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LIFETIME OF $(\pi^+\pi^-)$ ATOM: ANALYSIS OF THE ROLE OF STRONG INTERACTIONS

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Время жизни $(\pi^+\pi^-)$-атома: анализ роли сильных взаимодействий

Дается вывод формулы для времени жизни $(\pi^+\pi^-)$-атома в рамках формализма уравнений Бете-Солпитера, учитывая вклад сильных взаимодействий в первом приближении теории возмущений. Показано, что время жизни можно выразить через решения кулоновской задачи (волновая функция $1S$ состояния в нуле $\Psi_1(0)$, энергия связи S-состояния E_1), разность S-волновых длин $\pi\pi$-рассеяния и сдвиг энергии связи за счет сильных взаимодействий ΔE_1):

$$\frac{1}{\tau_1} \sim (a_0^0 - a_0^2)^2 \left|\Psi_1(0)\right|^2 \left(1 - \frac{9\Delta E_1}{8E_1}\right).$$

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Lifetime of $(\pi^+\pi^-)$ Atom: Analysis of the Role of Strong Interactions

The expression for the $(\pi^+\pi^-)$ atom lifetime is derived within the Bethe-Salpeter approach. First-order perturbative corrections due to the contribution of strong interactions are taken into account. It is demonstrated that the atom lifetime can be expressed in terms of the solutions of the Coulombic problem (the wave function of the $1S$ state at the origin $\Psi_1(0)$, the binding energy of the S-state E_1), the difference of the S-wave $\pi\pi$ scattering lengths and the energy shift ΔE_1 of the level due to the strong interactions:

$$\frac{1}{\tau_1} \sim (a_0^0 - a_0^2)^2 \left|\Psi_1(0)\right|^2 \left(1 - \frac{9\Delta E_1}{8E_1}\right).$$

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

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At the present time, the experiment is being prepared by the DIRAC collaboration at CERN (Experiment PS212) on measurement of the lifetime of $(\pi^+\pi^-)$ atoms $(A_{2\pi})$ with a 10% accuracy. The first estimation of the lifetime of the atom formed by π^+ and π^- in the ground $(1S)$ state $\tau_1 = 2.9^{+\infty}_{-2.1} \times 10^{-15} s$ was obtained in ref. [1]. From a physical point of view, interest in the experiment on measurement of the $(\pi^+\pi^-)$ atom lifetime stems primarily from the fact that it allows one to determine the difference of the S-wave $\pi\pi$ scattering lengths $a_0^0 - a_0^2$ with the total isospin 0 and 2 in a model-independent way with a 5% accuracy. The obtained experimental information about $\pi\pi$ scattering lengths can provide a decisive test of predictions of the chiral theory [2]. Recently, high precision experiments on the measurement the characteristics of both the pionic hydrogen [3] and pionic deuterium [4] have also been performed.

For the first time the expression relating τ_n to the combination of the S-wave hadronic scattering lengths has been obtained in the paper [5]. In this paper in the framework of the nonrelativistic quantum mechanics the atoms formed of the proton and π^- meson were considered. The main idea of that paper consisted in the factorization of the strong and electromagnetic contributions to the width of the $(p\pi^-)$ atom decay into the pair $n - \pi^0$. Namely, it was assumed that the energy spectrum of the $(p\pi^-)$ bound state was almost fully determined by the Coulomb potential as the Borh radius of the atom $r_B \simeq$ 222 fm was much larger than the strong interaction range. On the other hand, strong interactions were responsible for the decay of the atom. In the lowest-order approximation in the fine-structure constant α the atom decay width was written in a form of the product of the square of the pure Coulombic wave function (w.f.) at the origin and the square of the difference of the S-wave πN scattering lengths [5]. In ref. [6] the analogous formula has been obtained for the case of $\pi^+\pi^-$ atom decay in the nS state (see [7] for the corrected expression)

$$\frac{1}{\tau_n} = \frac{16\pi}{9}\sqrt{\frac{2\Delta m_\pi}{m_\pi}}\frac{(a_0^0 - a_0^2)^2}{1 + 2/9 m_\pi \Delta m_\pi (2a_0^2 + a_0^0)^2}|\Psi_n(0)|^2 \qquad (1)$$

Here the isotopic invariance of pure strong interactions was assumed, relating the charge exchange amplitude at threshold $A(\pi^+\pi^- \to \pi^0\pi^0) = (32\pi/3)m_\pi \times$

$(a_0^0 - a_0^2)$ to the scattering lengths in the $I = 0$ and $I = 2$ isotopic channels. The denominator $1 + 2/9 m_\pi \Delta m_\pi (2a_0^2 + a_0^0)^2$ in Eq. (1) arises via the unitarization procedure. Further, $\Psi_n(0) = (m_\pi^3 \alpha^3 / 8\pi n^3)^{1/2}$ is the nonrelativistic Coulombic w.f. of $A_{2\pi}$ in the nS state at the origin, Δm_π is the π^+-π^0 meson mass difference and m_π is the charged pion mass.

In refs. [8, 9] the strong interaction corrections to the atom Coulombic w.f. $\Psi_n(0)$ have been estimated, taking into account the contribution coming only from the discrete spectrum. It was demonstrated that the strong correction $\Delta\Psi_1(0)$ to $\Psi_1(0)$ was of an order of 10^{-3}. However, if in the calculations the continuous Coulombic spectrum is taken into account, it is easy to demonstrate that this leads to the drastic modification of the atom w.f. at the origin. The first - order perturbative estimate yields the result $\Delta\Psi_1(0)/\Psi_1(0) \approx (2a_0^0 + a_0^2)/(2R) \sim 1/R \geq 35\%$ assuming that the range of strong potential $R \leq 1$ fm.

Consequently, the inclusion of strong interactions leads to the essential modification of the Coulombic w.f. of the atom at the origin. This does not contradict the statement that strong interactions give a small contribution to the parameters of the $(\pi^+\pi^-)$ bound state, since the latter implies that the matrix elements of the strong interaction potential are small compared to the matrix elements of the Coulombic potential. The w.f., in its turn, is not an integral characteristic of the system, and it is essentially modified near the origin where, as is expected from the beginning, strong interactions should give rise to a sizeable contribution. On the other hand, it is the Coulombic w.f. that enters into the expression (1) whereas the entire contribution from the strong interactions is concentrated in the $\pi\pi$ scattering lengths. Consequently, the inclusion of strong interactions in the atom w.f. can be regarded as "double counting" and leads to the erroneous predictions for the $A_{2\pi}$ lifetime.

In the framework of the multichannel potential theory the strong and electromagnetic corrections to the observable characteristics of $\pi\pi$ atom have recently been calculated in ref. [10]. The strong corrections to the formula (1) were calculated in the effective range approximation (see, also [11]) and given in a form of the series with an expansion parameter equal to A/r_B where A

denotes the strong $\pi\pi$ scattering amplitude at threshold. Since r_B is inversely proportional to the fine structure constant, these series, in some sense, can be thought to be an expansion in this constant of the strong amplitude in the presence of Coulombic interaction. Further, in the chiral theory the $\pi\pi$ scattering amplitude is obtained in the limit $m_{\pi^+} = m_{\pi^0}$. Consequently, for the comparison of the chiral theory predictions with the high-precision experimental data it was necessary to evaluate the effect which stems from the finite $m_{\pi^+} - m_{\pi^0}$ mass difference. In these calculations as well as during the evaluation of the electromagnetic corrections the knowledge of the explicit form of strong interaction potential was required. Moreover, it turned out that the corrections are rather sensitive to the particular choice of strong interaction potential [10]. However, in view of the forthcoming experiment on $A_{2\pi}$ which will provide a consistent test of the chiral theory predictions, it is necessary to calculate "strong" corrections directly with the use of the chiral Lagrangian, without any reference to the concept of phenomenological $\pi\pi$ potential which is a source of an additional ambiguity in the evaluation of the atom observables.

On the other hand, in ref. [12] electromagnetic correction to the $\pi^+\pi^-$ atom lifetime formula has been calculated. This correction is caused by the dynamical retardation effect in the one-photon exchange kernel of Bethe-Salpeter equation for the atomic wave function. It turned out that this pure electromagnetic correction $(4\alpha/\pi)$ is of the same order of magnitude as the strong corrections and thus can not be neglected (in the lowest order these corrections enter additively into the formula for the atom lifetime).

The aim of the present investigation is to present a self-consistent field-theoretical framework for the description of the strong decay of the $\pi^+\pi^-$ atom on the basis of the Bethe-Salpeter (BS) approach. In this framework, an unambiguous factorization of the strong and electromagnetic contributions to the expression for the $A_{2\pi}$ lifetime is achieved. The first-order perturbative corrections due to strong interactions in the expression for the atom lifetime are calculated without specifying a concrete form of the strong $\pi\pi$ interaction.

The $(\pi^+\pi^-)$ atom lifetime is calculated according to the well-known for-

mula [13]

$$\frac{1}{\tau_A} = \frac{\lambda^{1/2}(M^2, m_{\pi^0}^2, m_{\pi^0}^2)}{32\pi M^3} |T(A \to \pi^0\pi^0)|^2 \qquad (2)$$

where λ is the well-known kinematic triangle function, $M = 2m_\pi - E_B \approx 2m_\pi$ is the atom mass, E_B being the binding energy of the $A_{2\pi}$.

After simple transformations we have

$$\frac{1}{\tau_A} = \frac{1}{64\pi m_\pi} \sqrt{\frac{2\Delta m_\pi}{m_\pi}} \sqrt{1 - \frac{\Delta m_\pi}{2m_\pi}} |T(A \to \pi^0\pi^0)|^2 \qquad (3)$$

In the calculations of $T(A \to \pi^0\pi^0)$, we start from the standard expression for the transition amplitude for the reactions involving bound states [14]-[16]

$$T(A \to \pi^0\pi^0) =$$

$$= \lim_{P^2 \to M^2} i \int \frac{d^4q_1}{(2\pi)^4} \int \frac{d^4q_2}{(2\pi)^4} \bar{\chi}_P(q_1) \left[G_0^{-1}(P; q_1, q_2) - V^{\pi^+\pi^- \to \pi^+\pi^-}(P; q_1, q_2) \right] \times$$

$$\times G^{\pi^+\pi^- \to \pi^0\pi^0}(P; q_2, k) \left[m_{\pi^0}^2 - \left(\frac{P}{2} + k \right)^2 \right] \left[m_{\pi^0}^2 - \left(\frac{P}{2} - k \right)^2 \right] \qquad (4)$$

where P_μ denotes the total 4-momentum of the $\pi^+\pi^-$ atom and k_μ is the relative 4-momentum of two π^0 mesons produced in the decay process. In the c.m.f. $P_\mu = (P_0, \vec{0})$, $k_0 = 0$, $|\vec{k}| = \sqrt{P_0^2/4 - m_{\pi^0}^2}$. Here G_0 denotes the free Green function of charged π mesons, $G^{\pi^+\pi^- \to \pi^0\pi^0}$ is the full Green function for the reaction $\pi^+\pi^- \to \pi^0\pi^0$, and $V^{\pi^+\pi^- \to \pi^+\pi^-}$ denotes the sum of all irreducible diagrams for the process $\pi^+\pi^- \to \pi^+\pi^-$. The operator $G_0^{-1} - V^{\pi^+\pi^- \to \pi^+\pi^-}$ acting on the full Green function G, "excludes" all redundant diagrams which have already been taken into account in the w.f. $\bar{\chi}_P$, thus resolving the "double counting" problem (see, e.g. [16]). The w.f. of the $\pi^+\pi^-$ atom obeys the BS equation

$$\bar{\chi}_P(q) G_0^{-1}(P; q) = \int \frac{d^4k}{(2\pi)^4} \bar{\chi}_P(k) V^{\pi^+\pi^- \to \pi^+\pi^-}(P; k, q), \qquad P^2 = M^2 \qquad (5)$$

The reason why expression (4) is not convenient for our purpose is twofold. First, the w.f. $\bar{\chi}_P$ contains the strong interaction contributions. Second, the irreducible kernel $(G_0^{-1} - V^{\pi^+\pi^- \to \pi^+\pi^-}) G^{\pi^+\pi^- \to \pi^0\pi^0}$ for the transition $\pi^+\pi^- \to \pi^0\pi^0$ does not contain all strong $\pi\pi$ interaction diagrams and, therefore, cannot be directly related to the experimentally measured charge exchange amplitude. In order to overcome this difficulty, we transform (4) into the form which is more convenient for further investigations. Namely, we "transfer" all diagrams corresponding to the strong $\pi^+\pi^-$ interaction from the BS w.f. $\bar{\chi}_P$ to the irreducible kernel for the $\pi^+\pi^- \to \pi^0\pi^0$ transition. To this end, we split the kernel of Eq. (5) into two parts: $V^{\pi^+\pi^- \to \pi^+\pi^-}(P; k, q) = V_e + V'$ where V_e denotes the instantaneous Coulombic potential and V' stands for the rest including, in particular, all strong interaction diagrams and the piece of one photon exchange diagram responsible for the dynamical retardation effect. It should be pointed out that this decomposition is rather arbitrary; however, for our purpose it is convenient to choose it in the form given above. From a physical point of view, this corresponds to a picture in which the instantaneous Coulombic interaction is basically responsible for the formation of the bound state whereas all other contributions are small and can be taken into account perturbatively.

Let us now define the new w.f.

$$\bar{\chi}_P(p) = \lim_{\substack{P^2 \to M^2 \\ P^{*2} \to M^{*2}}} C \int \frac{d^4q_1}{(2\pi)^4} \int \frac{d^4q_2}{(2\pi)^4} \bar{\psi}_{P^*}(q_1) \left(G_0^{-1}(P^*; q_1, q_2) - V_e \right) \times$$

$$\times G^{\pi^+\pi^- \to \pi^+\pi^-}(P; q_2, p) \qquad (6)$$

Here $M^* = 2m_\pi - E_1 + O(\alpha^3)$ is the mass of the bound state calculated taking into account of only the instantaneous Coulombic interaction and C is the normalization constant, which will be defined below. Substituting (6) into (5), it is easy to verify that the new w.f. $\bar{\psi}_{P^*}$ obeys the BS equation (5) with the displacements $V^{\pi^+\pi^- \to \pi^+\pi^-} \to V_e$ and $M \to M^*$. The result of the action of the operators in (6) depends on the order of limiting procedures. The correct result is obtained if we assume, e.g., $P^2 = M^2 + \lambda$, $P^{*2} = M^{*2} + \lambda$, $\lambda \to 0$. Substituting the expression (6) into (4), we get

$$T(A \to \pi^0\pi^0) = \lim_{\substack{P^2 \to M^2 \\ P^{*2} \to M^{*2}}} iC \int \frac{d^4q_1}{(2\pi)^4} \frac{d^4q_2}{(2\pi)^4} \bar{\psi}_{P^*}(q_1) G_e^{-1}(P^*; q_1, q_2) \times$$

$$\times G^{\pi^+\pi^- \to \pi^0\pi^0}(P; q_2, k) \left[m_{\pi^0}^2 - \left(\frac{P}{2} + k \right)^2 \right] \left[m_{\pi^0}^2 - \left(\frac{P}{2} - k \right)^2 \right] \qquad (7)$$

Expression (7) is better suited for our purpose than (4). The irreducible transition kernel entering into the integrand in (7) contains the total contribution from strong interactions and $\bar{\psi}_{P^*}$ is the Coulombic BS w.f. In the lowest order in α we neglect the difference between G_e^{-1} and G_0^{-1} and define the strong transition amplitude, according to the well-known relation $G^{\pi^+\pi^-\to\pi^0\pi^0}_{strong} = G_0^{\pi^+\pi^-} T^{\pi^+\pi^-\to\pi^0\pi^0}_{strong} G_0^{\prime\pi^0\pi^0}$. With the use of the explicit expression for the BS w.f.

$$\bar{\psi}_{P^*}(p) = m_\pi^{-1/2} w(M^{*2} - 4w^2) G_0(P^*; p) \tilde{\Psi}_1(\vec{p}),$$

where $w = \sqrt{m_\pi^2 + \vec{p}^2}$ and the 3-dimensional Coulombic w.f. in the lowest order in α is written in the following form [17]

$$\tilde{\Psi}_1(\vec{p}) = \frac{(\alpha m_\pi)^{3/2}}{(8\pi)^{1/2}} \frac{4\pi\alpha m_\pi}{(\vec{p}^2 + m_\pi^2\alpha^2/4)^2}, \tag{8}$$

we get

$$T(A \to \pi^0\pi^0) = \lim_{\substack{P^2\to M^2 \\ P^{*2}\to M^{*2}}} C m_\pi^{-1/2} \int \frac{d^3\vec{q}_1}{(2\pi)^3} \tilde{\Psi}_1(\vec{q}_1) w(M^{*2} - 4w^2) \times$$

$$\times \int \frac{dq_1^0}{2\pi i} G_0(P; q_1) T^{\pi^+\pi^-\to\pi^0\pi^0}_{strong}(P; q_1, k)$$

The w.f. $\tilde{\Psi}_1(\vec{q}_1)$ rapidly decreases at the momenta $\vec{q}_1^{\,2} > m_\pi^2\alpha^2/4$ so the main contribution to the integral comes from the area $|\vec{q}_1| \approx 0$, where the expression $(M^{*2} - 4w^2)$ is small. In the vicinity of the bound-state pole in the integral over dq_1^0, only the poles of the Green function $G_0(P, q_1)$ can be taken into account. Integrating over dq_1^0 and then over $d^3\vec{q}_1$, we get

$$T(A \to \pi^0\pi^0) = C\, T^{\pi^+\pi^-\to\pi^0\pi^0}_{strong}\Big|_{thresh.} m_\pi^{-1/2} \int \frac{d^3\vec{q}_1}{(2\pi)^3} \tilde{\Psi}_1(\vec{q}_1) \frac{M^{*2} - 4w^2}{M^2 - 4w^2} \approx$$

$$\approx C\, T^{\pi^+\pi^-\to\pi^0\pi^0}_{strong}\Big|_{thresh.} m_\pi^{-1/2} \Psi_1(0)(1 + \delta) \tag{9}$$

where $\delta = -\Delta E_1/(4E_1) + O(\Delta E_1^2/E_1^2)$, and $\Psi_1(0)$ is the Schrödinger Coulombic w.f. at the origin (1) and ΔE_1 is the energy shift of the 1S Coulombic level (E_1) due to the strong interactions.

The amplitude $T^{\pi^+\pi^-\to\pi^0\pi^0}_{strong}$ at the elastic threshold is expressed through the S-wave $\pi\pi$ scattering lengths (the isotopic invariance of pure strong interactions is assumed when the scattering amplitudes are expressed in terms of scattering lengths in $I = 0$ and $I = 2$ channels)

$$T^{\pi^+\pi^-\to\pi^0\pi^0}_{strong}\Big|_{thresh.} = \frac{32\pi}{3} m_\pi \frac{a_0^0 - a_0^2}{1 + i(a_0^0 + 2a_0^2)\sqrt{\frac{2}{9} m_\pi (m_\pi - m_{\pi^0})}}$$

The normalization constant C is calculated perturbatively. To this end, we substitute (6) into the normalization condition for the w.f. $\bar{\chi}_P(p)$. As a result, we get

$$C = -2iM^* \left[\int \frac{d^4q_1}{(2\pi)^4} \frac{d^4q_2}{(2\pi)^4} \bar{\chi}_P(q_1) \left[\frac{\partial}{\partial P_0^*} G_0^{-1}(P^*; q_1, q_2) \right] \psi_{P^*}(q_2) \right]^{-1}_{P_0^* = M^*} \tag{10}$$

Further, we write the Green function $G^{ch} = G^{\pi^+\pi^-\to\pi^+\pi^-}$ in Eq. (6) in the following form:

$$G^{ch} = \left[G_0^{-1}(P^*) - V_e - V'(P^*) + (M - M^*) \frac{\partial G_0^{-1}(P^*)}{\partial P_0^*} + \cdots \right]^{-1} \tag{11}$$

By taking account of (11), the w.f. $\bar{\chi}_P(p)$ in the first perturbative approximation has the following form

$$\bar{\chi}_P(p) = C \lim_{\lambda\to 0} \left[\bar{\psi}_{P^*}(p) + \int \frac{d^4q_1}{(2\pi)^4} \frac{d^4q_2}{(2\pi)^4} \bar{\psi}_{P^*}(q_1) \times \right.$$

$$\left. \times \left[V'(P^*; q_1, q_2) - (M - M^*) \frac{\partial}{\partial P_0^*} G_0^{-1}(P^*; q_1, q_2) \right] G_e(P^*; q_2, p) \right] \tag{12}$$

where $G_e \equiv G_0^{-1} - V_e$.

Next, we multiply Eq. (12) from the right by $\int \frac{d^4k}{(2\pi)^4} \frac{\partial}{\partial P_0^*} G_0^{-1}(P^*; p, k) \psi_{P^*}(k)$ and integrate over d^4p. Taking account of the normalization condition for the w.f. $\bar{\psi}_{P^*}$, the explicit expression (10) for the constant C and the expression for the energy level shift in the first perturbative approximation (see, e.g., [17])

$$-2iM^*(M - M^*) = \left[\int \frac{d^4q_1}{(2\pi)^4} \frac{d^4q_2}{(2\pi)^4} \bar{\psi}_{P^*}(q_1) V'(P^*; q_1, q_2) \psi_{P^*}(q_2) \right]_{P_0^* = M^*} \tag{13}$$

and after simple transformations we get

$$\left[\int \frac{d^4q_1}{(2\pi)^4}\int \frac{d^4q_2}{(2\pi)^4}\bar{\chi}_P(q_1)\left[\frac{\partial}{\partial P_0^*}G_0^{-1}(P^*;q_1,q_2)\right]\psi_{P^*}(q_2)\right]^2_{P_0^*=M^*} =$$

$$= -2iM^*\left[2M^* + i\int \frac{d^4q_1}{(2\pi)^4}\frac{d^4q_2}{(2\pi)^4}\bar{\psi}_{P^*}(q_1)\left[\frac{\partial}{\partial P_0^*}V'(P^*;q_1,q_2) - \right.\right.$$

$$\left.\left. - (M-M^*)\frac{\partial^2}{\partial P_0^{*2}}G_0^{-1}(P^*;q_1,q_2)\right]\psi_{P^*}(q_2)\right]_{P^{*2}=M^{*2}} \qquad (14)$$

It is easy to verify that in expression (14) the main contribution to the integral comes from the term containing the free Green function $\frac{\partial^2}{\partial P_0^{*2}}G_0^{-1}(P^*)$. Having neglected the dependence of the $\pi^+\pi^-$ strong interaction potential on the energy in the vicinity of the elastic threshold and calculating the integral in (14) containing $\frac{\partial^2}{\partial P_0^{*2}}G_0^{-1}(P^*)$, we get

$$\frac{i}{m_\pi}\int \frac{d^3\vec{q}}{(2\pi)^3}\tilde{\Psi}_1^2(\vec{q})w^2(M^{*2}-4w^2)\int \frac{dq_0}{2\pi i}G_0(P^*;q)\frac{\partial^2}{\partial P_0^{*2}}G_0^{-1}(P^*,q)G_0(P^*;q) =$$

$$= -\frac{10i}{\alpha^2}(1+O(\alpha)) \quad \Longrightarrow \quad C = 1 - \frac{5}{16}\frac{\Delta E_1}{E_1} + O\left(\left[\frac{\Delta E_1}{E_1}\right]^2\right)$$

Finally, the expression for the $\pi^+\pi^-$ atom lifetime takes the following form:

$$\frac{1}{\tau_A} = \frac{16\pi}{9}\sqrt{\frac{2\Delta m_\pi}{m_\pi}}\sqrt{1-\frac{\Delta m_\pi}{2m_\pi}}\frac{(a_0^0-a_0^2)^2}{1+\frac{2}{9}m_\pi(m_\pi-m_{\pi^0})(a_0^0+2a_0^2)^2}\Psi_1^2(0)\times$$

$$\times\left[1-\underbrace{2\left(\frac{1}{4}+\frac{5}{16}\right)}_{=9/8}\frac{\Delta E_1}{E_1}\right]$$

where we have separately indicated the corrections coming from the energy level shift ($-\frac{\Delta E_1}{2E_1}$) and from the change of the w.f. normalization ($-\frac{5\Delta E_1}{8E_1}$). Note that the correction due to the change of the w.f. normalization is a genuine relativistic effect and arises due to the fact that the free Green function in the BS equation depends on the bound state mass in a power more than two. In

the quantum mechanics, where $G_0^{-1}(E) = E - H_0$, $\frac{\partial^2}{\partial E^2}G_0^{-1}(E) = 0$ and the potential does not depend on energy, it is well known that the normalization of the w.f. does not change in the first order of perturbation theory.

Thus, we have obtained the correction to the formula for the $\pi^+\pi^-$ atom lifetime [6] due to strong interactions in the leading order of the perturbation theory within the field-theoretical framework based on the Bethe-Salpeter approach. This correction is expressed in terms of the ratio $\Delta = \Delta E_1/E_1$. For the estimation of the size of Δ we use the well-known formula $\Delta E_1 = (4\pi a_S)/m_\pi \cdot \Psi_1^2(0)$ [5, 8], relating the energy level shift ΔE_1 to the $\pi\pi$ scattering singlet length $a_S = 2/3a_0^0 + 1/3a_0^2$. Consequently, $\Delta = 9\Delta E_1/(8E_1) = 9/4\cdot a_S m_\pi \alpha \sim 10^{-3}$ is negligible. Strong corrections to the $A_{2\pi}$ lifetime formula, obtained in the present paper are of the same order of magnitude (but have the opposite sign) as the corrections obtained within the potential picture [10, 11] but not the result from ref. [18] where an unphysically large value of this correction was obtained. The small size of the pure strong first-order corrections indicates that it is important to evaluate the electromagnetic corrections as well as to take into account the dynamical retardation effect [12] which stems from the noninstantaneous nature of the one-photon exchange interaction in the 4-dimensional BS approach. These corrections can perturbatively be taken into account in the irreducible kernel (4), corresponding to the $A_{2\pi}$ decay.

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