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ON THE CALCULATION OF THE INTERQUARK POTENTIAL GENERATED BY A STRING WITH MASSIVE ENDS

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К расчету межкваркового потенциала, генерируемого струной с массами на концах

Предложена итерационная процедура решения системы вариационных уравнений, определяющих стационарную точку действия струны с массами на концах. Полученное итерационное решение используется для вычисления струнного функционального интеграла и оценки поправок к межкварковому потенциалу, генерируемому струной. Расчеты показали, что высшие поправки малы. Нулевое приближение достаточно хорошо воспроизводит межкварковый потенциал, генерируемый струной с массами на концах.

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Nesterenko V.V., Pirozhenko I.G. E2-96-376
On the Calculation of the Interquark Potential Generated by a String with Massive Ends

It is proposed to solve the variational equations, defining the stationary point of the effective action of a string with massive ends, by means of iterative procedure. The obtained solution is used to evaluate the string functional integral. After that the corrections to interquark potential generated by a string with masses at the ends are calculated. The calculations show that the higher corrections prove to be small and zero approximation fairly well reproduces the interquark potential generated by the Nambu-Goto string with massive ends.

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1. Interquark potential generated by the Nambu-Goto string with point-like masses at ends was at first calculated in [1]. A standard method [3,4] based on variational estimation of the string functional integral in $D \to \infty$ limit was applied (D is the space-time dimension). However, the variational equations obtained in [1] were not solved exactly. Some approximation was used there. The aim of the present note is to justify the applicability of such approximation.

2. Let us remind briefly the derivation of the variational equations in the considered problem. The starting point is the well-known formula

$$\exp\left[-\beta V(R)\right] = \int [Du] \exp\left\{-S^{\beta}[u]\right\}, \quad \beta \to \infty, \tag{1}$$

relating interquark potential $V(R)$ with the string functional integral. The Euclidean action of the string with masses at its ends is given by the formula

$$S^{\beta}[u] = {M_0}^2 \int\limits_0^{\beta} dt \int\limits_0^{R} dr \sqrt{\det(\delta_{ij} + \partial_i \mathbf{u} \partial_j \mathbf{u})}$$

$$+ \sum_{a=1}^{2} m_a \int\limits_0^{\beta} \sqrt{1 + \dot{\mathbf{u}}^2(t,r)} \qquad i,j = 0,1, \tag{2}$$

where ${M_0}^2$ is the string tension, $\beta = T^{-1}$ is the inverse temperature, and string coordinates $\mathbf{u}(t,r)$ satisfy the periodicity condition

$$\mathbf{u}(0,r) = \mathbf{u}(\beta,r). \tag{3}$$

In the present case $1/(D-2)$ expansion is obtained by means of the following procedure [3]. The composite fields σ_{ij} for $\partial_i \mathbf{u} \partial_j \mathbf{u}$ are introduced, and the constraint $\delta_{ij} = \partial_i \mathbf{u} \partial_j \mathbf{u}$ is taken into account through the lagrange multipliers $\alpha^{ij}, i,j = 0,1$. To this end, the following presentation of a unit is inserted into the string functional integral (1)

$$1 = \int [D\sigma] \delta\left(\sigma_{ij} - \partial_i \mathbf{u} \partial_j \mathbf{u}\right).$$

After using the integral representation of δ-function we obtain

$$\exp\left[-\beta V(R)\right] = \int [D\mathbf{u}][D\alpha][D\sigma]\exp\left\{-S^\beta[\mathbf{u},\alpha,\sigma]\right\}, \quad (4)$$

where

$$S^\beta\left[\mathbf{u},\alpha,\sigma_i\right] = \frac{M_0^2}{2}\int_0^\beta dt \int_0^R dr \left[2\sqrt{\det(\delta_{ij}+\sigma_{ij})} + \alpha^{ij}(\partial_i \mathbf{u}\partial_j\mathbf{u} - \sigma_{ij})\right]$$

$$+ \sum_{a=1}^{2} m_a \int_0^\beta dt\sqrt{1+\dot{\mathbf{u}}^2(t,r_a)} \quad (5)$$

$$i,j = 0,1, \quad r_1 = 0, \quad r_2 = R.$$

Functional integral over $\mathbf{u}$ can be done exactly as the action is quadratic in string coordinates. Functional integrals over α and σ are evaluated by variational method and in $(D-2)\to\infty$ limit they are equal to the integrand value at the stationary point of the action (5). The functional variables α_{ij} and σ_{ij} are diagonal matrices independent of t and r, $\alpha^{ij} = \delta^{ij}\alpha^j, \sigma_{ij} = \delta_{ij}\sigma_j$.

The action (5) leads to the following equations of motion

$$\Delta_\alpha = \alpha^0\ddot{\mathbf{u}} + \alpha^1\mathbf{u}'' = 0 \quad (6)$$

and boundary conditions

$$\frac{m_1}{\sqrt{1+\sigma_0}}\ddot{\mathbf{u}} = -{M_0}^2\alpha^1\acute{\mathbf{u}} \quad r = 0, \quad (7)$$

$$\frac{m_2}{\sqrt{1+\sigma_0}}\ddot{\mathbf{u}} = {M_0}^2\alpha^1\acute{\mathbf{u}} \quad r = R. \quad (8)$$

Integrating by parts the second term in (5) and taking into consideration the boundary conditions (7),(8) we arrive at the effective action

$$S^\beta\left[\mathbf{u},\alpha,\sigma\right] = \frac{M_0^2}{2}\int_0^\beta dt\int_0^R dr\, \mathbf{u}(-\Delta_a)\mathbf{u}$$

$$+ M^2\beta R\left[\sqrt{(1+\sigma_0)(1+\sigma_1)} - \frac{1}{2}(\alpha^0\sigma_0 + \alpha^1\sigma_1)\right], \quad (9)$$

which is quadratic in string coordinates. This action does not include exactly the string ends contributions. The latter are taken into account through the definition of the operator $(-\Delta_\alpha)$. Therefore after functional integration over $\mathbf{u}$ the term

$$\frac{D-2}{2}\, Tr\ln(-\Delta_\alpha) = \frac{1}{2}\sum_{k=1}^{\infty}\omega_k, \quad (10)$$

appears in the effective action [1]. Here ω_k are solutions of the frequency equation

$$\tan(\omega R) = \frac{2mM_0^2\omega\alpha^0\sqrt{1+\sigma_0}}{m^2\omega^2 - M_0^2\alpha^0\sqrt{1-\sigma_0}}, \quad (11)$$

following from (6)-(8). Taking into account (10) we obtain the effective string action

$$S^\beta = M_0^2\beta R\left\{\sqrt{(1+\sigma_0)(1+\sigma_1)} - \frac{1}{2}(\alpha^0\sigma_0 + \alpha^1\sigma_1)\right\}$$

$$+ \beta(D-2)\sqrt{\frac{\alpha^1}{\alpha^0}}E_c(\alpha^0,\sigma_0), \quad (12)$$

where

$$E_c(\alpha^0,\sigma_0) = \frac{1}{2}\sum_k \omega_k =$$

$$\frac{1}{2\pi}\int_0^\infty dy\ln\left\{1 - e^{-2Ry}\left(\frac{my - M_0^2\alpha^0\sqrt{1+\sigma}}{my + M_0^2\alpha^0\sqrt{1+\sigma_0}}\right)\right\} \quad (13)$$

is renormalised Casimir energy [1,2], depending on variational parameters $\alpha^0; \sigma_0$

The stationary point of this action is defined from the variational equations

$$\alpha^0 = \sqrt{\frac{1+\sigma_1}{1+\sigma_0}} + \frac{D-2}{M_0^2 R}\sqrt{\frac{\alpha^1}{\alpha^0}}\frac{\partial E_c(\alpha_0,\sigma_0)}{\partial\sigma_0}, \quad (14)$$

$$\alpha^1 = \sqrt{\frac{1+\sigma_0}{1+\sigma_1}}\,, \tag{15}$$

$$\sigma_0 = -\frac{D-2}{M_0^2 R\alpha^0}\sqrt{\frac{\alpha^1}{\alpha_0}}E_c(\alpha^0,\sigma_0) + 2\frac{D-2}{M_0^2 R}\sqrt{\frac{\alpha^1}{\alpha^0}}\frac{\partial E_c(\alpha^0,\sigma_0)}{\partial\alpha^0}\,, \tag{16}$$

$$\sigma_1 = \frac{D-2}{M_0^2 R}\frac{E_c(\alpha^0,\sigma_0)}{\sqrt{\alpha^0\alpha^1}}\,. \tag{17}$$

Solution of the nonlinear equations (14)-(17) should be substituted in (12). This enables one to get the interquark potential in the string model under consideration.

3. When solving the variational equations (14)-(17) in paper [1], the following approximation has been used. The Casimir energy $E_c(\alpha^0,\sigma_0)$, entering the right-hand sides of Eqs. (14)-(17) was calculated at fixed values of variational parameters α^0 and σ_0: $\alpha^0 = 1$ and $\sigma_0 = 0$. In definite sense this corresponds to the nonrelativistic approximation for quarks dynamics (1). In this approximation the derivatives of the Casimir energy over variational parameters α^0 and σ_0 are obviously equal to zero. As a result, the solution of the system (14)-(17) is

$$\alpha^0 = \sqrt{1-2\lambda},\quad \alpha^1 = \frac{1}{\sqrt{1-2\lambda}},\quad \sigma_0 = \frac{\lambda}{1-2\lambda},\quad \sigma_1 = -\lambda, \tag{18}$$

where $\lambda = -(D-2)\,E_c/(M_0^2 R)$ [3].

In general case the following expressions

$$\frac{\partial E_c(\alpha^0,\sigma_0)}{\partial\alpha^0} = \frac{2M_0^2\sqrt{1+\sigma_0}}{\pi m}\int\limits_0^\infty dy\,\frac{(y-\eta)e^{-2y}}{(y-\eta)^3 - e^{-2y}(y-\eta)^2(y+\eta)},$$

$$\frac{\partial E_c(\alpha^0,\sigma_0)}{\partial\sigma_0} = \frac{\alpha^0 M_0^2}{\pi m\sqrt{1+\sigma_0}}\int\limits_0^\infty dy\,\frac{(y-\eta)e^{-2y}}{(y-\eta)^3 - e^{-2y}(y-\eta)^2(y+\eta)}, \tag{19}$$

should be substituted in the right-hand sides of (14)-(17). Here $\eta = \alpha^0\sqrt{1+\sigma_0}M_0^2 R/m$.

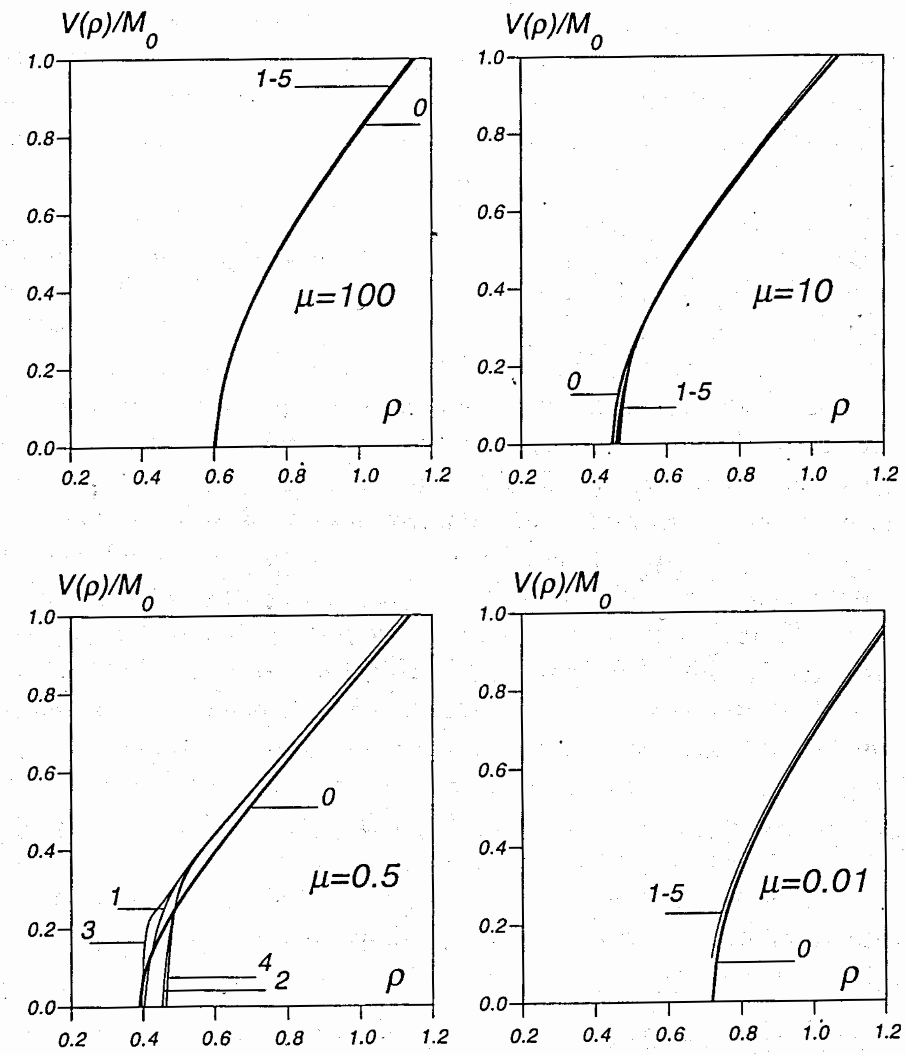

Fig.1. Interquark potential obtained by means of the iterative solution of variational equations system (14)-(17). Curve's numbers are equal to the number of iterations. The curve **0** corresponds to zero approximation ($\alpha^0 = 1, \sigma_0 = 0$). The following dimensionless parameters are used: $\rho = M_0 R$, $\mu = m/M_0$.

Now we are going to construct an iterative solution of the exact equations (14)–(17). To this end we shall treat the terms $\partial E_c/\partial\sigma_0$ and $\partial E_c/\partial\alpha_0$ there as small corrections. To start with, we take the solution (18) as a zero approximation. Substituting it into the right-hand sides of (14)–(17), we obtain some values of α^0, α^1, σ_0, σ_1. We put them into the right-hand sides of (14)–(18) and so on. At each step we shall calculate the effective string action (12) obtaining the corresponding interquark potential and comparing it with the zero approximation.

The calculation results for different quark masses are presented in Fig.1. At large interquark distances the potential rises linearly and corrections to zero approximation are small. For heavy quarks ($\mu = m/M_0 = 100$) the iterative solution coincides with zero approximation at all range of available distances $R > R_c$. For average masses ($\mu \sim 1 - 10$) and not far from $R = R_c$ the calculated plots oscillate near zero approximation with increasing number of iteration. In the limit $R \to R_c$ the corrections are not small and the proposed iterative procedure may give no convergent result. For light quarks $\mu \sim 0.01$ iterative solution insignificantly displaces as a whole to smaller distances in comparison with zero approximation. It is important to note that consideration of Casimir energy dependence on α^0, σ_0 does not lead to significant modification of the critical radius R_c.

Taking all this into account, we infer that approximation (18) works sufficiently well for any quark mass and practically for all distances between quarks.

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