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STRING MODEL WITH NEGATIVE STIFFNESS — TEMPERATURE BEHAVIOR AND THERMAL DECONFINEMENT

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Струнная модель с отрицательной жесткостью —
термальные свойства и температура деконфайнмента

В работе исследуются термальные свойства новой модели релятивистской струны с отрицательной жесткостью в пределе $d \to \infty$, где d — размерность пространства-времени. Эта модель предложена нами, чтобы обойти серьезные проблемы, возникающие в прежней струнной модели Полякова—Кляйнерта с положительной жесткостью. Вместе с тем она сохраняет многие привлекательные свойства ранее предложенных струнных моделей. Новая модель имеет естественный масштаб, где отрицательная жесткость становится существенной. Для низких до умеренных значений температур получены температурная зависимость эффективного натяжения струны и приближенное значение температуры деконфайнмента. Эти результаты хорошо согласуются с ранее полученными в струнных моделях Намбу-Гото и Полякова—Кляйнерта.

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String Model with Negative Stiffness —
Temperature Behavior and Thermal Deconfinement

We examine the thermal properties of a string model with a negative stiffness in the limit of infinite spacetime dimension. The model was proposed by us to overcome serious consistency problem arising in an earlier string model, in which the curvature stiffness is positive. The new model possesses a natural length scale where the negative stiffness becomes important. We derive the temperature dependence of the effective string tension and an approximate deconfinement temperature for small to moderate temperatures. The results turn out to be similar to those obtained for both the Nambu-Goto and the previous rigid-string models.

The investigation has been performed at the Laboratory of Computing Techniques and Automation and at the Bogoliubov Laboratory of Theoretical Physics, JINR.

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1 Introduction

The inclusion of a higher-derivative extrinsic curvature term into the usual string has led to a so-called rigid-string model which seems to be more relevant for a description of the quark forces in QCD than the former Nambu-Goto string. The rigid string is asymptotically free [1,2] and generates spontaneously a string tension, a realistic interquark potential. It also possesses a reasonable deconfinement temperature (see, for example, Refs. [3,4]).

Unfortunately, the extrinsic curvature produces a serious consistency problem [5-7]. It is well known that higher-derivative theories must be quantized with an indefinite norm, to avoid energies unbounded from below. The reason lies in an unphysical "ghost" pole in the propagator of such theories. In the rigid-string model, this pole arises from the second derivatives with respect to the string coordinates in the action.

In order to overcome this difficulty, a new string action was proposed in Ref. [8]. It has the form

$$\mathcal{A} = \frac{1}{2}M^2 \int d^2\xi \sqrt{g}\, g^{ij}\, D_i x^\lambda \frac{1}{1 - D^2/\mu^2}\, D_j x_\lambda\,, \tag{1.1}$$

where g_{ij} is the metric tensor of the string world sheet $x^\lambda(\xi^i)$ $(i = 0,1)$ in a d-dimensional euclidean spacetime $(\lambda = 0, 1, \dots, d-1)$ and the symbol D_i denotes the covariant derivative, while $D^2 = D_i D^i$ being Laplace-Beltrami operator. The physics of such strings is governed by two constants M^2 and μ^2 with dimensionality of a squared mass.

The propagator arising from the action (1.1) has no unphysical "ghost" pole, thus avoiding the main unphysical feature of the standard rigid-string model. In the large-μ^2 limit, we obtain from (1.1), after expanding the denominator in $1/\mu^2$ up to the first order, an action

$$\mathcal{A} = M^2 \int d^2\xi \sqrt{g} + \frac{1}{2\alpha} \int d^2\xi \sqrt{g} \left(D^2 x_\lambda\right)^2 , \tag{1.2}$$

with the stiffness parameter $\alpha = -\mu^2/M^2$. In contrast to the model of Refs.[1,2], thus the stiffness parameter is *negative*. Physically, the length scale $1/\mu$ is of the order of the thickness of a color-electric flux tube between quarks. The core of this flux is not completely confined to the string but reaches out into the vacuum up to a distance $1/\mu$.

In the one-loop approximation the string model (1.1) generates a linearly rising interquark potential with the universal Lüscher correction at large distances [8]. By studying the model in the limit of infinite spacetime dimension d, the results were obtained which were valid to all orders in perturbative theory.

In this note we examine the temperature behavior of the new string model in the same limit. In particular, we are interested in the partition function near the deconfinement transition in the presence of two static quarks separated by a large extrinsic distance $R_{\rm ext}$. The world sheet has a finite extent in the imaginary-time direction and is periodic with a period $\beta_{\rm ext} = T_{\rm ext}^{-1}$, where $T_{\rm ext}$ is the extrinsic temperature. In momentum space the integrals diverge in ultraviolet, thus requiring the regularization. To account for the dynamical content of the action (1.1), we cut all momentum integrals off at a physical momenta $|q| = \mu$. The length scale $1/\mu$ specifies where the model in Eq.(1.1) becomes unphysical. Such cutoff dependence makes our theory only an effective ones what is also obvious from the nonlocal character of the action (1.1).

The rest of the paper is arranged as follows. In Sec.2, the effective action of the model is derived in the framework of $1/d$-expansion. In Sec.3, the saddle point equations is solved in the analytic approximation valid for small to moderate temperatures. In Sec.4, the results are discussed.

2 Action and free energy at finite temperature

Following a standard procedure of the $1/d$-expansion, we introduce an independent metric field $g_{ij}(\xi)$ and the Lagrange multipliers $\lambda^{ij}(\xi)$ which enforce $g_{ij}(\xi)$ to be equal to the induced metric

$$g_{ij} = \partial_i x^\lambda \partial_j x_\lambda . \tag{2.1}$$

Then we rewrite the action (1.1) as

$$\mathcal{A} = \frac{1}{2} M^2 \int d^2\xi \sqrt{g} \Big[g^{ij} \left(\delta_{ij} + D_i x^a \frac{1}{1 - D^2/\mu^2} D_j x_a \right) + \lambda^{ij} \left(\partial_i x^a \partial_j x_a - g_{ij} + \delta_{ij} \right) \Big] . \tag{2.2}$$

where $x^a(\xi)$ $(a = 2, 3, \dots, d-1)$ are the transverse string coordinates in the parametrization $x^\lambda(\xi) = (\xi^0, \xi^1, x^a(\xi))$. After integrating out the quadratic x^a fluctuations, the expression (2.2) takes the purely intrinsic form

$$\mathcal{A}_{\rm eff} = \frac{(d-2)}{2} \mathrm{Tr} \ln \left(-\frac{D_i g^{ij} D_j}{1 - D^2/\mu^2} - D_i \lambda^{ij} D_j \right) + \frac{M^2}{2} \int d^2\xi \sqrt{g} \left(g^{ii} - \lambda^{ij} g_{ij} + \lambda^{ii} \right) . \tag{2.3}$$

with fluctuating g_{ij} - and λ^{ij} - fields. In the limit $d \to \infty$, a saddle point of the action (2.3) is expected with ξ–independent metric g_{ij} and Lagrange multipliers λ^{ij}. For symmetry reasons, the extremal values have the diagonal forms $g_{ij} = \rho_i \delta_{ij}$, $\lambda^{ij} = \lambda_i g^{ij}$, and the effective action becomes

$$\mathcal{A}_{\rm eff} = \frac{d-2}{2} R_{\rm ext} \beta_{\rm ext} \sqrt{\rho_0 \rho_1} \left[f^{T=0} + \Delta f^T + \Delta f^{\rm an} \right] . \tag{2.4}$$

The first term $f^{T=0}$ in brackets is the "free energy" density of the infinite system at $T = 0$:

$$f^{T=0} = f_0^{T=0} + \frac{\tilde{M}^2}{2} \left(\frac{\lambda_0 + 1}{\rho_0} + \frac{\lambda_1 + 1}{\rho_1} \right) . \tag{2.5}$$

where

$$f_0^{T=0} = \int \frac{d^2 q}{(2\pi)^2} \ln \left(\frac{q^2}{1 + q^2/\mu^2} + \tilde{\lambda} q^2 \right) - \tilde{M}^2 \tilde{\lambda} . \tag{2.6}$$

and we have introduced the notation $\tilde{M}^2 = 2M^2/(d-2)$. The second term Δf^T is the finite-size correction for a hypothetical isotropic average gap $\tilde{\lambda} = (\lambda_0 + \lambda_1)/2$:

$$\Delta f^T = \left(T \sum_{m=-\infty}^{\infty} - \int_{-\infty}^{\infty} \frac{d\omega_m}{2\pi} \right) \int_{-\infty}^{\infty} \frac{dq_1}{2\pi} \ln \left[\frac{\omega_m^2 + q_1^2}{1 + (\omega_m^2 + q_1^2)/\mu^2} + \tilde{\lambda} \left(\omega_m^2 + q_1^2 \right) \right] . \tag{2.7}$$

The third term $\Delta f^{\rm an}$, finally, is the correction due to the gap anisotropy $\delta = (\lambda_1 - \lambda_0)/2\tilde{\lambda}$:

$$\Delta f^{\rm an} = -T \sum_{m=-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{dq_1}{2\pi}\left\{ \ln\left[\frac{\omega_m^2 + q_1^2}{1 + (\omega_m^2 + q_1^2)/\mu^2} + \tilde{\lambda}\left(\omega_m^2 + q_1^2\right)\right]\right.$$

$$\left. - \ln\left[\frac{\omega_m^2 + q_1^2}{1 + (\omega_m^2 + q_1^2)/\mu^2} + \tilde{\lambda}\left(\omega_m^2 + q_1^2\right) - \tilde{\lambda}\,\delta\left(\omega_m^2 - q_1^2\right)\right]\right\}. \qquad (2.8)$$

The temporal components q_0 are summed over all thermal Matsubara frequences

$$q_0 = \omega_m \equiv 2\pi T m, \quad m = 0, \pm 1, \pm 2, \ldots, \qquad (2.9)$$

owing to the periodicity of the world-sheet of the string, with a period $\beta_{\rm ext} = T_{\rm ext}^{-1} = (\sqrt{\rho_0}\,T)^{-1}$ along the imaginary-time axis.

The ultraviolet divergences in (2.6) can be regularized for an isotropic infinite system at $T = 0$ which has $\lambda_0 = \lambda_1 = \tilde{\lambda}$. In this system, Δf^T and $\Delta f^{\rm an}$ are absent and the total "free energy" density is defined by Eqs. (2.5) and (2.6).

At this point it is usful to establish the contact with the standard rigid-string model. For this, let us replace λ_i as follows

$$\lambda_i = -\frac{\eta_i}{\mu^2} - 1, \quad i = 1,2 \qquad (2.10)$$

and, similarly,

$$\tilde{\lambda} = -\frac{\tilde{\eta}}{\mu^2} - 1, \qquad (2.11)$$

where η_i are some new coefficients of the Lagrange matrix λ^{ij} and $\tilde{\eta} = (\eta_0 + \eta_1)/2$. Then Eqs. (2.5) and (2.6) become

$$f^{T=0} = f_0^{T=0} + \frac{1}{2\tilde{\alpha}}\left(\frac{\eta_0}{\rho_0} + \frac{\eta_1}{\rho_1}\right), \qquad (2.12)$$

with

$$f_0^{T=0} = \int \frac{d^2q}{(2\pi)^2} \ln\left[\frac{q^2}{1 + q^2/\mu^2} - q^2\left(\frac{\tilde{\eta}}{\mu^2} + 1\right)\right] + \tilde{M}^2 - \frac{\tilde{\eta}}{\tilde{\alpha}}, \qquad (2.13)$$

where $\tilde{\alpha} = (d-2)\alpha/2$. Going to large μ^2, we expand these expressions in powers of $1/\mu^2$ up to the first order and obtain exactly the "free energy" density of the infinite-size rigid string, if we insert there the negative stiffness parameter $\tilde{\alpha} = -\mu^2/\tilde{M}^2$ (compare Ref. [4]):

$$f^{T=0} = \int \frac{d^2q}{(2\pi)^2} \ln\left(q^4 + q^2\tilde{\eta}\right) + \tilde{M}^2 - \frac{\tilde{\eta}}{\tilde{\alpha}} + \frac{1}{2\tilde{\alpha}}\left(\frac{\eta_0}{\rho_0} + \frac{\eta_1}{\rho_1}\right). \qquad (2.14)$$

Note that the same limit is not straightforward after performing the integral in Eq.(2.13) via an analytic regularization. The reason lies in arising the term

$$\frac{\mu^2}{4\pi}\left(\ln\frac{\Lambda^2}{\mu^2} + 1\right),$$

where Λ is an ultraviolet cutoff, which would diverge for large μ^2. In contrast, if we take the limit of large μ^2 at fixed Λ no divergence appears, as it should be. In general, the limit $\mu^2 \to \infty$ and the integral over q in Eq.(2.13) do not commute. An immediate conclusion is that all momentum integrals must be limited to the momentum range

$$|q| \le \mu. \qquad (2.15)$$

(The physical meaning of the length scale $1/\mu$ was discussed in the Introduction). With the restriction (2.15) we neglect the quadratic infinity μ^2, as in analytic regularization, and obtain for the momentum integral in Eq.(2.13) the expression

$$\int_{|q|<\mu} \frac{d^2q}{(2\pi)^2} \ln\left[\frac{q^2}{1 + q^2/\mu^2} - q^2\left(\frac{\tilde{\eta}}{\mu^2} + 1\right)\right] = \frac{\tilde{\eta}}{4\pi(1 + \tilde{\eta}/\mu^2)}\left[\ln\frac{\mu^2(1 + \tilde{\eta}/\mu^2)}{\tilde{\eta}} + 1\right]. \qquad (2.16)$$

In the limit $\mu^2 \to \infty$, the right-hand side of Eq.(2.16) tends to

$$\frac{\tilde{\eta}}{4\pi}\left(\ln\frac{\mu^2}{\tilde{\eta}} + 1\right), \qquad (2.17)$$

which is precisely the answer with the same cutoff for the momentum integral in Eq.(2.14). Of course, our theory together with such cutoff dependence is an effective theory what is also obvious from the nonlocal character of the action (1.1).

To exhibit the limit of the rigid-string model we replace the average gap $\tilde{\eta}$ as follows

$$\tilde{\eta} \to \tilde{\zeta} = \frac{\tilde{\eta}}{1 + \tilde{\eta}/\mu^2}. \qquad (2.18)$$

With use of (2.18) and (2.16), Eq. (2.13) becomes

$$f_0^{T=0} = \tilde{M}^2 - \frac{\tilde{\zeta}/\tilde{\alpha}}{(1 - \tilde{\zeta}/\mu^2)} + \frac{\tilde{\zeta}}{4\pi}\left(\ln\frac{\mu^2}{\tilde{\zeta}} + 1\right). \qquad (2.19)$$

From (2.19) we see that the ultraviolet divergences cannot be absorbed into a redefinition of the coupling constant $\tilde{\alpha}$. Renormalizability requires the limit $\mu^2 \to \infty$. Nevertheless, we can introduce an analog of the usual dimensionally transmuted coupling constant:

$$\bar{\zeta} = \mu^2 \exp\left[-\frac{2}{(d-2)}\frac{4\pi}{\left(1 - \bar{\zeta}/\mu^2\right)}\frac{1}{\alpha} + 1\right], \qquad (2.20)$$

and rewrite Eq. (2.19) as follows

$$\begin{aligned} f_0^{T=0} &= \tilde{M}^2 + f_0(\tilde{\zeta}) - \left(1 - \frac{\tilde{\zeta}}{\mu^2}\right)\frac{\tilde{\zeta}}{4\pi} + \frac{\tilde{\zeta}}{\tilde{\alpha}} - \frac{\tilde{\zeta}/\tilde{\alpha}}{(1 - \tilde{\zeta}/\mu^2)} \\ &= \tilde{M}^2 - \frac{\tilde{\zeta}}{4\pi}\ln\frac{\tilde{\zeta}}{\bar{\zeta}} + \frac{\tilde{\zeta}/\tilde{\alpha}}{(1 - \tilde{\zeta}/\mu^2)} - \frac{\bar{\zeta}/\tilde{\alpha}}{(1 - \bar{\zeta}/\mu^2)}. \end{aligned} \tag{2.21}$$

Then the action of the infinite system has the form

$$\mathcal{A}^{T=0} = \frac{d-2}{2} R_{\text{ext}}\beta_{\text{ext}}\sqrt{\rho_0\rho_1}\, f^{T=0}, \tag{2.22}$$

where $\eta_0 = \eta_1 = \tilde{\eta}$ at $T = 0$, and $\tilde{\eta}$ is replaced by $\tilde{\zeta}$ according to (2.18).

For a moment, we shall drop the first (Nambu-Goto-like) term $\tilde{M}^2$ in Eq.(2.21). Then an extremization of (2.22) in ρ_0, ρ_1, and $\tilde{\zeta}$ gives $f^{T=0}(\tilde{\zeta}) = f_0(\bar{\zeta})$ and $\tilde{\zeta} = \bar{\zeta}$. The $T = 0$ - values $\rho_0 = \rho_1 \equiv \bar{\rho}$ satisfy

$$\frac{1}{\tilde{\alpha}\bar{\rho}} = \frac{1}{\tilde{\alpha}} + \left(1 - \frac{\bar{\zeta}}{\mu^2}\right)^2\left[\frac{1}{4\pi} - \frac{1/\tilde{\alpha}}{(1 - \bar{\zeta}/\mu^2)}\right]. \tag{2.23}$$

Making a further renormalization $\bar{\rho} \to 1$ at $T = 0$, we conclude that with $\tilde{\zeta} = \bar{\zeta} \neq 0$, the surface has acquired spontaneously a string tension

$$M_{\text{sp}}^2 = \frac{d-2}{2}\frac{\bar{\zeta}}{4\pi}. \tag{2.24}$$

When the Nambu-Goto-like term $\tilde{M}^2$ is added to $f^{T=0}$, the extremization of (2.22) in ρ_0, ρ_1 and $\tilde{\zeta}$ gives

$$f^{T=0}(\tilde{\zeta}) = 2\tilde{M}^2 + f_0(\tilde{\zeta}) + \frac{\tilde{\zeta}}{\tilde{\alpha}}.$$

The remaining gap equation

$$\tilde{M}^2 - \frac{\tilde{\zeta}}{4\pi}\ln\frac{\tilde{\zeta}}{\bar{\zeta}} + \frac{\tilde{\zeta}/\tilde{\alpha}}{(1 - \tilde{\zeta}/\mu^2)} - \frac{\bar{\zeta}/\tilde{\alpha}}{(1 - \bar{\zeta}/\mu^2)} = 0$$

can be solved by $\tilde{\zeta} = \bar{\zeta}e^{\nu}$, where ν is the "normality" of the string. In the ordinary rigid-string model, $\nu\, e^{\nu}$ measures the relative amount of $\tilde{M}^2$ with respect to the spontaneously generated string tension $\tilde{M}_{\text{sp}}^2 = \bar{\zeta}/4\pi$. Here the parameter ν brings the above equation to the form

$$\tilde{M}^2 = \frac{\bar{\zeta}}{4\pi}e^{\nu}\,\frac{\nu\left(1 - \bar{\zeta}/\mu^2\right) + (e^{\nu} - 1)\,\bar{\zeta}/\mu^2}{1 - (\bar{\zeta}/\mu^2)\,e^{\nu}}. \tag{2.25}$$

From the full action (2.22), we obtain the total string tension

$$M_{\text{tot}}^2 = \frac{(d-2)}{2}\,\frac{\bar{\zeta}e^{\nu}}{4\pi}\,(\nu + 1)\frac{\left[1 - \bar{\zeta}/\mu^2\right]}{\left[1 - (\bar{\zeta}/\mu^2)\,e^{\nu}\right]}. \tag{2.26}$$

We are now ready to calculate the finite-temperature correction to $f^{T=0}$ at an isotropic average gap λ, which is given by the expression (2.7). The q_1 - integration can be performed by using the cutoff (2.15) again and replacing λ according to (2.11). Then the integral over q_1 in Eq.(2.7) becomes after analytic regularization

$$\begin{aligned} &\int_{-\infty}^{\infty}\frac{dq_1}{2\pi}\ln\left[\frac{\omega_m^2 + q_1^2}{1 + (\omega_m^2 + q_1^2)/\mu^2} - (\omega_m^2 + q_1^2)\left(1 + \frac{\tilde{\eta}}{\mu^2}\right)\right] = \\ &= \sqrt{\omega_m^2} + \sqrt{\omega_m^2 + \tilde{\zeta}} = 2\pi T\left(\sqrt{m^2} + \sqrt{m^2 + \tilde{\zeta}_T}\right), \end{aligned} \tag{2.27}$$

having gone from the quantity $\tilde{\eta}$ to $\tilde{\zeta}$ according to (2.18). The quantity $\tilde{\zeta}_T$ is the dimensionless parameter

$$\tilde{\zeta}_T \equiv \frac{\tilde{\zeta}}{(2\pi)^2T^2}. \tag{2.28}$$

Making use of (2.27), we can write Δf^T as

$$\begin{aligned} \Delta f^T(\tilde{\zeta}, T) &= \left(T\sum_{m=-\infty}^{\infty} - \int_{-\infty}^{\infty}\frac{d\omega_m}{2\pi}\right)\left[\sqrt{\omega_m^2 + \tilde{\zeta}} + \sqrt{\omega_m^2}\right] = \\ &= 2\pi T^2\left(\sum_{m=-\infty}^{\infty} - \int_{-\infty}^{\infty}dm\right)\left[\sqrt{m^2 + \tilde{\zeta}_T^2} + \sqrt{m^2}\right]. \end{aligned} \tag{2.29}$$

Both the sum and the integral in (2.29) diverge, but the difference is finite. The expression (2.29) has been introduced and calculated in Ref. [4]. The answer can be represented in the different forms depending on which the size of $\tilde{\zeta}_T$. For small $\tilde{\zeta}_T$ (large T), $\Delta f^T(\tilde{\zeta}, T)$ is given by

$$\Delta f^T(\tilde{\zeta}, T) = -\frac{2\pi T^2}{3} + T\sqrt{\tilde{\zeta}} - \frac{\tilde{\zeta}}{4\pi}\left[\ln\frac{16\pi^2T^2e^{-2\gamma}}{\tilde{\zeta}} + 1\right] + \frac{\tilde{\zeta}}{\pi}S_1(\tilde{\zeta}_T), \tag{2.30}$$

where $S_1(\tilde{\zeta}_T)$ denotes the sum over m

$$S_1(\tilde{\zeta}_T) \equiv \frac{1}{\tilde{\zeta}_T}\sum_{m=1}^{\infty}\left[\sqrt{m^2 + \tilde{\zeta}_T} - m - \frac{\tilde{\zeta}_T}{2m}\right], \tag{2.31}$$

which converges rapidly for small $\tilde{\zeta}_T$. Adding $f_0(\tilde{\zeta})$ from (2.21) to (2.30) gives the free-energy density for an isotropic gap $\tilde{\zeta}$:

$$f_{iso}^T(\tilde{\zeta}, T) = f_0 + \Delta f^T = -\frac{2\pi}{3}T^2 + T\sqrt{\tilde{\zeta}} + \frac{\tilde{\zeta}}{\pi}S_1 -$$

$$\frac{\bar{\zeta}}{4\pi}\ln\frac{T^2}{\bar{T}^2} - \frac{\bar{\zeta}}{\tilde{\alpha}} + \frac{\bar{\zeta}/\tilde{\alpha}}{(1-\bar{\zeta}/\mu^2)} - \frac{\bar{\zeta}^2}{4\pi\mu^2}. \tag{2.32}$$

Here we have introduced the natural temperature scale of the system:

$$\bar{T} \equiv \frac{\sqrt{\bar{\zeta}}}{4\pi\, e^{-\gamma}}. \tag{2.33}$$

When $\bar{\zeta}_T$ becomes large, it is better to use another representation for Δf^T:

$$\Delta f^T(\bar{\zeta}, T) = -\frac{\pi}{3}T^2 + \frac{\bar{\zeta}}{\pi}\tilde{S}_1(\bar{\zeta}_T), \tag{2.34}$$

where $\tilde{S}_1(\bar{\zeta}_T)$ is the dimensionless sum

$$\tilde{S}_1(\bar{\zeta}_T) = -2\sum_{m=1}^{\infty}\frac{K_1\left(2\pi m\sqrt{\bar{\zeta}_T}\right)}{2\pi m\sqrt{\bar{\zeta}_T}}. \tag{2.35}$$

Equation (2.34) allows us to calculate the behavior of $\Delta f^T(\bar{\zeta}, T)$ for small T. Since $K_1(z)$ decreases exponentially fast in large z, this limit reads

$$\Delta f^T(\bar{\zeta}, T) \underset{T\to 0}{\approx} -\frac{\pi}{3}T^2. \tag{2.36}$$

For large $\bar{\zeta}_T$ (small T), there exist yet another useful formula for $\Delta f^T(\bar{\zeta}, T)$. With the help of the integral representation for $K_1(z)$ we can rewrite (2.34) as

$$\Delta f^T(\bar{\zeta}, T) = -\frac{\pi}{3}T^2 + 2T\int_{-\infty}^{\infty}\frac{dq_1}{2\pi}\ln\left[1 - e^{-(q_1^2+\bar{\zeta})^{1/2}/T}\right]. \tag{2.37}$$

Note that any of the representations (2.30), (2.34), or (2.37) for Δf^T has the functional form

$$\Delta f^T(\bar{\zeta}\, T) = \bar{\zeta}\, g(\bar{\zeta}_T). \tag{2.38}$$

This observation will be useful in the following study of the approximation of an isotropic gap.

3 Isotropic gap approximation

In this section the variational equations for the stationary point of the action (2.4) will be derived in an approximation in which we assume the solution to be isotropic in the space and imaginary-time direction. Thus we set $\delta = 0$, $\bar{\zeta} \equiv \zeta$, and consider the action

$$\mathcal{A}_{\rm iso} = \frac{d-2}{2}R_{\rm ext}\beta_{\rm ext}\sqrt{\rho_0\rho_1}\, f_{\rm iso}, \tag{3.1}$$

with

$$f_{\rm iso} = f^{T=0} + \Delta f^T(\zeta, T) = \tilde{M}^2 + f_0(\zeta) - \frac{\zeta}{4\pi}\left(1 - \frac{\zeta}{\mu^2}\right) + \frac{\zeta}{\tilde{\alpha}}$$
$$- \frac{\zeta/\tilde{\alpha}}{(1-\zeta/\mu^2)} + \zeta\, g(\zeta_T) + \frac{\zeta/\tilde{\alpha}}{(1-\zeta/\mu^2)}\left(\frac{1}{\rho_0} + \frac{1}{\rho_1}\right). \tag{3.2}$$

For simplicity, we drop the Nambu-Goto-like term $\tilde{M}^2$ ($\nu = 0$). In extermizing this simplified action we must keep in mind that according to (2.28) the dimensionless variable ζ_T depends on ρ_0 via

$$\zeta_T = \frac{\zeta\,\rho_0}{4\pi^2 T_{\rm ext}^2}, \tag{3.3}$$

and $T_{\rm ext}$ should be fixed. Therefore, the extremization of $\mathcal{A}_{\rm iso}$ yields, upon applying the derivative $\sqrt{\rho_0}\,\partial/\partial\sqrt{\rho_0}$:

$$f_0(\zeta) - \frac{\zeta}{4\pi}\left(1 - \frac{\zeta}{\mu^2}\right) + \frac{\zeta}{\tilde{\alpha}} - \frac{\zeta/\tilde{\alpha}}{(1-\zeta/\mu^2)} +$$
$$\zeta\, g(\zeta_T) + 2\zeta\,\zeta_T\, g'(\zeta_T) + \frac{\zeta/\tilde{\alpha}}{2(1-\zeta/\mu^2)}\left(-\frac{1}{\rho_0} + \frac{1}{\rho_1}\right) = 0, \tag{3.4}$$

upon applying $\sqrt{\rho_1}\partial/\partial\sqrt{\rho_1}$:

$$f_0(\zeta) - \frac{\zeta}{4\pi}\left(1 - \frac{\zeta}{\mu^2}\right) + \frac{\zeta}{\tilde{\alpha}} - \frac{\zeta/\tilde{\alpha}}{(1-\zeta/\mu^2)} +$$
$$\zeta\, g(\zeta_T) + \frac{\zeta/\tilde{\alpha}}{(1-\zeta/\mu^2)}\left(\frac{1}{\rho_0} - \frac{1}{\rho_1}\right) = 0, \tag{3.5}$$

and upon applying $\partial/\partial\zeta$:

$$\frac{\partial}{\partial\zeta}f_0(\zeta) + \frac{\zeta}{2\pi\mu^2} - \frac{1}{4\pi} + \frac{1}{\tilde{\alpha}} - \frac{1/\tilde{\alpha}}{(1-\zeta/\mu^2)^2} +$$
$$g(\zeta_T) + \zeta_T\, g'(\zeta_T) + \frac{1/\tilde{\alpha}}{2(1-\zeta/\mu^2)^2}\left(\frac{1}{\rho_0} + \frac{1}{\rho_1}\right) = 0. \tag{3.6}$$

The prime denotes the derivative with respect to ζ_T. Adding and subtracting Eqs. (3.4) and (3.5) results in the equations

$$f_0(\zeta) - \frac{\zeta}{4\pi}\left(1 - \frac{\zeta}{\mu^2}\right) + \frac{\zeta}{\tilde{\alpha}} - \frac{\zeta/\tilde{\alpha}}{(1-\zeta/\mu^2)} + \zeta\, g(\zeta_T) + \zeta\,\zeta_T\, g'(\zeta_T) = 0, \tag{3.7}$$

$$\zeta\,\zeta_T\, g'(\zeta_T) = \frac{\zeta/\tilde{\alpha}}{2(1-\zeta/\mu^2)}\left(\frac{1}{\rho_0} - \frac{1}{\rho_1}\right). \tag{3.8}$$

The second equation determines the difference between the extremal ρ_0 and ρ_1. Substracting (3.7) from (3.6) we find that the minimum lies at

$$\frac{\zeta}{4\pi}\left(1-\frac{\zeta}{\mu^2}\right)-\frac{\zeta}{\tilde\alpha}+\frac{\zeta/\tilde\alpha}{(1-\zeta/\mu^2)}=\frac{\zeta/\tilde\alpha}{2(1-\zeta/\mu^2)}\left(\frac{1}{\rho_0}+\frac{1}{\rho_1}\right)=0. \qquad (3.9)$$

Inserting this into the gap equation (3.6) gives

$$\frac{\partial}{\partial\zeta}\left(f_0+\Delta f^T\right)=-\frac{1}{\tilde\alpha}-\frac{\zeta}{2\pi\mu^2}+\frac{1/\tilde\alpha}{(1-\zeta/\mu^2)}, \qquad (3.10)$$

and the free-energy density is simply

$$f_{\rm iso}=f_0(\zeta)+\zeta\, g(\zeta_T)=f_0(\zeta)+\Delta f^T(\zeta,T)\equiv f^T_{\rm iso}. \qquad (3.11)$$

Hence, the set of the variational equations for the stationary point $(\rho_0,\ \rho_1,\ \zeta)$ of (3.1) is reduced to (3.8)–(3.10). Depending on which of the forms of $f^T_{\rm iso}$ (2.32) or (2.34) we prefer to use, the gap equation (3.10) reads

$$-\frac{1}{4\pi}\ln\frac{T^2}{\bar T^2}+\frac{T}{2\sqrt{\zeta}}+\frac{1/\tilde\alpha}{(1-\bar\zeta/\mu^2)}-\frac{1/\tilde\alpha}{(1-\zeta/\mu^2)}+\frac{1}{2\pi}S_2(\zeta_T)=0, \qquad (3.12)$$

$$-\frac{1}{4\pi}\ln\frac{\zeta}{\bar\zeta}+\frac{1/\tilde\alpha}{(1-\bar\zeta/\mu^2)}-\frac{1/\tilde\alpha}{(1-\zeta/\mu^2)}+\frac{1}{2\pi}\tilde S_2(\zeta_T)=0. \qquad (3.13)$$

Here we have introduced the sum

$$S_2(\zeta_T)=\sum_{m=1}^{\infty}\left[\frac{1}{\sqrt{m^2+\zeta_T}}-\frac{1}{m}\right], \qquad (3.14)$$

in analogy to S_1 of (2.31), and the sum

$$\tilde S_2(\zeta_T)=2\sum_{m=1}^{\infty}K_0(2\pi m\sqrt{\zeta_T}), \qquad (3.15)$$

in analogy to (2.35).

From (3.7) and the other two equations (3.8) and (3.9), we can find the extremal ρ_0 and ρ_1. First, we reexpress $\zeta_T\, g'(\zeta_T)$ in Eq. (3.7) as follows:

$$\zeta_T\, g'(\zeta_T)=-\frac{1}{\zeta}f^T_{\rm iso}+\frac{1}{4\pi}\left(1-\frac{\zeta}{\mu^2}\right)-\frac{1}{\tilde\alpha}+\frac{1/\tilde\alpha}{(1-\zeta/\mu^2)}. \qquad (3.16)$$

Substituting this into (3.8) and solving the set of equations (3.8) and (3.9) with respect to ρ_0 and ρ_1, we derive

$$\frac{1/\tilde\alpha}{(1-\zeta/\mu^2)}\frac{1}{\rho_0}=\frac{1}{2\pi}\left(1-\frac{\zeta}{\mu^2}\right)-\frac{2}{\tilde\alpha}+\frac{2/\tilde\alpha}{(1-\zeta/\mu^2)}-\frac{1}{\zeta}f^T_{\rm iso}, \qquad (3.17)$$

$$\frac{1/\tilde\alpha}{(1-\zeta/\mu^2)}\frac{1}{\rho_1}=\frac{1}{\zeta}f^T_{\rm iso}. \qquad (3.18)$$

These equations determine the ratio

$$\frac{\rho_1}{\rho_0}=\left[\frac{1}{2\pi}\left(1-\frac{\zeta}{\mu^2}\right)-\frac{2}{\tilde\alpha}+\frac{2/\tilde\alpha}{(1-\zeta/\mu^2)}-\frac{1}{\zeta}f^T_{\rm iso}\right]\Big/\frac{1}{\zeta}f^T_{\rm iso}. \qquad (3.19)$$

At low temperatures

$$\frac{1}{\zeta}f^T_{\rm iso}\big|_{T\to 0}\to\frac{1}{4\pi}\left(1-\frac{\zeta}{\mu^2}\right)-\frac{1}{\tilde\alpha}+\frac{1/\tilde\alpha}{(1-\zeta/\mu^2)},$$

and the two equations (3.17) and (3.18) have a common limit which is equal to (2.23). Extracting from (2.23) the coupling constant

$$\frac{1}{\tilde\alpha}=\frac{\bar\rho}{4\pi}\frac{\left(1-\bar\zeta/\mu^2\right)^2}{\left(1-\bar\rho\,\bar\zeta/\mu^2\right)}, \qquad (3.20)$$

and inserting (3.20) into (3.17) and (3.18) gives

$$\frac{1}{\hat\rho_0}=2\frac{\zeta_{\rm ext}}{\mu^2}+\frac{2(1-\zeta/\mu^2)(1-\bar\zeta_{\rm ext}/\mu^2)}{(1-\bar\zeta/\mu^2)^2}\left[\left(1-\frac{\zeta}{\mu^2}\right)-\frac{2\pi}{\zeta}f^T_{\rm iso}\right], \qquad (3.21)$$

$$\frac{1}{\hat\rho_1}=\frac{(1-\zeta/\mu^2)(1-\bar\zeta_{\rm ext}/\mu^2)}{(1-\bar\zeta/\mu^2)^2}\frac{4\pi}{\zeta}f^T_{\rm iso}. \qquad (3.22)$$

Here we have gone over to an extrinsic value $\bar\zeta_{\rm ext}$ of $\bar\zeta$ defined with the help of the metric factor $\bar\rho$ which is the common limit of ρ_0 and ρ_1 for $T\to 0$:

$$\bar\zeta_{\rm ext}\equiv\bar\rho\,\bar\zeta. \qquad (3.23)$$

We also have introduced the reduced quantities

$$\hat\rho_0=\frac{\rho_0}{\bar\rho},\quad \hat\rho_1=\frac{\rho_1}{\bar\rho}. \qquad (3.24)$$

We can now calculate the effective string tension as a function of the temperature $T_{\rm ext}$. As before in Eq. (2.26), this quantity is simply defined by dropping the extrinsic area factor $R_{\rm ext}\beta_{\rm ext}$ in the action (3.1) which is now given by:

$$M^2_{\rm tot}(T_{\rm ext})=\frac{(d-2)}{2}\bar\rho\sqrt{\hat\rho_0\hat\rho_1}\,f^T_{\rm iso}. \qquad (3.25)$$

Inserting (3.21) and (3.22) into (3.25), we obtain

$$M^2_{\rm tot}(T_{\rm ext})=M^2_{\rm tot}(0)\,\hat M^2_{\rm tot}(T_{\rm ext}). \qquad (3.26)$$

Here $M^2_{\rm tot}(0)$ is the spontaneously-generated string tension

$$M^2_{\rm tot}(0) = M^2_{\rm sp} = \frac{(d-2)}{2}\frac{\zeta_{\rm ext}}{4\pi}\frac{1-\bar\zeta/\mu^2}{1-\bar\zeta_{\rm ext}/\mu^2}. \tag{3.27}$$

The reduced tension $\hat M^2_{\rm tot}$ which is normalized to unity at $T_{\rm ext} = 0$ is given by

$$\hat M^2_{\rm tot} = \frac{\zeta_{\rm ext}}{\bar\zeta_{\rm ext}}\frac{(1-\bar\zeta/\mu^2)}{(1-\zeta/\mu^2)}\left[\frac{f^T_{\rm iso}/\zeta}{\frac{\zeta_{\rm ext}}{2\pi\mu^2}\frac{(1-\zeta/\mu^2)^2}{(1-\zeta/\mu^2)(1-\zeta_{\rm ext}/\mu^2)} + \frac{1}{2\pi}(1-\zeta/\mu^2) - f^T_{\rm iso}/\zeta}\right]^{1/2}. \tag{3.28}$$

The extremal ζ is determined by the gap equation (3.12) or (3.13). With use of (3.20), the latter equation can be rewritten as

$$-\ln\frac{\zeta_{\rm ext}}{\bar\zeta_{\rm ext}} + \frac{\bar\zeta_{\rm ext}}{\mu^2}\frac{(1-\bar\zeta/\mu^2)(1-\zeta_{\rm ext}/\bar\zeta_{\rm ext})}{(1-\zeta/\mu^2)(1-\bar\zeta_{\rm ext}/\mu^2)} + 2\tilde S_2(\zeta_T) = 0. \tag{3.29}$$

We see that Eqs.(3.28) and (3.29), after removing the dependence of the coupling constant $\bar\alpha$, cannot be expressed completely in terms of extrinsically renormalized quantities. This is a consequence of the nonlocality of the theory. A full numerical treatment of these equations requires therefore a specification of the scale μ where the model breaks down. However, the temperature dependence of the string tension and the deconfinement temperature are completely determined in this model. The temperature dependence of the string tension can easily be calculated as analytic approximate expressions valid for small to moderate temperatures. We observe that for small T, the argument z of the Bessel function $K_0(z)$ becomes large and, since $K_0(z)$ decrease exponentially for large z, the gap equation (3.29) loses the last term. Then the obvious solution of (3.29) is $\zeta \approx \bar\zeta$ with only exponentially small corrections. In the limit of small T, also $K_1(z)$ is exponentially small and, with the gap ζ being close to $\bar\zeta$, we find from (3.11) right away the approximation

$$f^T_{\rm iso}(\zeta, T) \simeq \frac{\bar\zeta}{4\pi}\frac{1-\bar\zeta/\mu^2}{1-\bar\rho\,\bar\zeta/\mu^2} - \frac{\pi}{3}T^2. \tag{3.30}$$

Therefore, if T exceeds the value

$$T^{\rm d} = \sqrt{\frac{3}{4\pi^2}}\sqrt{\bar\zeta}\sqrt{\frac{1-\bar\zeta/\mu^2}{1-\bar\zeta_{\rm ext}/\mu^2}} \tag{3.31}$$

which because of (3.27) is the same as

$$\sqrt{\bar\rho}\,T^{\rm d} = \sqrt{\frac{6}{\pi(d-2)}}\,M_{\rm tot}(0), \tag{3.32}$$

the string tension turns negative and the confinement is destroyed.

In order to compare (3.32) with experiment, we have to go to an extrinsically renormalized quantity. For this we calculate

$$T^{\rm d}_{\rm ext} = \sqrt{\rho_0}\,T^{\rm d} = \sqrt{\hat\rho_0}\sqrt{\bar\rho}\,T^{\rm d} = \sqrt{\hat\rho_0}\sqrt{\frac{6}{\pi(d-2)}}\,M_{\rm tot}, \tag{3.33}$$

where the quantity $\hat\rho_0|_{T^{\rm d}}$ at $T^{\rm d}$ can be determined from (3.21) as follows

$$\left.\frac{1}{\hat\rho_0}\right|_{T^{\rm d}} \doteq 2. \tag{3.34}$$

Putting (3.34) back into (3.33), we find the approximate deconfinement temperature

$$T^{\rm d}_{\rm ext} = \sqrt{\frac{3}{\pi(d-2)}}\,M_{\rm tot}(0) \simeq 0.69\,M_{\rm tot}(0). \tag{3.35}$$

This value is not far from what is found in Monte Carlo simulations of lattice gauge models (see, for example, Refs.[3,4]).

The full temperature dependence of the string tension in the approximation of neglecting the Bessel function follows from the the expression (3.28), where $f^T_{\rm iso}$ is given by (3.30). Therefore, we obtain

$$\hat M^{2\,\rm app}_{\rm tot}(T_{\rm ext}) = \left[\frac{1}{4\pi}\frac{(1-\bar\zeta/\mu^2)}{(1-\bar\zeta_{\rm ext}/\mu^2)} - \frac{\pi T^2}{3\bar\zeta}\right]^{1/2}\left[\frac{1}{4\pi}\frac{(1-\bar\zeta/\mu^2)}{(1-\bar\zeta_{\rm ext}/\mu^2)} + \frac{\pi T^2}{3\bar\zeta}\right]^{-1/2}. \tag{3.36}$$

Here T is related to the extrinsic temperature $T_{\rm ext}$ as follows

$$T^2_{\rm ext} = \rho_0 T^2 = \hat\rho_0\,\bar\rho\,T^2 = \bar\rho\,T^2\left[1 + \frac{4\pi^2}{3\bar\zeta}\frac{(1-\bar\zeta_{\rm ext}/\mu^2)}{(1-\bar\zeta/\mu^2)}T^2\right]^{-1}. \tag{3.37}$$

Solving (3.37) with respect to T^2 and inserting it into (3.36), we find the simple approximate analytic expression for the string tension

$$\hat M^{2\,\rm app}_{\rm tot}(T_{\rm ext}) = \left[1 - \frac{\pi\,(d-2)T^2_{\rm ext}}{3\,M^2_{\rm tot}(0)}\right]^{1/2}. \tag{3.38}$$

In this approximation, the temperature behavior of the string tension of our model coincides precisely with that of the Nambu-Goto string and of the previous rigid-string model.

In this approximation, there exist yet another representations for the string tension $\hat M^{2\rm app}_{\rm tot}(T_{\rm ext})$ and an approximate deconfinement temperature $T^{\rm d}_{\rm ext}$. Inserting (3.27) into (3.38) and (3.33), we obtain

$$\hat M^{2\rm app}_{\rm tot}(T_{\rm ext}) = \left(1 - \frac{8\pi^2}{3\bar\zeta_{\rm ext}}\,h\,T^2_{\rm ext}\right)^{1/2}. \tag{3.39}$$

This shows the deconfinement temperature to lie at

$$T^{\rm d}_{\rm ext} = \sqrt{6}\, e^{-\gamma}/\sqrt{h}\,, \tag{3.40}$$

where h is the dimensionless factor

$$h = (1 - \bar\zeta_{\rm ext}/\mu^2)/(1 - \bar\zeta/\mu^2)\,. \tag{3.41}$$

The string tension (3.39) and the deconfinement temperature (3.40) are rather complicated functions of the temperature scale $\bar T_{\rm ext} = \sqrt{\bar\rho}\,\bar T$, where $\bar T$ is defined by (2.33). This scale depends on the string tension at zero temperature $\tilde M^2_{\rm tot}(0) = 2M^2_{\rm tot}(0)/(d-2)$ in the complicated way:

$$\frac{\bar T^2_{\rm ext}}{\mu^2} = \frac{\bar\zeta_{\rm ext}/\mu^2}{16\pi^2 e^{-2\gamma}} = \frac{\bar\rho e^{2\gamma}}{32\pi^2}\left[\left(1 + 4\pi\,\frac{\tilde M^2_{\rm tot}}{\mu^2}\right) - \sqrt{\left(1 + 4\pi\,\frac{\tilde M^2_{\rm tot}}{\mu^2}\right)^2 - 16\pi\,\frac{\tilde M^2_{\rm tot}}{\mu^2\bar\rho}}\,\right] \tag{3.42}$$

It is therefore better to use another temperature scale

$$\bar T_{\rm ext,h} = \sqrt{\bar\zeta_{\rm ext}}/4\pi\, e^{-\gamma}\sqrt{h} = \bar T_{\rm ext}/\sqrt{h}\,. \tag{3.43}$$

Then the expressions (3.39) and (3.40) take the following form:

$$\tilde M^{2\rm app}_{\rm tot}(T_{\rm ext}) = \left[1 - \frac{e^{2\gamma}}{6}\left(\frac{T_{\rm ext}}{\bar T_{\rm ext,h}}\right)^2\right]^{1/2}, \tag{3.44}$$

where

$$T^{\rm d}_{\rm ext}/\bar T_{\rm ext,h} = \sqrt{6}\, e^{-\gamma} \simeq 1.38\,. \tag{3.45}$$

In order to compare the two temperature scales (3.43) and (2.33), we introduce the dimensionless constant

$$\mu^2_{\bar T} = \frac{\mu^2}{(4\pi\, e^{-\gamma})^2\bar T^2} = \frac{\mu^2}{\bar\zeta}\,. \tag{3.46}$$

For large μ^2 and small $\bar T$, the value of $\mu^2_{\bar T}$ is very large. Inserting (3.46) into (3.41), and expanding h in powers of $1/\mu^2_{\bar T}$, gives with good accuracy

$$h \simeq 1 - (\bar\rho - 1)/\mu^2_{\bar T} = 1 - 1/\mu^2_{\bar T,\rm ext}\,, \tag{3.47}$$

where we have gone over to an extrinsic value of $\mu^2_{\bar T,\rm ext}$ with the help of a metric factor $\bar\rho$. In the large-$\mu^2_{\bar T,\rm ext}$ limit, the factor (3.47) is close to unity, slightly increasing ($\bar\rho > 1$) or decreasing ($\bar\rho < 1$) the value of the model's string tension (3.39), in comparison with that of the rigid-string model with a positive curvature stiffness. In the case $\bar\rho = 1$, the two versions coincide exactly.

In Fig. 1 we show the string tension (3.39) as a function of the reduced temperature $T_{\rm ext}/\bar T_{\rm ext}$ in the large $\mu^2_{\bar T,\rm ext}$ limit for various values of $\bar\rho$ (dots), where they are compared with the positive rigid-string curve (solid line). In the isotropic gap approximation the two curves coincide.

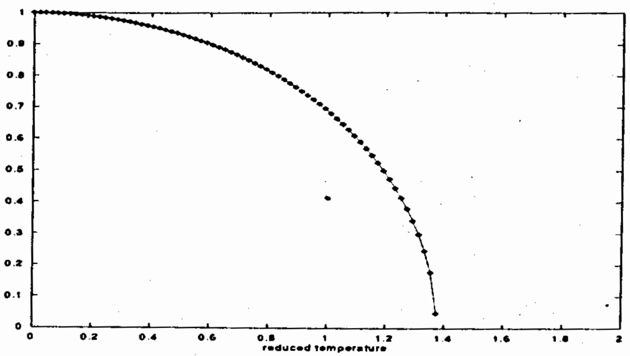

Figure 1: String tension of Eq.(3.39) as a function of the reduced temperature $T_{\rm ext}/\bar T_{\rm ext}$. Dots correspond to our tension for $\bar\rho > 1$ or $\bar\rho < 1$, respectively, with $\mu^2_{\bar T,\rm ext} = 100$. The solid line shows the string tension in the rigid-string model with a positive curvature stiffness [4]. The curves, as given by an analytic approximation neglecting the gap anisotropy, coincide.

4 Conclusion

In this paper we have evaluated exactly the finite-temperature string tension for the new string model with negative stiffness in the limit of large spacetime dimension d. For the region of small to moderate temperatures we have found with large accuracy an analytic approximation to the solution up to the deconfinement temperature. The derived results for the effective string tension and an approximate deconfinement temperature coincide precisely with the corresponding ones of a pure Nambu-Goto and spontaneous string. They are also not far from those extracted from the Monte-Carlo simulations of the lattice gauge models. Presumably, the answer to the question about which of the models provides for a better surface representation of the string between quarks in QCD should be found in the study of the high-temperature limit of the corresponding string tensions.

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