{equation*}
L=\int d \eta_{0}(\chi|Q| \chi\rangle \tag{3.9}
\end{equation*}
$$

To be physical, lagrangian $L$ must have zero ghost number. It means that vectors $|\chi\rangle$ and $\left|\chi_{1}\right\rangle$ must have zero ghost numbers as well. In this case the ghost number of $\left|\chi_{2}\right\rangle$ is minus one. The most general expressions for such vectors are

$$
\begin{align*}
& \left|\chi_{1}\right\rangle=\left|S_{1}\right\rangle+\eta_{1}^{+} \mathcal{P}_{1}^{+}\left|S_{2}\right\rangle,  \tag{3.10}\\
& \left|\chi_{2}\right\rangle=\mathcal{P}_{1}^{+}\left|S_{3}\right\rangle \tag{3.11}
\end{align*}
$$

with vectors $\left|S_{i}\right\rangle$ having ghost number zero and depending only on bosonic creation operators $a_{\mu}^{+}, b^{+}$

$$
\begin{equation*}
\left|S_{i}\right\rangle=\sum \phi_{\mu_{1}, \mu_{2}, \ldots \mu_{n}}^{n}(x) a_{\mu_{1}}^{+} a_{\mu_{2}}^{+} \ldots a_{\mu_{n}}^{+}\left(b^{+}\right)^{n}|0\rangle \tag{3.12}
\end{equation*}
$$

Integration over the $\eta_{0}$ leads to the following lagrangian in terms of $\left|S_{i}\right\rangle$

$$
\begin{align*}
L= & \left\langle S_{1}\right| \tilde{L}_{0}\left|S_{1}\right\rangle-\left\langle S_{2}\right| \tilde{L}_{0}\left|S_{2}\right\rangle-\alpha^{\prime}\left\langle S_{1}\right|\left|S_{1}\right\rangle-\alpha^{\prime}\left\langle S_{2}\right|\left|S_{2}\right\rangle-  \tag{3.13}\\
& -\left\langle S_{1}^{\prime}\right| \tilde{L}_{1}^{+}\left|S_{3}\right\rangle-\left\langle S_{3}\right| \tilde{L}_{1}\left|S_{1}\right\rangle+\left\langle S_{2}\right| \tilde{L}_{1}\left|S_{3}\right\rangle+\left\langle S_{3}\right| \tilde{L}_{1}^{+}\left|S_{2}\right\rangle \tag{3.14}
\end{align*}
$$

For comparison with the massless case we write down the lagrangian of [13] in the same terms

$$
\begin{align*}
L= & -\left\langle S_{1}\right| p^{2}\left|S_{1}\right\rangle+\left\langle S_{2}\right| p^{2}\left|S_{2}\right\rangle+\left\langle S_{3}\right|\left|S_{3}\right\rangle-  \tag{3.15}\\
& -\left\langle S_{1}\right| L_{1}^{+}\left|S_{3}\right\rangle-\left\langle S_{3}\right| L_{1}\left|S_{1}\right\rangle+\left\langle S_{2}\right| L_{1}\left|S_{3}\right\rangle+\left\langle S_{3}\right| L_{1}^{+}\left|S_{2}\right\rangle \tag{3.16}
\end{align*}
$$

The nilpotency of the BRST - charge leads to the invariance of the lagrangian (3.9) under the following transformations

$$
\begin{equation*}
\delta|\chi\rangle=Q|\Lambda\rangle \tag{3.17}
\end{equation*}
$$

The parameter of transformation must have ghost number -1 and can be written as $|\Lambda\rangle=\mathcal{P}_{1}^{+}|\lambda\rangle$, where $|\lambda\rangle$ belong to the Fock space generated by $a_{\mu}^{+}, b^{+}$and depends from the space - time coordinates. On the component level such invariance leads to the invariance of the lagrangian (3.13) under the following transformations

$$
\begin{align*}
\delta\left|S_{1}\right\rangle & =\tilde{L}_{1}^{+}|\lambda\rangle  \tag{3.18}\\
\delta\left|S_{2}\right\rangle & =\tilde{L}_{1}|\lambda\rangle  \tag{3.19}\\
\delta\left|S_{3}\right\rangle & =\tilde{L}_{0}|\lambda\rangle \tag{3.20}
\end{align*}
$$

One can show that using together (3.18) and equations of motion for the fields $\left|S_{i}\right\rangle$

$$
\begin{align*}
\left.\left(\tilde{L}_{0}-\alpha^{\prime}\right) \mid S_{1}\right) & =\tilde{L}_{1}^{+}\left|S_{3}\right\rangle,  \tag{3.21}\\
\left(\tilde{L}_{0}+\alpha^{\prime}\right)\left|S_{2}\right\rangle & =\tilde{L}_{1}\left|S_{3}\right\rangle,  \tag{3.22}\\
\tilde{L}_{1}\left|S_{1}\right\rangle & =\tilde{L}_{1}^{+}\left|S_{2}\right\rangle, \tag{3.23}
\end{align*}
$$

one can eliminate the fields $\left|S_{2}\right\rangle$ and $\left|S_{3}\right\rangle$. Firstly we solve the equation

$$
\begin{equation*}
\left|S_{3}\right\rangle+\tilde{L}_{0}|\lambda\rangle=0 \tag{3.24}
\end{equation*}
$$

using decompositions

$$
\begin{equation*}
\left|S_{i}\right\rangle=\sum\left(b^{+}\right)^{n}\left|S_{i n}\right\rangle,|\lambda\rangle=\sum\left(b^{+}\right)^{n}\left|\lambda_{n}\right\rangle \tag{3.25}
\end{equation*}
$$

The equation (3.24) does not fix parameter $|\lambda\rangle$ completely. There will be residual invariance with parameter $\left|\lambda^{\prime}\right\rangle$ under the condition $\tilde{L}_{0}\left|\lambda^{\prime}\right\rangle=0$. After the elimination of the field $\left|S_{2}\right\rangle$ with the help of the equation $\left|S_{2}\right\rangle+\tilde{L}_{1}\left|\lambda^{\prime}\right\rangle=0$ the new parameter $\left|\lambda^{\prime \prime}\right\rangle$ will satisfy two conditions $\tilde{L}_{0}\left|\lambda^{\prime \prime}\right\rangle=\tilde{L}_{1}\left|\lambda^{\prime \prime}\right\rangle=0$. With the help of this parameter all fields $\left|S_{1 n}\right\rangle$, except zero mode $\left|S_{10}\right\rangle$ can be eliminated. The residual gauge invariance $\delta\left|S_{10}\right\rangle=L_{1}^{+}\left|\lambda^{\prime \prime \prime}\right\rangle$ allows to impose constraint

$$
\begin{equation*}
L_{1}\left|S_{10}\right\rangle=0 \tag{3.26}
\end{equation*}
$$

which together with the equation of motion

$$
\begin{equation*}
L_{0}\left|S_{10}\right\rangle=0 \tag{3.27}
\end{equation*}
$$

describe the spectrum of Regge trajectory. The constraint (3.26) kills unphysical degrees of freedom and equation (3.27) fixes linear dependence between spin and square of mass. The absence of the condition

$$
\begin{equation*}
L_{2}\left|S_{10}\right\rangle=0 \tag{3.28}
\end{equation*}
$$

means that the wavefunctions $\phi_{\mu_{1}, \mu_{2}, \ldots \mu_{n}}$ belong to the reducible representations of the $D$ - dimensional Lorentz group. In turn it means that the daughter Regge trajectories belong to the spectrum as well.

## 4 The simple example

To describe the Regge trajectory without its daughter trajectories we must take into account additional constraint (3.28) on the wavefunctions. It means that the second class constraints $L_{ \pm 2}$ must be included in the BRST charge. Following the line of [15] - [17] we can transform these constraints into commuting ones by introducing additional freedoms like $b, b^{+}$. This procedure is rather simple for the classical case when Poisson brackets are used instead of commutators. It leads to the finite system of differentional equations which can be solved without troubles. In the quantum case the corresponding system of equations is infinite due to accounting of repeated commutators.

In this section we describe the modification of the procedure of [15]-[17] which works well in the case of two second class constraints $L_{2}$ and $L_{-2}$ (2.4). We modify this system of constraints by introduction of two additional operators $b_{1}, b_{2}$ together with their conjugates $b_{1}^{+}, b_{2}^{+}:\left[b_{i}, b_{k}^{+}\right]=\delta_{i k}$. New constraints are

$$
\begin{align*}
\tilde{L}_{2} & =L_{2}+b_{1}^{+} b_{2}  \tag{4.1}\\
\tilde{L}_{-2} & =L_{-2}+b_{2}^{+} b_{1} . \tag{4.2}
\end{align*}
$$

Together with new operator $\tilde{G}_{0}=G_{0}+b_{2}^{+} b_{2}-b_{1}^{+} b_{1}$ they form an $S U(2)$ algebra

$$
\begin{align*}
& {\left[\tilde{L}_{2}, \tilde{L}_{-2}\right]=\tilde{G}_{0}}  \tag{4.3}\\
& {\left[\tilde{G}_{0}, \tilde{L}_{ \pm 2}\right]=\mp 2 \tilde{L}_{ \pm 2}} \tag{4.4}
\end{align*}
$$

and can be considered as first class constraints. The counting of physical degrees of freedom shows equal number for both systems of constraints. Indeed, we have introduced four additional variables $b_{i}, b_{k}^{+}$, but instead of two second class constraints each killing one degree of freedom we get three first class constraints which kill together six degrees of freedom.

To illustrate the BRST - approach to this simple system we introduce additional set of anticommuting variables $\eta_{0}, \eta_{2}, \eta_{2}^{+}$having ghost number one and corresponding momenta $\mathcal{P}_{0}, \mathcal{P}_{2}^{+}, \mathcal{P}_{2}$ with commutation relations:

$$
\begin{equation*}
\left\{\eta_{0}, \mathcal{P}_{0}\right\}=\left\{\eta_{2}, \dot{\mathcal{P}}_{2}^{+}\right\}=\left\{\eta_{2}^{+}, \mathcal{P}_{2}\right\}=1 \tag{4.5}
\end{equation*}
$$

The standard prescription gives the following nilpotent BRST - charge

$$
\begin{equation*}
Q=\eta_{2}^{+} \tilde{L}_{2}+\eta_{2} \tilde{L}_{2}^{+}+\eta_{0} \tilde{G}_{0}+\eta_{2} \eta_{2}^{+} \mathcal{P}_{0}+2 \eta_{0} \eta_{2}^{+} \mathcal{P}_{2}-2 \eta_{0} \eta_{2} \mathcal{P}_{2}^{+} \tag{4.6}
\end{equation*}
$$

Consider the total Fock space generated by creation operators $a_{\mu}^{+}, b_{i}^{+}, \eta_{2}^{+}, \mathcal{P}_{2}^{+}$. In addition each vector of the Fock space depends linearly on the real grassmann variable $\eta_{0}$ ( $\mathcal{P}_{0}$ considered as corresponding derivative $\mathcal{P}_{0}=\partial / \partial \eta_{0}$ )

$$
\begin{equation*}
|\chi\rangle=\left|\chi_{1}\right\rangle+\eta_{0}\left|\chi_{2}\right\rangle \tag{4.7}
\end{equation*}
$$

Ghost numbers of $\left|\chi_{1}\right\rangle$ and $\left|\chi_{2}\right\rangle$ are different if the state $|\chi\rangle$ have some definite one.

The BRST - invariant lagrangian in such Fock space can be written as

$$
\begin{equation*}
L=\int d \eta_{0}\langle\chi| Q|\chi\rangle \tag{4.8}
\end{equation*}
$$

To be physical, lagrangian $L$ must have zero ghost number. It means that vectors $|\chi\rangle$. and $\left|\chi_{1}\right\rangle$ must have zero ghost number as well. In this case the ghost number of $\left|\chi_{2}\right\rangle$ is minus one. The most general expressions for such vectors are

$$
\begin{align*}
& \left|\chi_{1}\right\rangle=\left|S_{1}\right\rangle+\eta_{2}^{+} \mathcal{P}_{2}^{+}\left|S_{2}\right\rangle,  \tag{4.9}\\
& \left|\chi_{2}\right\rangle=\mathcal{P}_{2}^{+}\left|S_{3}\right\rangle, \tag{4.10}
\end{align*}
$$

with vectors $\left\langle S_{i}\right\rangle$ having ghost number zero and depending only on bosonic creation operators $a_{\mu}^{+}, b_{i}^{+}$

$$
\begin{equation*}
\left|S_{i}\right\rangle=\sum \phi_{\mu_{1}, \mu_{2}, \ldots \mu_{n}}^{n_{1}, \mu_{2}} a_{\mu_{1}}^{+} a_{\mu_{2}}^{+} \ldots a_{\mu_{n}}^{+}\left(b_{1}^{+}\right)^{n_{1}}\left(b_{2}\right)^{n_{2}}|0\rangle \tag{4.11}
\end{equation*}
$$

In general the vawefunction $\phi_{\mu_{1}, \mu_{2}, \ldots \mu_{n}}^{n_{1}, n_{2}}$ can depend on other physical variables of the theory such as space - time coordinates etc.

Integration over the $\eta_{0}$ leads to the following lagrangian in terms of $\left|S_{i}\right\rangle$

$$
\begin{aligned}
L= & \left(S_{1}\left|\tilde{G}_{0}\right| S_{1}\right\rangle-2\left(S_{1}| | S_{1}\right\rangle-\left\langle S_{2}\right| \tilde{G}_{0}\left|S_{2}\right\rangle-2\left\langle S_{2}\right|\left|S_{2}\right\rangle+\left\langle S_{3}\right|\left|S_{3}\right\rangle- \\
& -\left\langle S_{1}\right| \tilde{L}_{2}^{+}\left|S_{3}\right\rangle-\left\langle S_{3}\right| \tilde{L}_{2}\left|S_{1}\right\rangle+\left\langle S_{2}\right| \tilde{L}_{2}\left|S_{3}\right\rangle+\left\langle S_{3}\right| \tilde{L}_{2}^{+}\left|S_{2}\right\rangle .
\end{aligned}
$$

Owing to the nilpotency of the BRST - charge - $Q^{2}=0$, the lagrangian (4.8) is invariant under the transformation

$$
\begin{equation*}
\delta|\chi\rangle=Q|\Lambda\rangle \tag{4.13}
\end{equation*}
$$

with $|\Lambda\rangle=\mathcal{P}_{2}^{+}|\lambda\rangle$. In turn it means the invariance of the lagrangian (4.12) under the following transformation

$$
\begin{align*}
\delta\left|S_{1}\right\rangle & =\tilde{L}_{2}^{+}|\lambda\rangle  \tag{4.14}\\
\delta\left|S_{2}\right\rangle & =\tilde{L}_{2}|\lambda\rangle  \tag{4.15}\\
\delta\left|S_{3}\right\rangle & =\tilde{G}_{0}|\lambda\rangle \tag{4.16}
\end{align*}
$$

The field $\left|S_{3}\right\rangle$ is auxiliary. Using its equation of motion

$$
\begin{equation*}
\left|S_{3}\right\rangle=\tilde{L}_{2}\left|S_{1}\right\rangle-\tilde{L}_{2}^{+}\left|S_{2}\right\rangle, \tag{4.17}
\end{equation*}
$$

the lagrangian (4.12) can be rewritten in terms of two fields $\left|S_{1}\right\rangle$ and $\left|S_{2}\right\rangle$

$$
\begin{align*}
L= & \left\langle S_{1}\right|\left(\tilde{G}_{0}-2-\tilde{L}_{2}^{+} \tilde{L}_{2}\right)\left|S_{1}\right\rangle-\left\langle S_{2}\right|\left(\tilde{G}_{0}+2+\tilde{L}_{2} \tilde{L}_{2}^{+}\right)\left|S_{2}\right\rangle+  \tag{4.18}\\
& +\left\langle S_{1}\right| \tilde{L}_{2}^{+} \tilde{L}_{2}^{+}\left|S_{2}\right\rangle+\left\langle S_{2}\right| \tilde{L}_{2} \tilde{L}_{2}\left|S_{1}\right\rangle
\end{align*}
$$

with the following equations of motion:

$$
\begin{align*}
& \left(\tilde{G}_{0}-2-\tilde{L}_{2}^{+} \tilde{L}_{2}\right)\left|S_{1}\right\rangle+\tilde{L}_{2}^{+} \tilde{L}_{2}^{+}\left|S_{2}\right\rangle=0  \tag{4.19}\\
& \left(\tilde{G}_{0}+2+\tilde{L}_{2} \tilde{L}_{2}^{+}\right)\left|S_{2}\right\rangle-\tilde{L}_{2} \tilde{L}_{2}\left|S_{1}\right\rangle=0 \tag{4.20}
\end{align*}
$$

Following the line of preceding section one can show that the gauge freedom (4.14)-(4.16) is sufficient to kill the $b_{2}^{+}$dependence in $\left|S_{1}\right\rangle$ and get the following conditions on the reduced field $\left|\tilde{S}_{1}\right\rangle$ :

$$
\begin{equation*}
\frac{1}{2} a a\left|\tilde{S}_{1}\right\rangle=0,\left(-a^{+} a+\frac{D}{2}-b_{1}^{+} b_{1}\right)\left|\tilde{S}_{1}\right\rangle=0 \tag{4.21}
\end{equation*}
$$

The first of the conditions (4.21) means tracelessness of the wavefunctions $\phi_{\mu_{1}, \mu_{2}, \ldots}^{n_{1}, 0}$ The second one connects $n$ and $n_{1}: n_{1}=n+\frac{D}{2}$ killing effectively the $b_{1}^{+}$dependence in $\left|S_{1}\right\rangle$. So, the system contains no additional degrees of freedom and describes traceless wavefunctions.

## 5 Conclusions

In this paper we have applied the BRST approach to the description of the free Regge trajectory. We have described also the simple modification of the conversion procedure for the second class constraints which can be used for elimination of daughter trajectories from the physical spectrum. The corresponding modification of the BRST approach to the single Regge trajectory will be given elsewhere.

Acknowledgments. This investigation has been supported in part by the Russian Foundation of Fundamental Research, grants 96-02-17634 and 96-02-18126, joint grant RFFR-DFG 96-02-00186G, and INTAS, grant 94-2317 and grant of the Dutch NWO organization.

## References

[1] M.Fierz, W.Pauli. Proc.Roy.Soc., A173 (1939) 211
[2] L.P.H.Singh, C.R.Hagen. Phys.Rev., D9 (1974) 898; Ibid D9 (1974) 910
[3] S.J.Chang. Phys.Rev., 161 (1967) 1308
[4] C.Fronsdal. Phys.Rev., D18 (1978) 3624
[5] J.Fang, C.Fronsdal. Phys.Rev., D18 (1978) 3630.
[6] T. Curtright. Phys.Lett., B85 (1979) 219
[7] A.K.H.Bengtsson, I.Bengtsson, L.Brink. Nucl.Phys., B227 (1983) 31
[8] F.A.Berends, G.J.H.Burgers, H.F.A.van Dam. Nucl.Phys., B271 (1986) 429
[9] Y.S. Kim, Marilyn E. Noz. Phys.Rev., D12 (1975) 129;ibid D15 (1977) 335
[10] A.Barducci, D.Dominici. Nuovo Cim., A37 (1977) 385
[11] V.D.Gershun, A.I.Pashnev. Theor.Math.Phys., 73 (1987) 1227
[12] A.I.Pashnev. Theor.Math.Phys., 78 .(1989) 424
[13] S.Ouvry, J.Stern., Phys.Lett., B177 (1986) 335
[14] R.Marnelius. Nucl.Phys., B294 (1987) 685
[15] L.D. Faddeev, S.L. Shatashvili. Phys.Lett., B167 (1986) 225
[16] E.S. Fradkin, T.E. Fradkina. Phys.Lett., B72 (1978) 343
[17] E.T. Egoryan, R.P. Manvelyan. Theor. Math.Phys., 94 (1993) 241

## Пашнев А:И., Цулая М.М.

О БРСТ-подходе к описанию траектории Редже
В рамках метода БРСТ-квантования описана свободная теория поля для траектории Редже. Наряду с самой траекторией физйческий спектр включает дочериие траектории. Обсуждается применимость БРСТ-подхода к описанию одной траектории Редже без ее дочерних траекторий Простой пример иллюстрирует соответствуюцую модификацию. БРСТ построения для требуемых связей второго рода.

Работа выполнена в Јаборатории теоретической физики им Н.Н.Боголюбова ОИЯИ.

Препринт Объединенвого института ядерных исследораний. Дубна, 1996

## Pashnev A, Tsulaia M.

E2-96-408 On the BRST Approach to the Description of a Regge Trajectory

The free field trajectory is described in the framework of the BRST-quantization method. The physical spectrum includes daughter trajectories along with parent one. The applicability of the BRST approach to the description of a single Regge trajectory without its daughter trajectories is described. The simple example illustrates the appropriately modified BRST construction for the needed second class constraints.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.


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