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ELECTROMAGNETIC WAVES IN SPACE WITH THREE-DIMENSIONAL TIME

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1. — Introduction

The Einsteinean theory of relativity was the first one to reveal a symmetry in the properties of space and time, the subsequent generalizations introducing a proper time for each particle [1] and for each space point $\mathbf{x}$ [2] equalized in right space and time still more. It is interesting to take a next step on this way and to consider a more consistent from relativistic viewpoint theory with the equal number of space and time co-ordinates when

$$(\hat{\mathbf{x}})_\mu = (-\mathbf{x}, \hat{t})^T_\mu \quad , \quad (\hat{\mathbf{x}})^\mu = ((\mathbf{x}, \hat{t})^{T\mu}$$

(Here and in what follows the superscript "T" denotes the transpose, three-dimensional vectors in x- and t-subspaces will be marked respectively by bold symbols and by a hat, six-dimensional vectors we denote by bold symbols with the hat; values of greek and latin indices $\mu \leq 6, k \leq 3$). The most satisfactory variant of such a theory was developed by E.A.B.Cole [3 – 5]. His theory has an elegant mathematical form, but at present stage it has serious difficulties due to a presence of objects with negative energy, to a spontaneous creation of particles, what results in vacuum non-stability and to a possibility of exotic decays in which the secondaries mass excels the decaying particle mass. It is not clear, is that an essential feature of the multi-time approach restricting its applicability only by a ultra-small space-time regions as it is emphasized by Cole [5] or one may sidestep the difficulties using some additional physical conditions.

The goal of our paper is to consider the peculiarities of the multi-time theory by means of a simple example of a plane electromagnetic wave motion in order that to more exactly understand the physical meaning of equation solutions with negative energy and to point out conditions excepting vacuum non-stability and exotic decays. It will be shown, one can gain that by means of the causality principle restricting acceptable types of time trajectories.

Section 2 considers the plane wave solutions of the generalized multi-time Maxwell equations. In Sec. 3 the six-dimensional momentum-energy vector of plane wave is derived, and in Sec. 4 we discuss conditions by which the total energy of any plane wave remains always positive defined.

2. — Plane waves in multi-dimensional time

We shall use a multi-time generalization of Maxwell equations found by Cole [6, 7]. In empty space, in absence of charges and currents, it comes to a consideration of an equation

$$(1) \qquad \hat{\boldsymbol{\nabla}}^2 \hat{\mathbf{A}} - \boldsymbol{\nabla}(\hat{\boldsymbol{\nabla}} \hat{\mathbf{A}}) = 0,$$

where $(\hat{\mathbf{A}})_\mu = (-\mathbf{A}, \hat{A})^T_\mu$ is the six-dimensional vector potential, $(\hat{\boldsymbol{\nabla}})_\mu = (-\boldsymbol{\nabla}, \hat{\nabla})_\mu$ and $\hat{\nabla}_i = -\partial/\partial t_i$ are del operators. The last term of equation (1) vanishes due to the extended Lorentz condition

$$(2) \qquad \hat{\boldsymbol{\nabla}} \hat{\mathbf{A}} = 0.$$

As a rectilinear world-line of plane wave can be defined by the vector $\hat{\mathbf{n}} = (1, 0, 0, 1, 0, 0)^T$, the equations (1), (2) can be written in the form

(3) $$\partial^2\hat{\mathbf{A}}/\partial t^2 - c^2\partial^2\hat{\mathbf{A}}/\partial x^2 = 0$$

(4) $$\partial A_4/\partial t + \partial A_1/\partial x = 0.$$

Further, just as in the known one-time theory, the first term in (4) is turned into zero by means of a gauge and we get then from the equation (3):

$$\partial A_1/\partial t = \text{const},$$

i.e. the component A_1 bears no relation to a wave process and can be removed. It means out that the potentials $\mathbf{A}$ and $\hat{A}$ are transverse vectors: $\hat{\mathbf{A}} \cdot \hat{\mathbf{n}} = 0$.

The electric field tensor looks now as

(5) $$\hat{\mathbf{E}} = \mathbf{A}\hat{\nabla} - \boldsymbol{\nabla}\hat{A} = c^{-1}\begin{pmatrix} 0 & \dot{A}_5 & \dot{A}_6 \\ -\dot{A}_2 & 0 & 0 \\ -\dot{A}_3 & 0 & 0 \end{pmatrix}$$

(The solution of a wave equation corresponding to a wave moving along the vector $\mathbf{n}$ satisfies the condition $\partial A_\mu/\partial x = -\dot{A}_\mu/c$). Let us introduce the three-dimensional electric fields $\mathbf{E}_k = (\hat{\mathbf{E}}_{1k}, \hat{\mathbf{E}}_{2k}, \hat{\mathbf{E}}_{3k})$ and $\hat{E}_k = (\hat{\mathbf{E}}_{k1}, \hat{\mathbf{E}}_{k2}, \hat{\mathbf{E}}_{k3})$. It is easy to prove that the fields $\mathbf{E}_k$ and $\hat{E}_k$ for $k = 2,3$ are longitudinal vectors, but $\mathbf{E}_1$ and $\hat{E}_1$ are transversal ones: $\mathbf{E}_1 \cdot \mathbf{n} = \hat{E}_1 \cdot \hat{n} = 0$.

From the formal point of view the appearance of the longitudinal electric field is stipulated by the impossibility to remove by means of the gauge transformation more than one component of the three-vector $\hat{A}$.

Using for $\mathbf{E}_i$ and $\hat{E}_i$ the expressions from (5), one can prove that the magnetic field

$$\mathbf{H} = \boldsymbol{\nabla} \times \mathbf{A} = -c^{-1}\frac{\partial}{\partial t}\mathbf{n} \times \mathbf{A} = \mathbf{n} \times \mathbf{E}_1$$

and the "time-magnetic" field

$$\hat{G} = -\hat{\nabla} \times \hat{A} = c^{-1}\frac{\partial}{\partial t}\hat{n} \times \hat{A} = \hat{n} \times \hat{E}_1$$

are also transverse ones: $\mathbf{H} \cdot \mathbf{n} = \hat{G} \cdot \hat{n} = 0$. The absolute values $H = |\mathbf{E}_1|$, $G = |\hat{E}_1|$.

As we see, in the multi-dimensional world the electromagnetic wave becomes apparent in two essence: in x-subspace it looks as a superposition of the transverse wave $(\mathbf{E}_1, \mathbf{H})$ and the longitudinal wave $\mathbf{E}_L$, in t-subspace it is a sum of the transverse wave $(\hat{E}_1, \hat{G})$ and the longitudinal one with the vector $\hat{E}_L$. The structure of the plane wave in x- and t-subspaces is completely symmetrical — that part of the wave which in x-subspace is transversal in t-subspace becomes longitudinal and conversely (see Fig.). Depending on their signs the longitudinal field strengthens $\mathbf{E}_k$ and $\hat{E}_k (k = 2, 3)$ can be in parallels or antiparallels to the direction of the space and time vectors $\mathbf{n}$ and $\hat{n}$.

3. — Momentum-energy of a plane wave

The derived with the help of the canonical rules momentum-energy tensor of electromagnetic field

$$T^{\mu\nu} = \frac{1}{4\pi}\left(\mathcal{F}^{\nu}_{\delta}\mathcal{F}^{\delta\mu} + \frac{1}{4}g^{\mu\nu}\mathcal{F}_{\lambda\gamma}\mathcal{F}^{\lambda\gamma}\right)$$

distinguishes from the respective tensor of one-time theory only by a number of components. If the electromagnetic field tensor $\mathcal{F}_{\mu\nu} = \partial A_\mu/\partial x^\nu - \partial A_\nu/\partial x^\mu$ is represented in the form

$$\hat{\mathbf{F}} = \begin{pmatrix} -\hat{\mathbf{H}} & \hat{\mathbf{E}} \\ -\hat{\mathbf{E}}^T & \hat{\mathbf{G}} \end{pmatrix},$$

where $\hat{\mathbf{H}}$, $\hat{\mathbf{G}}$, $\hat{\mathbf{E}}$ are 3×3 matrixes then $T^{\mu\nu}$ can be written in the similar matrix form also:

$$T^{\mu\nu} \equiv \begin{pmatrix} \hat{\mathbf{T}}_1 & \hat{\mathbf{T}}_2 \\ \hat{\mathbf{T}}_3 & \hat{\mathbf{T}}_4 \end{pmatrix}^{\mu\nu} =$$

$$\frac{1}{4\pi}\begin{pmatrix} -\hat{\mathbf{H}}\hat{\mathbf{H}} - \hat{\mathbf{F}}\hat{\mathbf{E}} & -\hat{\mathbf{H}}\hat{\mathbf{E}} - \hat{\mathbf{E}}\hat{\mathbf{G}} \\ \hat{\mathbf{E}}^T\hat{\mathbf{H}} + \hat{\mathbf{G}}\hat{\mathbf{E}}^T & \hat{\mathbf{E}}^T\hat{\mathbf{E}} + \hat{\mathbf{G}}\hat{\mathbf{G}} \end{pmatrix}^{\mu\nu} + \frac{1}{8\pi}g^{\mu\nu}\left(\mathbf{H}^2 + \hat{G}^2 - \hat{\mathbf{E}}^2\right),$$

where

$$\hat{\mathbf{E}}^2 = \sum_{i=1}^{3}\mathbf{E}_i^2 = \sum_{i=1}^{3}\hat{E}_i^2.$$

Taking into account the relations

$$\begin{aligned}
&(\hat{\mathbf{H}}\hat{\mathbf{H}})_{ik} = H_iH_k - \delta_{ik}\mathbf{H}^2 \quad , \quad (\hat{G}\hat{G})_{ik} = G_iG_k - \delta_{ik}\hat{G}^2 \\
&(\hat{\mathbf{E}}\hat{\mathbf{E}}^T)_{ik} = \hat{E}_i\hat{E}_k \quad , \quad (\hat{\mathbf{E}}^T\hat{\mathbf{E}})_{ik} = \mathbf{E}_i\mathbf{E}_k \\
&(\hat{\mathbf{E}}^T\hat{\mathbf{H}})_{ik} = -(\hat{\mathbf{H}}\hat{\mathbf{E}})_{ik} = (\mathbf{E}_i \times \mathbf{H}) \\
&(\hat{\mathbf{E}}\hat{\mathbf{G}})_{ik} = -(\hat{\mathbf{G}}\hat{\mathbf{E}}^T)_{ik} = (\hat{E}_i \times \hat{G}_k),
\end{aligned}$$

which can be proved by means of the explicit expressions for the matrixes $\hat{\mathbf{E}}, \hat{\mathbf{H}}$ and $\hat{\mathbf{G}}$ we get the following expressions for the tensors $\hat{\mathbf{T}}_i$:

$$\begin{aligned}
4\pi\hat{\mathbf{T}}_1^{ik} &= -\hat{E}_i\hat{E}_k - H_iH_k + \tfrac{1}{2}\delta_{ik}(\mathbf{H}^2 - \hat{G}^2 + \hat{\mathbf{E}}^2) \\
4\pi\hat{\mathbf{T}}_4^{ik} &= \mathbf{E}_i\mathbf{E}_k + G_iG_k + \tfrac{1}{2}\delta_{ik}(\mathbf{H}^2 - \hat{G}^2 - \hat{\mathbf{E}}^2) \\
4\pi\hat{\mathbf{T}}_3^{ik} &= 4\pi\hat{\mathbf{T}}_2^{ik} = (\mathbf{E}_k \times \mathbf{H})_i - (\hat{E}_i \times \hat{G})_k
\end{aligned}$$

Owing to the symmetry $T^{\mu\nu} = T^{\nu\mu}$ the tensors $\hat{\mathbf{T}}_l^{ik}$ are linked by the relations

$$\hat{\mathbf{T}}_1^{ik} = \hat{\mathbf{T}}_1^{ki}, \qquad \hat{\mathbf{T}}_4^{ik} = \hat{\mathbf{T}}_4^{ki}, \qquad \hat{\mathbf{T}}_2^{ik} = \hat{\mathbf{T}}_3^{ki}$$

. It must be noted also that up to a sign the expressions for $\hat{\mathbf{T}}_l^{ik}$ are symmetrical with respect to pairs of the fields $(\mathbf{E}_k, \mathbf{H})$ and $(\hat{E}_k, \hat{G})$: $\hat{\mathbf{T}}_1^{ik} \to \hat{\mathbf{T}}_4^{ik}, \hat{\mathbf{T}}_2^{ik} \to \hat{\mathbf{T}}_2^{ik}$ under the change $\mathbf{E}_k \to i\hat{E}_k, \mathbf{H} \to i\hat{G}$ and backward.

It is easy to satisfy oneself that by

$$\mathbf{E}_i \to \mathbf{E}\delta_{1k} \quad , \quad (\hat{E}_i)_k \to E_i\delta_{1k} \quad , \quad \hat{G} \to 0$$

all above written expressions turn into the known Maxwell's theory formula.

Let us define a six-dimensional momentum-energy vector

$$P^\mu = \int T^{\mu\nu} ds_\nu$$

where $ds_\nu = n_\nu dV$ and dV is an element of a three-dimensional hypersurface. In particular case of a plane wave when the direction of its time-trajectory is taken as the t_1-axis

$$P^\mu = \int T^{\mu k}\tau_k dV = \begin{cases} \int \hat{\mathbf{T}}_3^{1k} dV & \mu = k \le 3 \\ \int \hat{\mathbf{T}}_4^{1k} dV, & \mu = 3 + k \end{cases}$$

So, the field momentum-energy density

(6) $$\hat{\mathbf{p}} = (W_T + W_L)\hat{\mathbf{n}}$$

where

$$W_T = (\dot{A}_2^2 + \dot{A}_3^2/\pi c^2 = (\mathbf{E}_1^2 + \mathbf{H}^2)/8\pi$$

and

$$W_L = -(\dot{A}_5^2 + \dot{A}_6^2)/8\pi = -(\hat{E}_1^2 + \hat{G}^2)/8\pi$$

are the energies of the transverse and the longitudinal fields.

As we see the three-dimensional energy and momentum vectors of a plane wave are directed along its world-line $\hat{\mathbf{x}}$ and their transverse components $p_k, k \neq 1, 4$, equal to zero always. The momentum-energy of the transverse field component $(\mathbf{E}_1, \mathbf{H})$ is positive, meanwhile not only the momentum but also the energy of the longitudinal field $\mathbf{E}_L$ are negative.

4. — Discussion

It would seem strange, that a part of the wave energy is negative, however, an analogues situation takes place in the Maxwell electrodynamics also where a "scalar photon" energy is negative. Because the multi-component electromagnetic wave is an unitary object, only its summary energy is physically significant and its negative part is never observed. In the six-dimensional theory the total wave energy (6) must be positive also, i. e. $W_T \geq |W_L|$ and the transverse component prevails always over the longitudinal one: $|\mathbf{E}_1| \geq |\hat{E}_1|$. In this case the plane wave momentum is directed along the x_1-axis and the wave is developing along t_1-axes from the past into the future.

The mathematically acceptable solution with negative energy $W_T + W_L < 0$ corresponding to an incoming wave with backward space and time directions must be rejected

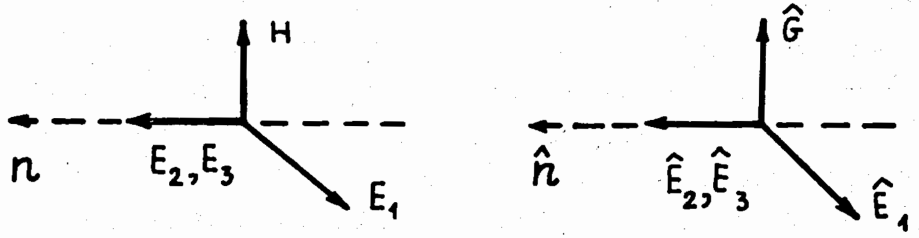
if we take into account the causality principle.

In the boundary case of equal amplitudes $\mathbf{E}_1^2 = \hat{E}_1^2 = \mathbf{H}^2 = \hat{G}^2$ the complete compensation of the transverse and the longitudinal components occurs (similar to the compensation of the scalar and longitudinal components of a plane wave in the one-time theory). the momentum-energy (6) turns into zero and the wave disappears.

So, as in the usual Maxwell electrodynamics the energies of all outgoing plane waves in six-dimensional world are positive and these waves develop along the positive directions of all time axes t_i.

The considered example of a plane electromagnetic waves promotes that an analogues situation take place in the general cases: due to the causality principle all wave solutions describing the motion of particles correspond to time trajectories with positive projections $\tau_i \geq 0$ and, respectively. to energy vectors with positive components. That forbids spontaneous creations of groups of particles from vacuum and exotic decays in which the mass of secondaries exceeds the decaying particle mass, since every such an event is accompanied always by a time-reverse motion along even if one axis t_i and by a violation of causation.

It should be emphasized that the time reversibility is only an approximate property of theories with the finite number of particles and interconnections meanwhile due to a non-exhaustive huge number of real interconnections in Nature time reversibility doesn't realizes, strictly speaking. even in microscopic processes, since it would demand the time turning of all these innumerable interconnections.



Figure

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Баращенков В.С., Юрьев М.З. E2-96-3

Электромагнитные волны в пространстве с трехмерным временем

Рассматриваются свойства плоских волн в шестимерном пространстве-времени Минковского с равным числом пространственных и временных координат. В x-подпространстве волна характеризуется тремя ортогональными друг к другу векторными полями, два из которых являются поперечными, а третье — продольным по отношению к направлению распространения волны. Энергия поперечных и продольной полей-компонент оказывается, соответственно, положительной и отрицательной, однако в силу принципа причинности, разрешающего эволюцию событий вдоль всех трех временных осей только из прошлого в будущее, полная энергия волны всегда положительная, что предотвращает рождение волн из вакуума и аномальные распады, в которых масса вторичных больше массы распадающейся частицы.

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Electromagnetic Waves in Space with Three-Dimensional Time

Properties of plane waves in sixdimensional Minkowski space-time with the equal number of space and time coordinates are considered. In x-subspace the wave is characterized by three orthogonal one to other vector field two of which are transversal and the third is longitudinal with respect to the direction of the wave motion. The energy of transverse and longitudinal field-components is, respectively, positive and negative, however, due to the causality principle permitting the development along all time axes only from the past to the future, the total wave energy is positive always, what prevents a spontaneous creation of waves from vacuum and exotic processes in which the final mass exceeds the initial one.

The investigation has been performed at the Laboratory of Computing Techniques and Automation, JINR.

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