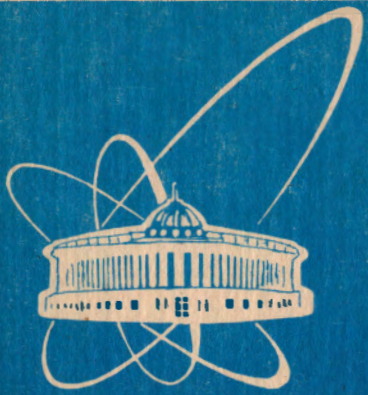

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Дубна

E2-96-342

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COMPARISON OF TWO FORMS OF VLASOV-TYPE RELATIVISTIC KINETIC EQUATIONS IN HADRODYNAMICS

Submitted to «Il Nuovo Cimento»

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1996

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Сравнение двух форм релятивистского кинетического уравнения типа Власова в адродинамике

На примере простейшей σω-версии квантовой адродинамики проведено сравнение двух подходов в релятивистской кинетической теории ферми-систем. Показано, что релятивистское кинетическое уравнение типа Власова (РКУВ), полученное с использованием на промежуточном этапе процедуры квадрирования уравнения движения, приводит к нефизическим особенностям. Предложен прямой метод получения кинетических уравнений. Этот метод не содержит такого недостатка и позволяет получить непротиворечивые РКУВ в адродинамике, с учетом спиновых эффектов и состояний с положительной и отрицательной энергиями.

Работа выполнена в Лаборатории теоретической физики им. Н.Н.Боголюбова ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна, 1996

Mashnik S.G. et al. E2-96-342

Comparison of Two Forms of Vlasov-Type Relativistic Kinetic Equations in Hadrodynamics

A comparison of two methods in the relativistic kinetic theory of the Fermi systems is carried out assuming, as an example, the simplest σω-version of quantum hadrodynamics with allowance for strong mean meson fields. It is shown that the Vlasov-type relativistic kinetic equation (VRKE) obtained by means of the procedure of squaring at an intermediate step is responsible for unphysical features. A direct method of derivation of kinetic equations is proposed. This method does not contain such drawback and gives rise to VRKE in hadrodynamics of a non-contradictory form in which both spin degrees of freedom and states with positive and negative energies are taken into account.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna, 1996

1. Introduction

The kinetic approach is one of the main theoretical instruments in the modern relativistic nuclear physics. It has been successfully applied at intermediate energies to the description of both heavy–ion collisions [1] and nuclear reactions initiated by hadrons and other projectiles in the framework of different cascade–type models [2]. During the last years, a tendency toward the change from a semiphenomenological level of description of the kinetic stage of nuclear matter evolution to a more sophisticated and dynamically justified theory, is evident. The relativistic kinetic equation of the Vlasov type (VRKE) for nucleon component of the spin–saturated nuclear matter was one of the first results in this direction [3]–[7]

$$\hat{D}(xP)\mathcal{F}(xP) \equiv \left\{ P\frac{\partial}{\partial x} + \left(M\frac{\partial M}{\partial x^{\nu}} + g_V P^{\mu}\widetilde{F}_{\mu\nu} \right) \frac{\partial}{\partial P_{\nu}} \right\} \mathcal{F}(xP) = 0 \,. \qquad (1)$$

This equation was derived in the quasi–classical approach within the framework of the simplest $\sigma\omega$–version of quantum hadrodynamics. In eq.(1), $\mathcal{F}(xP)$ is the scalar part of the expansion of the covariant Wigner function in terms of Clifford algebra (see details in §2 below); $M = M(x) = m_N - g_S\widetilde{\varphi}(x)$ is the effective nucleon mass in the nuclear matter, $P = p - g_V\tilde{\omega}$, $\widetilde{F}_{\mu\nu} = \partial_\mu\widetilde{\omega}_\nu - \partial_\nu\widetilde{\omega}_\mu$, and $\widetilde{\varphi}(x)$ and $\widetilde{\omega}^\mu(x)$ are the average scalar and vector meson fields, respectively.

In the recent years, more general VRKEs with consideration for spin degrees of freedom and states with positive and negative energies were obtained [8]–[12]. In spite of the fact that methods used in these researches are numerous and various, they may be classified under two types. In the more popular approaches of the first type, one introduces the procedure of squaring the Dirac equation written in the mean–field approximation. The methods of the second type do not use this procedure. The results of calculations performed according to these two approaches are distinctive in some details. It is worth remarking that VRKE derived using the procedure of squaring contains some source terms which violate barion number conservation and lead to entropy production even by mean–field dynamics [10]. The authors of the fundamental work [10] have frankly pointed out that they did not have a simple interpretation of this constraint (see Appendix 3 of Ref. [10]).

In this paper, we provide a detailed analysis of these two types of VRKE obtained using the same approximations in both approaches. In §2, it is shown that the procedure of squaring leads to appearance of the above–mentioned anomalous sources in the VRKE and, therefore, such a VRKE has to be removed from the class of dynamically justified kinetic equations. Therewith, not only the scalar part of VRKE casts some doubts but also the VRKE as a whole, including the spin degrees of freedom and states with positive and negative energies. Alternatively, in §3, a VRKE is derived by means of the direct method of kinetic theory. It does not imply the difficulties of the VRKE of the first type and, therefore, is dynamically justified. Finally, in §4, the results of our work are summarized.

Our analysis is based on the simplest $\sigma\omega$–version of the Walecka model with the Lagrangian density

$$\begin{aligned}\mathcal{L}(x) &= \frac{i}{2}\overline{\Psi}\overleftrightarrow{\partial_\mu}\gamma^\mu\Psi - m_N\overline{\Psi}\Psi + \frac{1}{2}\left[(\partial_\mu\varphi)\partial^\mu\varphi - m_S^2\varphi^2\right] - \\ &- \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}m_V^2\omega_\mu\omega^\mu + g_S\overline{\Psi}\varphi\Psi - g_V\overline{\Psi}\gamma^\mu\omega_\mu\Psi\,, \qquad (2)\end{aligned}$$

where m_S and m_V are the masses of scalar (φ) and vector (ω_μ) mesons, respectively, while g_S and g_V are the relevant coupling constants. For a kinetic description of the nucleon subsystem of the model, let us write the one–particle covariant Wigner function

$$f_{\alpha\beta}(xp) = (2\pi)^{-4}\int dy\, e^{-ipy}\left\langle\overline{\Psi}_\beta(x+y/2)\Psi_\alpha(x-y/2)\right\rangle, \qquad (3)$$

where $< \cdots > = Sp\cdots\rho$ denotes the procedure of statistical averaging with the density matrix in the Heisenberg representation.

In order to derive the VRKE, we adopt the dynamical approach proposed in refs. [8, 9, 12] that allows us to use alternatively at an intermediate step either the procedure of squaring (§2), or the direct method (§3). Such a way is convenient for comparison's sake as far as the results of the two approaches are concerned. Since no consideration will be given here to collision processes, it is sufficient to use the shortened version of the theory [8, 9, 12] which — in its complete amount — represents an extension to the relativistic region of the well–known Zubarev's method based on the non–equilibrium statistical operator.

2. VRKE derived by means of the squaring procedure

Let us write according to the definition (3) the one-body operator

$$P_{\alpha\beta}(xy) = \overline{\Psi}_\beta(x+y/2)\Psi_\alpha(x-y/2)\,.$$

Since in a space–uniform and stationary–state case the correlation function $< P_{\alpha\beta}(xy) >$ does not depend on x^μ, we assume that, for a small deviation from equilibrium, function $< P_{\alpha\beta}(xy) >$ depends slowly on x^μ in comparison with a "fast" dependence on y^μ. This behaviour gives reason to introduce a "slow" invariant time, $\tau = nx$, where n^μ is — for the moment — an arbitrary unit time–like vector directed towards the future. After differentiating the Wigner function (3) with respect variable τ and using the Liouville equation, $d\rho/d\tau = 0$, we have:

$$\frac{df(xp)}{d\tau} = u\frac{\partial f(xp)}{\partial x} = (2\pi)^{-4}\int dy\, e^{-ipy}\left\langle\frac{d}{d\tau}P(xy)\right\rangle\,. \qquad (4)$$

Here, the arbitrariness in choosing the direction of the unit-vector n^μ has been eliminated as well, assuming $n^\mu = u^\mu = p^\mu/\sqrt{p^2}$ [8, 9, 12]. Further manipulations of the right–hand–side of eq. (4) can be performed using alternatively one of the methods stressed above.

The direct method is based on the motion equations in the Heisenberg representation as a starting point. In this case, we have

$$p\frac{\partial f(xp)}{\partial x} = -i\sqrt{p^2}(2\pi)^{-4}\int dy\, e^{-ipy}\left\langle[P(xy), H]\right\rangle\,. \qquad (5)$$

If the Heisenberg operator, H, is defined as a uniform two–linear form relative to the field operators, Ψ and $\overline{\Psi}$, as this is actually the case of our model in the mean–field approximation, eq. (5) gives rise to a closed VRKE after calculation of the commutator. We will make use of this procedure in §3.

Let us consider now another version of the theory based on the technique of squaring. In the right–hand–side of eq. (4), we take into account the fact that $d/d\tau = u_\mu\partial^\mu(x)$ and use the relation:

$$p^\mu e^{-ipy} = i\partial^\mu(y)e^{-ipy}\,. \qquad (6)$$

After integration by parts we have ($x_\pm \equiv x \pm y/2$)

$$p\frac{\partial f_{\alpha\beta}(xp)}{\partial x} = \frac{i}{2}(2\pi)^{-4}\int dy\, e^{-ipy}\left\langle \overline{Q}_\beta(x_+)\Psi_\alpha(x_-) - \overline{\Psi}_\beta(x_+)Q_\alpha(x_-)\right\rangle \ , \qquad (7)$$

where $\overline{Q} = Q^+\gamma^0$ and Q is the operator–source in the wave equation $(\Box - m_N^2)\Psi = Q$. The increasing of the order of differential operator when passing from eq. (4) to eq. (7) is a consequence of relation (6).

For the model (2), in the mean–field approximation, we have

$$Q = \left\{-2g_s\widetilde{\varphi}M + g_s^2\widetilde{\varphi}^2 + i\gamma^\mu(\partial_\mu M) + \frac{g_V}{2}\sigma^{\mu\nu}\widetilde{F}_{\mu\nu} + 2ig_V\widetilde{\omega}^\mu\partial_\mu - g_V^2\widetilde{\omega}^2\right\}\Psi \ , \qquad (8)$$

where $\sigma^{\mu\nu} = \frac{i}{2}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)$. The substitution of this expression into eq. (7) gives rise to a VRKE of a non–Markovian type. The restriction of our consideration to minimal orders of the gradient expansion [15] allows us to write a VRKE in a local form

$$P\frac{\partial f}{\partial x} + M\frac{\partial M}{\partial x}\frac{\partial f}{\partial P} + g_V P^\mu\widetilde{F}_{\mu\nu}\frac{\partial f}{\partial P_\nu} - \frac{1}{2}\frac{\partial M}{\partial x^\mu}\{\gamma^\mu, f\} - \frac{g_V}{4i}\widetilde{F}_{\mu\nu}[\sigma^{\mu\nu}, f] = 0 \ . \qquad (9)$$

In order to obtain a closed system of equations, it is necessary to complement this equation by those defining the mean meson fields:

$$\begin{aligned}\widetilde{\varphi}(x) &= g_S\int dy\, D(x-y)\int dP\, Sp\, f(yP) \ , \\ \widetilde{\omega}^\mu(x) &= g_V\int dy\, D^\mu_\nu(x-y)\int dP\, Sp\,\gamma^\nu f(yP) \ , \end{aligned} \qquad (10)$$

where $D(x)$ and $D_{\mu\nu}(x)$ are the free scalar and vector meson–field Green functions, respectively. The relations (10) are true under the assumption of the absence of mean meson fields when the interaction with the nucleon component of nuclear matter is "witched off".

A VRKE like (9) was derived for the first time in ref. [10] by means of the contour Green function technique (with the replacement $P \to -P$ and taking into account a misprint in ref. [10]) using the procedure of squaring.

To analyze the VRKE (9), let us separate out in the Wigner function (3) the states with positive and negative energies $f = f^{(+)} + f^{(-)}$, and let us perform a transition from the spinor representation to the spin one [14]

$$\begin{aligned} f^{(+)}_{\alpha\beta}(xP) &= \sum_{s,r=1}^{2}\int\frac{d^3k}{2k^0}\,\overline{u}^s_\beta(k)F^{(+)}_{rs}(xk)u^r_\alpha(k)\delta(P-k) \ , \\ f^{(-)}_{\alpha\beta}(xP) &= -\sum_{s,r=1}^{2}\int\frac{d^3k}{2k^0}\,\overline{v}^s_\beta(k)F^{(-)}_{sr}(xk)v^r_\alpha(k)\delta(P+k) \ , \end{aligned} \qquad (11)$$

where $P^0 = (M^2 + \boldsymbol{P}^2)^{1/2}$. The spinors of nucleon states with positive and negative energies $u(k)$ and $v(k)$ play an important rolè here. It is assumed that in the mean–field approximation they satisfy the "quasi-free" equations of motion:

$$(\hat{P} - M)u(P) = 0, \qquad (\hat{P} + M)v(P) = 0, \qquad \text{where} \qquad \hat{P} = p_\mu\gamma^\mu \ .$$

This fact means that meson fields may be quite large in amplitude ($m_N \geq g_S\widetilde{\varphi}$), but adiabatically slow (in the scale $1/m_N$). Moreover, the adoption of such a spinor basis leads to the introduction of the mass–shell condition $P^2 = M^2$ which was absent before the transformation to the spin representation. This circumstance hinders the transition to the spin representation for quasi–particle excitations like Δ–isobars with distributed masses [17].

Let us rewrite the VRKE (9) in a symbolic form in order to perform the transformation to the spin representation,

$$\hat{L}f \equiv \{\hat{L}^{(1)} + \hat{L}^{(0)}\}f = 0 \ , \qquad (12)$$

where $\hat{L}^{(1)}$ and $\hat{L}^{(0)}$ are homogeneous differential operators of the first and second order, respectively.

$$\begin{aligned} \hat{L}^{(1)}_{\alpha\beta,\alpha'\beta'}(xP) &= \hat{D}(xP)g_{\alpha\alpha'}g_{\beta\beta'} \ , \\ \hat{L}^{(0)}_{\alpha\beta,\alpha'\beta'}(xP) &= \frac{1}{2}\frac{\partial M}{\partial x^\mu}\left[\gamma^\mu_{\alpha\alpha'}g_{\beta\beta'} + \gamma^\mu_{\beta'\beta}g_{\alpha\alpha'}\right] - \frac{g_V}{4i}\widetilde{F}_{\mu\nu}\left[\sigma^{\mu\nu}_{\alpha\alpha'}g_{\beta\beta'} - \sigma^{\mu\nu}_{\beta'\beta}g_{\alpha\alpha'}\right] \ . \end{aligned} \qquad (13)$$

Operator $\hat{D}(xP)$ is specified by the relation (1). According to the definition (11), the transition to the spin representation is carried out by means of the following matrices:

$$R^{(+)sr}_{\alpha\beta}(P) = \overline{u}^s_\beta(P)u^r_\alpha(P) \qquad R^{(-)rs}_{\alpha\beta}(P) = \overline{v}^s_\beta(P)v^r_\alpha(P) \ .$$

The contraction of the operator, $\hat{L}$ (see eqs. (12) and (13)), with one of these matrices results in projection of VRKE (9) on states with positive or negative energies.

Due to the formal charge symmetry, it will suffice to analyze only one of resulting equations. In particular, after integration over P^0, we have

$$\hat{L}_{rr',ss'}(xP)F^{(+)}_{s's}(xP) \equiv \frac{1}{2P^0} R^{(+)rr'}_{\beta\alpha}(P)\hat{L}_{\alpha\beta,\beta'\alpha'}(xP)R^{(+)ss'}_{\alpha'\beta'}(P)\hat{F}^{(+)}_{s's}(P) = 0 \,. \qquad (14)$$

Here, the allowance for $\left[\hat{L}^{(1)}, R^{(+)}\right] = 0$ is taken into account. It is also worth pointing out that the local character of the mass-shell condition by its dependence on the point of observation, x^μ, does not contribute to the drift-movement of the nucleon liquid in the sense that

$$\int dP^0\, R^{(+)rr'}_{\beta\alpha}(P)R^{(+)ss'}_{\alpha'\beta'}(P)\hat{L}^{(1)}_{\alpha\beta,\beta'\alpha'}(xP)\left\{\frac{1}{2P^0}\delta\left[P^0 - \sqrt{M^2+\boldsymbol{P}^2}\,\right]\right\} = 0 \,.$$

Further manipulations of operator $\hat{\mathrm{L}}$, eq. (14), are based on a number of formulae for contractions of different combinations of γ-matrices with spinors. We recall here some necessary formulae

$$\widetilde{F}_{\mu\nu}\overline{u}^r(P)\gamma^\mu\gamma^\nu u^s(P) = 4i\left\{P^0\left(\boldsymbol{H}\boldsymbol{\sigma}_{rs}\right) - \frac{1}{M+P^0}\left(\boldsymbol{PH}\right) - \boldsymbol{E}\left[\boldsymbol{P}\boldsymbol{\sigma}_{rs}\right]\right\} \,,$$

$$\overline{u}^r(P)\frac{\partial}{\partial P^k}u^s(P) = -\frac{\partial\overline{u}^r(P)}{\partial P^k}u^s(P) = \frac{i}{M+P^0}\left[\boldsymbol{P}\boldsymbol{\sigma}_{rs}\right]^k \,,$$

where $k = 1,2,3$, σ^k are the Pauli matrices, $E^k = \widetilde{F}_{0k}$, and $H^k = -\frac{1}{2}\varepsilon^{kil}\widetilde{F}_{il}$ are the vectors of "electric" and "magnetic" intensities of the vector meson field.

The resulting VRKE in the spin representation reads as:

$$\left\{P\frac{\partial}{\partial x} - \frac{1}{M}P\frac{\partial M}{\partial x}\right\}F^{(\pm)} - \left\{M\frac{\partial M}{\partial x^k} + g_V\left(P^0E^k + [\boldsymbol{PH}]^k\right)\right\}\frac{\partial F^{(\pm)}}{\partial P^k} \;+$$
$$+\frac{i}{2(M+P^0)}\left\{\left(g_VE^k - \frac{\partial M}{\partial x^k}\right)\left[[\boldsymbol{P}\boldsymbol{\sigma}]^k, F^{(\pm)}\right] + g_V\frac{\boldsymbol{P}^2}{M}\boldsymbol{H}\left[\boldsymbol{\sigma}, F^{(\pm)}\right]\right\} = 0 \,. \qquad (15)$$

The Wigner functions, $F^{(\pm)}$, are defined on the mass-shell in a seven-dimensional phase space. Let us carry out their expansion in terms of the Pauli algebra basis

$$F^{(\pm)}_{rs}(xP) = F^{(\pm)}(xP)\delta_{rs} + F^{(\pm)}_k(xP)\sigma^k_{rs} \,. \qquad (16)$$

In order to analyze the character of the problems noted in the Introduction and to find out their sources to VRKE (15), it will suffice to consider the case of spin-saturated nuclear matter which is consistent with VRKE relevant to the scalar Wigner

functions ($k = 1,2,3$)

$$P\frac{\partial F^{(\pm)}}{\partial x} - \left\{M\frac{\partial M}{\partial x^k} + g_V\left(P^0E^k + [\boldsymbol{PH}]^k\right)\right\}\frac{\partial F^{(\pm)}}{\partial P^k} = \frac{1}{M}P\frac{\partial M}{\partial x}F^{(\pm)} \,. \qquad (17)$$

As one could expect, these equations are reversible in time. The left-hand sides of VRKEs (17) have the usual structure for the Vlasov equation describing a drift-movement of nucleon liquid in presence of mean meson fields. On the contrary, in their right-hand sides VRKEs have out-of-range sources with a non-drift character leading to unphysical results. This result can be immediately proved by writing the barion current density as follows:

$$\begin{aligned}\langle j^\mu(k)\rangle &= \left\langle\overline{\Psi}(x)\gamma^\mu\Psi(x)\right\rangle = \int dP\, Sp\,\gamma^\mu f(xP) = \\ &= 2\int d^3P\,\frac{P^\mu}{P^0}\left\{F^{(+)}(xP) - F^{(-)}(xP)\right\} \qquad (18)\end{aligned}$$

and the entropy flow density as

$$S^\mu(x) = -\int d^3P\,\frac{P^\mu}{P^0}\left\{F^{(+)}(xP)\ln F^{(+)}(xP) + F^{(-)}(xP)\ln F^{(-)}(xP)\right\} \,. \qquad (19)$$

Then, employing the VRKE (17), the corresponding continuity equations can be easily obtained:

$$\partial_\mu(x)j^\mu = -\frac{1}{M}j^\mu\partial_\mu(x)M \,,$$

$$\partial_\mu(x)S^\mu = -\frac{1}{M}\partial_\mu(x)M\int d^3P\,\frac{P^\mu}{P^0}\left\{F^{(+)}\left[1+F^{(+)}\right] + F^{(-)}\left[1+F^{(-)}\right]\right\} \,.$$

Hence, the VRKEs (17) lead really to non-conservation of the barion charge and the entropy (we refer here just to the change of entropy without collisions, but not to its monotone increasing due to them).

The anomalous source in the right-hand sides of VRKEs (17) can be formally eliminated by resorting to the modified Wigner functions

$$\mathcal{F}^{(\pm)}(xP) = \xi(x)F^{(\pm)}(xP) \,, \qquad \xi(x) = m_N/M(x) \,. \qquad (20)$$

Actually, this transformation leads to a conventional form of VRKEs which reads as:

$$P\frac{\partial\mathcal{F}^{(\pm)}}{\partial x} - \left\{M\frac{\partial M}{\partial x^k} + g_V\left(P^0E^k + [\boldsymbol{PH}]^k\right)\right\}\frac{\partial\mathcal{F}^{(\pm)}}{\partial P^k} = 0 \qquad (21)$$

(some distinctions of this equation from VRKE (1) are caused by the definition of functions $\mathcal{F}^{(\pm)}$ on the mass-shell). But local quantities like as densities of physical values (18) and (19) are defined as before in terms of usual Wigner functions (3), so that the transformation (20) has a character of pure camouflage.

Now, it is easy to find out the reason of appearance of the anomalous source in the right-hand side of eq. (17). Tracing the way of derivation of this equation, one can see that the term $\sim \partial_\mu(x) M \{\gamma^\mu, f\}$ of VRKE (9) leads to an anomalous source in VRKEs (17). This factor is generated in its turn by term $i\gamma^\mu\partial_\mu(x)$ in the source, $Q(x)$ (8), of the wave equation. As a consequence, the anomalous term appears when passing from eq. (4) to relation (7) by means of eq. (6) and the the order of differential operator in the right-hand side of eq. (7) increases as a result of the squaring procedure.

3. VRKE derived without the squaring procedure

It is possible to take advantage of eq. (5) and derive a VRKE without using the squaring procedure. Let us define the covariant Hamiltonian operator featured in it by the relation

$$H = \int_\sigma d\sigma(x|n)\, n^\mu T^{\mu\nu} n_\nu \ ,$$

where n^μ is a normal time-like vector which defines the orientation of the space-like hyperplane $\sigma(x|n)$ passing through point x. Here, $T^{\mu\nu}$ is the energy-momentum tensor of the system. In the nucleon sector of Walecka model (2), in the mean-field approximation, we have

$$H = -\int d\sigma(x|n)\, \overline{\Psi} \left\{ \frac{i}{2}\gamma^\mu \overleftrightarrow{\partial}^\perp_\mu(x) - M - g_V\gamma^\mu\widetilde{\omega}_\mu \right\} \Psi \ . \qquad (22)$$

Here, $\partial^{\leftrightarrow}_\mu = \partial^{\rightarrow}_\mu - \partial^{\leftarrow}_\mu$, and the arrows show the directions of action of operator $\partial_\mu(x)$, and $\partial^\perp_\mu(x) = \Delta^\nu_\mu \partial_\nu(x)$. By means of the projection operator $\Delta_{\mu\nu} = g_{\mu\nu} - n_\mu n_\nu$, one eliminates the differentiation in the time-like direction. In other words, the operator, $\partial^\perp_\mu(x)$, works on the hyperplane, $\sigma(x|n)$. This fact is essential in calculations with formula (5) when we have to integrate by parts integrals given on hyperplane $\sigma(x|n)$.

One takes also into account that the Hamiltonian operator (22) does not depend on the particular choice of hyperplane $\sigma(x|n)$.

In derivation of the right-hand side of formula (5), it is useful to use the following expression (see, e.g., [8, 9]):

$$\int \sigma(x'|n)\, S_{\alpha\beta}(x-x')\Phi_\beta(x') = i\, n_\mu \gamma^\mu_{\alpha\beta}\Phi_\beta(x) \ , \qquad x \in \sigma(x'|n) \ , \qquad (23)$$

where $\Phi_\alpha(x)$ is a function depending on field operators, and

$$S(x-x') = i\left[\Psi(x), \overline{\Psi}(x')\right]_+ \ . \qquad (24)$$

The relation (23) is based on a covariant extension of the known feature of the reposition function (24), $S(\overline{x}, 0) = i\gamma^0\delta^{(3)}(\overline{x})$. At the end of our calculation we assume that $n^\mu = u^\mu$.

The outlined recipes will suffice for derivation of VRKE on the basis of eq. (5). In the minimal order of gradient expansion [15], we obtain

$$\begin{aligned} & P^\mu\partial_\mu(x) f + \frac{1}{2}\partial^\nu(x) M \{\hat{\gamma}, \partial_\nu(P) f\} + g_V P^\mu \widetilde{F}_{\mu\nu}\partial^\nu(P) f \ + \\ & + \frac{1}{2}[\hat{\gamma}\gamma^\mu, \partial_\mu(x) f] + iM[\hat{\gamma}, f] - \frac{1}{2} g_V \widetilde{F}_{\mu\nu}[\hat{\gamma}\gamma^\mu, \partial^\nu(P) f] \ = \ 0 \ , \end{aligned} \qquad (25)$$

where $\hat{\gamma} = P_\mu\gamma^\mu$.

The same set of approximations was used in the derivation of both VRKEs (9) and (25): Namely, the mean-field approximation and the limitation to only the lowest orders of the gradient expansion. However, the resulting VRKEs (9) and (25) are found significantly different.

In order to analyze this difference, let us turn to the spin representation and confine ourselves to the case of spin-saturated nuclear matter. Then, VRKE (25) reduces to the following equation: $(k = 1, 2, 3)$:

$$P\frac{\partial F^{(\pm)}}{\partial x} - \left\{ M\frac{\partial M}{\partial x^k} + g_V \left(P^0 E^k + [\boldsymbol{P}\boldsymbol{H}]^k \right) \right\} \frac{\partial F^{(\pm)}}{\partial P^k} = 0 \ . \qquad (26)$$

The comparison of this VRKE with the analogous expressions (17) and (21), obtained on the basis of the squaring procedure, shows that VRKE (26) does not contain the anomalous sources intrinsic to VRKE (17) and, therefore, does not require a transition to the modified Wigner functions, as in the case of VRKE (21).

We turn back now to comparisons of more general VRKEs, eqs. (9) and (25), which take into account also spin effects. It should be noticed that a simple elimination of the term $\sim \partial_\mu(x) M \{\gamma^\mu, f\}$ leading to anomalous sources in VRKE (17) from the VRKE (9), is not sufficient to give identical VRKEs in eqs. (9) and (25). This fact clearly shows that VRKEs (9) and (25) describe spin effects in a very different manner. In this instance, the non-contradiction of VRKE (26) suggests us the assumption that the more general VRKE (25) be correct as well.

It should be also observed that anomalous effects generically dictated by the presence of mean scalar fields in the Walecka model helped us to reveal evidence of incorrectness of VRKE (9). The elimination of the scalar field would make difficult the problem of choosing a non-contradictory kinetic description of polarization effects within the framework of one of the two different approaches previously discussed.

4. Summary

The comparison of two dynamical approaches in the relativistic kinetic theory of Fermi systems, namely, the direct method (§3) and that based on the squaring procedure (§2), has shown that, at least in the Hartree approximation for models containing scalar fields, the second method leads to unphysical features in VRKE (violation of the barion number conservation and entropy production without collisions), as well as to an incorrect description of spin effects. This form is conditioned by an artificial increasing of the order of differential operators used at intermediate steps in the derivation of relativistic kinetic equations, but presumably — from a formal point of view — it does not affect the structure of collision integrals (of the Boltzman–Uehling–Uhlenbeck or the Bloch types). At the same time, the direct method of derivation of VRKE is devoid of any contradiction and allows us to avoid these unpleasant peculiarities.

Thus, the available dynamical methods in the relativistic kinetic theory based on the squaring procedure would require a careful analysis of the relevant results and, perhaps, need some corrections which would permit to avoid such a dangerous procedure. On the other hand, this situation gives priority to the direct methods of the relativistic

kinetic theory and, above all, to the Zubarev's method of the non-equilibrium statistical operator as one of the most sequential and universal tools for the description of non-equilibrium systems.

Acknowledgement This work was partly supported by the State Committee of the Russian Federation for Higher Education under grant No. 2-61-1-2.

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Received by Publishing Department
on September 20, 1996.