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ONE-LOOP POTENTIAL IN THE NEW STRING MODEL WITH NEGATIVE STIFFNESS

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Однопетлевой потенциал в новой модели релятивистской струны с отрицательной жесткостью

Конфигурации глюонных полей представляют собой трубки хромоэлектрического потока между кварками (струны) конечной толщины. Поэтому они оказывают конечное сопротивление изменению внешней кривизны. В отличие от прежней модели релятивистской струны с жесткостью Полякова—Кляйнерта и аналогично свойствам магнитных потоков в сверхпроводниках второго рода, в работе предполагается, что жесткость квантовомеханических струн является отрицательной. Предложена новая модель релятивистской струны с отрицательной жесткостью, в которой пропагатор не имеет лишнего нефизического полюса, как в модели Полякова—Кляйнерта. В случае больших расстояний получена однопетлевая поправка к линейнорастущему межкварковому потенциалу, генерируемому новой моделью струны, которая имеет вид универсального люшеровского члена.

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Kleinert H., Chervyakov A.M., Nesterenko V.V. E2-96-380
One-loop Potential in the New String Model with Negative Stiffness

The color-electric flux tube between quarks has a finite thickness therefore also a finite curvature stiffness. Contrary to earlier rigid-string proposal by Polyakov and Kleinert and motivated by the properties of a magnetic flux tube in a type-II superconductor, we put forward the hypothesis that the stiffness is negative. We set up and study the properties of an idealized string model with such negative stiffness. In contrast to the rigid string, the propagator in the new model has no unphysical pole. One-loop calculations show that the model generates an interquark potential which does not contain the square root singularity even for moderate values of a negative stiffness. At large distances, the potential has usual linearly rising term with the universal Lüscher correction.

The investigation has been performed at the Laboratory of Computing Techniques and Automation and at the Bogoliubov Laboratory of Theoretical Physics, JINR.

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1 Introduction

The Nambu-Goto string which is characterized only by tension and cannot represent the color-electric flux tube between quarks. First, it is consistent only in unphysical space dimensions. Second, the interquark potential has an imaginary part at short distances, this being a manifestation of a tachyonic state in the mass spectrum of the string. Third, the flux tube has a finite thickness and a nonzero curvature stiffness. For these reasons, Polyakov [1] and Kleinert [2] have proposed a rigid-string model (also called smooth, or spontaneous string with stiffness). Investigations of this model have revealed a number of appealing properties [1–9]: a realistic interquark potential, asymptotic freedom at short distances, and a reasonable estimate of the deconfinement temperature. Unfortunately, these positive results have married by an unphysical " ghost" pole in the propagator. This pole is generated by the second derivatives with respect to the string coordinates in the rigid-string action. This drawback has been emphasized in Ref. [2], and no way for overcoming this difficulty has yet been found [10–14].

In the present paper, a possible solution to this problem is proposed. A new string action is set up which contains a simple rational function of the Laplace-Beltrami operator on the string world surface. The new action is non-local and contains an infinite series of derivatives to an arbitrary order. The form of the string action was suggested by studies of the energy spectrum of a magnetic flux tube in a type-II superconductor in the London limit [15-17]. The results was then simplified to consist the essential features of this spectrum. It contains only one parameter and exhibits a negative stiffness. The propagator arising from the new action has no unphysical poles in contrast with the model of Refs. [1,2]. The absence of such poles is intimately related with the negative sign of the stiffness.

Note, however, that it is impossible to avoid the consistency problem of the rigid-string theory via a formal changing of the sign of a parameter corresponding to stiffness. The action for such rigid string with a negative stiffness was derived effectively from a field theory with spontaneous symmetry breaking as a first-order correction to the Nambu-Goto action in powers of the tube width [18–20]. Even though the propagator in this model does not contain an unphysical pole, nevertheless, contrary to our proposal, it still suffers from the lack of lower-energy bound which is independent of the sign of stiffness.

The new model provides a basis for consistent perturbative calculations to be done in this paper. In the one-loop approximation we obtain a linearly rising interquark potential with the universal Lüscher correction at large distances. In contrast with the usual rigid-string model and with the Nambu-Goto string the potential has no the square root singularity even for moderate values of a negative stiffness. Certainly, the advantages of the new string model should also be investigated beyond the scope of perturbation theory.

The layout of the paper is as follows. In Section 2, the new string action is introduced and its basic properties are discussed. In Section 3, the action is expanded in powers of the transverse displacements, and a perturbation theory is developed. In the one-loop approximation, the interquark potential generated by this string is calculated. The Conclusion (Section 4) discusses the results and perspectives. Some mathematical details in calculating the potential are given in Appendix A.

2 New string action

The new string model is described by the following action

$$\mathcal{A} = \frac{M_0^2\,\Lambda_0^2}{2}\int d^2\xi\,\sqrt{g}\,g^{ij}\,D_i x^\mu\,\frac{1}{\Lambda_0^2 - D^2}\,D_j x^\mu\,, \tag{2.1}$$

where $x^\mu(\xi)$ with $\mu = 0, 1, \ldots, d-1$ are the string coordinates in a d-dimensional spacetime parametrized by ξ^i, $i = 0, 1$. We shall investigate the euclidean version of

the model. The induced metric on the string world surface is $g_{ij} = \partial_i x^\mu\,\partial_j x_\mu$ and g^{ij} is its inverse, while $g = \det(g_{ij})$. Covariant differentiation with respect to ξ^i is denoted by D_i, and $D^2 = D_i D^i$ is the Laplace-Beltrami operator

$$D^2 = \frac{1}{\sqrt{g}}\,\frac{\partial}{\partial\xi^i}\left(\sqrt{g}\,g^{ij}\,\frac{\partial}{\partial\xi^j}\right). \tag{2.2}$$

The parameters M_0 and Λ_0 have the dimension of a mass.

Neglecting D^2 in the denominator in Eq. (2.1), we immediately obtain the Nambu-Goto string with tension M_0^2. Expanding $(\Lambda_0^2 - D^2)^{-1}$ in powers of D^2 and retaining only the first two terms, we arrive at the rigid-string model with a *negative* stiffness $-M_0^2/\Lambda_0^2$. Note that despite this property, no instability problem arises since the quadratic action is positive definite.

The length scale $1/\Lambda_0$ plays the role of a thickness of the color-electric flux tube between quarks. Another length scale, an *intrinsic thickness*, arises naturally in our model after performing a loop expansion of the action (2.1) and restricts the range of the internal string excitations to be short, since the long-range modes are forbidden, and thus unobservable. The string has an intrinsic thickness due to virtual loops, just like a magnetic flux tube in a type-II superconductor.

Obviously, the action (2.1) is nonlocal due to the operator D^2 in the denominator. This is not a disqualifying property – we only remind the reader that the action of Maxwell-Dirac electrodynamics is nonlocal in the Coulomb gauge. We could easily introduce a set of auxiliary fields $\sigma_i^\mu(\xi)$ into the functional integral of the partition function creating local action:

$$\mathcal{A}_{\rm loc} = \int d^2\xi\sqrt{g}\left\{\sigma_i^\mu(\xi)\,D^i x^\mu(\xi) + \frac{1}{2\,M_0^2\,\Lambda_0^2}\,\sigma_i^\mu(\xi)\,(\Lambda_0^2 - D^2)\,\sigma_\mu^i(\xi)\right\}, \tag{2.3}$$

which is completely equivalent to (2.1). In fact, there are local models in the physics of biomembranes which give rise to action like (2.1) [15, 16]. However, for our purpose the initial nonlocal action (2.1) will be more convenient.

3 One-loop interquark potential

For perturbative calculations we employ a Gauss parametrization of the string world surface

$$x^\mu(\xi) = \left(\xi^0, \xi^1, x^2(\xi), \ldots, x^{d-1}(\xi)\right) = \left(\xi^i, \mathbf{u}(\xi)\right), \quad i = 0, 1. \tag{3.1}$$

The vector field $u^a(\xi)$ with $a = 2, 3, \ldots, d-1$ describes $(d-2)$ - transverse displacements of the string position vector $x^2, x^3, \ldots, x^{d-1}$. The components of the two-dimensional vector $\xi^i = (\xi^0, \xi^1)$ will be also denoted by t and r, respectively. In the parametrization (3.1), the induced metric g_{ij} on the world surface of the string is

$$g_{ij} = \delta_{ij} + (\mathbf{u}_i\mathbf{u}_j), \quad g^{ij} = g^{-1}[(1 + \mathbf{u}_k^2)\,\delta_{ij} - (\mathbf{u}_i\mathbf{u}_j)]\,,$$
$$\mathbf{u}_i \equiv \partial\mathbf{u}/\partial\xi^i\,, \tag{3.2}$$
$$g = \det(g_{ij}) = 1 + \mathbf{u}_i^2 + \frac{1}{2}\mathbf{u}_j^2\mathbf{u}_k^2 - \frac{1}{2}(\mathbf{u}_i\mathbf{u}_j)^2\,.$$

In the following, we treat the derivatives of $\mathbf{u}(\xi)$ as small quantities. Expanding up to the fourth order in $\mathbf{u}$, we obtain [21]

$$\sqrt{g} \simeq 1 + \frac{1}{2}\mathbf{u}_i^2 + \frac{1}{8}\mathbf{u}_i^2\mathbf{u}_k^2 - \frac{1}{4}(\mathbf{u}_i\mathbf{u}_j)^2\,,$$
$$\frac{1}{\sqrt{g}} \simeq 1 + \frac{1}{2}\mathbf{u}_i^2 + \frac{1}{8}\mathbf{u}_i^2\mathbf{u}_k^2 - \frac{1}{4}(\mathbf{u}_i\mathbf{u}_j)^2\,, \tag{3.3}$$
$$D^2 \simeq \partial^2 - \Big[(\mathbf{u}_{ii}\mathbf{u}_k) - (\mathbf{u}_{ii}\mathbf{u}_k)\,\mathbf{u}_j^2 + (\mathbf{u}_i\mathbf{u}_{ik})\,\mathbf{u}_j^2 -$$
$$-(\mathbf{u}_i\mathbf{u}_j)(\mathbf{u}_{ik}\mathbf{u}_j) - (\mathbf{u}_i\mathbf{u}_{ij})(\mathbf{u}_j\mathbf{u}_k)\Big]\,\partial_k\,,$$

where ∂^2 stands for the two-dimensional Laplace operator $\partial^2 = \partial^2/\partial t^2 + \partial^2/\partial r^2$. Summation over repeated indices is assumed everywhere. Making use of (3.2) and (3.3), we expand

$$\frac{1}{2}\sqrt{g}\,g^{ij}\,D_i\,\xi^k(\Lambda_0^2 - D^2)^{-1}D_j\xi^k \simeq \frac{1}{2\,\Lambda_0^2}\sqrt{g}\,g^{kk}$$
$$\simeq \frac{1}{\Lambda_0^2}\left(1 - \frac{1}{8}\mathbf{u}_i^2\,\mathbf{u}_i^2 + \frac{1}{4}(\mathbf{u}_i\mathbf{u}_j)^2\right)\,,$$
$$\sqrt{g}\,g^{ij}\,D_i u^a\,(\Lambda_0^2 - D^2)^{-1}\,D_j u^a \simeq \tag{3.4}$$
$$\simeq \left[(1 + \frac{1}{2}\mathbf{u}_i^2)\,u_j^a - (\mathbf{u}_i\mathbf{u}_j)\,u_i^a\right](\Lambda_0^2 - \partial^2)^{-1}u_j^a - u_k^a\,(\Lambda_0^2 - \partial^2)^{-2}(\mathbf{u}_{ii}\mathbf{u}_l)\,u_{kl}^a\,.$$

After substitution of the expansion (3.3) and (3.4) into (2.1), we obtain up to the fourth order in $\mathbf{u}$:

$$\mathcal{A} = M_0^2\int d^2\xi\,\Big\{1 + \frac{1}{2}\Lambda_0^2\,\mathbf{u}_i\,(\Lambda_0^2 - \partial^2)^{-1}\mathbf{u}_i - \frac{1}{8}[\mathbf{u}_i^4 - 2\,(\mathbf{u}_i\mathbf{u}_j)^2] +$$
$$\frac{\Lambda_0^2}{4}[\mathbf{u}_i^2\,\mathbf{u}_j - 2\,(\mathbf{u}_i\mathbf{u}_j)\,\mathbf{u}_i]\,(\Lambda_0^2 - \partial^2)^{-1}\mathbf{u}_j - \frac{\Lambda_0^2}{2}\,\mathbf{u}_i\,(\Lambda_0^2 - \partial^2)^{-2}(\mathbf{u}_{kk}\mathbf{u}_j)\,\mathbf{u}_{ij}\Big\}\,. \tag{3.5}$$

The corresponding free energy contains the contributions from Feynman diagrams depicted in Fig.1.

In the quadratic approximation, the field $\mathbf{u}(\xi)$ obeys the wave equation

$$(\Lambda_0^2 - \partial^2)^{-1}\,\partial^2\,u^a(\xi) = 0\,. \tag{3.6}$$

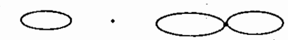
$\langle \mathcal{A} \rangle$ = 

Figure 1: The Feynman diagrams corresponding to the loop expansion (3.5).

It is an essential point that this equation has the same plane-wave solutions as the usual D'Alembert equation:

$$\partial^2\,u^a(\xi) = 0 \quad \text{or} \quad k^2\,u^a(k) = 0\,, \tag{3.7}$$
$$a = 2,\,3,\,\ldots,\,d-1\,.$$

The correlation functions of the equations (3.7) and (3.8) are, however, quite different. In momentum space, this function has the form

$$G(\mathbf{k}) = \frac{\Lambda_0^2 + k^2}{k^2}\,, \tag{3.8}$$

in contrast to the usual string propagator $1/k^2$.

A quantum field theory in euclidean space with the wave equation (3.6) contains two kinds of quanta, one with $k^2 = 0$, and another one with $k^2 = \infty$. An important property of the latter quanta is that they do not exist as free states. They influence the system only as internal lines in Feynman diagrams. As will be shown below, these quanta give a *negative contribution* to the total energy of the system. Their absence in free states prevents the energy from the lack of its lowest bound, thus avoiding an unstability problem which arising usually in the higher-derivative theories associated with nonlocality (see, for example, Ref. [22] and references therein).

The existance of the internal quanta in our model leads immediately to the conclusion that the description of string fluctuations with the action (3.5) is necessarily restricted to the long wavelengths. This implies that, in addition to the thickness $1/\Lambda_0$, the string has yet another length scale, an intrinsic thickness, due to virtual loops, just like a magnetic flux tube in a type-II superconductor. The internal string excitations are restricted within a small range around the flux tube, since the long-range interacted string modes are forbidden, and thus unobservable. This description of the quantum string fluctuations are very close to that of the fluctuations in a type-II superconductor [23].

In the following it will be sufficient that the correlation function (3.8), fortunately, has only a single pole at $k^2 = 0$ and no unphysical poles in contrast to the previous rigid-string model with a positive curvature stiffness. At large k^2, the new correlation function does not vanish but goes to unity. This requires some care in the loop calculations. We define the interquark potential in the usual way

$$\exp[-T\,V(R)] = \int Du\,\exp(-\mathcal{A}^{\mathrm{TR}}),\ T \to \infty,$$

where A^{TR} is the euclidean action (3.5) for finite ranges $(0,\, T)$ and $(0,\, R)$ of "time" $x^0 = \xi^0$ and coordinate $x^1 = r$, respectively. The functional integral should be done with "field" variables $\mathbf{u}(\xi)$ periodic in ξ^0: $\mathbf{u}(\xi^0,\, \xi^1) = \mathbf{u}(\xi^0 + T,\, \xi^1)$.

Confining ourselves to the quadratic approximation in (3.5), which is sufficient to determine the interquark potential generated by the new string model in the one-loop approximation, we obtain

$$V(R) = \lim_{T\to\infty} \frac{1}{T}\left[\int_0^T dt \int_0^R dr\, M_0^2 + \frac{d-2}{2}\,\mathrm{Tr}\,\ln G^{-1}\right], \tag{3.9}$$

where G^{-1} is the operator generated by the quadratic part of the action (3.5)

$$G^{-1} = \frac{\partial^2}{\Lambda_0^2 - \partial^2}\,.$$

This involves evaluating a single bubble diagram in Fig.1. In momentum space, the trace Tr reduces to the following operations

$$\mathrm{Tr}\,\ldots = \frac{T}{2}\int \frac{d\omega}{2\pi}\sum_n \ldots\,, \quad \omega = k^0,$$

where the sum runs over the proper discrete values of the spatial component of k_n^1. Now Eq. (3.9) acquires the form

$$V(R) = \lim_{T\to\infty} \frac{1}{T}\int_0^T dt \int_0^R dr\left[M_0^2 + \frac{d-2}{2}\, f^{\mathrm{R}}\right]. \tag{3.10}$$

Here $(1/2)f^{\mathrm{R}}$ is the density of the free energy of a single scalar field $u^a(\xi)$ with the propagator $(\Lambda_0^2 + k^2)/k^2$ in the euclidean space

$$f^{\mathrm{R}} = \frac{1}{2R}\sum_n \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \ln \frac{\omega^2 + k_n^2}{\omega^2 + k_n^2 + \Lambda_0^2}\,. \tag{3.11}$$

The admissable values of the wave vector k_a for the field $\mathbf{u}(\xi)$ are determined by the boundary conditions. We shall treat a string connecting infinitely heavy quarks. Then the field $\mathbf{u}(\xi)$ must vanish at the string ends. Therefore

$$k_n = \frac{n\pi}{R}, \quad n = \pm 1, \pm 2, \ldots\,. \tag{3.12}$$

For the regularization of the divergences and their absorption in the renormalized parameters of the theory, it is convenient to replace f^{R} by the expression $f^\infty + (f^{\mathrm{R}} - f^\infty)$, where f^∞ means the same density of the free energy but for an infinite space interval

$$f^\infty = \int \frac{d^2 k}{(2\pi)^2} \ln \frac{k^2}{k^2 + \Lambda_0^2} = -\frac{\Lambda_0^2}{4\pi^2}(1 + 4\pi L_0)\,. \tag{3.13}$$

Here L_0 is a logarithmically divergent integral

$$L_0 = \int \frac{d^2 k}{(2\pi)^2}\frac{1}{k^2 + \Lambda_0^2} = \frac{1}{4\pi}\ln\frac{K^2}{\Lambda_0^2}\,,$$

and K is an ultraviolet cutoff.

We now write for f^{R} the representation

$$f^{\mathrm{R}} = f^\infty + (f^{\mathrm{R}} - f^\infty) = -\frac{\Lambda_0^2}{4\pi}(1 + 4\pi L_0) +$$

$$\frac{1}{2R}\left(\sum_{n=-\infty}^{\infty}{}' - \int_{-\infty}^{\infty} dn\right)\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\ln\frac{\omega^2 + k_n^2}{\omega^2 + k_n^2 + \Lambda_0^2}\,, \tag{3.14}$$

where the prime in the sum means that the term with $n = 0$ is absent. The sum in (3.14) gives f^{R} and the integral over n reproduces f^∞. In the analytical regularization, the ω-integration in (3.14) can be performed by the following formula

$$\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\ln(\omega^2 + a^2) = \sqrt{a^2}\,. \tag{3.15}$$

Upon integration, the difference $f^{\mathrm{R}} - f^\infty$ reads

$$\Delta f = f^{\mathrm{R}} - f^\infty = \frac{\pi}{2R^2}\left(\sum_{n=-\infty}^{\infty}{}' - \int_{-\infty}^{\infty} dn\right)\left(\sqrt{n^2} - \sqrt{n^2 + \Lambda_0^2}\right), \tag{3.16}$$

where Λ_{0R}^2 is the dimensionless constant, $\Lambda_{0R}^2 = \Lambda_0^2 R^2/\pi^2$.

In fact, equation (3.16) is a regularized energy of the zero-point oscillations of two kinds with frequencies $\overset{(1)}{\omega}_n = \sqrt{n^2}$ and $\overset{(2)}{\omega}_n = \sqrt{n^2 + \Lambda_{0R}^2}$. The contribution of the frequencies $\overset{(2)}{\omega}_n$ to the energy is *negative*. Nevertheless, this does not ruin the consistency of the model, because the quanta with $\overset{(2)}{\omega}_n$ do not exist as free states (see analysis of the solutions to the wave equation (3.6) in the preceding Section).

For the difference $\Delta f = f^{\mathrm{R}} - f^\infty$, equation (3.16) gives a finite result because the divergences in the sum and integral cancel with each other. After some calculations, whose details are presented in Appendix A, we cast the formula (3.10) for the interquak potential into the form

$$V(R) = \lim_{T\to\infty}\frac{1}{T}\int_0^T dt\int_0^R dr\left[M_0^2 + \frac{d-2}{2}(f^\infty + \Delta f)\right] =$$

$$= M^2 R - \frac{d-2}{2}\left[\frac{\pi}{12R} - \frac{\Lambda_0}{2} - \frac{\Lambda_0}{\pi}\sum_{n=1}^{\infty} n^{-1} K_1(2n\Lambda_0 R)\right]. \tag{3.17}$$

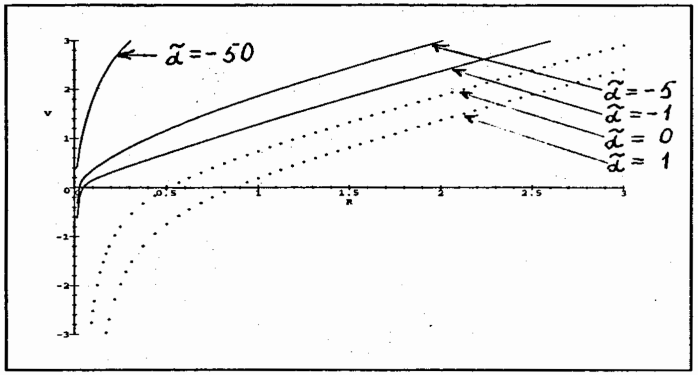


Figure 2: The reduced potential (3.19), as a dimensionless function of $\tilde{M}R$, is shown for $\tilde{\alpha} = -1, -5, -50$. The dotted curves represent the potentials of the usual rigid string with $\tilde{\alpha} = 1$ and the Nambu-Goto string, where $\tilde{\alpha} = 0$, respectively. The curves with a negative stiffness are seen to lie upper than that with a positives ones. The square root singularity vanishes at the value $\tilde{\alpha} = -50$.

Here M^2 is the renormalized string tension

$$M^2 = M_0^2 - \frac{d-2}{2}\frac{\Lambda_0^2}{4\pi}(1 + 4\pi L_0), \tag{3.18}$$

whereas, the coupling constant Λ_0^2 receives no correction in one-loop level. The linear term and its first $1/R$ - correction in Eq. (3.17) are analogous completely to that in the Nambu-Goto model and in the usual rigid string. In contrast with these models, however, the contributions to the interquark potential (3.17) from the last terms are positive. Therefore the increasing of the absolute value of a negative stiffness pushes the point of a square root singularity, where the potential becomes zero, to more smaller values until its removing finally. The numerical computations show that the square root singularity in the interquark potential (3.17) vanishes at the value of a negative stiffness $\tilde{\alpha} = -50$ (see Fig.2).

In order to display graphically the behavior of $V(R)$ we introduce the negative stiffness $\tilde{\alpha} = -\Lambda_0^2/\tilde{M}^2$, where $\tilde{M}^2 = 2M^2/(d-2)$, and define the reduced potential

$$\tilde{V} \equiv \frac{2}{(d-2)}\frac{1}{\tilde{M}} V(\tilde{M}, R), \tag{3.19}$$

which is a dimensionless function of $\tilde{M}R$. This potential is plotted in Fig.2 for various values of $\tilde{\alpha}$.

Equation (3.17) for the interquark potential is a convenient starting point for the investigation of the behaviour at large R, since the modified Bessel function $K_1(z)$ decays exponentially for large values of z. In this limit, the first correction to

a linearly rising part has the form $-(d-2)\pi/(24R)$ that is exactly the universal Lüscher term while the other terms vanish exponentially.

4 Conclusion

The new string model proposed here reproduces, to lowest orders in the coupling constant $1/\Lambda_0^2$, the properties of the Nambu-Goto model. The string possesses a linearly rising interquark potential with the standard Lüscher correction at large distances. The model has a negative extrinsic curvature stiffness, and no unphysical pole in the propagator generated, despite the presence of a k^4-term in an expression (3.5).

Certainly, it is of great interest to study the new string model nonperturbatively in order to find out the critical dimension of spacetime. These issues will be considered in forthcoming publications.

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Appendix A
One-loop calculation of the interquark potential

Here we present some mathematical details of the one-loop calculation of the potential generated by the new string. For shortening the formulas, it is convenient to introduce the function

$$S(x) = \frac{\pi}{2R^2}\left(\sum_{n=-\infty}^{\infty} - \int_{-\infty}^{\infty} dn\right)\sqrt{n^2+x}. \tag{A.1}$$

In terms of $S(x)$ the difference Δf is written as

$$\Delta f = S(0) - S(\Lambda_{0R}^2) + \frac{\pi}{2R^2}\Lambda_{0R}. \tag{A.2}$$

The last term in (A.2) removes the term with $n = 0$ in the sum in (A.1). Now we represent the square root in (A.1) using the following identity

$$\frac{1}{(n^2+x)^s} = \frac{1}{\Gamma(s)}\int_0^{\infty} t^{s-1}e^{-(n^2+x)t}dt, \tag{A.3}$$

where $\Gamma(s)$ is the Euler gamma function, $\Gamma(-1/2) = -2\sqrt{\pi}$. Equation (A.1) acquires the form

$$S(x) = \frac{\pi}{2R^2\,\Gamma(-1/2)}\int_0^{\infty} dt\, t^{-3/2}e^{-tx}\left(\sum_{n=-\infty}^{\infty} - \int_{-\infty}^{\infty} dn\right)e^{-n^2t}. \tag{A.4}$$

Integration over n is easily performed but the sum should be rewritten by employing the duality transformation [24]

$$\sum_{n=-\infty}^{+\infty} e^{-n^2t} = \sqrt{\frac{\pi}{t}}\sum_{n=-\infty}^{\infty}\exp\left\{-\frac{\pi^2}{t}n^2\right\}. \tag{A.5}$$

The result is

$$S(x) = \frac{\pi^{3/2}}{R^2\,\Gamma(-1/2)}\sum_{n=1}^{\infty}\int_0^{\infty} dt\, t^{-2}\exp\left(-tx - \frac{n^2\pi^2}{t}\right) \tag{A.6}$$

Using here the following integral representation for the modified Bessel function

$$K_\nu(2\sqrt{\beta\gamma}) = \frac{1}{2}\left(\frac{\gamma}{\beta}\right)^{\frac{\nu}{2}}\int_0^{\infty} x^{\nu-1}\exp\left(-\gamma x - \frac{\beta}{x}\right)dx \tag{A.7}$$

we finally obtain [24, 25]

$$S(x) = -\frac{\sqrt{x}}{R^2}\sum_{n=1}^{\infty}\frac{K_1(2\pi n\sqrt{x})}{n}. \tag{A.8}$$

To calculate $S(0)$ one should take into account that $K_1(z) \to z^{-1}$ when $z \to 0$. Hence,

$$S(0) = -\frac{1}{2\pi R^2}\sum_{n=1}^{\infty}\frac{1}{n^2} = -\frac{\zeta(2)}{2\pi R^2}, \tag{A.9}$$

where $\zeta(s)$ is the Riemann zeta function, $\zeta(2) = \pi^2/6$.

Now the difference (A.2) assumes the form

$$\Delta f = -\frac{\pi}{12R^2} + \frac{\Lambda_0}{2R} + \frac{\Lambda_0}{\pi R}\sum_{n=1}^{\infty} n^{-1}K_1(2n\Lambda_0 R), \tag{A.10}$$

from which the final formula (3.17) for the potential $V(R)$ follows immediately.

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